Title
Line integral representation of the modal radiation for an open-ended waveguide

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TABLE I
CONSTRAINTS FOR CROSS-POLARIZATION MINIMIZATION

Constraints	$SP(\%)$	G_{co}(dB)	ΔG(dB)
Dualmode Horns	95.0	49.0	25.0
Trimode Horns	90.0	49.0	35.0

TABLE II
NET RESULTS AFTER MINIMIZATION

Modes	$SP(\%)$	G_{co}(dB)	G_{cr}(dB)	ΔG(dB)
TE_{11}	93.535	48.614	27.823	20.791
Dualmode Horns	95.897	49.346	23.881	25.465
Trimode Horns	95.607	49.390	9.578	39.812

constraints for this example are summarized in Table I and the net results based on the algorithm described in Fig. 2 are summarized in Table II. It is observed from Table II that the applications of dual-mode horns and trimode horns with optimum ratios improve the cross-pol performance by 4.674 and 19.021 dB, respectively, compared to the horns with single-mode excitations. These results are consistent with the findings published in [2], which presented results for a single-feed horn.

III. CONCLUSIONS

A systematic algorithm for suppressing the cross-polarized component of single-offset reflector antennas illuminated by a cluster of multimode horn feeds using a constrained minimization routine has been developed. The algorithm allows for design constraints to be imposed during the minimization process. The trimode-type horns provide for the greatest improvement in cross-polarization performance. While shown here for a scanned beam case, the algorithm would also be useful for minimizing cross polarization in shaped-beam applications as well.

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Line Integral Representation of the Modal Radiation for an Open-Ended Waveguide

S. Maci, F. Capolino, and F. Mioc

***Abstract*—A line integral representation of the Kirchhoff-type aperture integration is derived for an open-ended waveguide with arbitrary cross section, excited by an arbitrary mode. The problem is formulated by taking advantage of the equivalence between the radiation of the aperture and the radiation of the modal currents along the semi-infinite waveguide walls.**

***Index Terms*—Aperture antennas.**

I. INTRODUCTION AND FORMULATION

The Kirchhoff-type aperture integration (AI) is the simplest way to calculate the radiation of an open-ended waveguide (OEW) excited by a mode. Reducing the AI to a line integration (LI) may provide a significant improvement in terms of computational efficiency. This is particularly useful in the framework of a method of moments (MoM) procedure, which is formulated in terms of mode-shaped basis functions. Furthermore, a line integral representation of the aperture field is particularly well suited for introducing fringe augmentation that may be provided by incremental techniques such as the physical theory of diffraction (PTD) [1] and the incremental theory of diffraction (ITD) [2].

In this letter, a method for asymptotically reducing the AI to a LI is presented. One formulation starts from the radiation in free-space of the corresponding semi-infinite distribution of the electric wall currents associated with the unperturbed mode. Recently, the equivalence between the field predicted by AI and the above wall current integration has been rigorously demonstrated for OEW's with arbitrary cross-section [3]. Since this latter approach resembles the physical optics (PO) method for scattering problems, it will be referred to as PO as well.

Let us consider an OEW of arbitrary cross section, which is excited by either a TE or a TM mode of arbitrary order; the field of this mode is denoted by $\vec{E}^{\rm mod}, \vec{H}^{\rm mod}$ and its propagation constant by $k_z^{\rm mod}$. It is useful to define a reference system with its origin inside the aperture and its z axis parallel to the waveguide generatrix; its relevant spherical coordinate system is denoted by (r, θ, ψ). The integration point on the wall is indicated by $\mathrm{P}' \equiv (\ell', z')$ where ℓ' is a curvilinear parameter that describes the waveguide contour in the transverse plane [Fig. 1(a)]. A local coordinate system (x'', y'', z'') is also introduced with its origin at the point P', its z'' axis parallel to z, and its y'' axis tangent to the surface at P' [Fig. 1(a)]; a spherical coordinate system (R, Θ, Ψ) and its pertinent unit vectors $(\hat{R}, \hat{\Theta}, \hat{\Psi})$ is associated with it.

In [3], it has been demonstrated that the AI field is exactly equal to the PO field when observing outside the waveguide. Thus, it is useful to represent the AI field in terms of the spherical components E_ξ^{po} of the PO field; i.e.,

$$E_\xi^{po} = \int_{\ell'} F_\xi^{po}(\ell')\, d\ell'; \qquad \xi = r, \theta, \psi \tag{1}$$

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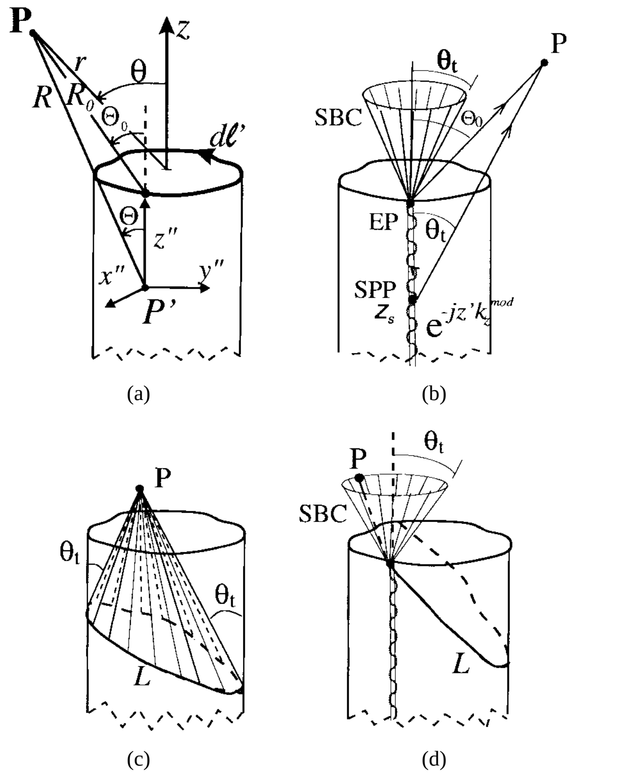

Fig. 1. (a) Geometry of an open-ended waveguide and its relevant reference systems. (b) Dominant asymptotic contributions associated to the radiation from an elementary strip. (c) Incremental geometrical optics contribution (IGOC) along the line L. (d) Incremental end-point contribution (IEC) at the intersection between L and the edge.

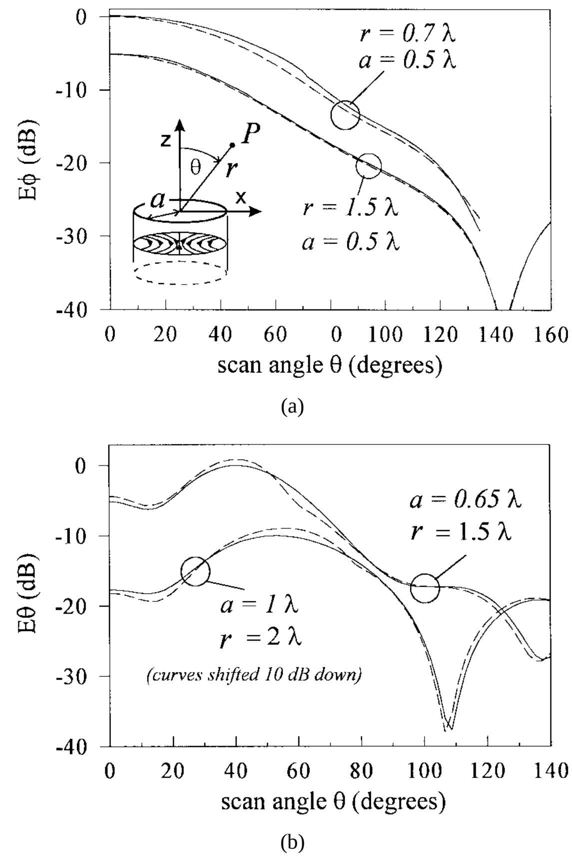

Fig. 2. Field radiated at finite distance from an open-ended circular waveguide (H plane); aperture integration (continuous line), line integration (dashed line). (a) TE_{11}-mode excitation, E_ψ component. (b) TM_{11} excitation, E_θ component.

where

$$F_\xi^{po}(\ell') = \int_{-\infty}^{0} f_\xi(z', \ell') \frac{e^{-jkR(z',\ell')}}{4\pi R(z', \ell')} e^{-jz' k_z^{\mathrm{mod}}}\, dz' \tag{2}$$

and

$$f_\xi(z', \ell') = -jk\zeta a \hat{\xi} \cdot (\hat{\Theta}\hat{\Theta} + \hat{\Psi}\hat{\Psi}) \cdot (\hat{n}_w \times \vec{H}^{\mathrm{mod}}). \tag{3}$$

In (2), the functional dependence of R on the integration variables has been explicitly indicated. In (3), $\hat{n}_w$ is the normal to the wall (pointing inside the OEW), and the gradient operator ∇ has been replaced by its relevant asymptotic approximation $-jk\hat{R}$. The term $F_\xi^{po}(\ell')\, \mathrm{d}\ell'$ in (1) represents the radiation of an elementary, semi-infinite strip of traveling-wave current, parallel to the z axis [Fig 1(b)].

For each ℓ', the integrand in (1) is asymptotically evaluated by its stationary phase point (SPP) contribution $F_\xi^{go}(\ell')$ and its end-point (EP) contribution $F_\xi^{e}(\ell')$, i.e.,

$$F_\xi^{po}(\ell') \sim F_\xi^{go}(\ell') + F_\xi^{e}(\ell') \tag{4}$$

where

$$F_\xi^{go}(\ell') = U(\Theta_0 - \theta_t) f_\xi(z_s, \ell') \frac{1}{4j} \cdot \sqrt{\frac{2j}{\pi k_t R_0 \sin\Theta_0}}\, e^{-jkR_0 \cos(\Theta_0 - \theta_t)} \tag{5}$$

and

$$F_\xi^{e}(\ell') = f_\xi(0, \ell') \frac{e^{-jkR_0}}{4\pi R_0} \frac{\mathcal{F}[2kR_0 \sin^2(\frac{1}{2}(\theta_t - \Theta_0))]}{jk(\cos\theta_t - \cos\Theta_0)}. \tag{6}$$

In (5) and (6), $(R_0, \Theta_0) \equiv (R, \Theta)|_{z'=0}$ [Fig. 1(a)]; θ_t is the incident-mode ray angle that is defined by $k_z^{\mathrm{mod}} = k\cos\theta_t$; $\mathcal{F}(x)$ is the transition function of the uniform theory of diffraction (UTD) [4]; $z_s = z - R_0 \sin\Theta_0 \cot\theta_t$ is the SPP, and $U(x)$ is a Heavyside unit-step function, which is unity when the SPP lies on the strip and zero elsewhere. The above expressions have been calculated by applying to the integral in (2) a simplified version of the method presented in [5].

In the following, F_ξ^{go} and F_ξ^{e} will be referred to as incremental geometrical optics contribution (IGOC) and incremental end-point contribution (IEC), respectively. As expected from the radiation of a traveling-wave current, each IGOC is a conical wave propagating in the direction θ_t. According to $U(\Theta_0 - \theta_t)$ the conical wave exists within the region $\Theta_0 > \theta_t$ and has its shadow boundary cone (SBC) at $\Theta_0 = \theta_t$ [Fig. 1(b)]. Each IEC is a spherical wave arising from a point of the edge; close to SBC of the corresponding IGOC, the IEC exhibits a transition into a conical wave behavior that allows compensation for the discontinuity of the IGOC. The sum of the IGOC plus the IEC is continuous everywhere and easily integrable.

The IGOC's can also be interpreted as incremental fields that arise from the SPP's $\mathrm{P}_s \equiv (\ell', z_s)$ defined by $\Theta = \theta_t$. These points belong to a line L, which is the intersection between the waveguide and a ray cone with vertex in P and aperture angle θ_t [Fig. 1(c)]. When L does not intersect the waveguide rim, the integration of IGOC's asymptotically reconstructs the field of an infinite waveguide, namely the modal field inside the infinite waveguide and zero outside. To speed-up practical calculations this latter condition (null field outside the infinite waveguide) can be enforced *a priori*. If the line L intersects the rim [Fig. 1(d)], only a part of L has to be integrated;

the integration end-points are determined by the condition $z_s = 0$ and described by the unit step function in (5).

It is worth noting that for large apertures in terms of a wavelength, $F_{\xi}^{po}(\ell')$ is a rapidly oscillating function of ℓ'. Consequently, the integration in (3) can be asymptotically evaluated by its stationary phase point contributions, thus, leading to a UTD-type ray-field representation. However, this latter fails in describing the field close to and at the axial caustic and it has also been found less accurate with respect to the present numerical line integration for moderate sized apertures.

II. Numerical Results

Numerical results from AI (continuous line) have been compared with those from LI (dashed line) for the case of a circular OEW with radius a. In particular, Fig. 2(a) shows results for a waveguide with radius $a = 0.5\lambda$, excited by the TE_{11} mode. The ψ component of the electric field in the H plane is plotted at a distance $r = 1.5\lambda$ and $r = 0.7\lambda$, respectively. Both curves are normalized with respect to the maximum value obtained in the case $r = 0.7\lambda$; furthermore, the field is calculated in the region external to the waveguide; i.e., $\theta < 130^\circ$ for $r = 0.7\lambda$ and $\theta < 160^\circ$ for $r = 1.5\lambda$. Normalized near field patterns for TM_{11}-mode excitation are presented in Fig. 2(b). The θ component of the electric field in the H plane is plotted for the two cases $a = 0.65\lambda$, $r = 1.5\lambda$, and $a = \lambda$, $r = 2\lambda$, respectively. The curves corresponding to this latter case are shifted 10 dB down to render the figure more readable.

In spite of the moderate size of the apertures, the agreement between the AI and its corresponding LI has been found quite satisfactory over the total 40-dB dynamic range. The small glitches arise from the fact that the IGCO integration has been turned off when L does not intersect the edge [see Fig. 1(c)]. The result presented here also suggests an effective method to speed-up practical calculations of the interaction between modes [6].

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An Efficient Formulation for Calculating the Modal Coupling for Open-Ended Waveguide Problems

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***Abstract*—A double line integral representation of the mutual coupling between open-ended waveguides of arbitrary cross section is presented, which is useful to speed up calculations inside the framework of a Galerkin method of moments.**

***Index Terms*—Aperture antennas, electromagnetic coupling.**

I. Introduction and Formulation

The theorem demonstrated in [1] establishes a rigorous equivalence between the field predicted by the Kirchhoff-type aperture integration (AI) and that radiated by the physical optics (PO) wall current. By using this equivalence, a formulation has been presented [2] to asymptotically reduce the AI into a line integration (LI) along the waveguide edge.

In this paper, the equivalence between PO and AI [1] is applied to modal coupling between OEW's of arbitrary cross-sections. This allows for the derivation of a convenient double integral expression for the modal coupling to be applied in the framework of a method of moments (MoM) for arrays of OEW's and horns. Note that a MoM Galerkin mutual impedance is generally given in terms of a quadruple integral in the space domain and the reduction to a double line integral is possible only for rectangular coplanar apertures [3]. When closed-form Fourier transform representations of modal coplanar distributions are available, one can resort to a spectral-domain approach; however, the resulting double spectral integrals are improper and slowly convergent. The method presented here is independent from the waveguide cross sections and, although it is developed here only for coplanar apertures, it can be easily extended to the noncoplanar case.

Let us consider two open-ended waveguides OEW1 and OEW2 of arbitrary cross sections. For the sake of simplicity, but without loss of generality, we will assume the two axes of the OEW's to be parallel. Two reference systems (x', y', z') and (x'', y'', z'') are introduced in which $z' < 0$ and $z'' < 0$ are the semi-axes of OEW1 and OEW2, respectively (Fig. 1). Let us denote by $(\vec{e}_1^{\,n} \exp(jk_z^n z'), \vec{h}_1^{\,n} \exp(jk_z^n z'))$ the field propagating into OEW1 toward negative z' and by $(\vec{e}_2^{\,m} \exp(-jk_z^m z''), \vec{h}_2^{\,m} \exp(-jk_z^m z''))$ the field of the mth mode propagating toward the positive z axis of OEW2, where k_z^n and k_z^m are the z-propagation constant of the relevant modes. These fields are normalized in such a way that the integration of $\vec{e}_i^{\,p} \cdot \vec{h}_i^{\,q} (i = 1, 2)$ on the aperture is equal to $(-1)^i \delta_{pq}$ (Kronecker's delta). Magnetic $\vec{m}_i^{\,m} = (-1)^i \vec{e}_i^{\,m} \times \hat{z}$, and electric $\vec{j}_i^{\,m} = (-1)^i \hat{z} \times \vec{h}_i^{\,m}$ current distributions associated to the unperturbed modes are defined on each aperture. The field radiated in the near zone by these aperture distributions is denoted by $(\vec{E}_i^{\,m}, \vec{H}_i^{\,m})$. The mutual coupling coefficient between the two

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