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ABSTRACT

Analysis of eddy currents in solid core magnets shows that for the two extremes of large and small field level changes, very different consequences of the eddy currents are of concern for the user. It is shown that in the former case, irreproducibility and deterioration of magnet performance can be caused by the eddy currents, and it is indicated what steps can be taken to prevent this from occurring. In the latter case, the main concerns are field inhomogeneities that decay with time. This may necessitate long waiting periods before the magnet can be used again to its full capability, and it is shown how that waiting time can be reduced.

1. Introduction

This article is a summary of a part of the work done on eddy current effects that were considered important for the design and operation of the LASL High Resolution Spectrometer (HRS)[1] magnets. While the topics discussed here are of importance for a larger class of magnets than only precision spectrometer magnets, effects that are demonstrably unimportant for the HRS magnets (for instance eddy current-caused energy deposition in the steel for reasonable operating procedures) are not described.

The HRS magnets are equipped with field correction windings imbedded in the steel[2] that allow to obtain a field homogeneity of the order 10^{-5}. Since it is very time consuming to determine the correction current setting at a given field level, it is imperative that all relevant magnet properties are reproducible. To insure reproducibility of the magnetic properties of the steel, the basic operating cycle for the magnets will consist of (with the exception of the procedure described in sect. 4.2) increasing the magnet current monotonically from zero to the maximum value, and then reducing it monotonically to zero. This cycle puts a minor restraint on the typical experiment, where one wants to take data at a fairly large number of slightly differing field levels: the field levels have to be set in monotonical order. Unless proper precautions are taken, eddy currents can cause basically two undesirable effects:

1) When making small field level changes, eddy currents directly cause field inhomogeneities, and one may have to wait for a long time until the eddy currents have decayed to such a level that the associated field inhomogeneity is small enough.

2) When making large field level changes, such as are, for instance, necessary when reducing the field level from the highest value (required by the cycling procedure) to a substantially lower working level, eddy currents can generate DC aftereffects that can lead to a deterioration of the field homogeneity.

Although some explicit prescriptions will be given that allow one to reduce or avoid these damaging consequences of eddy currents, the main aim is to give a good understanding of the eddy currents, their secondary effects, and their consequences. To this end, it will be attempted to give a model that contains the essential physics content, but is still simple enough to allow an analytical mathematical description of the eddy currents that in turn gives still a little more insight, and allows also to make estimates of the important quantities involved.

All descriptions and calculations in this paper are carried out using MKS units.

2. Eddy-currents and eddy current-caused fields

2.1. QUALITATIVE DESCRIPTION OF EDDY CURRENT MODEL

To simplify the introduction of the basic concepts, the magnet discussed first will be a long, straight, solid core, symmetric H-type dipole magnet. The upper half of its two dimensional cross section is schematically shown in fig. 1. The discussion is restricted at first to the parts of the magnet that are far from the ends, i.e. only two dimensional effects will be discussed. Because of the assumed symmetry, a time varying excitation of the magnet leads to an eddy current pattern that is symmetric to the line a b c in fig. 1, and the eddy current density j at a given point (n) is proportional to the flux change per unit length between that point and the point (n') located symmetrically with respect to line a b c. From this follows that $j \equiv 0$ along a b c, and also along e' d' c d e to the extent that the flux going through the air region outside the magnet can be neglected. Since that flux will in all but extremely strongly saturated magnets be very small compared to the flux carried by the yoke, this air flux will be neglected in the following discussions.

For the purposes of the discussions of this paper, one can distinguish between three classes of eddy current phenomena. They are, listed in order of increasing length of time required for the phenomena to develop:

a) For extremely fast changes, eddy currents will flow only in a very thin skin on the steel surface, and only directly adjacent to the coils, just as if the air-iron interfaces were coated with a superconductor.

b) On a slower time scale, eddy currents will be flowing everywhere in the steel adjacent to the air-iron interface (except the region immediately adjacent to point b), and flux will enter the pole face region i b i'. The amount of flux that is carried by the steel is much larger than in case a, and although it is spread over a much wider strip of the steel, the fields have not yet penetrated significantly to the outer contour of the magnet, i.e. the flux- and eddy current pattern is practically independent of the yoke thickness (distance between lines m k and d e, or k j and d c).

c) On a still slower time scale, the flux penetrates the bulk of steel, i.e. the yoke thickness (d) becomes an essential part of the problem.

No attempt is made to discuss or adequately describe the phenomena listed in groups a and b, since it will be shown that even when the flux penetrates all of the steel, damaging effects may occur unless proper precautions are taken. This means that it is certainly not desirable to produce the conditions leading to the phenomena described in a and b. Despite this argument, in some of the calculations made in this paper, a time behavior of the excitation current is assumed that would lead during the early part of the described process to the conditions described under a and/or b. This is done only for mathematical simplicity, since in all of these cases the fact that condition c is violated during some time is not relevant to the specific topic under discussion. But, as stated above, the conditions leading to a or b have to be avoided in actual operation of a magnet.

2.2. QUALITATIVE DESCRIPTION OF THE EDDY-CURRENT AND -FIELD DISTRIBUTION, PART ONE

To obtain an understanding of the process that leads to the eddy-current and -field distribution, it is convenient to assume that the Ampère turns (I) carried by the coil have a constant time derivative, and that at the time of the following consideration, $\dot{I}$ has such a magnitude (and has had that value for a sufficiently long time) so that the conditions set forth in class c of sect. 2.1 are satisfied.

A quantity of primary importance is the magnetic flux per unit length that is carried by the steel, and how it is modified by the eddy currents. To obtain this it is useful to consider first $\int \mu_0 \vec{H} \cdot \vec{ds}$ along a closed contour such as f g i j k l m f in fig. 1, with path g – m lying just inside the steel along the air-iron interface. One obtains

$$\int_f^g B_y \, dy = \mu_0 I - \int_g^m \frac{B_{\parallel}}{\mu} ds \quad . \tag{1}$$

Ignoring eddy currents for the moment, for most magnets the integral on the right side of eq. (1) is of the order of not more than a few percent of the integral on the left side of eq. (1), since the longer integration path, and the possibility of $B_{\parallel} > B_y$ is strongly overcompensated by the factor $1/\mu$. Taking now eddy currents into account, it is clear that one of their effects will be a change of $B_{\parallel}$ on the right side of eq. (1). However, for reasons given below, one will always want to keep this change of $B_{\parallel}$ within moderate limits. Consequently one comes to the conclusion that the left side of eq. (1) will not change by more than a few percent because of the eddy currents, and this holds for virtually every path f g within the air gap region. This is the reason for the ability to state that, within something of the order of one per cent, the flux carried by the steel is independent of the eddy currents.

Applying this now to the cross section e – m of the yoke, one can state that the average B value ($\overline{B}$) then has the same value that B would have had with the same coil excitation, but without eddy currents. It is furthermore clear that the polarity of the eddy current density in the steel is opposite to the current density in the closest coil. Consequently, the field must decrease as one goes from m toward e, leading to a field distribution as schematically indicated in fig. 2. That figure also shows the average value of this distribution, which, as mentioned above, is for all practical purposes identical with the constant field that one would have there without the eddy currents for the same excitation of the coils. The important conclusion is, of course, that even on a time scale that allows penetration of the field throughout the steel, the field can exceed its ultimate DC level over a substantial part of the steel volume.

2.3. SEMIQUANTITATIVE DESCRIPTION OF THE EDDY-CURRENT AND -FIELD DISTRIBUTION IN THE YOKE

To simplify the problem, it is assumed that for some distance above (and below) line e m, $\vec{B}$ has only a y component B, and that B depends only on x. Then $\vec{E}$ has only a z component E, which also depends on x only. Since the concern is with transients, the DC reference level for B and I is arbitrary. Since there is little chance for confusion, the same letter B is used for $B(x,t)$ or its Laplace transform $B(x,p) = \int_0^\infty B(x,t)\, e^{-pt}\, dt$, and the same notation applies to E or the current density $j = \sigma E$. For the relationship between $\vec{B}$ and $\vec{H}$, $\vec{B} = \mu_0 \mu \vec{H}$, it is assumed that μ is constant.

One then obtains for the Laplace transforms of

$$\text{curl}\ \vec{H} = \vec{j} = \sigma\vec{E} \tag{2a}$$

and

$$\text{curl}\ \vec{E} = -\dot{\vec{B}}\ : \tag{2b}$$

$$dB/dx = \mu_0\mu\sigma\ E \tag{3a}$$

and

$$dE/dx = p\ B\ . \tag{3b}$$

It follows that

$$d^2B/dx^2 = k^2B\ ;\quad k^2 = p\ \mu_0\mu\sigma\ . \tag{4}$$

The solution to eq. (4) that gives $j(0,p) = 0$ is

$$B(x,p) = B_{00}\cosh(kx)\ . \tag{5}$$

From this follows, with

$$\phi = k\ d = \sqrt{p\tau}\ ;\quad \tau = \mu_0\mu\sigma\ d^2 \tag{6a}$$

and

$$\varepsilon = x/d\ : \tag{6b}$$

$\overline{B}(p) = B_{00}\sinh(\phi)/\phi$, and consequently

$$B(\varepsilon,p) = \overline{B}(p)\cdot\phi\cosh(\phi\varepsilon)/\sinh(\phi). \tag{7}$$

According to the discussion in sect. 2.2, one can use

$$\overline{B}(p) = K \cdot I(p) \; ; \quad K = \text{const.} \tag{8}$$

without committing a serious error. To make this explicitly clear, one can take the loss of Ampère turns in the steel roughly into account as follows: Describing the field in the air gap by $B_0(p)$, and the total flux entering the pole with the help of an effective halfwidth W, one obtains

$$\overline{B} = B_0(p) \cdot W/d \tag{9}$$

Considering, as in sect. 2.2, $\int \vec{H}\cdot\vec{dl}$ along path f g i j k l m f, one obtains, analogously to eq. (1):

$$B_0(p) \cdot h = \mu_0 I - B(l,p) \cdot L/\mu \quad . \tag{10}$$

The approximation made here, describing $\int \mu_0 H_{\|} \, ds$ along path g i j k l m by $B(l,p)L/\mu$, where L is an effective path length, is admittedly crude, but does contain the most essential features that are of interest here. Eliminating $B(l,p)$ and $B_0(p)$ with eqs. (7) and (9) gives, with

$$\eta = \frac{1}{\mu} \cdot \frac{L}{h} \cdot \frac{W}{d} \quad : \tag{11}$$

$$\overline{B} = \frac{\mu_0 I}{h} \; \frac{W}{d} \Big/ \left(1 + \eta \cdot \frac{\phi}{\tanh(\phi)}\right) . \tag{12}$$

It is evident that in most cases $\eta \ll 1$ will be satisfied, so that it is not very important that one knows the values for W, and particularly L, only approximately. It is also clear how one could refine the theory further. This seems, however, hardly worthwhile because the effect approximately described by eq. (10) is fairly weak, whereas it would be hard to find a more accurate way to describe the more important relationship between B and H, without losing the analytical manageability.

Substituting eq. (12) into eq. (7) gives

$$B(\varepsilon,p) \doteq \frac{\mu_0 I}{h} \cdot \frac{W}{d} \cdot \frac{\cosh(\varepsilon\phi)}{\sinh(\phi)/\phi + \eta \cosh(\phi)} \tag{13}$$

To obtain $B(\varepsilon,t)$ for ramp excitation, i.e. $I(t) = 0,\ t < 0;\ I(t) = \dot{I} \cdot t,\ t > 0$, one has to use $I(p) = \dot{I}/p^2$, and then take the inverse Laplace transform of eq. (13). It is, of course, not possible to represent $B(\varepsilon,t)$ in closed form with elementary transcendental functions; this is, however, not a serious drawback, since $B(\varepsilon,t)$ is really only of interest for times large enough so that the turn-on transients (which would not be very accurately described by eq. (13) anyway) have decayed. For the asymptotic behavior of $B(\varepsilon,t)$ for long times, one needs to know the singularities of $B(\varepsilon,p)$. They are, in order of increasing distance from the imaginary p-axis: 1) from $I(p) = \dot{I}/p^2$ comes a double singularity at $p_0 = 0$, 2) From $\sinh(\phi)/\phi + \eta\cosh(\phi) = 0$ comes, to a very good approximation for the first few n:

$$p_n = -n^2\pi^2(1-\eta)^2/\tau\,, \quad n = 1, 2 \ldots \tag{14}$$

The contributions from the singularities at p_n for $n = 1, 2, \ldots$ are proportional to $\exp(p_n t)$; therefore, even the term with the slowest decay ($n = 1$) has decayed after a time of the order τ/π sufficiently that it can be ignored.

The contribution from the double singularity at p = 0 is $(d(B(\varepsilon,p)e^{pt})/dp)_{p=0}$; this gives for the asymptotic behavior:

$$B(\varepsilon,t) = \frac{\mu_0 \dot{I}}{h} \cdot \frac{W}{d} \cdot \frac{1}{1+\eta} \left(t + \frac{\tau}{2}\left(\varepsilon^2 - \frac{1}{3} \cdot \frac{1+3\eta}{1+\eta}\right)\right) \quad .$$

As expected, the last equation as well as eq. (14) show explicitly how unimportant η is for the problem under discussion; therefore, the last equation can be rewritten, for $\eta = 0$ and ramp excitation ($\overline{B}(t) = \dot{\overline{B}} \cdot t$):

$$B(\varepsilon,t) = \overline{B}(t) + \dot{\overline{B}} \cdot \tau \cdot (3\varepsilon^2 - 1)/6 \quad . \tag{15}$$

Inserting in eq. (6a) for σ the value $1/12.6 \cdot 10^8$ $(\Omega m)^{-1}$, corresponding approximately 1010 steel, gives for τ:

$$\tau = 10 \cdot \mu \cdot d^2 \quad . \tag{16}$$

Although, as mentioned before, the assumption μ = const. is somewhat crude, eqs. (15) and (6a) or (16) give probably quite an adequate picture if in the equation for τ one uses for μ the value $\mu = dB/d\mu_0 H$ for the field level existing in the yoke at the time of interest.

If the ramp excitation is suddenly terminated by keeping $\overline{B}$ constant from time t on, eq. (15) shows that the fraction of the steel where B is at that time larger than the value $\overline{B}$ that one would have without eddy currents is 42.5%. The maximum value by which $\overline{B}$ is exceeded amounts to $\dot{\overline{B}} \cdot \tau/3$, a field that can be quite substantial for a large magnet unless $\dot{\overline{B}}$ is kept very low.

2.4. QUALITATIVE DESCRIPTION OF EDDY-CURRENT AND -FIELD DISTRIBUTION, PART TWO

It is instructive to obtain an understanding of the eddy-current-caused field perturbation with a line of reasoning that is different from the one used in sect. 2.2: The statement made in sect. 2.1 about the eddy current density j in the steel can be rephrased to say that j is proportional to the flux change/unit length between the point under consideration and any point on line a b c or e' d' c d e in fig. 1. From this follows that j increases if one moves from the outer air-iron boundary of the magnet to its inner air-iron boundary. The field lines caused by these eddy currents are in the yoke parts of the magnet, roughly speaking, parallel or antiparallel to the field lines caused directly by the coils. In the pole region, the pattern is quite different: reluctance considerations obviously favor closing of the eddy current-caused field lines in the steel parallel to the pole face, rather than going accross the air gap, and this is of course another way to see why the previously mentioned quantity $\overline{B}$ is practically not changed by the eddy currents. One will therefore obtain a pattern of the field lines caused by the eddy currents alone in one quarter of a symmetrical dipole magnet such as the one shown in fig. 3. That particular pattern was computed by the program POISSON[3] for a slow ramp excitation and shows all features of interest here. Considering the sign of the eddy currents in the yoke region, it is clear that for $\dot{I} > 0$, the eddy current-caused fields are, again roughly speaking, parallel to the coil-caused fields adjacent to the inner air-iron boundary, and antiparallel at the outer air-iron boundary. This leads again to a total field distribution in the yoke such as is schematically shown in fig. 2.

3. DC aftereffects of eddy currents

3.1. A SECONDARY EFFECT RESULTING FROM EDDY CURRENTS

Although shown explicitly only for ramp excitation, it is clear from the previous sections that unless specific precautions are taken when making large field level changes, a large part of the steel in the yoke will temporarily be excited substantially beyond the flux density $\overline{B}$, the value of B that one would have throughout the cross section under discussion if one had no eddy currents. Restricting now for simplicity's sake the discussion to the case of an increase in field level, the hysteresis in the relationship between B and H in the steel leads to conditions schematically represented in fig. 4: Without eddy current effects, all parts of the steel under consideration would have moved up monotonically from lower field levels through point 1 on the B(H) curve to the final point 2. For some parts of the steel, this pattern of monotonic change is not affected by eddy currents. For other parts of the steel, however, B will temporarily be higher than it would have been without eddy currents. If the maximum in a particular part of the steel corresponds to point 3 on the B(H) curve in fig. 4, the decay of the eddy currents results in a subsequent lowering of B. Because of the hysteresis, the properties of this steel will be described during the eddy current decay by curve 3-4, with point 4 describing the final state of this particular part of the steel after the complete decay of the eddy currents. This DC aftereffect of the eddy currents on the steel properties has consequences for the performance of the magnet that will be discussed in the following three sections.

3.2. DC AFTEREFFECT ON THE FIELD LEVEL OF THE MAGNET

As a consequence of the effect discussed in the previous section, the steel in the yoke is a little bit "better", i.e. for the same flux carried by the steel, the associated value of H is a little smaller. This is immediately clear by considering cross section e-m in fig. 1, recognizing that H must be constant over that cross section after the eddy currents have decayed, and by taking into account the content of sect. 3.1 and fig. 4. It is furthermore clear that, although different in detail, essentially the same happens throughout the yoke of the magnet. One can formulate a simple theoretical model that allows one to make a quantitative estimate of the reduction of H. The model is not described here since one can prevent the DC aftereffects from occurring with the procedure described in section 3.7.

The conclusion regarding the DC aftereffect on the field level can be expressed as follows: If one uses a ramp to turn on a magnet to a given coil current excitation the final field level will be larger if one makes the ramp steeper. This is precisely what Cobb[4]) observed in 1965. Historically, the order is, of course, reverse: It was Cobb's description of this field level effect, together with the need to develop an understanding of the eddy currents for the HRS magnets, that led to this study. The size of the effect that Cobb reported can be understood in terms of the model given here, and were sizeable for his magnet: by changing the ramp steepness, field level changes of the order 0.2% were observed.

3.3. DC AFTEREFFECTS ON THE FIELD DISTRIBUTION ALONG THE AXIS OF A MAGNET (UMBRELLA EFFECT)

It has been observed[5,6] that when one energizes a long magnet and measures the field along its axis, the field at the ends decreases relative to the field in the bulk of the magnet if one turns the magnet on in a shorter time. Although it is difficult to construct a theoretical model that allows one to compute this effect quantitatively, the cause is quite obvious: Discussing the case of a field level increase, it was found in the previous section that one eddy current DC aftereffect consists of reducing the value of H in the yoke, leading to a higher field level. Considering now the three dimensional eddy current flow, it is clear that while the eddy currents in the bulk of the magnet flow parallel (or antiparallel) to the magnet axis, they must close over the top and bottom of the magnet at the ends. This must lead to a reduction of the "improvement" of the steel properties as one approaches the end, and consequently to a reduction in field level there relative to the level in the bulk of the magnet. As one would expect from this model, the size of the effect is comparable to the field level effect discussed[4] in the previous section.

3.4. DC AFTEREFFECT ON FIELD DISTRIBUTION PERPENDICULAR TO THE AXIS OF A MAGNET (DISH EFFECT)

Referring to sects. 2.4, 3.1, and 3.2 it is clear that when changing the field level without precautions, the outer parts of the pole, i.e. the regions adjacent to i j and i' j' in fig. 1, will have steel with modified magnetic properties. For too rapid a field increase, the μ in that region will be increased, leading to an increase of the field in the air region as one goes away from the centerline a b. This effect is not very large, but has been observed on an HRS test magnet[7]). The relative field inhomogeneity produced by a rapid field change from 2 T to 1.4 T was $\approx 10^{-4}$, not very much for most magnets, but intolerable for precision magnets.

The fact that the eddy currents cause temporarily tangential field components at the poleface (see fig. 3) has been invoked by Enge[8]) to explain the umbrella effect mentioned in the previous section. While this author finds that explanation very hard to understand, that temporary tangential field component can, in principle, contribute to the field inhomogeneity discussed here. However the mechanism, namely magnetic rotational hysteresis, seems to be too weak an effect to expect that this aspect of the eddy currents contributes significantly to any of the observed DC aftereffects.

It should be noted that fig. 3 is representative only for the eddy current-caused fields at a field level where $B/H \approx dB/dH$. At higher fields, where B/H is considerably larger than dB/dH, in the outer parts of the pole the region where the eddy current-caused field is essentially parallel to the coil-caused field comes considerably closer to the magnet gap, and this region extends also closer to the vertical symmetry line. Consequently the dish effect will be stronger at those field levels than one would expect from fig. 3.

3.5. GENERALIZATIONS

The simplifying assumptions introduced in order to develop more succinctly the basic concepts are obviously not essential for the processes that lead to effects described in this paper. To see how dropping of simplifications modifies the discussed effects, the following two cases will be discussed: A long C-magnet, and a yoke with circular cross section.

Discussing first the C-magnet, far away from its ends the eddy current density j is parallel to the axis of the magnet and the integral of j over the iron cross section has to be zero. That means that there must be a line in the two dimensional iron cross section along which $j \equiv 0$, corresponding in fig. 1 to the line a b c and the outer boundary of the symmetrical H-magnet. This $j = 0$ line must obviously be located approximately as indicated in fig. 5. From this follows that, except for minute details, a long C magnet should behave just as an equivalent H magnet does. Specifically, the distance d entering in the equation (6a) for the time constant τ is 1/2 of the yoke width in case of the C magnet, i.e. identical to the quantity to be used for an H magnet that uses the same amount of steel as the C magnet.

A yoke with circular cross section is discussed as a mathematically treatable example of a magnet that is short. While the basic concept, that the flux that has to be carried by the steel is essentially independent of the eddy current effects, is not altered, some details are affected: It is clear that as a magnet becomes shorter, the electric field associated with the return of the eddy currents at the ends of the magnet becomes more and more important, and ultimately changes the three dimensional eddy current pattern completely. For instance, while for a long magnet the eddy current flowing far away from the ends through point p in fig. 1 will return at the end over the top and then will flow through point p' far away from the end, for a magnet whose length is equal to the width d of the yoke, the current flowing in one direction at location p' will return through p".

To treat a yoke with circular cross section of area πr_0^2 in the vicinity of the midplane it is assumed that the magnetic field has only a component perpendicular to the midplane that depends only on the distance r from the center of the circle. Then one obtains, instead of eq. (4):

$$d^2B/dr^2 + dB/rdr = k^2 B \;; \quad k^2 = p\,\mu_0\mu\sigma \tag{17}$$

The solution, corresponding to eq. (5), is (J_n denotes Bessel functions of first kind and order n):

$$B(r,p) = B_{00} \cdot J_0(ikr) \quad . \tag{18}$$

With

$$\phi = kd = \sqrt{p\tau} \;; \quad \tau = \mu_0\mu\sigma\, r_0^2 \tag{19a}$$

and

$$\varepsilon = r/r_0 \tag{19b}$$

follows

$$\overline{B}(p) = B_{00} \cdot \frac{J_1(i\phi)}{i\phi/2} \;, \tag{20}$$

and finally:

$$B(\varepsilon,p) = \overline{B} \cdot \frac{i\phi/2 \cdot J_0(i\phi\varepsilon)}{J_1(i\phi)} \quad . \tag{21}$$

Considering again a ramp-like excitation and ignoring the effect described by η in sect. 2.3, one part of the long time asymptotic behavior is governed by the roots of $J_1(i\phi)/i\phi$. This gives, instead of eq. (14):

$$p_n = -a^2_{1,n}/\tau \; ; \quad J_1(a_{1,n}) = 0 \; ; \quad n = 1, 2, \ldots , \tag{22a}$$

$$a_{1,1} = 3.83; \quad a_{1,2} = 7.02 \; ; \; \ldots \tag{22b}$$

and the statements following eq. (14) apply with obvious modification. After the transients resulting from turn-on of the ramp have decayed, $B(\varepsilon,t)$ is obtained with a procedure equivalent to the one that gave eq. (15), and one obtains

$$B(\varepsilon,t) = \overline{B}(t) + \dot{\overline{B}}\tau(2\varepsilon^2-1)/8 \quad . \tag{23}$$

As one expects, the eddy-currents and fields are governed by shorter effective time constants, and a somewhat modified time constant pattern. It should be noted that the above treatment of a yoke with circular cross section applies equally to the legs of a symmetrical H magnet or a C magnet.

3.6. POSSIBLE BENEFITS OF DC AFTEREFFECTS

Since DC aftereffects can modify the field distribution in magnets, one can try to use this fact to improve the field distribution by using special turn on procedures. Although occasionally such prescriptions have been developed empirically, it has to be pointed out that they are practical only for one particular kind of use of a magnet, namely when that magnet is used for very long times at the same field level, or field levels that differ only very little from the original field level. When one wants to use a magnet for only a relatively short time at any particular field level, and intends to change frequently that level by small amounts, so that one changes the field level by a factor of 1.5 to 2 during the course of a day, utilizing DC aftereffects can become time consuming. It would obviously require one to "re-process" the steel by appropriate cycling several times during the course of the day, a procedure that would clearly be time consuming since one would always have to wait until the field inhomogeneities that are directly caused by eddy currents have sufficiently decayed. For this reason, it will often be advisable to obtain the required field distribution with other means, and to operate the magnet in such a way that the DC aftereffects are kept down to an unobjectionable level.

3.7. SUPPRESSION OF DC AFTEREFFECTS

One obvious method to avoid eddy currents affecting magnet performance in a significant way is to laminate the magnet in conventional fashion. Time constants can also be reduced by approximately a factor 4 if one splits a symmetrical H magnet (fig. 1) along its length, with an insulating gap along line b c. Since other considerations often rule out these solutions, one must find other ways to prevent "damaging" a magnet by excessive eddy current effects. Since the simple rule "make large field level changes sufficiently slowly" would require in the case of large magnets intolerably long times, one has to find a way to make large field level changes as fast as possible without producing DC aftereffects to such a degree that the performance of the magnet is noticeably affected. The method proposed here consists of measuring B (or a quantity directly related to B) in the steel at the location where one expects to find the strongest DC aftereffects, and controlling the current in the coils so that the field level at the B-probe location reaches the finally desired DC field level as fast as possible without overshooting that level by more than a safe amount. What that safe amount is depends, of course, on the detailed characteristics of the magnet and, for any given magnet, on the field level; but an upper limit between 0.05 T and 0.2 T seems reasonable, whereas an overshoot by more than 0.5 T will probably give noticeable DC aftereffects in most precision magnets.

The following is a particular and more detailed example of how the above given method could be carried out. One can use a power supply regulator that has inputs for both the signal from the probe that gives a measure of B in the steel, as well as from a probe that measures B in the air gap. If one wants to change the field to a new level in the gap, one would have a provision to enter that new level, as well as the corresponding signal level for the steel probe, but increased by the safe amount of overshoot. By an appropriate feedback system, the power supply would at first be controlled by the steel probe, giving there very soon the prescribed level. This means that the current would change in the beginning quite rapidly, then slow down more and more. While this is happening, the other probe will measure the field in the gap, and control of the power supply will switch to that probe as soon as the desired air gap field is reached. That will then result essentially in a "freezing" of the coil current. As a consequence, the eddy currents will decay, and the field at the steel probe location will decrease gradually by the safe overshoot amount to the DC level corresponding to the air gap field.

While the nonlinearity of the B(H) curve makes it difficult to calculate accurately how long the above mentioned process takes, a calculation with

μ = const. gives at least some idea, and allows a fairly accurate estimate how much additional time it takes when the amount of safe overshoot is changed. Referring to eq. (7), and ignoring the effect described by η in eq. (12), $\overline{B}$ is proportional to the excitation current. For the purpose of this discussion, B(1,p) has to be considered the primary quantity, since the behavior of B(1,t) is prescribed. Using for the sake of mathematical simplicity a step function with unit amplitude for B(1,t), B(1,p) = 1/p and

$$\overline{B}(p) = \frac{1}{p} \cdot \frac{\sinh(\phi)/\phi}{\cosh(\phi)} . \tag{24}$$

For the asymptotic behavior at long times, the singularity at p = 0 and the root of $\cosh(\phi)$ closest to p = 0 give the only contributions of interest, and one obtains

$$\overline{B}(t) = 1 - \frac{8}{\pi^2} \exp\left(-\frac{\pi^2}{4} \cdot \frac{t}{\tau}\right) . \tag{25}$$

It should be noted that the exponential function entering here decays four times slower than the dominant exponential function for the eddy current decay given in sect. 2.3.

As mentioned before, eq. (25) cannot be expected to give an accurate description of the whole period during which the current changes. However, if the appropriate value for $\mu = dB/\mu_0 dH$ is used in the expression for τ,

$$\overline{B(\infty)} - \overline{B}(t) = \overline{\Delta B(t)} = C \cdot \exp\left(-\frac{\pi^2}{4} \cdot \frac{t}{\tau}\right) \tag{26}$$

will describe with fair accuracy how the final value of $\overline{B}$ would be approached if the value of $\overline{B}$ was not frozen before. Since one freezes $\overline{B}$ at a value $\overline{\Delta B}$ (corresponding to the permissible overshoot) below the original asymptotic value, another form of eq. (26), namely

$$\Delta\overline{B}(t + \Delta t) = \Delta\overline{B}(t) \cdot \exp\left(- \frac{\pi^2}{4} \frac{\Delta t}{\tau}\right) \tag{27}$$

allows one to estimate at least how much longer the whole process lasts if the allowed value of $\overline{\Delta B}$ is changed. It follows that reduction of $\Delta\overline{B}$ by a factor two requires the additional time $\Delta t = 0.28\ \tau$.

For completeness, a few suggestions about methods to measure B in the steel are added. With regard to the size of the DC aftereffects the strongest effects may appear at the sides of the pole. However, the difference between the pole and the yoke will in most cases be small enough so that the probe can be placed on the appropriate location of the yoke.

Possibly the simplest way to measure a quantity directly related to B in the steel may consist in machining a slot with its long dimensions perpendicular to $\vec{B}$ into the surface and inserting a Hall probe into it. This method has the disadvantage that unless the thickness of the slot is small compared to its large dimension divided by the permeability of the steel, one measures essentially H in the steel. Although such a signal is usable in principle, it would lead to an extremely inconvenient calibration curve between the air gap field and the signal from the steel. Consequently, the following methods all use a flux measuring coil whose output signal is integrated. Because of the quality of operational amplifiers available today and the ease with which one can obtain a reasonable total coil area, drift considerations are not important for any of the probes described below.

Two practical geometries for a flux-measuring coil are:

a) At the appropriate location, iron is removed in the shape of one half of a circular cylinder, with its axis parallel to $\vec{B}$ there. A smaller half cylinder, preferably made of the same steel as the yoke, and with the flux measuring coil on it, is inserted into the above described void. The total gap at the top and bottom should be smaller than the length of the void divided by the permeability. This condition should be easily satisfied by grinding the surfaces involved, and possibly press-fitting.

b) Similarly to a), a short hole is drilled into the steel, with the axis perpendicular to $\vec{B}$. Then a steel disk with a coil, as shown in fig. 6, is press fitted into the hole.

The last two examples have the advantage that they do not interfere with the coils of the magnet. If the coils leave enough room, one can also attach a steel cylinder (with a coil on it) to the yoke surface, provided one can produce a low reluctivity connection between the cylinder and the yoke.

It is important to know under what circumstances it is most important to adhere very strictly to a DC aftereffect-suppression procedure such as the one described above. As a basis for this discussion it is assumed that one uses the cycling procedure described in the introduction, and that the magnet is used only during that part of the cycle when the field level is decreased. It is clear that it is most important to suppress DC aftereffects when one makes a large field change toward a working level; a good example is the change from the maximum field level, (required by the cycling prescription) to an often substantially lower working level. Next in order of importance is the turning off of the magnet. Since the field level used last before turning off will in general vary every time the magnet has been used, without suppression of DC aftereffects the amount by which some of the steel is driven into the third quadrant of the B(H) curve would vary, leading to an impairment of the reproducibility of the magnet. The degree of importance of DC aftereffect suppression during turn off depends on the magnitude of the maximum field level attainable during the cycling process. If the maximum field level is so high that the steel is driven strongly into saturation, the memory of the steel for past history is effectively destroyed, and correct turn off is not very important. If, on the other hand, a magnet is used only at moderate field levels, and the power supply does not allow strong saturation of the steel, proper turn off is much more important. DC aftereffect suppression is least important during the first part of the turn on procedure, namely when going from zero field level to its maximum value. In fact, since one is really only concerned with the reproducibility of the magnet, DC aftereffect suppression during this phase is not at all important as long as this initial field change is done in an identical way every time the magnet is used.

4. Temporary eddy current effects

In contrast to the topics discussed in sect. 3, the subjects scrutinized here are the direct temporary eddy current effects that occur when small field level changes are made, and how undesirable effects can be suppressed. Whether a particular field level change can and should be treated as described in sect. 4 depends on the amount of local temporary overshoot of the final B value associated with the prescriptions developed in sect. 4. If this overshoot exceeds the experimentally determined safe amount, the rules developed in sect. 3 are, by definition, of more importance than those developed here. If, on the other hand, the amount of overshoot is small enough so that the DC aftereffects are negligible, the treatment of the problem with the linear theory developed below will be valid in most cases.

4.1. PROPERTIES OF TEMPORARY EDDY CURRENT-CAUSED FIELD INHOMOGENEITIES

The discussion is restricted at first to the middle part of a long symmetric magnet, and it will be assumed throughout that the excitation changes are so small that the behavior of the magnet in time can be described by a linear system, i.e. the Laplace transforms of all quantities of interest are related by transfer functions of the Laplace variable p. In order to characterize the behavior of the magnet, its response to an excitation change in form of a step function will now be discussed. It follows from sects. 2.2, 2.3, and 2.4 that after some time, the eddy currents in the pole region would be well described by the first few terms of the asymptotic expansion.

$$j_{SF}(x, y, t) \sim C_0(x,y) + \sum_{n=1}^{\infty} C_n(x,y)\, e^{-t/T_n} \quad . \tag{28}$$

The important property of this expression is that while the coefficients C depend on location x, y, the time constants do not. This follows from the qualitative picture developed in sect. 2, and is implicitly contained in eq. (13). From that section (eq. (14)) one would also expect that the time constants T_n have, very roughly, the pattern $T_n \sim 1/n^2$. The fact that in eq. (28) there will be no terms corresponding to complex T_n, i.e. the absence of terms proportional to damped trigonometric functions, is qualitatively clear when one realizes that the whole system has only one kind of energy storage.

The magnetic fields generated by the eddy currents will produce a field line pattern similar to the one depicted in fig. 3. The component of that pattern that is parallel to the pole-air interface at that interface produces a field perturbation in the air region. The field change resulting from a step function-like excitation change must therefore have the same form in the air as eq. (28):

$$B_{SF}(x, y, t) = B_0(x,y) - \sum_{n=1} B_n(x,y)\, e^{-t/T_n} \quad . \tag{29}$$

The Laplace transform of this equation can be described by the Laplace transform of the current change times of the transfer-function F(x, y, p) relating the current to the field change at location x, y. Normalizing to one the amplitude of the current change that gives eq. (29), it follows for F(x, y, p):

$$F(x, y, p) = B_0(x,y) - p \sum_{n=1} \frac{B_n(x,y)}{p + 1/T_n} \quad . \tag{30}$$

The fact that the singularities of $F(x, y, p)$ are independent of x and y is only a reflection of the fact that the time constants T_n in eq. (29) are location independent. Whether this is really true for a particular magnet can easily be determined by measuring the first few T_n at a number of locations. It is the experience of this author and a number of other workers that T_n is indeed practically independent of location for a variety of magnets. But it is also easy to imagine magnets where this condition will be strongly violated, for instance a long magnet whose pole- and yoke-widths change significantly over its length.

4.2. SUPPRESSION OF TEMPORARY EDDY CURRENT EFFECTS

When one is dealing with a large precision magnet, the effect described in sect. 4.1 can make it necessary to wait for unpleasantly long times until a magnet can be used again productively after a small field level change has been made. It is fairly easy to determine the first few time constants over the range of field levels of interest, and it will be assumed from now on that they are known. When one wants to reduce the waiting period after a small field level change, one is really only interested in eliminating the contributions associated with T_1, or possibly with T_1 and T_2, because the T_n decrease very rapidly with increasing n. One way to express one particular form of a solution is to require that the Laplace transform of the current change, I(p), satisfies the condition

$$(I(p))_{p=-1/T_n} = 0 \quad \text{for } n = 1, \quad \text{or } n = 1, 2 \quad . \tag{31}$$

While this expression is very convenient when one wants to work out specific solutions, a better understanding of what cancellation of the T_1 term means for I(t) is obtained by going back to eq. (29). Since the field change at location x, y can be expressed as the convolution integral of $\dot{I}(t)$ and the right side of eq. (29), cancellation of the T_1 term means that

$$G = B_1(x,y) \int_0^t \dot{I}(\tau)\, e^{-(t-\tau)/T_1}\, d\tau \tag{32}$$

has to be zero for $t > t_0$ if t_0 is the time whence the current is constant. Normalizing that current to be one, the condition for cancellation of the T_1 term becomes

$$\int_0^{t_0} \dot{I}(\tau)\, e^{\tau/T_1}\, d\tau = 0 \quad .$$

Integration by part leads to

$$\int_0^{t_0} (I(t) - 1)\, e^{\tau/T_1}\, d\tau/T_1 = 1 \quad . \tag{33}$$

This equation means that it is unavoidable that the final value one of the current is temporarily exceeded, and that the maximum value (I_1) of the current is a minimum for given t_0 and T_1 if

$$\left.\begin{array}{lll} I(t) = 0 & & \text{for } t < 0 \\ I(t) = I_1 = 1/(1 - \exp(-t_0/T_1)) & & \text{for } 0 \le t < t_0 \\ I(t) = 1 & & \text{for } t_0 \le t \end{array}\right\} \tag{34}$$

The Laplace transform of this I(t) is

$$I(p) = (I_1 + (1 - I_1) \exp(- p\, t_0))/p \quad . \tag{35}$$

From this follows also that any field level change procedure gives $(I(p))_{p=-1/T_1} = 0$ if multiplied by the numerator of the right side of eq. (35). Expressing that in words gives the rule: initiate any suitable field change procedure with amplitude I_1 at $t = 0$, and linearly superimpose the same field change procedure, but with amplitude $1 - I_1$ (<0) and initiation time t_0.

Sometimes a different method is used to make field changes: the field is measured at a particular point (x_0, y_0), and by using the appropriate feedback system one can prescribe the field change there as a function of time. The feedback system then generates the current change

$$I(p) = B(x_0, y_0, p)/F(x_0, y_0, p) \quad ,$$

and the field change at other locations becomes

$$B(x, y, p) = B(x_0, y_0, p) \cdot F(x, y, p)/F(x_0, y_0, p) \quad . \tag{36}$$

Since the singularities of $F(x, y, p)$ are cancelled by those of $F(x_0, y_0, p)$, the long time behavior of $B(x, y, t)$ is dominated by the roots of $F(x_0, y_0, p)$ closest to the imaginary p-axis. As before, the most damaging contributions can be compensated by appropriate choice of $B(x_0, y_0, p)$.

According to the discussions above, singularities of $F(x, y, p)$ are fairly well understood, and these singularities and their residues can in fact be computed with reasonable accuracy with the POISSON program. Unfortunately the same is not true of the roots of $F(x_0, y_0, p)$, and although the author tried successfully the procedure described by eq. (34), the equipment to determine the roots of $F(x_0, y_0, p)$ was not available. The reason for the interest in the roots of $F(x_0, y_0, p)$ is the possibility that their locations might be such that the last described procedure allows faster small field level changes than the direct current change procedure discussed first. One could also imagine prescribing some other field quantity with a feedback system. However to be useful, the corresponding transfer function must have the same singularities as $F(x, y, p)$, and its roots must be more advantageously located than the singularities of $F(x, y, p)$.

Sometimes it can be desirable to cancel the first two singularities leading to temporary field inhomogeneities. The reason may be that they are not separated as far as eq. (14) suggests, or that some parts of the magnet, such as the ends, have a dominating time constant very different from the one in the bulk of the magnet. This cancellation of two time constants can easily be accomplished by generalizing eq. (34) or eq. (35) through the inclusion of one more field level during a certain period of time. The procedure to carry this through is so straightforward that it needs no further elaboration, and it is also clear how the methods presented in this paper can be applied to magnets other than dipole magnets.

Acknowledgment

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Figure Captions

1) 2D cross-section of upper half of symmetrical H-magnet.

2) Field distribution in yoke with (——) and without (-----) eddy currents.

3) Field distribution caused by eddy currents alone in 1/4 of symmetrical H-magnet.

4) High field part of hysteresis curve.

5) 1/2 of C-magnet with j = 0-line.

6) Shape of B-probe.

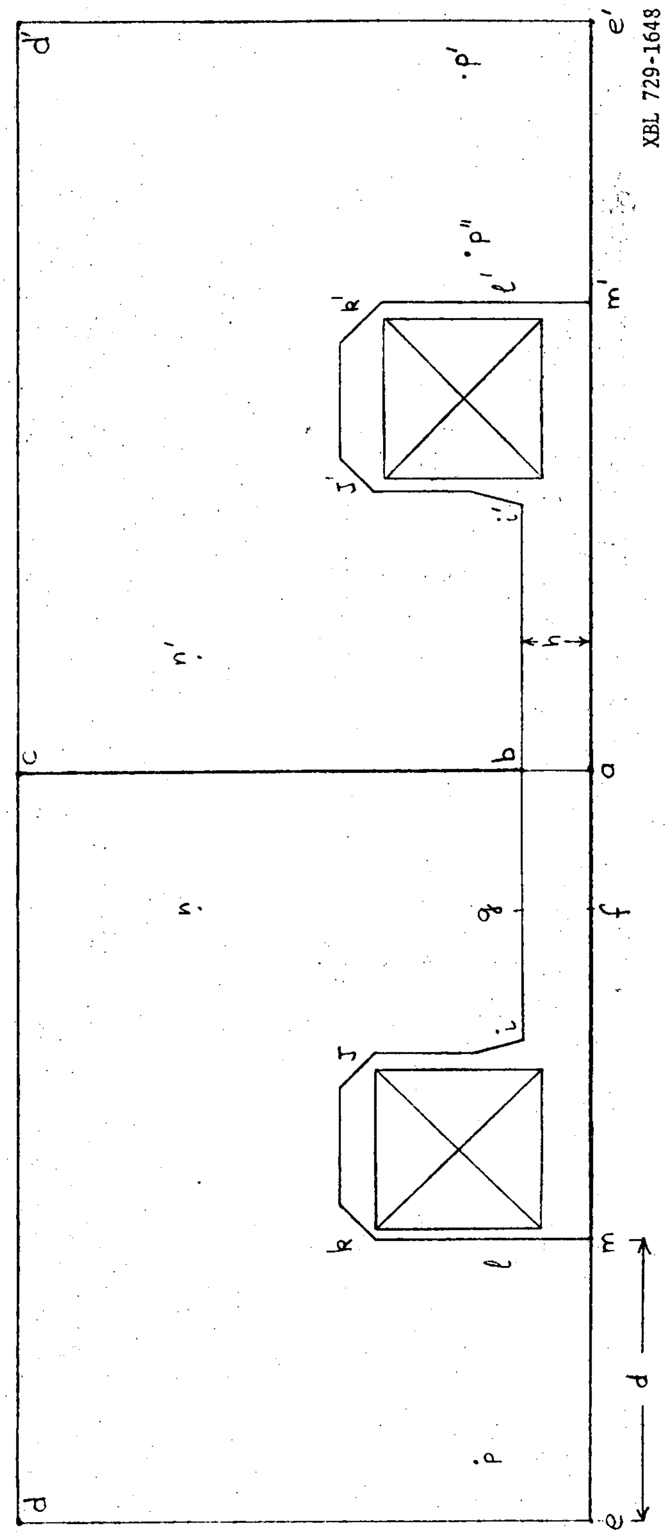

Fig. 1

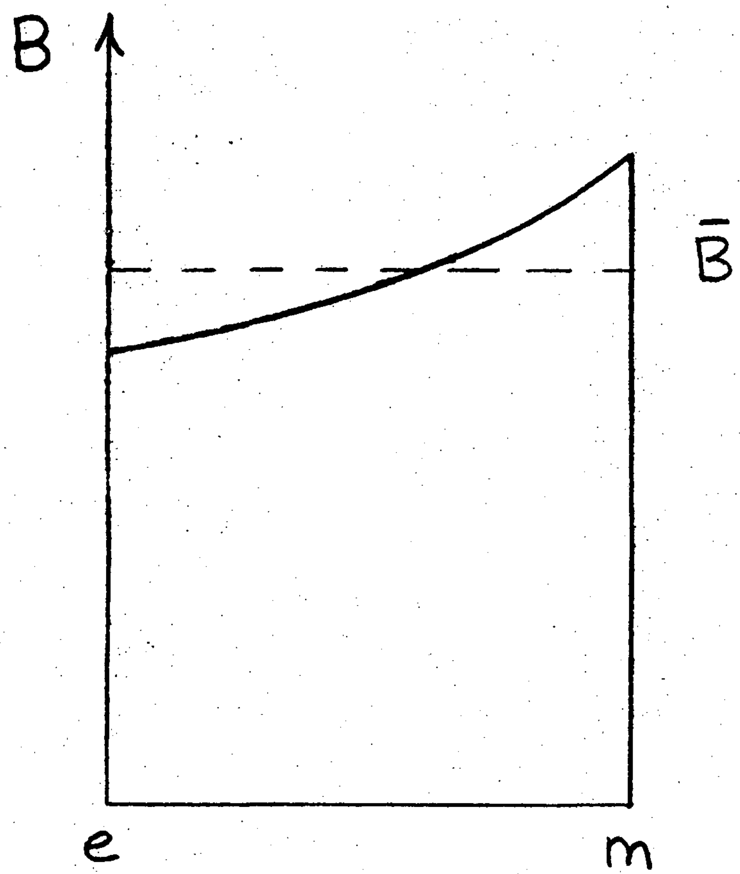

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Fig. 2

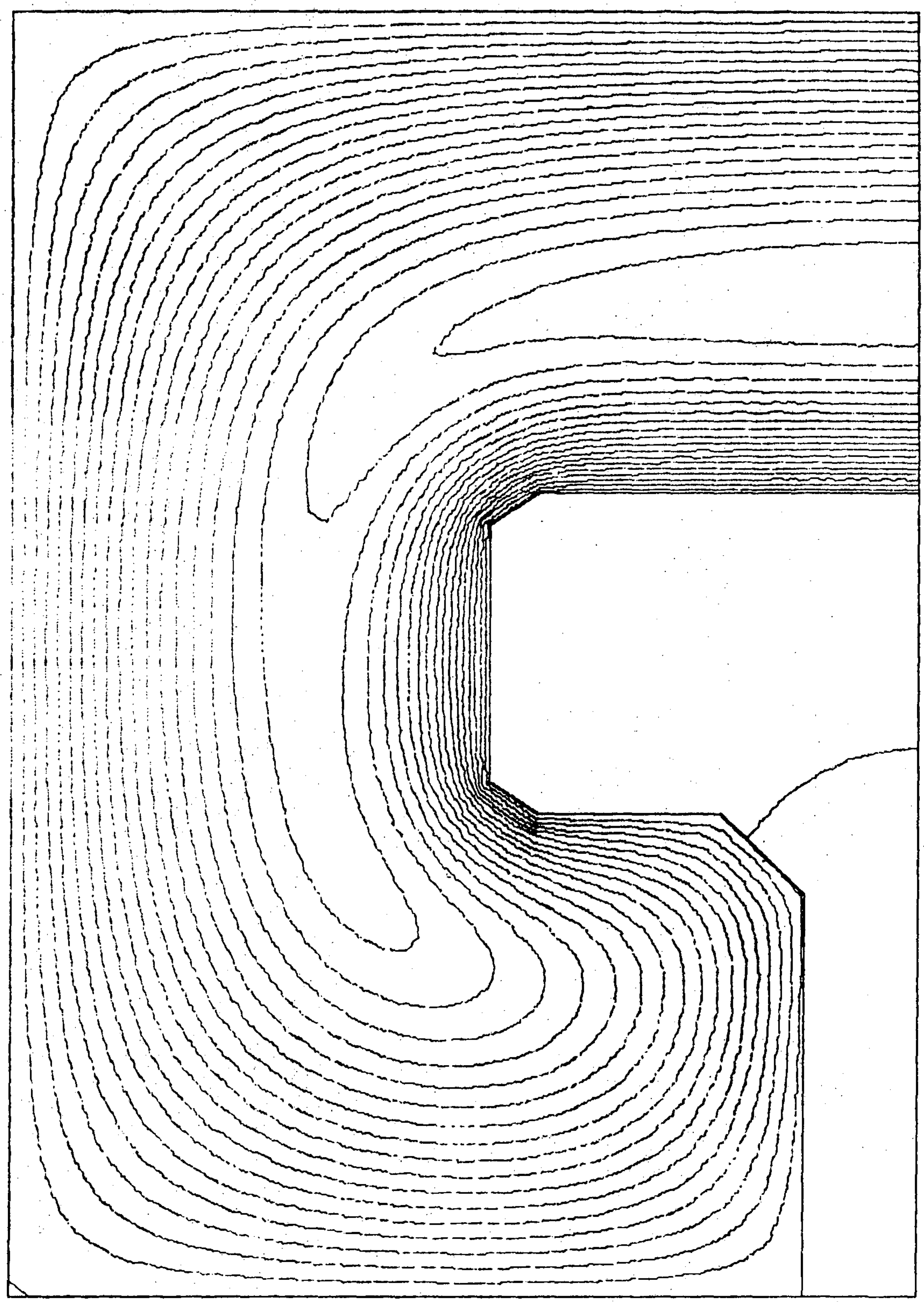

Fig. 3

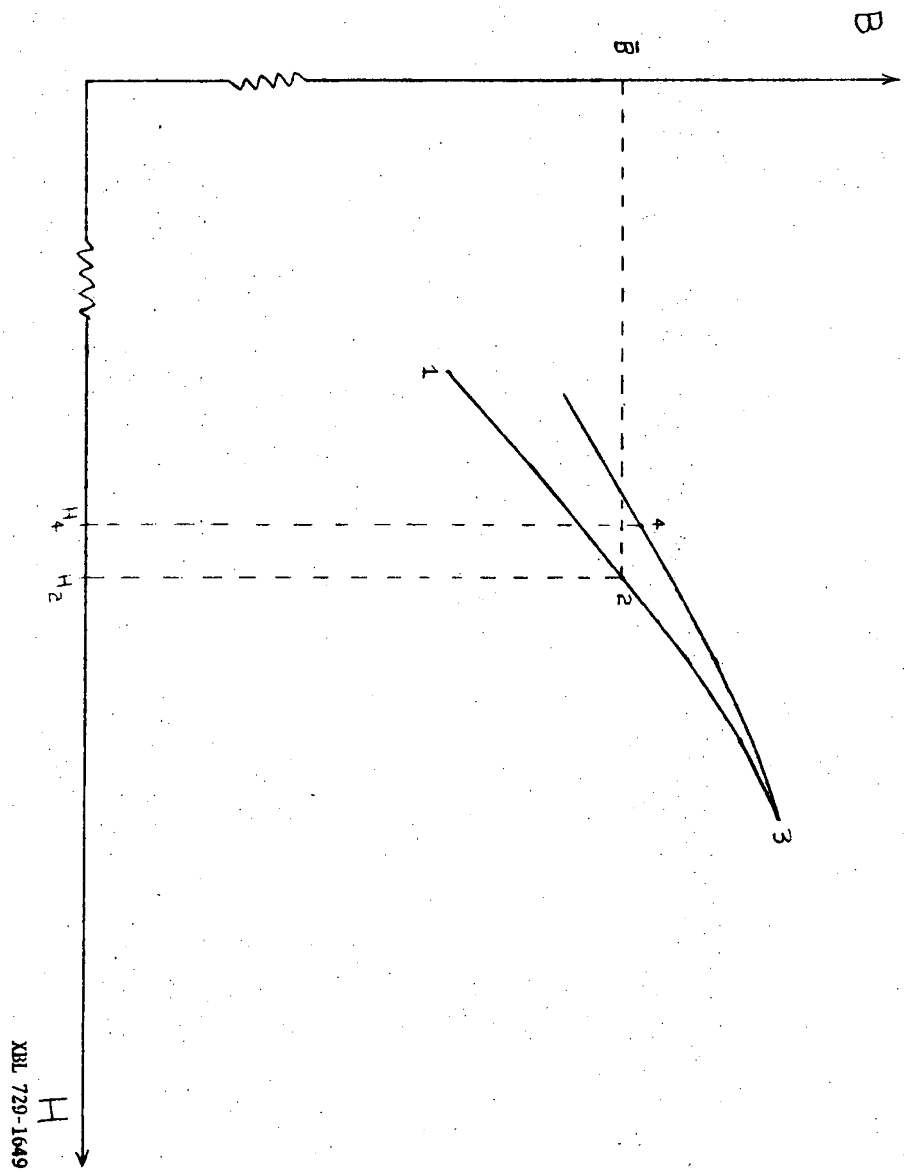

Fig. 4

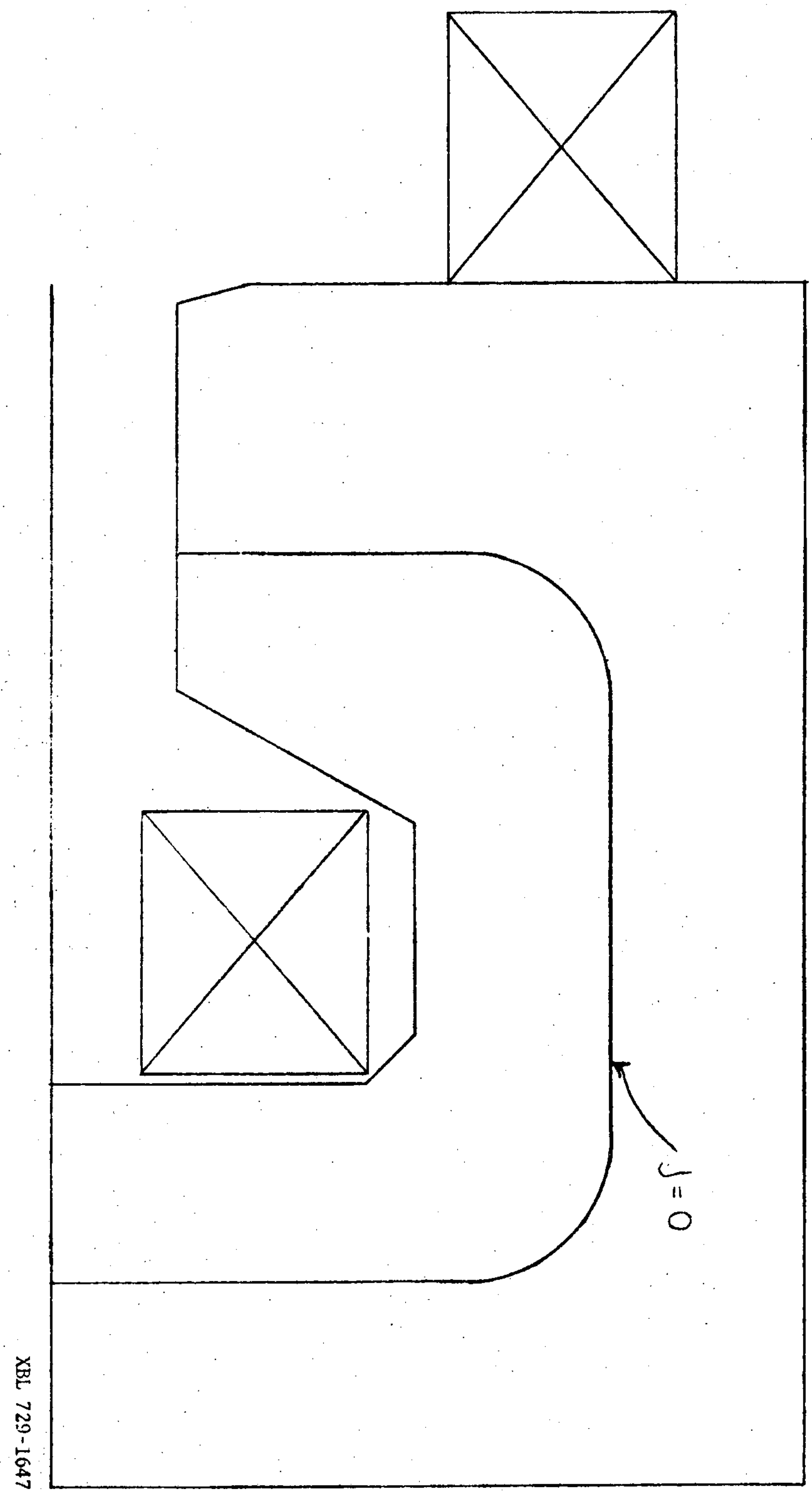

Fig. 5

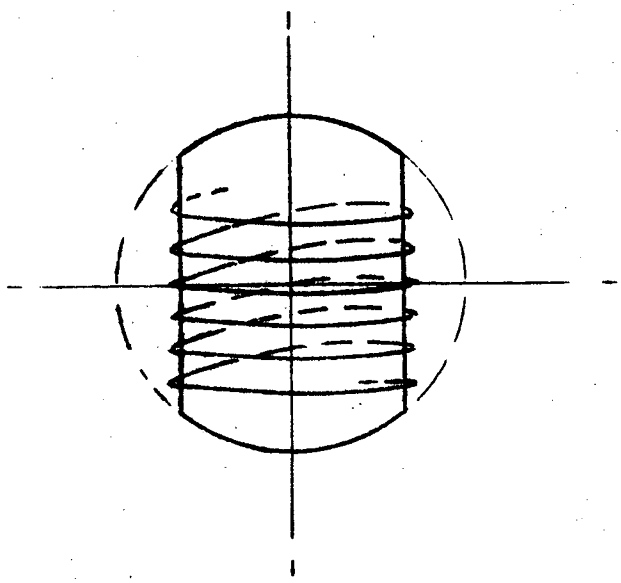

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Fig. 6

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