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### Title

An efficient formulation for calculating the modal coupling for open-ended waveguide problems

### Permalink

<https://escholarship.org/uc/item/83z9r9n6>

### Journal

IEEE Transactions on Antennas and Propagation, 45(12)

### ISSN

0018-926X

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### Publication Date

1997

### DOI

10.1109/8.650212

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the integration end-points are determined by the condition  $z_s = 0$  and described by the unit step function in (5).

It is worth noting that for large apertures in terms of a wavelength,  $F_\xi^{po}(\ell')$  is a rapidly oscillating function of  $\ell'$ . Consequently, the integration in (3) can be asymptotically evaluated by its stationary phase point contributions, thus, leading to a UTD-type ray-field representation. However, this latter fails in describing the field close to and at the axial caustic and it has also been found less accurate with respect to the present numerical line integration for moderate sized apertures.

## II. NUMERICAL RESULTS

Numerical results from AI (continuous line) have been compared with those from LI (dashed line) for the case of a circular OEW with radius  $a$ . In particular, Fig. 2(a) shows results for a waveguide with radius  $a = 0.5\lambda$ , excited by the  $TE_{11}$  mode. The  $\psi$  component of the electric field in the H plane is plotted at a distance  $r = 1.5\lambda$  and  $r = 0.7\lambda$ , respectively. Both curves are normalized with respect to the maximum value obtained in the case  $r = 0.7\lambda$ ; furthermore, the field is calculated in the region external to the waveguide; i.e.,  $\theta < 130^\circ$  for  $r = 0.7\lambda$  and  $\theta < 160^\circ$  for  $r = 1.5\lambda$ . Normalized near field patterns for  $TM_{11}$ -mode excitation are presented in Fig. 2(b). The  $\theta$  component of the electric field in the H plane is plotted for the two cases  $a = 0.65\lambda$ ,  $r = 1.5\lambda$ , and  $a = \lambda$ ,  $r = 2\lambda$ , respectively. The curves corresponding to this latter case are shifted 10 dB down to render the figure more readable.

In spite of the moderate size of the apertures, the agreement between the AI and its corresponding LI has been found quite satisfactory over the total 40-dB dynamic range. The small glitches arise from the fact that the IGCO integration has been turned off when  $L$  does not intersect the edge [see Fig. 1(c)]. The result presented here also suggests an effective method to speed-up practical calculations of the interaction between modes [6].

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## An Efficient Formulation for Calculating the Modal Coupling for Open-Ended Waveguide Problems

F. Mioc, F. Capolino, and S. Maci

**Abstract**—A double line integral representation of the mutual coupling between open-ended waveguides of arbitrary cross section is presented, which is useful to speed up calculations inside the framework of a Galerkin method of moments.

**Index Terms**—Aperture antennas, electromagnetic coupling.

## I. INTRODUCTION AND FORMULATION

The theorem demonstrated in [1] establishes a rigorous equivalence between the field predicted by the Kirchhoff-type aperture integration (AI) and that radiated by the physical optics (PO) wall current. By using this equivalence, a formulation has been presented [2] to asymptotically reduce the AI into a line integration (LI) along the waveguide edge.

In this paper, the equivalence between PO and AI [1] is applied to modal coupling between OEW's of arbitrary cross-sections. This allows for the derivation of a convenient double integral expression for the modal coupling to be applied in the framework of a method of moments (MoM) for arrays of OEW's and horns. Note that a MoM Galerkin mutual impedance is generally given in terms of a quadruple integral in the space domain and the reduction to a double line integral is possible only for rectangular coplanar apertures [3]. When closed-form Fourier transform representations of modal coplanar distributions are available, one can resort to a spectral-domain approach; however, the resulting double spectral integrals are improper and slowly convergent. The method presented here is independent from the waveguide cross sections and, although it is developed here only for coplanar apertures, it can be easily extended to the noncoplanar case.

Let us consider two open-ended waveguides OEW1 and OEW2 of arbitrary cross sections. For the sake of simplicity, but without loss of generality, we will assume the two axes of the OEW's to be parallel. Two reference systems  $(x', y', z')$  and  $(x'', y'', z'')$  are introduced in which  $z' < 0$  and  $z'' < 0$  are the semi-axes of OEW1 and OEW2, respectively (Fig. 1). Let us denote by  $(\vec{e}_1^n \exp(jk_z^n z'), \vec{h}_1^n \exp(jk_z^n z'))$  the field propagating into OEW1 toward negative  $z'$  and by  $(\vec{e}_2^m \exp(-jk_z^m z''), \vec{h}_2^m \exp(-jk_z^m z''))$  the field of the  $m$ th mode propagating toward the positive  $z$  axis of OEW2, where  $k_z^n$  and  $k_z^m$  are the  $z$ -propagation constant of the relevant modes. These fields are normalized in such a way that the integration of  $\vec{e}_i^p \cdot \vec{h}_i^q$  ( $i = 1, 2$ ) on the aperture is equal to  $(-1)^{i\epsilon_{pq}}$  (Kronecker's delta). Magnetic  $\vec{m}_i^m = (-1)^i \vec{e}_i^m \times \hat{z}$ , and electric  $\vec{j}_i^m = (-1)^i \hat{z} \times \vec{h}_i^m$  current distributions associated to the unperturbed modes are defined on each aperture. The field radiated in the near zone by these aperture distributions is denoted by  $(\vec{E}_i^m, \vec{H}_i^m)$ . The mutual coupling coefficient between the two

Manuscript received May 7, 1997; revised August 4, 1997. This work was developed in part by F. Mioc during her internship at the European Space Agency (ESA-ESTEC) Noordwijk, The Netherlands.

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Publisher Item Identifier S 0018-926X(97)09060-1.

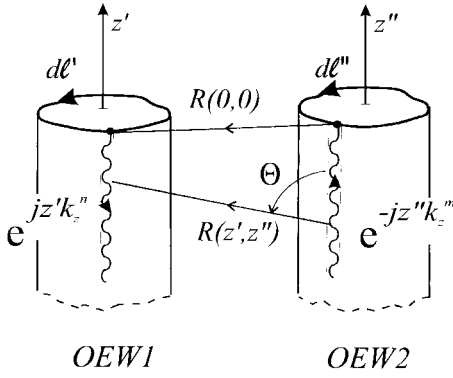


Fig. 1. Geometry for the problem of coupling between OEW's of arbitrary cross section.

modes is defined as

$$C_{12}^{nm} = \iint_{A_1} (\vec{E}_2^m \cdot \vec{j}_1^n - \vec{H}_2^m \cdot \vec{m}_1^n) dA_1 \quad (1)$$

or by reciprocity

$$C_{12}^{nm} = \iint_{A_2} (\vec{E}_1^n \cdot \vec{j}_2^m - \vec{H}_1^n \cdot \vec{m}_2^m) dA_2 \quad (2)$$

where  $A_i$  is the aperture of OEW $i$ . It is worth pointing out that calculating  $C_{12}^{nm}$  as in (1) or (2) implies a further two-dimensional integration to yield either  $(\vec{E}_1^n, \vec{H}_1^n)$  or  $(\vec{E}_2^m, \vec{H}_2^m)$  in the near zone. As demonstrated in [1],  $(\vec{E}_1^n, \vec{H}_1^n)$  is equal to the field produced by the radiation in free-space of the PO currents  $\vec{j}_{1w}^n = \hat{n}_{1w} \times \vec{h}_1^n$  of OEW1 (where  $\hat{n}_{1w}$  is the internal normal to the wall). When  $(\vec{E}_1^n, \vec{H}_1^n)$  is thought of as arising from  $\vec{j}_{1w}^n$ , the application of the reciprocity theorem leads to calculate (2) as the reaction integral between  $\vec{j}_{1w}^n$  and  $\vec{E}_2^m$ , i.e.,

$$C_{12}^{nm} = \iint_{S_1} \vec{E}_2^m \cdot \vec{j}_{1w}^n e^{jk_z^n z'} dS_1 \quad (3)$$

where the surface  $S_1$  is now the wall of OEW1. By inspection, it is apparent that (1) and (3) demonstrate the equivalence between the aperture currents and the PO currents in finding the field received by OEW1—also, when it is illuminated by a completely arbitrary incident field; indeed, the assumption that  $(\vec{E}_2^m, \vec{H}_2^m)$  arises from OEW2 is not at all necessary.

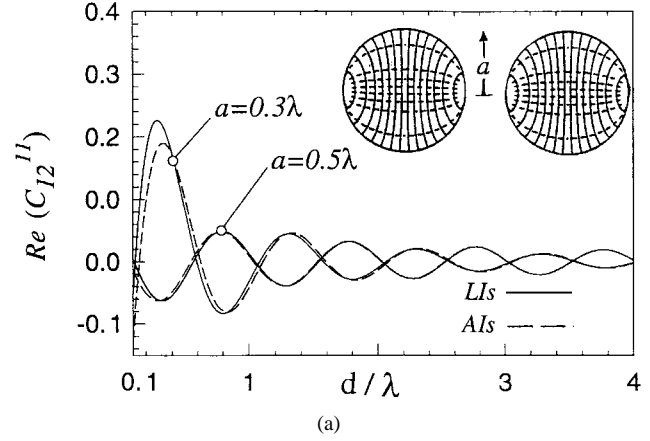
Again, by invoking the equivalence between PO and AI for OEW2,  $(\vec{E}_2^m, \vec{H}_2^m)$  may also be thought of as produced by the PO currents  $\vec{j}_{2w}^m = \hat{n}_{2w} \times \vec{h}_2^m$  on the OEW2 wall current so that (3) may be reformulated as

$$C_{12}^{nm} = \iint_{S_1} \iint_{S_2} \vec{j}_{1w}^n(\vec{r}_1) \cdot \overline{\overline{G}}_J^E(|\vec{r}_1 - \vec{r}_2|) \cdot \vec{j}_{2w}^m(\vec{r}_2) e^{jk_z^m z''} = e^{-jk_z^m z''} dS_1 dS_2 \quad (4)$$

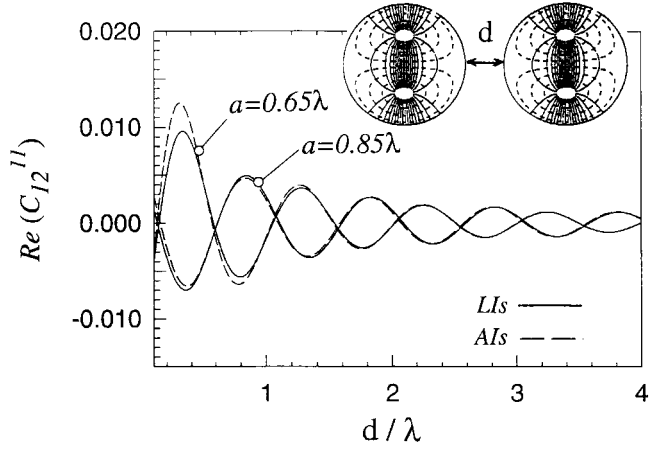
where  $S_2$  denotes the wall of OEW2,  $\overline{\overline{G}}_J^E(|\vec{r}_2 - \vec{r}_1|)$  is the pertinent free-space dyadic Green's function and  $\vec{r}_i$  is the position vector of  $dS_i$ .

For large values of  $k|\vec{r}_2 - \vec{r}_1|$  where  $k$  is the free-space wavenumber, the quadruple integral in (4) is asymptotically reduced to a double integral. For this purpose, the integration is first performed along strips parallel to the  $z$  axis (Fig. 1) and then along the edges of the apertures; this leads to

$$C_{12}^{nm} = \int_{\ell'} \int_{\ell''} c(\ell', \ell'') d\ell' d\ell'' \quad (5)$$



(a)



(b)

Fig. 2. Real part of the coupling coefficient between two circular OEW's with radius  $a$  against the distance  $d$  in wavelength; double line integral (continuous line), quadruple aperture integral [dashed line, (1)]. (a) TE<sub>11</sub> modes. (b) TM<sub>11</sub> modes.

where

$$c(\ell', \ell'') = \int_{-\infty}^0 \int_{-\infty}^0 g(z', z'') \frac{e^{-jkR(z', z'')}}{4\pi R(z', z'')} \cdot e^{jk_z^n z'} e^{-jk_z^m z''} dz' dz'' \quad (6)$$

represents the mutual coupling between two elementary strips of traveling wave current (Fig. 1). In (6),  $R(z', z'') = |\vec{r}_1 - \vec{r}_2|$  and

$$g(z', z'') = jk\zeta \vec{j}_{1w}^n \cdot (\hat{\Theta} \hat{\Theta} + \hat{\Psi} \hat{\Psi}) \cdot \vec{j}_{2w}^m \quad (7)$$

where  $\zeta$  is the free-space impedance,  $\hat{\Psi} = \hat{z} \times \hat{R}$ , and  $\hat{\Theta} = \hat{\Psi} \times \hat{R}$  with  $\hat{R} = (\vec{r}_1 - \vec{r}_2)/|\vec{r}_1 - \vec{r}_2|$ . In (7), only the dominant term of the asymptotic expansion for  $kR$  large of the dyadic free-space Green's function has been retained. It is also worth noting that in (7) only the dyade  $(\hat{\Theta} \hat{\Theta} + \hat{\Psi} \hat{\Psi})$  depends on  $(z', z'')$ .

The double integral is asymptotically dominated by the contributions from three two-dimensional critical points, i.e., one double end-point (EP) at  $(z', z'') = (0, 0)$  and two EP-stationary phase points (SPP's) at  $(z', z'') \equiv (\bar{z}_1, 0)$  and at  $(z', z'') \equiv (0, \bar{z}_2)$ . The integration along  $\ell'$  of all the contributions at  $(\bar{z}_1, 0)$  asymptotically reconstructs the field radiated by the modal current of waveguide 1 when the latter is supposed to be infinite. Since these modal currents are exact for the infinite structure, their radiation rigorously produces a zero field outside the waveguide. This means that practically we need to calculate only the contribution  $c_{00}(\ell', \ell'')$  from the double EP.

The latter can be evaluated as in [4], thus leading to

$$c_{00}(\ell', \ell'') = g(0, 0) \frac{e^{-jk} R(0, 0)}{4\pi k R(0, 0)} \frac{1}{\cos \theta_{t1} \cos \theta_{t2}} \cdot \left( \frac{\cos \theta_{t2} \mathcal{F}(\delta_1) - \cos \theta_{t1} \mathcal{F}(\delta_2)}{\cos \theta_{t2} - \cos \theta_{t1}} \right) \quad (8)$$

where  $\theta_{t1} = \cos^{-1}(k_z^n/k)$  and  $\theta_{t2} = \cos^{-1}(k_z^m/k)$  are the ray-mode angles in OEW1 and OEW2, respectively, and  $\mathcal{F}(x)$  is the Fresnel transition function of the uniform theory of diffraction (UTD) [2] with argument  $\delta_i = kR(0, 0)[1 - \sin \theta_{ti}]$  ( $i = 1, 2$ ). The coupling coefficient  $C_{12}^{n,m}$  is calculated by using  $c_{00}(\ell', \ell'')$  in (5).

The transition function in parentheses in (8) reduces to one for large  $kR(0, 0)$  and far from the shadow boundary cones of each strip [2]. In this case, the contribution  $c_{00}(\ell', \ell'')$  can be interpreted as the coupling coefficient between two elementary electric dipoles distributed on the two rims and oriented as the local modal currents. In practical applications, the presence of the factor into parentheses of (8) becomes practically relevant close to the cut-off condition of one of the two coupled modes, where the associated ray-mode angle approaches  $90^\circ$ .

In Fig. 2, the real part of the coupling coefficient  $C_{12}^{n,m}$  calculated via (1) (quadruple integral, dashed line) is compared to the one calculated with the present method (double integral, dashed line) for two equal circular OEW's with various radii  $a$ . The coupling coefficient has been calculated as a function of the minimum distance between the aperture rims for a pair of  $TE_{11}$  [Fig. 2(a)] and  $TM_{11}$  [Fig. 2(b)] mode distributions (shown in the insets). The calculation time for the double line integration is reduced by a factor 34 with respect to the corresponding quadruple integral. In spite of the moderate size of the apertures, the agreement has been found quite satisfactory even for small distances between the OEW's.

In conclusion, it should be noted that owing to the exponential factor  $\exp[-jkR(0, 0)]$ , the incremental coupling contribution  $c_{00}(\ell', \ell'')$  (8) is a rapidly oscillating function of both  $\ell'$  and  $\ell''$ . Consequently, the integration in (5) could be asymptotically evaluated by its stationary phase point contributions for larger  $d$ , thus leading to a complete closed-form representation.

#### ACKNOWLEDGMENT

The authors would like to thank M. Sabbadini, R. Tiberio, and P. Ya. Ufimtsev for useful and stimulating discussions on this topic.

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## Dual-Frequency Triangular Microstrip Antenna with a Shorting Pin

Shan-Cheng Pan and Kin-Lu Wong

**Abstract**—This letter presents a novel design of triangular microstrip antenna with dual-frequency operation. In this design the microstrip patch is short-circuited using a shorting pin and fed by a single probe feed. By varying the shorting-pin position in the microstrip patch, such a design can provide a large tunable frequency ratio of about 2.5–4.9 for the two operating frequencies. Experimental results are presented and discussed.

**Index Terms**—Microstrip antennas.

#### I. INTRODUCTION

Although dual-frequency operations of microstrip patch antennas have received much attention and many related designs have been demonstrated, most of the designs provide only a moderate frequency ratio (FR) between the two operating frequencies (about 1.3–2). The reports on dual-frequency operation with FR greater than two are relatively limited. Typical designs that use a single-layer, single-patch structure and provide a large frequency ratio include loading the rectangular microstrip patch with two varactor diodes [1], short-circuiting the rectangular microstrip patch with a shorting pin [2], etc. The former design can provide a dual-frequency operation with FR about 5.0 but it needs external circuitry to supply the bias voltage for the diodes, which makes the structure relatively complicated. As for the latter, the structure is much simpler and requires only proper selection of the shorting-pin position in the microstrip patch. Moreover, the impedance matching for operating at the two frequencies can be achieved using a single probe feed. The reported FR for such a design using a rectangular patch is tunable in the range 2.0–3.2 [2] and from the study on compact triangular microstrip antennas that use a similar shorting-pin technique [3], which shows a greater resonant frequency reduction than the rectangular or circular patches, it is expected that the tunable FR range can also be greater for a triangular patch than for a rectangular one. This motivates the present study. The design for a dual-frequency triangular microstrip antenna with a shorting pin is described and typical experimental results are presented.

#### II. ANTENNA DESIGN AND EXPERIMENTAL RESULTS

The antenna geometry is given in Fig. 1. The equilateral triangular patch of side length  $d$  is considered. For comparison with the dual-frequency rectangular microstrip antenna in [2], the same FR4 substrate ( $\epsilon_r = 4.4, h = 1.6$  mm) of  $7.5$  cm  $\times$   $7.5$  cm in size was used in this study and the triangular patch was designed to resonate at 1.9 GHz, which gives a side length of 50 mm. The shorting pin of radius ( $r_s = 0.32$  mm) and probe feed of radius ( $r_p = 0.63$  mm) are placed along the line segment between the triangle tip and the bottom side of the patch, as shown in Fig. 1. By varying the shorting-pin position ( $d_s$ ), a strong dependence of the first two resonant frequencies on  $d_s$  is observed. The results are shown in

Manuscript received May 2, 1997; revised September 4, 1997. This work was supported by the National Science Council of the Republic of China under Grant NSC86-2221-E-110-005.

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Publisher Item Identifier S 0018-926X(97)09061-3.