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J. D. Jackson and C. Quigg

March 25, 1970

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REMARKS ON PION PHOTOPRODUCTION⁺J. D. Jackson^{††} and C. QuiggDepartment of Physics and Lawrence Radiation Laboratory
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March 25, 1970

We record some predictions for charged pion photo-production implicit in our previous work. These include extension of the pseudomodel to somewhat larger t values and a discussion of photoproduction from a polarized target. Some remarks are also made on the predictions of gauge-invariant perturbation theory.

1. INTRODUCTION

Elsewhere we observed^{1, †} that finite energy sum rules (FESR) provide a more or less model-independent connection between low energy and high energy cross sections for charged pion photoproduction. In this paper we extend the predictions of the FESR pseudomodel to larger values

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[†] Three misprints of importance occur in ref. 1; for completeness we list them here: 1) In eq. (2) the argument of the exponential should be multiplied by a . 2) In the definition of the CMSR the argument of the exponential should be multiplied by $1/2$. 3) In eq. (10), the constant on the RHS of the σ_1 expression should be $(4\pi\mu^2)^{-1}$.

of $-t$. After summarizing the relations between several useful sets of amplitudes in section 2, we review in section 3 the formulation of the pseudomodel. Rather reliable predictions are possible for the gross features of the data: differential cross sections for unpolarized or linearly polarized photons incident. However, more delicate features, such as the left-right asymmetry from a polarized target are not predictable at the present stage of low energy analysis. We give expressions for the left-right asymmetry A , and the recoil nucleon polarization, and calculate the asymmetry A in some plausible Regge cut models in section 4. Section 5 is a digest of results which follow from gauge-invariant perturbation theory. These are mostly well known, with the possible exception of the asymmetry, Σ , for linearly polarized photons. For $|t| < 2\mu^2$ not only the cross section but also the polarized photon asymmetry Σ for charged pions are given quantitatively by the electric Born results.

2. INVARIANT AMPLITUDES, t -CHANNEL AND s -CHANNEL HELICITY AMPLITUDES;

CROSS-SECTION FORMULAS

2.1. Ball's Invariant Amplitudes

Let k, p, q, p' be the four-momenta of the photon, initial nucleon, pion, and final nucleon, respectively, and define $P = \frac{1}{2}(p + p')$. The nucleon mass is m , the pion mass is μ . The invariant Feynman amplitude for a photon of polarization ϵ incident on a nucleon of helicity λ leading to the production of a pion and a nucleon of helicity λ' is written as²

$$\mathcal{M} = \bar{u}_{\lambda'}(p') [A_1 \sigma_1 + A_2 \sigma_2 + A_3 \sigma_3 + A_4 \sigma_4] u_{\lambda}(p) \quad (1)$$

where the A_i are four invariant amplitudes and the four σ_i are gauge-invariant Dirac operators,

$$\left. \begin{aligned}
 \mathcal{O}_1 &= i \gamma_5 \gamma \cdot \epsilon \gamma \cdot k \\
 \mathcal{O}_2 &= 2i \gamma_5 (P \cdot \epsilon q \cdot k - P \cdot k q \cdot \epsilon) \\
 \mathcal{O}_3 &= \gamma_5 (\gamma \cdot \epsilon q \cdot k - \gamma \cdot k q \cdot \epsilon) \\
 \mathcal{O}_4 &= 2\gamma_5 (\gamma \cdot \epsilon P \cdot k - \gamma \cdot k P \cdot \epsilon - im \gamma \cdot \epsilon \gamma \cdot k)
 \end{aligned} \right\} \quad (2)$$

The invariant amplitudes A_i are Lorentz scalars, functions of s and t , and free of kinematic singularities.

An isospin decomposition of the amplitude is frequently useful. For the production of a pion with isospin component a the invariant amplitude (an operator in the 2×2 isospin space of the nucleons) is

$$A_i = A_i^{(+)} \delta_{a,3} + A_i^{(-)} \frac{1}{2} [\tau_a, \tau_3] + A_i^{(0)} \tau_a. \quad (3)$$

The amplitudes $A_i^{(+)}$ and $A_i^{(-)}$ correspond to the absorption of isovector (rho-like) photons, while $A_i^{(0)}$ is the contribution of the isoscalar photons. In tables I and IV below the various possibilities are itemized.

2.2. t-Channel Amplitudes

The t-channel helicity amplitudes $f_{\lambda_\gamma 0; \lambda \lambda'}$ for the process $NN' \rightarrow \gamma\pi$ are customarily related to the invariant amplitudes through the intermediary of the so-called parity-conserving t-channel amplitudes F_i , defined to be free of kinematic singularities. The F_i are related to the helicity amplitudes according to

$$F_1 = \frac{i\sqrt{2}}{(t - \mu^2)\sin \theta_t} \left(f_{10; ++}^t + f_{-10; ++}^t \right) \quad (4)$$

$$F_2 = \frac{i(2t)^{\frac{1}{2}}}{(t - 4m^2)^{\frac{1}{2}} \sin \theta_t} \left(f_{10; ++}^t - f_{-10; ++}^t \right) \quad (5)$$

$$F_3 = \frac{-i(2t)^{\frac{1}{2}}}{t - \mu^2} \left(\frac{f_{10; +-}^t}{\cos \theta_t + 1} + \frac{f_{-10; +-}^t}{\cos \theta_t - 1} \right) \quad (6)$$

$$F_4 = \frac{i\sqrt{2}}{(t - \mu^2)(t - 4m^2)^{\frac{1}{2}}} \left(\frac{f_{10; +-}^t}{\cos \theta_t + 1} - \frac{f_{-10; +-}^t}{\cos \theta_t - 1} \right). \quad (7)$$

The amplitudes F_1 and F_2 receive contributions from natural and unnatural parity sequences in the t -channel, respectively, and for charged pion photoproduction the contributions are further restricted to trajectories satisfying $\tau G = -1$. (τ is the signature of the trajectory.) In leading order (in powers of $\cos \theta_t$) F_3 and F_4 involve natural and unnatural parity sequences, respectively. Also, F_3 has the same signature-- G -parity relation--as F_1 and F_2 , while for F_4 it is opposite. The contributions of well-known particles are given in table I.

The connections of the F_i to the A_i of eq. (1) are

$$F_1 = -A_1 + 2mA_4 \quad (8)$$

$$F_2 = (t - \mu^2)(A_1 + tA_2) \quad (9)$$

$$F_3 = 2mA_1 - tA_4 \quad (10)$$

$$F_4 = -A_3. \quad (11)$$

We note in passing that eqs. (8) - (11) imply certain relations among the F_i at special values of t , whatever the value of s . [At $t = 4m^2$, $F_3 = -2mF_1$ (a threshold relation); at $t = 0$, $2mF_2 + \mu^2 F_3 = 0$ (a conspiracy relation).][†]

[†] For a discussion of the physical meaning of these relations see, for example, ref. 3.

2.3 s-Channel Amplitudes

In the s-channel we choose the four CMS helicity amplitudes $g_{0\lambda'; \lambda_\gamma \lambda} = g_j(s, t)$, where $j = 1, 2, 3, 4$ stands for the photon, initial nucleon, and final nucleon helicities $(\lambda_\gamma, \lambda_N; \lambda_{N'}) = (-1, -\frac{1}{2}; \frac{1}{2})$, $(1, -\frac{1}{2}; -\frac{1}{2})$, $(1, \frac{1}{2}; \frac{1}{2})$, $(1, -\frac{1}{2}; \frac{1}{2})$, respectively. [The careful reader may have noticed that the choices for helicities for g_1 and g_4 are different (all helicities reversed) from the definitions in ref. 1. These changes, instituted in order to conform to the phase conventions of ref. 4 concerning "particle 2", make the present g_1 and g_4 opposite in sign to those of ref. 1. As can be inferred from eqs. (14a) and (14b) below, none of the results of ref. 1 are affected by such sign changes.] The corresponding net s-channel helicity flips $n(j) = |\lambda_\gamma - \lambda_N + \lambda_{N'}|$ are 0, 1, 1, and 2. At high energies and small momentum transfers these amplitudes are related to the F_i by

$$g = (\nu/\sqrt{2})X \cdot F \quad (12)$$

where $\nu = (s - m^2)/2m$ is the photon energy in the laboratory and the crossing matrix X is

$$X = \begin{pmatrix} -\tau/2m & -2m/(\mu^2 + \tau) & 1 & 0 \\ (\tau)^{\frac{1}{2}} & 0 & (\tau)^{\frac{1}{2}}/2m & 2m(\tau)^{\frac{1}{2}} \\ (\tau)^{\frac{1}{2}} & 0 & (\tau)^{\frac{1}{2}}/2m & -2m(\tau)^{\frac{1}{2}} \\ -\tau/2m & 2m/(\mu^2 + \tau) & 1 & 0 \end{pmatrix} \quad (13)$$

In eq. (13) we have introduced $\tau = -t$, and have neglected terms of relative order $(\tau/4m^2)$, as well as nonleading powers of v .

2.4. Cross Sections

Cross sections for linearly polarized incident photons are most easily expressed in terms of s-channel amplitudes. The expressions are

$$(s - m^2)^2 d\sigma_{\parallel}/dt = (1/32\pi)(|g_2 - g_3|^2 + |g_1 - g_4|^2) \quad (14a)$$

(photons polarized in the reaction plane)

$$(s - m^2)^2 d\sigma_{\perp}/dt = (1/32\pi)(|g_2 + g_3|^2 + |g_1 + g_4|^2) \quad (14b)$$

(photons polarized perpendicular to the reaction plane).

The cross section for unpolarized photons is the average of (14a) and (14b):

$$(s - m^2)^2 d\sigma/dt = (1/32\pi) \sum_{i=1}^4 |g_i|^2. \quad (15)$$

For a polarized target with unpolarized photons incident the differential cross section in the laboratory takes the form,

$$\frac{d\sigma}{dt} \propto 1 + A \langle \underline{\sigma} \rangle \cdot \underline{n}$$

where $\langle \sigma \rangle$ is the target polarization vector, \underline{n} is the normal to the production plane defined according to the Basel convention⁵ (positive normal in the direction $\underline{k} \times \underline{q}$), and A is the left-right asymmetry parameter, given by

$$A = 2 \operatorname{Im}(g_1 g_2^* - g_3 g_4^*) / \sum_{i=1}^4 |g_i|^2. \quad (16)$$

If the target and photon beam are unpolarized, the recoiling nucleon possesses a polarization in the direction of the normal \underline{n} of magnitude

$$P = 2 \operatorname{Im}(g_1 g_3^* - g_2 g_4^*) / \sum_{i=1}^4 |g_i|^2. \quad (17)$$

This polarization is in general different from the asymmetry A .

However, for the circumstance where $F_4 = 0$ (a common assumption in the theory) the crossing matrix (13) shows that at high energies and small momentum transfers $g_2 = g_3$. Under these conditions, $A = P$.

3. PSEUDOMODEL PREDICTIONS

The low energy side of the zero-moment finite energy sum rule is defined as

$$\phi_i(\tau, \bar{\nu}) = -(\mu/\pi) \int_0^{\bar{\nu}} d\nu \operatorname{Im}(F_i(\nu, \tau)) . \quad (18)$$

That the FESR relate low energy data and high energy scattering amplitudes is well known. In fact, for pion photoproduction the connection can be made very direct; if all the Regge pole (and smeared Regge pole, i.e. Regge cut) contributions to F_i satisfy the condition on their (effective) trajectories, $|\alpha_{\text{eff}}| < 1$, it can be shown¹ that

$$\phi_i(\tau, \bar{\nu}) \approx \frac{1}{2} \mu \bar{\nu} \operatorname{Re}(F_i(\bar{\nu}, \tau)) . \quad (19)$$

This result is essentially model-independent.

As discussed in ref. 1, the direct connection between the FESR ϕ_i and the t-channel amplitudes F_i exhibited by eq. (19) provides a means for linking the low energy data and the high energy cross sections without an intervening model. We need one more assumption or empirical fact. It is well established⁶ that the energy dependence of the charged pion photoproduction amplitudes is consistent with a power-law behavior of $\nu^{\alpha_{\text{eff}}}$ with $\alpha_{\text{eff}} \approx 0$, at least for $|t| < 1 \text{ (GeV/c)}^2$. For π^0 photoproduction the exponent also seems close to zero, although it may

be somewhat positive in the region of the dip in the cross section ($|t| \sim 0.5(\text{GeV}/c)^2$). Now, independent of the possible presence of logarithmic variation in ν multiplying the power law, the Phragmén-Lindelöff theorem⁷ allows us to conclude that the phases of the amplitudes are given by $[\pm 1 + \exp(-i\pi\alpha)]$, depending on the behavior of the amplitude under crossing. For the amplitudes $F_i^{(-)}$ which dominate charged pion photoproduction in the forward direction the phase factor is $[1 + \exp(-i\pi\alpha)]$. With $\alpha_{\text{eff}} \approx 0$ these amplitudes are predominantly real. For charged pion photoproduction the high energy cross sections at small t can thus be written directly in terms of the ϕ_i of the FESR, by means of eq. (19). The cross sections for linearly polarized photons, (14a) and (14b) for example, are

$$(s - m^2)^2 \frac{d\sigma_{\parallel}}{dt} \approx \frac{2m^2}{\pi\mu^2} \left[\frac{(\phi_2)^2}{(\mu^2 + \tau)^2} + \tau(\phi_4)^2 \right],$$

$$(s - m^2)^2 \frac{d\sigma_{\perp}}{dt} \approx \frac{1}{2\pi\mu^2} \left[(\phi_3)^2 + \tau(\phi_1)^2 \right],$$
(20)

where the ϕ_i are actually the $\phi_i^{(-)}$ corresponding to isovector photons. Data on the π^-/π^+ ratio in deuterium⁸ indicate that for $\tau < 0.1(\text{GeV}/c)^2$ these contributions are dominant, but that for larger τ the isoscalar contributions (e.g. ρ -exchange) are appreciable. In the larger τ region, eq. (20) is expected to apply more closely to the average of π^+ and π^- photoproduction. The FESR results[†] show

† The sum rules were evaluated by G. C. Fox from the low-energy parameterization of ref. 9.

that for small τ the cross sections are given entirely by the first terms in eq. (20).

Comparison of the pseudomodel given by eq. (20), retaining only the contributions of $\phi_2^{(-)}$ and $\phi_3^{(-)}$, with the data^{10,11} on $\gamma p \rightarrow \pi^+ n$ at small τ is shown in fig. 1. This figure supplements fig. 1 of ref. 1 by extending the range of $\sqrt{\tau}$ from 0.4 to 0.7 GeV/c. Also shown here is a curve for the electric Born approximation. The rather remarkable agreement between the pseudomodel and the data has been commented on in ref. 1. The general success of the results of gauge-invariant perturbation theory, at least for $\tau < 0.1$ (GeV/c)², has been discussed some time ago by Harari.¹² We make some further comments about the perturbation theory in section 5.

A rather puzzling aspect of the excellent agreement shown in fig. 1 is made manifest if we examine the π^-/π^+ ratio from deuterium.⁸ This ratio has fallen from a value near 1 at $\tau = 0$ to ~ 0.3 at $\sqrt{\tau} = 0.6$ GeV/c. Such a ratio at $\sqrt{\tau} = 0.6$ implies an appreciable contribution from the isoscalar amplitudes $F_i^{(0)}$ and argues against the cross section for π^+ photoproduction being given entirely in terms of $F_i^{(-)}$, as is assumed in calculating the curve of fig. 1. Indeed, the pseudomodel does only moderately well in fitting the average of the π^+ and π^- cross sections beyond $\sqrt{\tau} = 0.2$ GeV/c, even though that is the cross section for which the isovector-isoscalar interference terms cancel. One may conjecture that the preferential success in fitting the π^+ data can be traced to a corresponding bias in Walker's fitting⁹ of the low-energy data from hydrogen and deuterium, but this is only speculation at this point.

Pseudomodel predictions for the asymmetry parameter for production of pions by linearly polarized photons,

$$\Sigma \equiv \frac{\sigma_{\perp} - \sigma_{\parallel}}{\sigma_{\perp} + \sigma_{\parallel}},$$

are displayed in fig. 2. There is quantitative agreement with the (rather low energy) data¹² on π^{\pm} photoproduction out to $\tau = 0.1(\text{GeV}/c)^2$ and qualitative agreement to $\tau \simeq 0.5(\text{GeV}/c)^2$. We emphasize that the predictions for $(s - m^2)^2 d\sigma/dt$ and for Σ are energy-independent (by virtue of our assumption that $\alpha \approx 0$ for each amplitude). All pseudomodel results are collected in table II.

The success of the pseudomodel at small $|t|$ encourages one to attempt further predictions, such as the asymmetry from a polarized target, but there are immediate difficulties. In terms of t-channel amplitudes, the asymmetry parameter A is

$$A = \frac{2(\tau)^{\frac{1}{2}} \text{Im} \left[\frac{F_1^* F_3}{4m^2} \left(1 + \frac{\tau}{4m^2} \right) + \frac{F_2^* F_4}{\mu^2 + \tau} \right]}{\left[\frac{\tau |F_1|^2 + |F_3|^2}{4m^2} \right] \left(1 + \frac{\tau}{4m^2} \right) + \left[\frac{|F_2|^2}{(\mu^2 + \tau)^2} + \tau |F_4|^2 \right]} \quad (21)$$

While the cross section [the denominator in (21)] is dominated at small $|t|$ by the amplitudes F_2 and F_3 , whose real parts are inferred to be much larger than their imaginary parts by the observation of $\alpha_{\text{eff}} \simeq 0$, the numerator in (21) depends crucially on the phase differences between F_1 and F_3 , and F_2 and F_4 . The phases of F_1

and F_4 can in principle be determined from the effective α values of their power-law behaviors, but the high-energy cross sections are no help for these amplitudes and the upper limit of integration $\bar{\nu} = 1.28$ GeV on the finite energy sum rules is too low to expect a reliable estimate of $\alpha(t)$ from them. Theoretical expectations are for the A_2 and ρ Regge poles to enter the isovector-photon and isoscalar-photon contributions to F_1 (and F_3), while A_1 -exchange occurs in F_4 . If the ratios of real to imaginary parts of the various amplitudes are evaluated by use of effective Regge trajectories deduced elsewhere, the FESR can be used to calculate A at high energies. The result[†] of such a

[†] Private communication from G. C. Fox.

calculation for photoproduction at 15 GeV is a small negative asymmetry of maximum magnitude ~ 0.15 at $\tau \simeq 0.08$ (GeV/c)². The magnitude of the effect cannot be taken too seriously because the sum rules for $F_1^{(-)}$ and $F_4^{(-)}$ are less accurate than those for $F_2^{(-)}$ and $F_3^{(-)}$; there is also the problem of $\bar{\nu} = 1.28$ GeV. The sign of the asymmetry seems less open to question, although even there modifications in some of the resonance contributions to the low-energy integrals could produce marked effects.

4. POLARIZATION PREDICTIONS FROM REGGE CUT MODELS

In ref. 1 we presented some non-unique Regge cut models for charged pion photoproduction. These models were required to (i) reproduce the near-forward differential cross section and (ii) satisfy the sum rules $\phi_2^{(-)}$ and $\phi_3^{(-)}$. Since our object was the construction of counterexamples to certain FESR "proofs" of the existence of a pion conspirator, we did not concern ourselves with the other six sum rules. Here we explore the range of predictions for the asymmetry parameter A given by these models and comment on the uniqueness or lack of it when related considerations about FESR and factorization are brought to bear. For convenience we exhibit the parameterization of ref. 1. The model involves only the isovector-photon amplitudes $F_1^{(-)}$, $F_2^{(-)}$, $F_3^{(-)}$ and has Regge pion and A_2 exchanges, modified by Regge cuts generated in the manner of the absorptive model [eq. (2) of ref. 1]. The input Regge pole amplitudes, expressed in terms of the s-channel amplitudes g_i of eq. (12), are

$$\begin{aligned}
 g_1^{\text{Regge}} &= g_\pi - g_A \\
 g_2^{\text{Regge}} &= g_3^{\text{Regge}} = h_A \\
 g_4^{\text{Regge}} &= -g_\pi - g_A
 \end{aligned}
 \tag{22}$$

where the individual Regge pole contributions are

$$\begin{aligned}
 g_{\pi} &= \text{eg } \alpha'_{\pi} \tau \Gamma(-\alpha_{\pi}(\tau)) [1 + \exp\{-i\pi\alpha_{\pi}(\tau)\}] (v/v_0)^{\alpha_{\pi}(\tau)}, \\
 g_A &= \text{eg } \frac{\tau}{4m^2} (\xi_1 + \xi_2) \Gamma(1 - \alpha_A(\tau)) [1 + \exp\{-i\pi\alpha_A(\tau)\}] (v/v_0)^{\alpha_A(\tau)}, \\
 h_A &= \text{eg } \frac{(\tau)^{\frac{1}{2}}}{2m} (\xi_1 - \tau\xi_2/4m^2) \Gamma(1 - \alpha_A(\tau)) [1 + \exp\{-i\pi\alpha_A(\tau)\}] (v/v_0)^{\alpha_A(\tau)}.
 \end{aligned}
 \tag{23}$$

The residues ξ_1 and ξ_2 of the A_2 Regge pole correspond to residues in $F_1^{(-)}$ and $F_3^{(-)}$, respectively, as can be inferred by inversion of (12). In ref. 1, and also here, these residues are taken as constants, an approximation suitable for the small $|t|$ region. The model has $F_4 = 0$ and so has the restriction, $A = P$.

Seven representative models are specified by the parameters given in table III. Model 4 has an elementary pion and no A_2 . It gives adequate fits to the CMSR and to the high-energy cross sections, as already mentioned in ref. 1. It gives $A = P = 0$, of course. Models 1, 2, 3 have $\xi_2/\xi_1 \simeq +6.6$ and differ in the absolute strength of the A_2 coupling and the strength of the cuts. Models 1', 2', 3' have a reversed sign for ξ_1 , while preserving the absolute values of the ratio $(\xi_1 + \xi_2)/\xi_1$ as in models 1, 2, 3. All six of these models give acceptable fits to the high-energy cross sections in the very small $|t|$ region (see Fig. 1 of ref. 1 for a comparison of model 1 with the data). The insensitivity of the cross sections to the sign of ξ_1 (negative in the unprimed models, positive in the primed models) can be understood by examining eqs. (14) and (22,23). The sign of

$(\xi_1 + \xi_2)$ in g_A is determined by the need for constructive interference between the A_2 and π contributions to g_1 in a model with reasonable strengths for the Regge cuts. The primed and unprimed models differ at small $|t|$ only in the sign of h_A , and this sign is irrelevant for the cross sections (14). At larger $|t|$ values the corresponding primed and unprimed models yield somewhat different magnitudes for $g_2 = g_3$, but the differences are not important for $\tau < 0.15$ (GeV/c)².

Inspection of eqs. (16) and (17) shows, on the other hand, that the sign of $A = P$ is determined at small $|t|$ by the relative signs (and phases) of g_1 and $g_2 = g_3$. Thus the corresponding primed and unprimed models give approximately equal and opposite predictions for the asymmetry A . These are plotted for models 1, 1', 2, and 2' in fig. 3.

How can we choose between the primed and unprimed classes of models, apart from by comparison with experimental data on A or P ? First we detail the things which unite the models: Models 1, 1', and 4 are in quantitative agreement with the CMSR constraints on $F_2^{(-)}$ and $F_3^{(-)}$ for small τ , while models 2, 2', 3, and 3' disagree by $\sim 10\%$. All six models fit the high-energy cross section for $\gamma p \rightarrow \pi^+ n$ for $(\tau)^{\frac{1}{2}} \lesssim 0.1$ and are $\sim 20\%$ low for larger τ . Confrontation of the models with the requirements of the FESR for $F_1^{(-)}$ is less satisfactory than for $F_2^{(-)}$ and $F_3^{(-)}$ (and perhaps less significant, as is indicated below). The high-energy side of the FESR for $F_1^{(-)}$ is assumed to be dominated by the A_2 meson at small t . The trajectory and residue function have been evaluated, assuming a single Regge pole,

by a number of authors.^{14,15,16} At $t = 0$ the FESR is consistent with the magnitude and sign of the input A_2 pole of model 1' ($\xi_1 \simeq +0.75$), but the corrections from the various Regge cut amplitudes are sufficiently great that model 1' gives a much smaller net value than the pole term alone. Since the magnitude of ξ_1 is smaller for models 2, 2', 3, 3', they also give a high-energy side of the FESR considerably smaller than that found from the low-energy data, and sometimes differing in sign.

The examples of parameterization in table III were found by trial and error, with a bias against solutions with very large Regge cut contributions. Obviously a search could be made for solutions with larger ξ_1 in order to try to satisfy the constraint of the FESR for $F_1^{(-)}$. We have not done this for several reasons. One is that the neglect of exchanges other than π and A_2 already makes the predictions for A shown in fig. 3 schematic, rather than quantitative. Secondly, the smallness of the upper limit of integration $\bar{\nu}$ makes suspect the assumption of the dominance of the high-energy side of the $F_1^{(-)}$ sum rule by a single effective pole. The same can be said of the sum rules for $F_2^{(-)}$ and $F_3^{(-)}$, but effective α values near zero make these FESR insensitive to the upper limit. If the low-energy data permitted integration up to $\bar{\nu} = 2$ or 2.5 GeV one would feel more satisfied about the determination of the parameters of the effective A_2 pole.

Factorization can indicate a preference for one class of models over the other. It is well known that the presence of Regge cuts restricts the application of factorization arguments. Nevertheless, if the cuts are not totally dominant, gross features can be expected to

exhibit approximate factorization. For the effective A_2 pole in meson-baryon scattering it has been shown¹⁷ that near $t = 0$ the residues of the t-channel spin-flip and nonflip amplitudes are such that $(\nu B/A')_{A_2} \simeq +10$. The numerical value of +10 is not known with any certainty, but $(\nu B/A')$ is large and positive. Comparison of the t-channel helicity amplitudes for meson-baryon scattering with the t-channel amplitudes (4) and (6) shows that factorization requires for the A_2 pole contributions the relation,

$$\left(\frac{\nu B}{A'}\right) \simeq - \frac{2m F_3^{(-)}}{tF_1^{(-)}} = - \frac{\xi_2}{\xi_1} \quad (24)$$

Now, eq. (24) is written as if there were no cuts. It can be expected, however, that cuts will not modify the order of magnitude and sign implied in (24). It therefore appears that factorization favors the primed models which have opposite signs for ξ_1 and ξ_2 , and requires negative values for the left-right asymmetry A from a polarized target.

In our particular models, with π and A_2 poles and cuts, the asymmetry A is predicted to have the energy dependence characteristic of $\pi - A_2$ interference, i.e. $\alpha_{\text{eff}}(\text{Asymmetry}) > 0$ for small $|t|$. If this should be verified experimentally, it is evidence against the hypothesis that photoproduction amplitudes are governed by fixed poles only at $j = 0$. Thus, for example, at somewhat higher energies (~ 30 GeV in these models) the differential cross section would be expected to behave as $\alpha > 0$ for small values of τ .

5. SUGGESTIVE PERTURBATION THEORY RESULTS

Among the puzzles in the photoproduction of pions at high energies are the apparent fixed power behavior⁶ which seems to hold for all one-meson photoproduction processes ($\alpha_{\text{effective}} = 0$ for forward and -0.5 for backward¹⁸ production), and the success of gauge-invariant perturbation theory in predicting certain features of differential cross sections. Harari¹² has remarked on the latter from the viewpoint of forward dispersion relations. The contribution to the dispersion integral (the continuum) is small compared to the nucleon pole terms. Since peripheral processes at high energies are normally thought of as dominated by t-channel exchanges, it is clear that gauge invariance, with its linking of s, t, and u-channel poles, is playing a profound role. An attempt has been made to rationalize the success of the simple perturbation results through use of Veneziano amplitudes,¹⁹ but the expected answer seems to have been built in and the mystery (if such it be) is as great as ever.

For completeness we summarize the results of the elementary calculation. The three Feynman graphs which comprise the gauge-invariant set, called (upon neglect of Pauli moment terms) the gauge-invariant electric Born approximation, are shown in fig. 4. The t-channel amplitudes that follow from this model are

$$\begin{aligned}
F_1^{(0)} &= F_1^{(+)} = \frac{1}{2}eg(t - \mu^2)/(s - m^2)(u - m^2) \\
F_2^{(0)} &= F_2^{(+)} = \frac{1}{2}eg(t - \mu^2)(t + \mu^2)/(s - m^2)(u - m^2) \\
F_3^{(0)} &= F_3^{(+)} = -2mF_1^{(+)} \\
F_4^{(0)} &= F_4^{(+)} = 0 \\
F_1^{(-)} &= \frac{1}{2}eg(s - u)/(s - m^2)(u - m^2) \\
F_2^{(-)} &= \frac{1}{2}eg(s - u)(t + \mu^2)/(s - m^2)(u - m^2) \\
F_3^{(-)} &= -2mF_1^{(-)} \\
F_4^{(-)} &= 0 .
\end{aligned} \tag{25}$$

The observables at small t values calculated from these amplitudes are listed in table IV. The predictions for the differential cross section are successful^{6,10,11} for the charged pion reactions, for $-t < 0.1$ (GeV/c)², while the predictions for neutral pion photoproduction are in obvious disagreement with the data.²⁰ The calculated asymmetry parameter, Σ , is seen in fig. 2 to be qualitatively similar to the pseudomodel result and to the data. The failure at large $(-t)$ is directly related to the failure there of the predicted differential cross section (see fig. 1). Note that pion exchange alone, which is not gauge-invariant, yields $\Sigma = -1$, in complete disagreement with the data.

A comment is necessary about the perturbation results for $\gamma p \rightarrow \pi^0 p$. Since the anomalous magnetic moment terms have been neglected and there is no pion current contribution, the nucleon pole terms alone give amplitudes $\frac{1}{\sqrt{2}} \frac{(\mu^2 - t)}{(m^2 - u)}$ times those for $\gamma p \rightarrow \pi^+ n$. At high energies the cross section from these contributions is of the order of $(t/s)^2$ times the π^+ cross section, a totally negligible value. The asymmetry for linearly polarized photons for this negligible cross section happens to be the same as that for charged pions. It is clear that such a result is meaningless. Any contribution from other diagrams will swamp the electric Born result. Vector meson exchange, for example, is gauge invariant by itself and gives $\Sigma = +1$ in agreement with existing data at small t values.⁶

The electric Born approximation gives no asymmetry A or recoil polarization P since all amplitudes are real.

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TABLE I: Contributions of Well-known Trajectories to t-Channel Amplitudes.

Amplitude	Spin (\overline{NN})	τP	τG	I	Trajectory	Isophoton
$F_1(-)$	1	+1	-1	1	π_c, A_2	V
$F_1(0)$	1	+1	-1	1	ρ	S
$F_1(+)$	1	+1	+1	0	ω, ϕ	V
$F_2(-)$	0	-1	-1	1	π	V
$F_2(0)$	0	-1	-1	1	B	S
$F_2(+)$	0	-1	+1	0	?	V
$F_3(-)$	1	+1	-1	1	π_c, A_2	V
$F_3(0)$	1	+1	-1	1	ρ	S
$F_3(+)$	1	+1	+1	0	ω, ϕ	V
$F_4(-)$	1	-1	+1	1	A_1	V
$F_4(0)$	1	-1	+1	1	?	S
$F_4(+)$	1	-1	-1	0	$h(?)$	V

S(V) denotes coupling to isoscalar (isovector) photons

TABLE II: Summary of Pseudomodel Results for $\gamma p \rightarrow \pi^+ n$.

$-t \text{ (GeV/c)}^2$	$(s - m^2)^2 \frac{d\sigma}{dt}$ $(\mu\text{b} - \text{GeV/c}^2)$	$\Sigma \equiv \frac{\sigma_{\perp} - \sigma_{\parallel}}{\sigma_{\perp} + \sigma_{\parallel}}$
0.001	307	~ 0.04
0.01	164	0.71
0.0225	143	0.96
0.04	138	0.91
0.09	129	0.70
0.10	128	0.69
0.16	101	0.485
0.20	97	0.47
0.30	70	0.32
0.40	45.6	0.195
0.50	32.5	0.115

TABLE III: Parameters of Regge Cut Models

Model	$a(\text{GeV}/c)^{-2}$	C	ξ_1	ξ_2	α_π	α_{A_2}
1	8	1.19	-0.75	-4.92	$-0.65(\tau + \mu^2)$	$0.45 - \tau$
1'	8	1.19	0.75	-6.42	$-0.65(\tau + \mu^2)$	$0.45 - \tau$
2	8	2.00	-0.25	-1.64	$-(\tau + \mu^2)$	$0.45 - \tau$
2'	8	2.00	0.25	-2.14	$-(\tau + \mu^2)$	$0.45 - \tau$
3	8	1.49	-0.50	-3.28	$-0.65(\tau + \mu^2)$	$0.45 - \tau$
3'	8	1.49	0.50	-4.28	$-0.65(\tau + \mu^2)$	$0.45 - \tau$
4	8	1.30	0	0	0	---

TABLE IV: Electric Born Approximation Predictions

Process	Amplitude	$(s-m^2)^2 d\sigma/dt$ ($\mu\text{b}\cdot\text{-GeV}^2$)	Asymmetry Σ
$\gamma\text{p} \rightarrow \pi^+\text{n}$	$-\sqrt{2}(F_i^{(-)} + F_i^{(0)})$	$262 \cdot \frac{(t^2 + \mu^4)}{(t - \mu^2)^2}$	$\frac{-2t\mu^2}{(t^2 + \mu^4)}$
$\gamma\text{n} \rightarrow \pi^-\text{p}$	$-\sqrt{2}(F_i^{(-)} - F_i^{(0)})$	$262 \cdot \frac{(t^2 + \mu^4)(s-m^2)^2}{(t-\mu^2)^2(u-m^2)^2}$	$\frac{-2t\mu^2}{(t^2 + \mu^4)}$
$\gamma\text{p} \rightarrow \pi^0\text{p}$	$(F_i^{(+)} + F_i^{(0)})$	$131 \cdot \frac{(t^2 + \mu^4)}{(u - m^2)^2}$	$\frac{-2t\mu^2}{(t^2 + \mu^4)}$
$\gamma\text{n} \rightarrow \pi^0\text{n}$	$(F_i^{(+)} - F_i^{(0)})$	0	-----

FIGURE CAPTIONS

- Fig. 1. Predictions of the pseudomodel (solid line) and of the electric Born approximation (broken line) for the differential cross section in $\gamma p \rightarrow \pi^+ n$. Data are from ref. 10 (DESY) and ref. 11 (SLAC).
- Fig. 2. Pseudomodel (solid line) and gauge-invariant perturbation theory (broken line) predictions for the polarized photon asymmetry parameter Σ . Data are from ref. 13.
- Fig. 3. Left-right asymmetry from a polarized target, A , or recoil neutron polarization, P , predicted by the Regge cut models of table III. Model 3 is not plotted because its prediction at 5 GeV is similar to that of Model 2 at 16 GeV, and its prediction at 16 GeV is similar to that of Model 1 at 5 GeV. Likewise, $3'$ is similar to $1'$ and $2'$.
- Fig. 4. Feynman diagrams for the electric Born approximation. Graphs a, b, and c contain s-, u-, and t-channel poles, respectively.

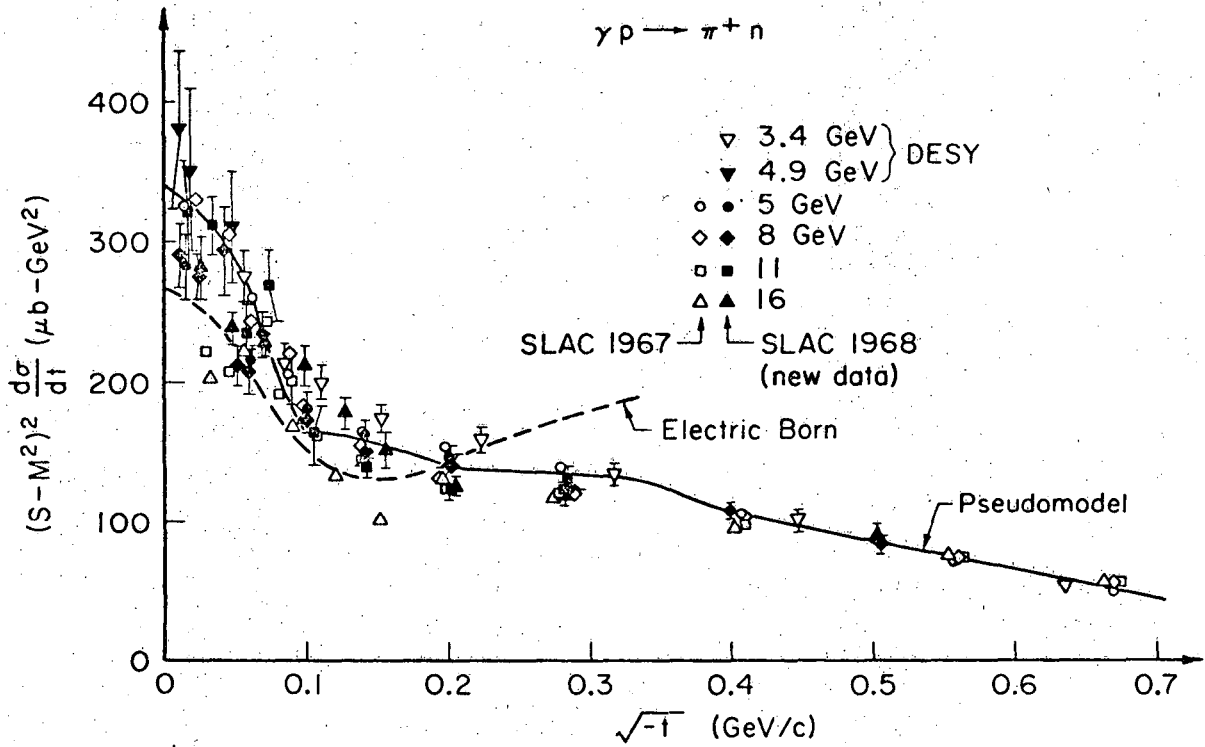
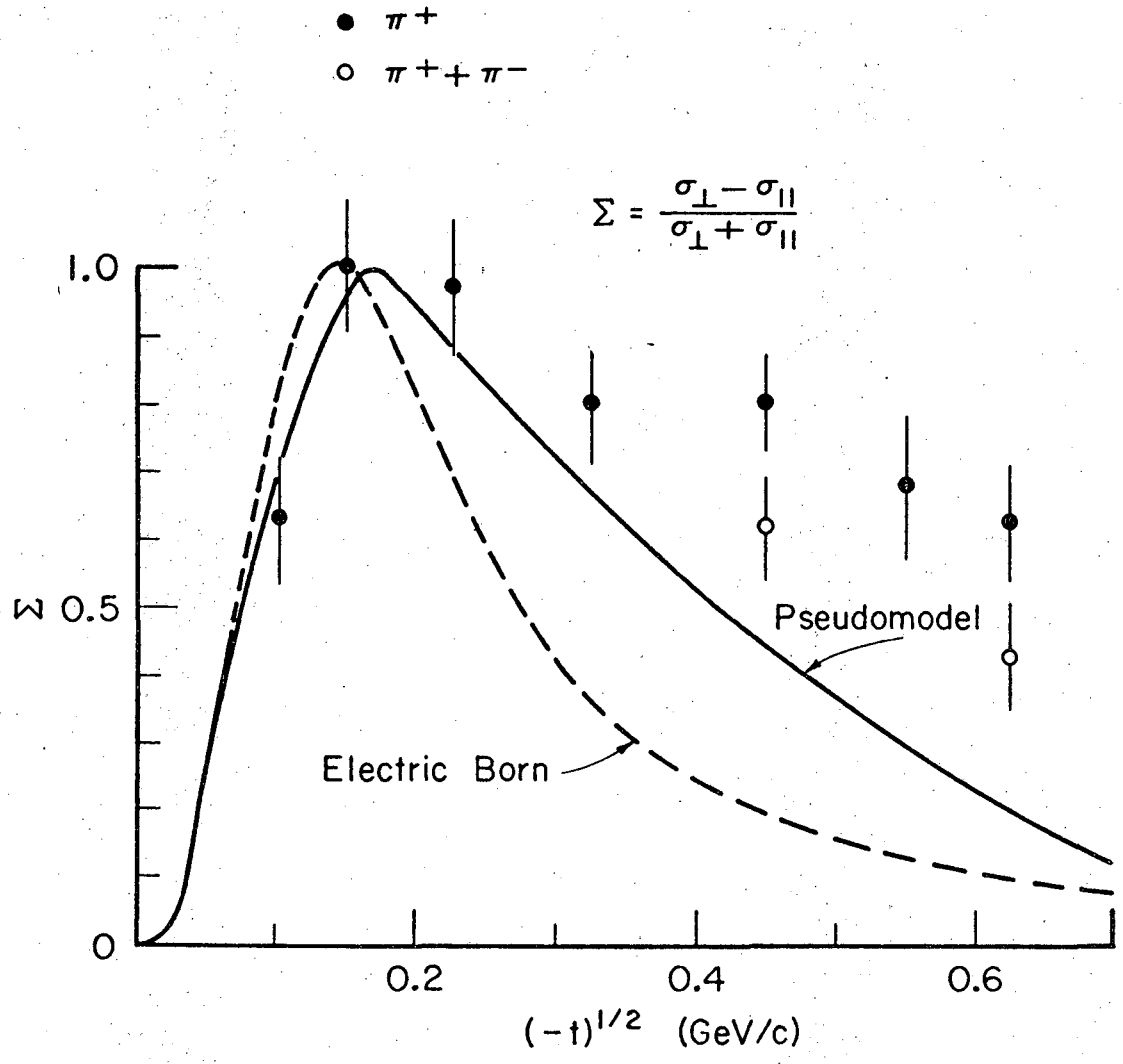
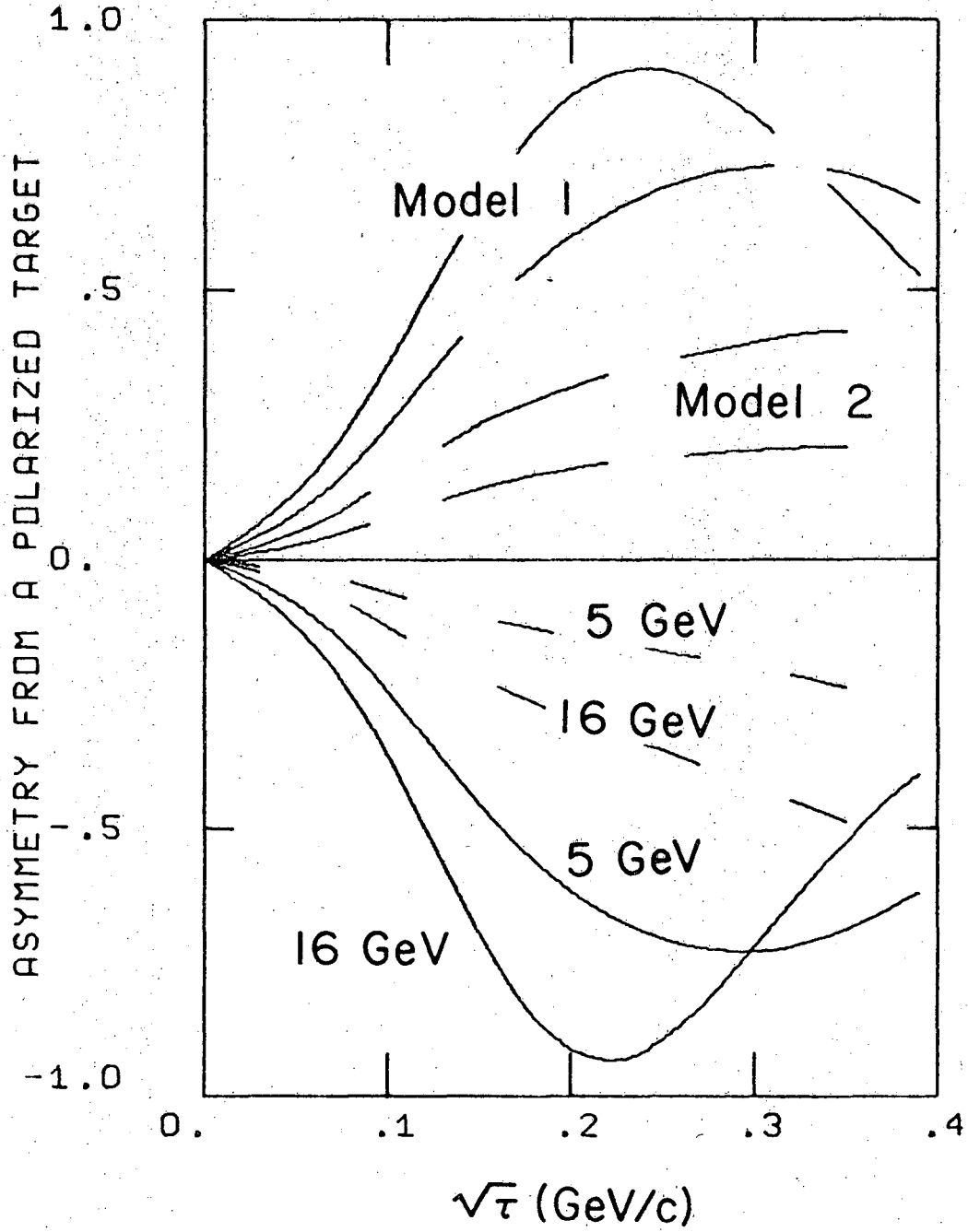


Fig. 1



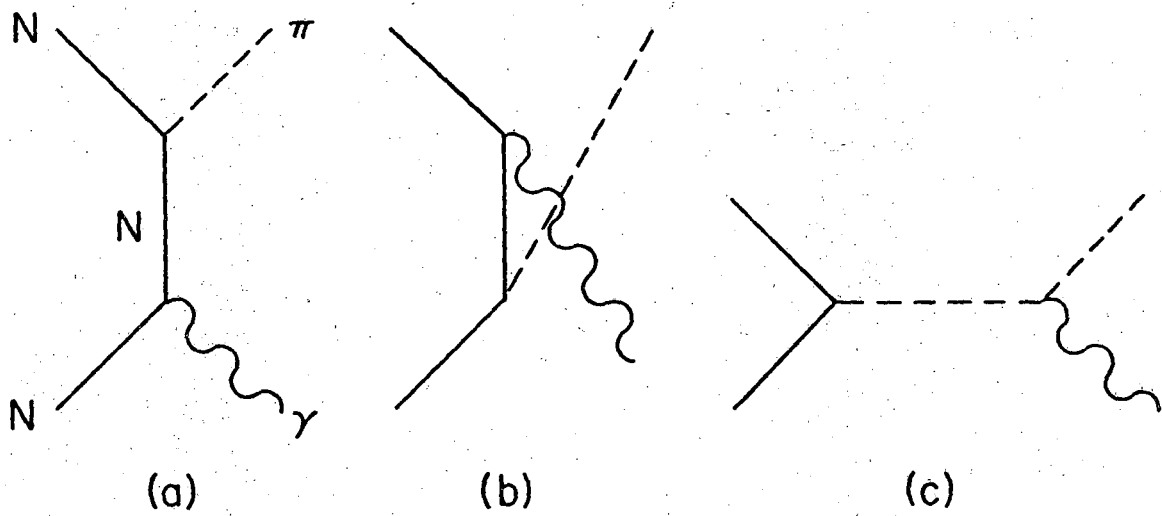
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Fig. 2



XBL 702-636

Fig. 3



XBL6911-6194

Fig. 4

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