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Performance Degradation of OFDM Systems Due to Doppler Spreading

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Abstract—The focus of this paper is on the performance of orthogonal frequency division multiplexing (OFDM) signals in mobile radio applications, such as 802.11a and digital video broadcasting (DVB) systems, e.g., DVB-CS2. The paper considers the evaluation of the error probability of an OFDM system transmitting over channels characterized by frequency selectivity and Rayleigh fading. The time variations of the channel during one OFDM symbol interval destroy the orthogonality of the different subcarriers and generate power leakage among the subcarriers, known as Inter-Carrier Interference (ICI). For conventional modulation methods such as phase-shift keying (PSK) and quadrature-amplitude modulation (QAM), the bivariate probability density function (pdf) of the ICI is shown to be a weighted Gaussian mixture. The large computational complexity involved in using the weighted Gaussian mixture pdf to evaluate the error probability serves as the motivation for developing a two-dimensional Gram-Charlier representation for the bivariate pdf of the ICI. It is demonstrated that its truncated version of order 4 or 6 provides a very good approximation in the evaluation of the error probability for PSK and QAM in the presence of ICI. Based on Jakes' model for the Doppler effects, and an exponential multipath intensity profile, numerical results for the error probability are illustrated for several mobile speeds.

Index Terms—OFDM, Doppler spreading, ICI, C/I ratio, Gaussian mixture, two-dimensional Gram-Charlier series.

I. INTRODUCTION

IN OFDM systems, a serial data stream is split into parallel streams that modulate a group of orthogonal sub-carriers. Compared to single carrier modulation, OFDM symbols have a relatively long time duration, but a narrow bandwidth. Consequently, OFDM is robust to channel multipath dispersion and results in a decrease in the complexity of equalizers for high delay spread channels or high data rates. However, the increased symbol duration makes an OFDM system more sensitive to the time variations of mobile radio channels. In particular, the effect of Doppler spreading destroys the orthogonality of the sub-carriers, resulting in inter-carrier interference (ICI) due to power leakage among subcarriers.

In several previous publications [1]-[4], the system performance for OFDM was analyzed based on the assumption

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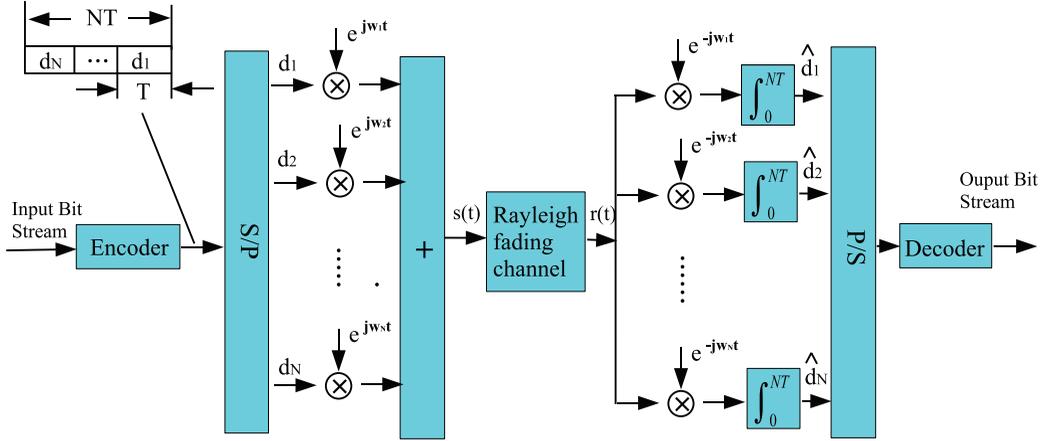
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that the ICI distribution is Gaussian by invoking the central limit theorem. In other related papers [5]-[9], efforts have been made to evaluate the effect of ICI by calculating its average power and comparing it with the power of the desired signal. In [5][6], the carrier to interference (C/I) ratio has been introduced to demonstrate the effect of the ICI under various maximum Doppler spreads and different Doppler spectra. Through numerical evaluations of the C/I ratios, it is reported in [7] that an OFDM system is robust to frequency selectivity but quite sensitive to time varying fading channels. Li and Cimini [8] provide universal bounds on the ICI in an OFDM system over Doppler fading channels, which are easier to evaluate and can provide useful insights compared with the exact ICI expression. Furthermore, the closed-form expression of ICI power is derived and evaluated in [9], where the normalized ICI power is represented as a function of the normalized Doppler spread. However, all these papers do not attempt to determine the underlying probability distribution function (pdf) of the ICI.

In this paper we focus on providing a statistical analysis for the ICI in an OFDM system that employs conventional PSK and QAM signal modulation in a frequency selective, Rayleigh fading time-varying channel. The channel, which is assumed to be wide-sense stationary with uncorrelated scattering (WSSUS), is modeled by a two-dimensional correlation function in time and frequency, representing the time variations and frequency selectivity of the channel. Each subchannel is assumed to be frequency flat and, based on the power series model developed by Bello [10], a two-term Taylor series expansion is used to model the time variations in an OFDM symbol. Jakes' model [14] is used as the model for the Doppler power spectrum and an exponential multipath intensity profile is the model adopted for the multipath effects. A cyclic prefix is assumed to remove the effects of inter-symbol interference. Based on this channel model, the ICI is expressed as the summation of leakage terms into each of the subcarriers and its pdf is shown to be characterized statistically by a bivariate pdf that is a weighted sum of Gaussian pdfs.

In deriving the probability of error for the OFDM system in the presence of ICI, the use of the weighted Gaussian pdf proves to be computationally intensive. This difficulty serves as the motivation to develop a two-dimensional Gram-Charlier series to represent the pdf of the ICI. A truncated version of the Gram-Charlier is used in the evaluation of the error probability for PSK and QAM signal modulations in an OFDM system.

The paper is organized as follows: In Section II we describe the model for the OFDM system. In Section III we describe

Fig. 1. Base-band OFDM transmission model with N subcarriers.

the channel model and use a Taylor series expansion for the time variations within an OFDM symbol. In Section IV an expression for the ICI and its power is presented. Section V provides a thorough analysis of the statistics of the ICI, its joint probability density, joint moments, and a two-dimensional Gram-Charlier representation. In Section VI, the error rate performance of BPSK and 16-QAM OFDM systems are presented and compared. Finally, concluding remarks are given in Section VII.

II. OFDM SYSTEM

An OFDM system with N subcarriers is represented in Fig. 1. In an OFDM system that employs M -ary digital modulation, a block of $\log_2 M$ input bits is mapped into a symbol constellation point d_k by a data encoder, and then N symbols are transferred by the serial-to-parallel converter (S/P). If $1/T$ is the symbol rate of the input data to be transmitted, the symbol interval in the OFDM system is increased to NT , which makes the system more robust against the channel delay spread. Each sub-channel, however, transmits at a much lower bit rate of $\frac{\log_2 M}{NT}$ bits/s. The parallel symbols (d_1, \dots, d_N) modulate a group of orthogonal subcarriers, which satisfy

$$\frac{1}{NT} \int_0^{NT} \exp(j2\pi f_i t) \exp(j2\pi f_j t) dt = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \quad (1)$$

where $f_i = \frac{i-1}{NT}$, $(i = 1, 2, \dots, N)$

Consider the system shown in Fig. 1. The baseband transmitted signal can be represented as

$$s(t) = \frac{1}{\sqrt{NT}} \sum_{k=1}^N s_k e^{j2\pi f_k t}, \quad 0 \leq t \leq NT, \quad f_k = \frac{k-1}{NT}. \quad (2)$$

We denote by $2E_s$ the average energy for the complex baseband symbol s_k . Then s_k is given by

$$s_k = \sqrt{2E_s} d_k \quad (3)$$

where $d_k = d_{k,r} + jd_{k,i}$, is the signal constellation point (e.g. BPSK, QPSK, QAM, etc.) with normalized variance $E[|d_k|^2] = 1$. Square M-QAM signal constellations may be viewed as two independent \sqrt{M} -PAM signals on orthogonal carriers. In this case, the real and imaginary parts $d_{k,r}$ and

$d_{k,i}$ are statistically independent, identically distributed and $E[d_{k,r}] = E[d_{k,i}] = 0$.

III. CHANNEL MODEL

We consider a frequency selective randomly varying channel with impulse response $h(t, \tau)$. Within the narrower bandwidth of each sub-carrier, compared with the coherence bandwidth of the channel, the sub-channel is modeled as a frequency nonselective Rayleigh fading channel. Hence, the channel impulse response $h_k(t, \tau)$ for the k^{th} subchannel is denoted as

$$h_k(t, \tau) = \beta_k(t) \delta(\tau) \quad (4)$$

where the process $\{\beta_k(t), -\infty < t < \infty\}$ is a stationary, zero mean complex-valued process described as follows: It is assumed that the processes $\{\beta_k(t), -\infty < t < \infty\}$, $k = 1, \dots, N$, are complex-valued jointly stationary and jointly Gaussian with zero means and cross covariance function

$$R_{\beta_k, \beta_l}(\tau) := E[\beta_k(t + \tau) \beta_l^*(t)], \quad k, l = 1, \dots, N. \quad (5)$$

For each fixed k , the real and imaginary parts of the process $\{\beta_k(t), -\infty < t < \infty\}$ are assumed independent with identical covariance function. We further assume that the correlation function $R_{\beta_k, \beta_l}(\tau)$ has the following factorable form

$$R_{\beta_k, \beta_l}(\tau) = R_1(\tau) R_2(k-l) \quad (6)$$

which has been frequently used in the literature, e.g., [4][15][16], and which is sufficient to represent the frequency selectivity and the time-varying effects of the channel. $R_1(\tau)$ gives the temporal correlation for the process $\{\beta_k(t), -\infty < t < \infty\}$ which is seen to be identical for all $k = 1, \dots, N$. $R_2(k)$ represents the correlation in frequency across subcarriers. We assume in this paper that the corresponding spectral density $\psi_1(f)$ to $R_1(\tau)$ is given by the Doppler power spectrum, modeled as in Jakes [14], i.e.,

$$\psi_1(f) = \begin{cases} \frac{1}{\pi F_d \sqrt{1 - (\frac{f}{F_d})^2}} & |f| \leq F_d \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

where F_d is the (maximum) Doppler bandwidth. Note that

$$R_1(\tau) = J_0(2\pi F_d \tau) \quad (8)$$

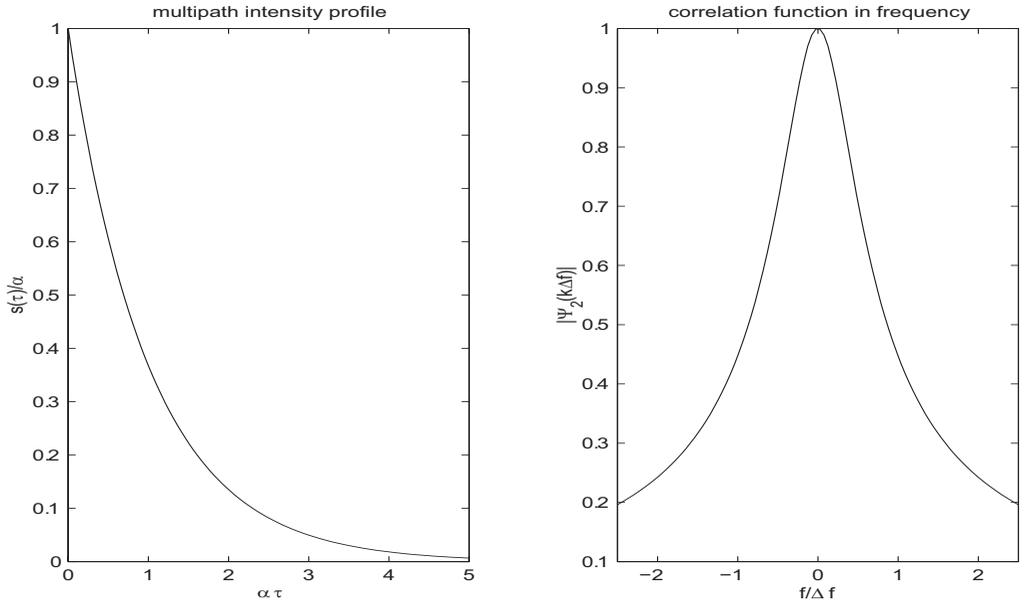


Fig. 2. Multipath delay profile and frequency correlation function.

where $J_0(\tau)$ is the zero-order Bessel function of the first kind. In order to specify the correlation in frequency across subcarriers, we adopt an exponential multipath power intensity of the form $S(\tau) = \alpha e^{-\alpha\tau}$, $\tau > 0, \alpha > 0$ where α is a parameter that controls the coherence bandwidth of the channel. The Fourier transform of $S(\tau)$ yields

$$\psi_2(f) = \frac{\alpha}{\alpha + j2\pi f} \quad (9)$$

which provides a measure of the correlation of the fading across the subcarriers as illustrated in Fig. 2. Then $R_2(k) = \psi_2(\Delta f k)$ where $\Delta f = \frac{1}{NT}$ is the frequency separation between two adjacent subcarriers. The 3dB bandwidth of $\psi_2(f)$ is defined as the coherence bandwidth of the channel and easily shown to be $f_{coherent} = \frac{\sqrt{3}\alpha}{2\pi}$.

The channel model described above is suitable for modeling OFDM signal transmission in mobile radio systems, e.g., cellular systems and broadcasting systems. For example, in DVB-CS2 with 2000 subcarriers, the symbol duration NT is $500\mu s$. In contrast, the delay spread of many fading channels is much smaller, which make it reasonable to view each subcarrier as a flat fading channel. However, compared with the entire OFDM system bandwidth $W = 1/T$, the coherence bandwidth $f_{coherent}$ is usually smaller, $f_{coherent} < W$, especially in an outdoor wireless communication environment. Hence, the channel is frequency-selective over the entire OFDM bandwidth.

We now turn our attention to modeling the time variations of the channel within an OFDM symbol interval. For most practical mobile radio fading channels, the time-varying effects in the channel are sufficiently slow, i.e., the coherence time is always much larger than the interval of an OFDM symbol [17][18]. For such slow fading channels, we use the two terms Taylor series expansion, first introduced by Bello [10], to represent the time-varying fading response $\beta_k(t)$ as

the following form:

$$\beta_k(t) = \beta_k(t_0) + \beta'_k(t_0)(t - t_0), \quad t_0 = \frac{NT}{2}, 0 \leq t \leq NT. \quad (10)$$

Therefore, the impulse response of the k^{th} subchannel is expressed as

$$h_k(t, \tau) = \beta_k(t)\delta(\tau) = [\beta_k(t_0) + \beta'_k(t_0)(t - t_0)]\delta(\tau). \quad (11)$$

Since $R_1(\tau)$ of (8) is infinitely differentiable, all mean-square derivatives exist and thus the differentiation above is justified. We use this model for the time variations of the channel within an OFDM symbol.

IV. EXPRESSION FOR THE ICI AND ITS POWER

Let $s(t)$ be the baseband signal transmitted over the channel with impulse response $h(t, \tau)$ as modeled above. Then the baseband received signal with additive noise may be expressed as

$$r(t) = h(t, \tau) \star s(t) + n(t) = \frac{1}{\sqrt{NT}} \sum_{k=1}^N \beta_k(t) s_k e^{j2\pi f_k t} + n(t) \quad (12)$$

where \star denotes convolution and $n(t)$ is the additive noise, which is modeled as a Gaussian process with zero mean and spectrally flat within the signal bandwidth, with one-sided spectral density N_0 watts/Hz. By using the Taylor series expansion for $\beta_k(t)$ as given in (11), we obtain

$$r(t) = \frac{1}{\sqrt{NT}} \sum_{k=1}^N \beta_k(t_0) s_k e^{j2\pi f_k t} + \frac{1}{\sqrt{NT}} \sum_{k=1}^N \beta'_k(t_0)(t - t_0) s_k e^{j2\pi f_k t} + n(t). \quad (13)$$

The received signal in a symbol interval is passed through a parallel bank of correlators, where each correlator is tuned

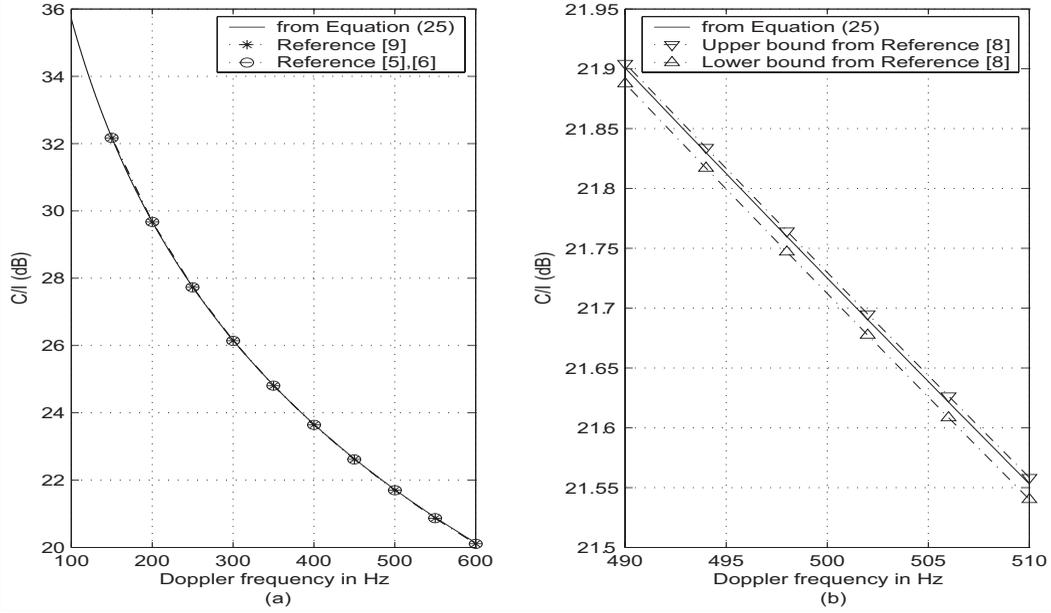


Fig. 3. C/I ratio curves of an OFDM system $N = 256$ subcarriers, subcarrier distance $\Delta f = 7.81 \text{ KHz}$, and carrier frequency $f_c = 2 \text{ GHz}$.

to one of the N subcarriers. The output of the i^{th} correlator is

$$\hat{d}_i = \frac{1}{\sqrt{2E_s}} \frac{1}{\sqrt{NT}} \int_0^{NT} r(t) e^{-j2\pi f_i t} dt. \quad (14)$$

Substituting (13) into (14), we obtain

$$\begin{aligned} \hat{d}_i &= \underbrace{\frac{1}{\sqrt{2E_s NT}} \int_0^{NT} \frac{1}{\sqrt{NT}} \sum_{k=1}^N \beta_k(t_0) s_k e^{j2\pi(f_k - f_i)t} dt}_{(1)} \\ &+ \underbrace{\frac{1}{\sqrt{2E_s NT}} \int_0^{NT} \frac{1}{\sqrt{NT}} \sum_{k=1}^N \beta'_k(t_0) (t - t_0) s_k e^{j2\pi(f_k - f_i)t} dt}_{(2)} \\ &+ \underbrace{\frac{1}{\sqrt{2E_s NT}} \int_0^{NT} n(t) e^{-j2\pi f_i t} dt}_{(3)} \end{aligned} \quad (15)$$

The first term yields

$$\sum_{k=1}^N \beta_k(t_0) d_k \left(\frac{1}{NT} \int_0^{NT} e^{j2\pi(f_k - f_i)t} dt \right) = \beta_i(t_0) d_i. \quad (16)$$

The second term yields

$$\begin{aligned} \sum_{k=1}^N \beta'_k(t_0) d_k \left(\frac{1}{NT} \int_0^{NT} (t - t_0) e^{j2\pi(f_k - f_i)t} dt \right) \\ = \sum_{\substack{k=1 \\ k \neq i}}^N \frac{\beta'_k(t_0) d_k}{j2\pi(f_k - f_i)} = \frac{NT}{2\pi j} \sum_{\substack{k=1 \\ k \neq i}}^N \frac{\beta'_k(t_0) d_k}{k - i}. \end{aligned} \quad (17)$$

Finally, the additive noise term is

$$n_i = \frac{1}{\sqrt{2E_s}} \frac{1}{\sqrt{NT}} \int_0^{NT} n(t) e^{-j2\pi f_i t} dt \quad (18)$$

where n_i is a complex Gaussian noise with zero mean and variance N_0/E_s . Thus we have

$$\hat{d}_i = \underbrace{\beta_i(t_0) d_i}_{\text{desired_signal}} + \underbrace{\frac{NT}{2\pi j} \sum_{\substack{k=1 \\ k \neq i}}^N \frac{\beta'_k(t_0) d_k}{k - i}}_{\text{ICI}} + n_i. \quad (19)$$

In Section V we establish the statistical properties of the ICI term. Here, we obtain the C/I ratio and we compare the result with those obtained in [5],[6],[8], and [9], which are based on different models for the time variations.

From Equation (19), the average power of the desired signal is

$$C = E[|\beta_i(t_0) d_i|^2] = E[|\beta_i(t_0)|^2] E[|d_i|^2] = 1. \quad (20)$$

Since $R_{\beta_k, \beta_k}(\tau) = R_1(\tau)$ is infinitely differentiable, all (mean-square) derivatives of the process $\{\beta_k(t), -\infty < t < \infty\}$ exist. In particular, the first-order derivative process $\{\beta'_k(t), -\infty < t < \infty\}$ is a zero mean complex-valued Gaussian process with correlation function $E[\beta'_k(t + \tau) \beta'_k(t)^*] = -R_1''(\tau)$ (identical for all k) with corresponding spectral density

$$\psi_3(f) = \begin{cases} \frac{(2\pi f)^2}{\pi F_d \sqrt{1 - (\frac{f}{F_d})^2}} & |f| \leq F_d \\ 0 & \text{otherwise} \end{cases} \quad (21)$$

Then,

$$\begin{aligned} E[|\beta'_k(t)|^2] &= \int_{-\infty}^{\infty} \psi_3(f) df \\ &= \int_{-F_d}^{F_d} \frac{(2\pi f)^2}{\pi F_d \sqrt{1 - (\frac{f}{F_d})^2}} df = 2\pi^2 F_d^2. \end{aligned} \quad (22)$$

Thus, the power of the interference (ICI) is

$$\begin{aligned}
I &= E \left[\left| \frac{NT}{2\pi j} \sum_{\substack{k=1 \\ k \neq i}}^N \frac{\beta'_k(t_0) d_k}{k-i} \right|^2 \right] \\
&= \left(\frac{NT}{2\pi} \right)^2 \sum_{\substack{k=1 \\ k \neq i}}^N \sum_{\substack{l=1 \\ l \neq i}}^N \frac{1}{(k-i)(l-i)} E[(\beta'_k(t_0) d_k)(\beta'_l(t_0) d_l)^*] \\
&= \left(\frac{NT}{2\pi} \right)^2 \sum_{\substack{k=1 \\ k \neq i}}^N \sum_{\substack{l=1 \\ l \neq i \\ k \neq l}}^N \frac{1}{(k-i)(l-i)} E[(\beta'_k(t_0) d_k)(\beta'_l(t_0) d_l)^*] \\
&\quad + \left(\frac{NT}{2\pi} \right)^2 \sum_{\substack{k=1 \\ k \neq i}}^N \frac{1}{(k-i)^2} E[|\beta'_k(t_0) d_k|^2] \\
&=: J_1 + J_2. \tag{23}
\end{aligned}$$

Note that $(\beta'_k(t_0), \beta'_l(t_0))$ is independent of (d_k, d_l) . Also, the d_k 's are i.i.d. with zero means. Thus $J_1 = 0$. It then follows that

$$\begin{aligned}
I &= \left(\frac{NT}{2\pi} \right)^2 \sum_{\substack{k=1 \\ k \neq i}}^N \frac{E[|\beta'_k(t_0)|^2] E[|d_k|^2]}{(k-i)^2} \\
&= \frac{(NTF_d)^2}{2} \sum_{\substack{k=1 \\ k \neq i}}^N \frac{1}{(k-i)^2}. \tag{24}
\end{aligned}$$

Thus the signal to interference ratio (C/I) ratio can be expressed as

$$\frac{C}{I} = \frac{1}{\frac{(NTF_d)^2}{2} \sum_{\substack{k=1 \\ k \neq i}}^N \frac{1}{(k-i)^2}} \tag{25}$$

The C/I curve of the middle subcarrier, i.e., subcarrier index $k = N/2$, is plotted versus the Doppler frequency F_d in Fig. 3 (a). The OFDM system is assumed to have $N = 256$ subcarriers, with subcarrier spacing $\Delta f = 1/NT = 7.81 \text{ KHz}$, and carrier frequency $f_c = 2 \text{ GHz}$. The C/I curve given in Fig. 3 (a) matches very well with the results given in [5][6][9], which were obtained without the use of Taylor series approximation. Furthermore, using an expanded scale in Fig. 3 (b), we compare our analytical result with the upper and lower bounds on C/I given by Li and Cimini [8].

In order to evaluate the approximation effect embodied in the two-term Taylor series expansion, we also demonstrate in Fig. 4 the comparison of the C/I ratio curves of the same OFDM system in very high Doppler frequency regimes. We can observe from Fig. 4 that the two-term Taylor series expansion model well approximates the actual fading channel up to a relatively large Doppler frequency 10000Hz, i.e., the two-term Taylor series expansion model becomes inaccurate only when F_d is larger than 10000Hz, which is equivalent to a normalized Doppler frequency of 1.28. This is larger than any practical wireless fading channel for OFDM applications. For example, if we consider 802.11a with carrier frequency of 5GHz, the terminal must be moving at a speed of 2160km/hr. Overall, the two term Taylor expansion channel model is a

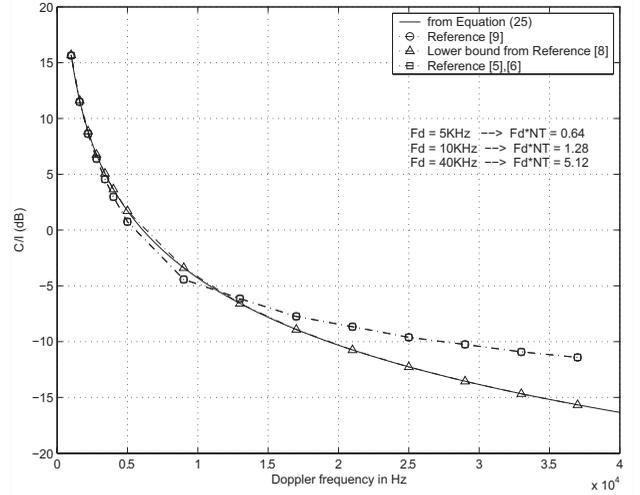


Fig. 4. C/I ratio curves of an OFDM system $N = 256$ subcarriers, subcarrier distance $\Delta f = 7.81 \text{ KHz}$, and carrier frequency $f_c = 2 \text{ GHz}$.

very good approximation of the time variations encountered in many physical channels.

V. THE DISTRIBUTION AND MOMENTS OF THE ICI

In this section we derive expressions for the bivariate probability density and joint moments of the ICI complex random variable Z including a two-dimensional Gram-Charlier expansion. We carry out this analysis in a fairly general setting so that the results can be applied to situations where the channel model and the signal constellation are different from those assumed in the previous sections. Let

$$Z := \sum_{k=1}^N a_k X_k d_k \tag{26}$$

where the a_k 's are real-valued constants and the sets of random variables $\{X_k\}_{k=1}^N$ and $\{d_k\}_{k=1}^N$ are independent. The random vector $\tilde{\mathbf{X}} := (X_1, \dots, X_N)^T$ is complex-valued circular Gaussian with zero mean and $N \times N$ complex-valued covariance matrix $\tilde{\Sigma} = E[\tilde{\mathbf{X}} \tilde{\mathbf{X}}^{*T}]$. We allow $\tilde{\Sigma}$ to be an arbitrary covariance matrix. It is readily evident from (26) that, given the $\{d_k\}$, Z is conditionally complex Gaussian random variable. We seek an explicit expression for the the joint probability density of its real and imaginary parts. Decompose X_k into its real and imaginary parts, $X_k = X_{k,r} + jX_{k,i}$, $k = 1, \dots, N$, and define the $2N \times 1$ real-valued Gaussian vector $\mathbf{X} := [X_{1,r} \dots X_{N,r} X_{1,i} \dots X_{N,i}]^T$. The $2N \times 2N$ real-valued covariance matrix $\Sigma = E[\mathbf{X} \mathbf{X}^T]$ is given by (see for example [19])

$$\Sigma = \frac{1}{2} \begin{bmatrix} \Re[\tilde{\Sigma}] & -\Im[\tilde{\Sigma}] \\ \Im[\tilde{\Sigma}] & \Re[\tilde{\Sigma}] \end{bmatrix} =: \begin{bmatrix} \Sigma_{11} & -\Sigma_{12} \\ \Sigma_{12} & \Sigma_{22} \end{bmatrix}. \tag{27}$$

Next the signal constellation points d_k 's are assumed to be i.i.d. complex-valued random variables and we set $d_k = d_{k,r} + jd_{k,i}$ for its real and imaginary parts. Again, in the general setting here, we do not assume that $d_{k,r}$ and $d_{k,i}$ are independent taking values with equal probabilities. Instead we assume that

$$P[d_{k,r} = b_l, d_{k,i} = c_m] = p_{l,m}, \quad l, m = 1, \dots, L \tag{28}$$

where b_l and c_m are real-valued. This allows general signal constellations. Finally the constants $\{a_k\}$ appearing in (26) are arbitrary; in the special case that will be considered later, we set $a_k = 1/(k-i)$, for $k \neq i$ and $a_k = 0$ for $k = i$. In this case, $ICI = \frac{NT}{2\pi J}Z$.

Decompose the complex-valued random variable Z of (26) into its real and imaginary parts,

$$Z := Z_r + jZ_i = \sum_{k=1}^N a_k [X_{k,r}d_{k,r} - X_{k,i}d_{k,i}] + j \sum_{k=1}^N a_k [X_{k,r}d_{k,i} + X_{k,i}d_{k,r}] \quad (29)$$

and let $f_{Z_r, Z_i}(u, v)$ be its joint probability density. We first seek an expression for this density and its joint moments.

It can be seen that $f_{Z_r, Z_i}(u, v)$ is a Gaussian mixture and one can write

$$f_{Z_r, Z_i}(u, v) = \sum_{l_1=1}^L \sum_{m_1=1}^L \cdots \sum_{l_N=1}^L \sum_{m_N=1}^L \left(\prod_{n=1}^N p_{l_n, m_n} \right) f_{Y_1, Y_2}(u, v) \quad (30)$$

where $f_{Y_1, Y_2}(u, v)$ is a bivariate Gaussian density with zero means and 2×2 dimensional covariance matrix whose entries depend on $(b_{l_1}, \dots, b_{l_N}; c_{m_1}, \dots, c_{m_N})$ (i.e., on the values of the signal constellation points $\{d_k\}_{k=1}^N$). Moreover, one can verify that under our assumptions we have $\text{cov}\{Y_1, Y_2\} = 0$ whereas the variance of Y_1 and Y_2 are identical and given by

$$\begin{aligned} \sigma_Y^2(b_{l_1}, \dots, b_{l_N}; c_{m_1}, \dots, c_{m_N}) \\ = \sum_{i=1}^N \sum_{k=1}^N a_i a_k \sigma_{11}(i, k) [b_{l_i} b_{l_k} + c_{m_i} c_{m_k}] \\ + \sigma_{12}(i, k) [-c_{m_i} b_{l_k} + b_{l_i} c_{m_k}] \end{aligned} \quad (31)$$

where $\sigma_{11}(i, k)$ is the (i, k) th entry of the matrix Σ_{11} and $\sigma_{12}(i, k)$ is the (i, k) th entry of the matrix Σ_{12} defined in (27). It follows that

$$f_{Z_r, Z_i}(u, v) = \sum_{l_1=1}^L \sum_{m_1=1}^L \cdots \sum_{l_N=1}^L \sum_{m_N=1}^L \left(\prod_{n=1}^N p_{l_n, m_n} \right) f_{Y_1}(u) f_{Y_2}(v) \quad (32)$$

where $f_Y(u)$ is a one-dimensional Gaussian density with zero mean and variance given by (31). Equation (32) allows the computation of the joint moments of (Z_r, Z_i) . We have

$$\begin{aligned} E[Z_r^{k_1} Z_i^{k_2}] \\ = \sum_{l_1=1}^L \sum_{m_1=1}^L \cdots \sum_{l_N=1}^L \sum_{m_N=1}^L \left(\prod_{n=1}^N p_{l_n, m_n} \right) E[Y_1^{k_1}] E[Y_2^{k_2}] \end{aligned} \quad (33)$$

and since

$$E[Y^k] = \begin{cases} 1 \times 3 \cdots (k-1) \sigma_Y^k, & k \text{ even} \\ 0, & k \text{ odd} \end{cases} \quad (34)$$

it follows that

$$\begin{aligned} E[Z_r^{k_1} Z_i^{k_2}] = \\ \begin{cases} \sum_{l_1=1}^L \sum_{m_1=1}^L \cdots \sum_{l_N=1}^L \sum_{m_N=1}^L \left(\prod_{n=1}^N p_{l_n, m_n} \right) [1 \cdot 3 \cdots (k_1 - 1)] \cdot \\ \quad \times [1 \cdot 3 \cdots (k_2 - 1)] \cdot \sigma_Y^{k_1 + k_2}, & k_1 \& k_2 \text{ even} \\ 0, & \text{otherwise} \end{cases} \end{aligned} \quad (35)$$

Note that when the real and imaginary parts of the signal constellation points are independent and equally probable, there is a significant simplification in (32) and (35) since then $p_{l, m}$ is a constant. Computationally, finding $f_{Z_r, Z_i}(u, v)$ from (32) is very intensive for large N and L since the dependence on the values of the signal constellation points appear in the exponent of the Gaussian density $f_Y(u)$ via its variance σ_Y^2 . On the other hand, computing the joint moments $E[Z_r^{k_1} Z_i^{k_2}]$ is considerably simpler as the dependence on the values of the constellation points appear directly in σ_Y and only in term of sums of products of two values as seen from (31). We therefore plan to use the joint moments $E[Z_r^{k_1} Z_i^{k_2}]$ to obtain an approximation of the joint probability density $f_{Z_r, Z_i}(u, v)$. This is carried out below using a two-dimensional Gram-Charlier expansion.

The one-dimensional Gram-Charlier expansion and its rate of convergence are as follows [20] [21]: Let $\phi(x)$ be the standard Gaussian probability density with zero mean and unit variance. The n th Hermite polynomial $H_n(x)$ is defined by $H_n(x) := (-1)^n e^{x^2/2} D^n [e^{-x^2/2}]$ where D stands for derivative. $H_n(x)$ is given explicitly as

$$H_n(x) = n! \sum_{k=0}^{[n/2]} \frac{(-1)^k x^{n-2k}}{k! 2^k (n-2k)!}. \quad (36)$$

The first few Hermite polynomials are given by

$$\begin{aligned} H_0(x) = 1, \quad H_1(x) = x, \quad H_2(x) = x^2 - 1, \\ H_3(x) = x^3 - 3x, \quad H_4(x) = x^4 - 6x^2 + 3. \end{aligned} \quad (37)$$

The Hermite polynomials satisfy the orthogonality relationship

$$\int_{-\infty}^{\infty} H_n(x) H_k(x) \phi(x) dx = n! \delta_{n,k} \quad (38)$$

where $\delta_{n,k}$ is the Kronecker delta. Let $g(x)$ be a probability density function of a real-valued random variable X . It can be expanded in a Gram-Charlier series

$$g(x) = \sum_{n=0}^{\infty} \theta_n H_n(x) \phi(x) \quad (39)$$

where the coefficient θ_n is given by

$$\theta_n = \frac{1}{n!} \int_{-\infty}^{\infty} g(x) H_n(x) dx = \frac{1}{n!} E[H_n(X)] \quad (40)$$

and by (36), can be expressed in terms of the moments of X up to order n :

$$\theta_n = \sum_{k=0}^{[n/2]} \frac{(-1)^k E[X^{n-2k}]}{k! 2^k (n-2k)!}. \quad (41)$$

The convergence properties of the Gram-Charlier series are presented in [21]: Set

$$g_K(x) := \sum_{n=0}^K \theta_n H_n(x) \phi(x) \quad (42)$$

then if $g(x)$ has s continuous derivatives satisfying certain integrability conditions ([21], Theorem II and Corollary I), we have

$$|g(x) - g_K(x)| \leq \frac{\text{constant}}{K^{s/2}} \quad (43)$$

uniformly in x . Thus, the smoother $g(x)$ is, the better the approximation of $g(x)$ by $g_K(x)$. In two-dimensions, the Gram-Charlier series takes the following form (see for example [22]): Let (X, Y) be jointly distributed random variables with probability density function $g(x, y)$. We can then represent $g(x, y)$ in a two-dimensional Gram-Charlier series

$$g(x, y) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \theta_{n,m} H_n(x) H_m(y) \phi(x) \phi(y) \quad (44)$$

where the coefficient $\theta_{n,m}$ is given by

$$\begin{aligned} \theta_{n,m} &= \frac{1}{n! m!} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H_n(x) H_m(y) g(x, y) dx dy \\ &= \frac{1}{n! m!} E[H_n(X) H_m(Y)]. \end{aligned} \quad (45)$$

In view of (36), it is seen that $\theta_{n,m}$ can be expressed in terms of the joint moments of (X, Y) :

$$\theta_{n,m} = \sum_{k=0}^{\lfloor n/2 \rfloor} \sum_{l=0}^{\lfloor m/2 \rfloor} \frac{(-1)^{k+l} E[X^{n-2k} Y^{m-2l}]}{k! l! 2^{k+l} (n-2k)! (m-2l)!}. \quad (46)$$

Applying the representation (44) to the bivariate density $f_{Z_r, Z_i}(u, v)$ we have

$$f_{Z_r, Z_i}(u, v) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \theta_{n,m} H_n(u) H_m(v) \phi(u) \phi(v) \quad (47)$$

where the coefficient $\theta_{n,m}$ is now given by

$$\theta_{n,m} = \sum_{k=0}^{\lfloor n/2 \rfloor} \sum_{l=0}^{\lfloor m/2 \rfloor} \frac{(-1)^{k+l} E[Z_r^{n-2k} Z_i^{m-2l}]}{k! l! 2^{k+l} (n-2k)! (m-2l)!}. \quad (48)$$

Note that by (35) $\theta_{n,m}$ is nonzero only if both n and m are even. In practice the two-dimensional Gram-Charlier series (47) is truncated at a total order K , i.e.,

$$f_{Z_r, Z_i}^{(K)}(u, v) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \theta_{n,m} H_n(u) H_m(v) \phi(u) \phi(v). \quad (49)$$

Thus if, say, $K = 4$, we only need to compute the marginal moments $E[Z_r^4]$ and the joint moment $E[Z_r^2 Z_i^2]$ in (35); in turn this only requires the computation of σ_Y^2 of (31).

It should be noted that the standard Gram-Charlier expansion is generated by a normalized Gaussian density $\phi(x)$ with zero mean and unit variance. Clearly, if the variables Z_r and Z_i in (49) have variances substantially different from one, we are likely to need large truncation level K for an acceptable approximation. This problem could be easily remedied as follows: Let s_1^2 and s_2^2 be the variances of the random variables Z_r and Z_i respectively. We can generate a two-dimensional Gram-Charlier expansion starting from Gaussian density functions with zero means and variances s_1^2 and s_2^2 . It is easy to see that instead of (47) we now have

$$f_{Z_r, Z_i}(u, v) = \frac{1}{s_1 s_2} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \bar{\theta}_{n,m} H_n\left(\frac{u}{s_1}\right) H_m\left(\frac{v}{s_2}\right) \phi\left(\frac{u}{s_1}\right) \phi\left(\frac{v}{s_2}\right) \quad (50)$$

with coefficients

$$\bar{\theta}_{n,m} = \sum_{k=0}^{\lfloor n/2 \rfloor} \sum_{l=0}^{\lfloor m/2 \rfloor} \frac{(-1)^{k+l} E[Z_r^{n-2k} Z_i^{m-2l}]}{k! l! 2^{k+l} s_1^{n-2k} s_2^{m-2l} (n-2k)! (m-2l)!} \quad (51)$$

and the representation is truncated at $n + m \leq K$.

We now specialize the above general results to the case introduced in the previous sections. For the covariance matrix of the vector \mathbf{X} , set

$$X_{k,r} := \Re[\beta'_k(t_0)], \quad X_{k,i} := \Im[\beta'_k(t_0)]. \quad (52)$$

It then follows from (6) that the (i, k) th component of the covariance matrix Σ_{11} is given by

$$\begin{aligned} \sigma_{11}(i, k) &:= E[X_{i,r} X_{k,r}] \\ &= -\frac{1}{2} R_1''(0) \Re[R_2(i-k)], \quad i, k = 1, \dots, N. \end{aligned} \quad (53)$$

Similarly, it is seen that the (i, k) th component of the covariance matrix Σ_{12} is given by

$$\sigma_{1,2}(l, k) := E[X_{l,r} X_{k,i}] = \frac{1}{2} R_1''(0) \Im[R_2(l-k)]. \quad (54)$$

Note that this vanishes for $l = k$. Next we specify the values and corresponding probabilities of the signal constellation points. Unlike the general formulation earlier, we now assume that $d_{k,r}$ and $d_{k,i}$ are independent taking values from a square M -point QAM signal constellation with equal probabilities

$$\begin{aligned} P[d_{k,r} = ld] &= P[d_{k,i} = ld] \\ &= \frac{1}{\sqrt{M}}, \quad l = \pm 1, \pm 3, \dots, \pm(\sqrt{M}-1). \end{aligned} \quad (55)$$

This corresponds to setting $p_{l,m} = \frac{1}{\sqrt{M}}$ with $L = \sqrt{M}$ in our general formulation. Finally, we set $a_k = 1/(k-i)$, for $k \neq i$ and $a_k = 0$ for $k = i$. Thus for the special case, the expression for σ_Y^2 of (31) remains applicable with a_k as specified above and $b_k = kd$, $c_m = md$ and the values of $\sigma_{11}(i, k)$ and $\sigma_{12}(i, k)$ are specified in (53) and (54) respectively. The expression for $f_{Z_r, Z_i}(u, v)$ of (32) simplifies to

$$f_{Z_r, Z_i}(u, v) = \frac{1}{M^{N-1}} \sum_{l_1} \sum_{m_1} \dots \sum_{l_N} \sum_{m_N} f_{Y_1}(u) f_{Y_2}(v) \quad (56)$$

where we use the compact notation

$$\sum_l := \sum_{\substack{l=-\sqrt{M}-1 \\ l \text{ odd}}}^{\sqrt{M}-1}. \quad (57)$$

The joint moments $E[Z_r^{k_1} Z_i^{k_2}]$ simplify to

$$\begin{aligned} E[Z_r^{k_1} Z_i^{k_2}] &= \frac{1}{M^{N-1}} \\ &\times \begin{cases} \sum_{l_1} \sum_{m_1} \dots \sum_{l_N} \sum_{m_N} [1 \times 3 \dots \times (k_1 - 1)] \\ \times [1 \times 3 \dots \times (k_2 - 1)] \sigma_Y^{k_1+k_2}, & k_1 \text{ and } k_2 \text{ even} \\ 0, & \text{otherwise} \end{cases} \end{aligned} \quad (58)$$

The truncated two-dimensional Gram-Charlier representation is again given by (49) with coefficients $\theta_{n,m}$ given by (48) involving the joint moments $E[Z_r^{k_1} Z_i^{k_2}]$ now given by (58). Thus, we only need to obtain an expression for σ_Y^2 . Straightforward calculations using (31) (53) and (54) show that

$$\begin{aligned} \sigma_Y^2 &= \left(\frac{NTFd}{2}\right)^2 \sum_{r=1}^N \sum_{\substack{s=1 \\ r \neq i \neq s}}^N \frac{\Re[\psi_2(\Delta f(r-s))](l_r l_s + m_r m_s)}{(r-i)(s-i)} \\ &+ \frac{\Im[\psi_2(\Delta f(r-s))](m_r l_s + l_r m_s)}{(r-i)(s-i)}. \end{aligned} \quad (59)$$

Similar computations hold for the scaled Gram-Charlier expansion (50).

The above Gram-Charlier approximation method can be easily applied to Rician fading statistics. The only difference in applying the above proposed approximation approach is to adjust the corresponding ICI joint moments. In this paper, our results are limited to Rayleigh fading statistics.

VI. EVALUATION OF OFDM SYSTEM PERFORMANCE

We now evaluate the error probability for an M -QAM system with coherent detection. In view of (19) and (26) we can write

$$\hat{d}_i = \beta_i(t_0)d_i + \frac{NT}{2\pi j}Z + n_i \quad (60)$$

where Z represents the ICI contribution whose statistics were thoroughly studied in Section V. We assume that we have perfect knowledge of $\beta_i(t_0)$ in each sub-channel and we form the decision variable

$$\hat{D}_i = \frac{\hat{d}_i \beta_i^*(t_0)}{|\beta_i(t_0)|^2} = d_i + \frac{NT}{2\pi j} \frac{Z \beta_i^*(t_0)}{|\beta_i(t_0)|^2} + \frac{n_i \beta_i^*(t_0)}{|\beta_i(t_0)|^2}. \quad (61)$$

Set $a := NT/(2\pi)$, $Z = Z_r + jZ_i$, $\beta_i(t_0) = W_1 + jW_2$, and $n_i = n_{i,1} + jn_{i,2}$ for their real and imaginary parts. Then

$$\Re[\hat{D}_i] = \Re[d_i] + \frac{a(Z_i W_1 - Z_r W_2)}{W_1^2 + W_2^2} + \frac{n_{i,1} W_1 + n_{i,2} W_2}{W_1^2 + W_2^2}. \quad (62)$$

$$\Im[\hat{D}_i] = \Im[d_i] - \frac{a(Z_r W_1 + Z_i W_2)}{W_1^2 + W_2^2} + \frac{n_{i,2} W_1 - n_{i,1} W_2}{W_1^2 + W_2^2}. \quad (63)$$

A final decision is made by comparing the location of random variable \hat{D}_i with the M -QAM constellation points and selecting the signal point that is nearest to d_i . The decision of the detector is based on (61): Assuming a symmetric rectangular QAM signal constellation, the detector performs independent decisions on the real and imaginary parts conditioned on a particular channel realization $\beta_i(t_0)$ and ICI realization Z . A symbol error occurs if either the real or imaginary components are in error. Let A_i be the event of making an error in the real (or imaginary) part in the i^{th} sub-channel. Then, conditioned on a particular channel realization $\beta_i(t_0)$ and ICI realization Z , we have the following equations for the probability of error in the x (real) and y (imaginary) parts:

$$\begin{aligned} P_s^x[A_i|Z = u + jv, \beta_i(t_0) = w_1 + jw_2] &= \frac{L-1}{L} \left[Q \left(\frac{d(w_1^2 + w_2^2) + a(vw_1 - uw_2)}{[(N_0/2E_s)(w_1^2 + w_2^2)]^{1/2}} \right) \right. \\ &\quad \left. + Q \left(\frac{d(w_1^2 + w_2^2) - a(vw_1 - uw_2)}{[(N_0/2E_s)(w_1^2 + w_2^2)]^{1/2}} \right) \right] \end{aligned} \quad (64)$$

$$\begin{aligned} P_s^y[A_i|Z = u + jv, \beta_i(t_0) = w_1 + jw_2] &= \frac{L-1}{L} \left[Q \left(\frac{d(w_1^2 + w_2^2) + a(uw_1 + vw_2)}{[(N_0/2E_s)(w_1^2 + w_2^2)]^{1/2}} \right) \right. \\ &\quad \left. + Q \left(\frac{d(w_1^2 + w_2^2) - a(uw_1 + vw_2)}{[(N_0/2E_s)(w_1^2 + w_2^2)]^{1/2}} \right) \right] \end{aligned} \quad (65)$$

where $d = \sqrt{\frac{3}{2(M-1)}}$ with $L^2 = M$. The conditional symbol error probability is then given by

$$\begin{aligned} P_s[A_i|Z = u + jv, \beta_i(t_0) = w_1 + jw_2] &= \\ &= 1 - \left((1 - P_s^x[A_i|Z = u + jv, \beta_i(t_0) = w_1 + jw_2]) \cdot \right. \\ &\quad \left. \times (1 - P_s^y[A_i|Z = u + jv, \beta_i(t_0) = w_1 + jw_2]) \right) \end{aligned} \quad (66)$$

In order to obtain unconditional probability of error, we average over the distributions of the ICI variable Z and over the distribution of $\beta_i(t_0)$. We first note that the bivariate probability density $f_{Z_r, Z_i}(u, v)$ of the ICI disturbance Z is given in (56) and its Gram-Charlier approximation by (49). We further note that the ICI complex random variable Z and the channel random variable $\beta_i(t_0)$ are independent. This follows from the following argument: The process $\{\beta_i(t), -\infty < t < \infty\}$ is complex Gaussian with covariance function $R_{\beta_i, \beta_i}(\tau) = R_1(\tau)$ given by (8). The derivative process $\{\beta'_i(t), -\infty < t < \infty\}$ is complex Gaussian with covariance function $R_{\beta'_i, \beta'_i}(\tau) = -R_1''(\tau)$. The cross covariance function $R_{\beta'_i, \beta_i}(\tau) = R_1'(\tau)R_2(k-l)$ which vanishes at $\tau = 0$. Since the processes are jointly Gaussian, the random variable $\beta_i(t_0)$ is independent of the random variables $\{\beta'_k(t_0)\}_{k=1}^N$ and thus independent of Z in view of the ICI term in (19). We finally note that the real and imaginary parts of $\beta_i(t_0)$ are independent zero mean Gaussian random variables with identical variance equals 1/2. Thus

$$\begin{aligned} P_s[A_i] &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_s[A_i|Z = u + jv, \beta_i(t_0) = w_1 + jw_2] \\ &\quad \times f_{Z_r, Z_i}(u, v) f_W(w_1) f_W(w_2) dudvdw_1 dw_2 \end{aligned} \quad (67)$$

where $f_W(w) = \frac{1}{\sqrt{\pi}} e^{-w^2}$. The above multidimensional integration can be evaluated by numerical integrations. The system performance is obtained by averaging over all sub-channels,

$$P_s = \frac{1}{N} \sum_{i=1}^N P_s[A_i]. \quad (68)$$

Since the probability density $f_{Z_r, Z_i}(u, v)$ of the ICI variable is quite involved (see (56)), we use the Gram-Charlier approximation developed in Section 5. We have

$$\begin{aligned} f_{Z_r, Z_i}^{(K)}(u, v) &= \frac{1}{s^2} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \bar{\theta}_{n,m} H_n(u/s) H_m(v/s) \phi(u/s) \phi(v/s). \end{aligned} \quad (69)$$

with coefficients $\bar{\theta}_{n,m}$ given by (51) and with joint moments given by (58). Here $s^2 = \text{var}[Z_r] = \text{var}[Z_i]$. This leads to an approximate probability of error,

$$P_s^{(K)} = \frac{1}{N} \sum_{i=1}^N P_s^{(K)}[A_i]. \quad (70)$$

Figs. 5 through 6 illustrate the symbol error probability of uncoded 16-QAM OFDM system with bandwidth $W = 1/T = 1\text{MHz}$ over a frequency-selective channel with an exponential multipath power intensity and channel coherence bandwidth (3dB bandwidth) of $W/4$ under different Doppler spreads,

TABLE I
PERFORMANCE DEGRADATION VS. DOPPLER SPREAD

	OFDM symbol (NT)	Carrier Frequency	Doppler Spread F_d at $v = 200km/hr$	SER $\times 10^{-4}$	SER No Doppler $\times 10^{-4}$
802.11a	$4\mu s$	$5GHz$	$1KHz$	3.5	1.8
DVBCS2	$500\mu s$	$480MHz$	$0.1KHz$	278	1.8

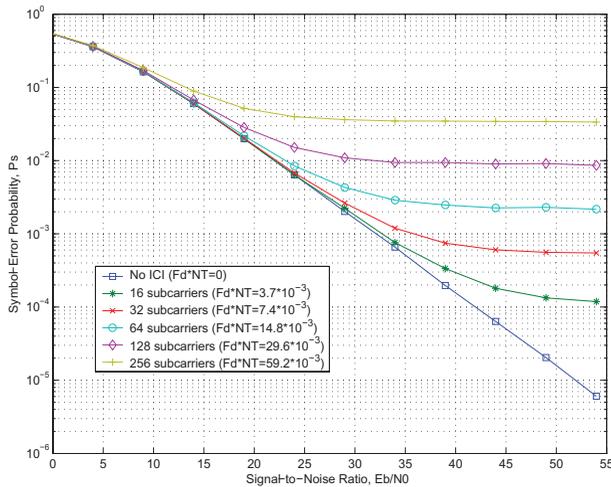


Fig. 5. Performance for 16-QAM OFDM system with $f_c = 5 GHz$, $T = 1\mu s$ ($W \approx 1MHz$), and speed of 50km/hr.

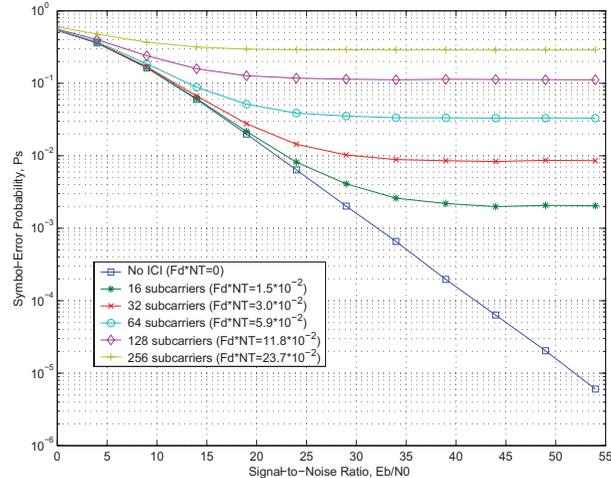


Fig. 6. Performance for 16-QAM OFDM system with $f_c = 5 GHz$, $T = 1\mu s$ ($W \approx 1MHz$), and speed of 200km/hr.

F_d , of 231.5 and 926.0Hz, corresponding to mobile speeds of 50 and 200km/hr. The carrier frequency is $f_c = 5GHz$. At the higher speeds, we observe that the ICI causes a large deterioration in performance as the number N of subcarriers increases. When the average SNR (E_b/N_0) is large, the ICI is ultimately the limiting factor in performance at any speed and for any N .

Fig. 7 provides a comparison in error rate performance between BPSK (bit error probability) and 16-QAM (symbol error probability) when the mobile speed is 100km/hr. In this case, we observe that the ICI causes a significantly higher

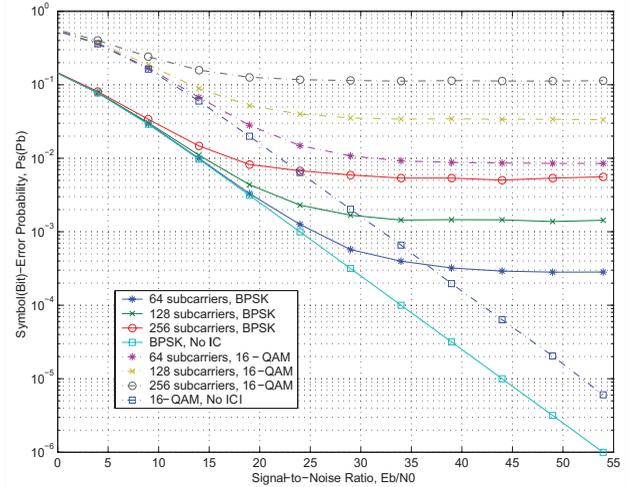


Fig. 7. Performance comparison for BPSK OFDM and 16-QAM OFDM system with $f_c = 5 GHz$, $T = 1\mu s$ ($W \approx 1MHz$), and speed of 100km/hr.

degradation in performance for 16-QAM compared with that for BPSK.

Next we compare the performance for two practical OFDM systems. One is an IEEE 802.11a system, the other is a Digital Video Broadcasting (DVB) system operating in the CS2 mode. Both systems are operating at a signal to noise ratio (SNR) of 40dB in a vehicle traveling at a speed of 200km/hr and both employ uncoded 16-QAM modulation. The SER degradation for the two systems is different for the same mobile Rayleigh fading channel. As can be seen in the Table I, the average error rates of 802.11a and DVB-CS2 increase by 1.94 times and 155 times respectively, compared to the case of no Doppler spread.

The error probability shown in Fig. 5 through 7 were evaluated by using equations (67), (69), and (70) with $K = 4$ in the Gram-Charlier expansion. We found that $K = 4$ provides a very close agreement with the error rate performance obtained from Monte Carlo simulation. For example, Fig. 8 illustrates the probability of a symbol error for 16-QAM and a mobile speed of 100 km/hr for three different orders of the Gram-Charlier series, $K = 2, 4, 6$, and the result obtained from simulation. We observe that $K = 4$ provides a very close approximation to the simulation results. We point out that Gram-Charlier approximations of orders $K = 0$ and $K = 2$ both result in a bivariate Gaussian pdf with uncorrelated (Z_r, Z_i) components. Consequently, the approximation for $K = 0$ and $K = 2$ corresponds to the case in which the ICI is modeled as bivariate Gaussian with uncorrelated real and imaginary components. Fig. 9 illustrates the comparison of the performance obtained with the Gaussian approximation (Gram-Charlier expansion of order 2) for the ICI and the non-

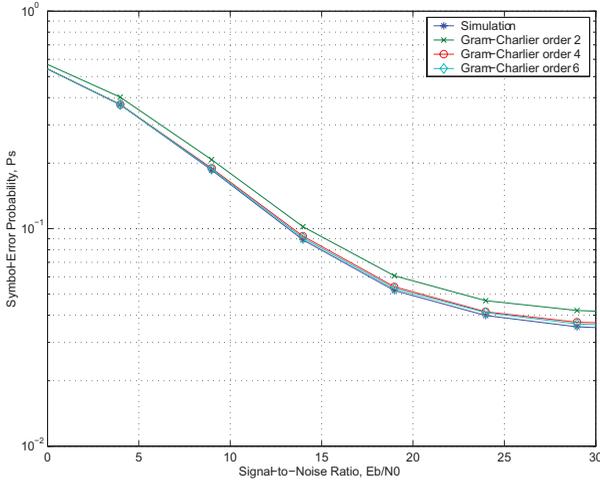


Fig. 8. Performance for 16-QAM system with 128 subcarriers, $f_c = 5$ GHz, $T = 1\mu s$ ($W \approx 1MHz$), and speed of 100km/hr.

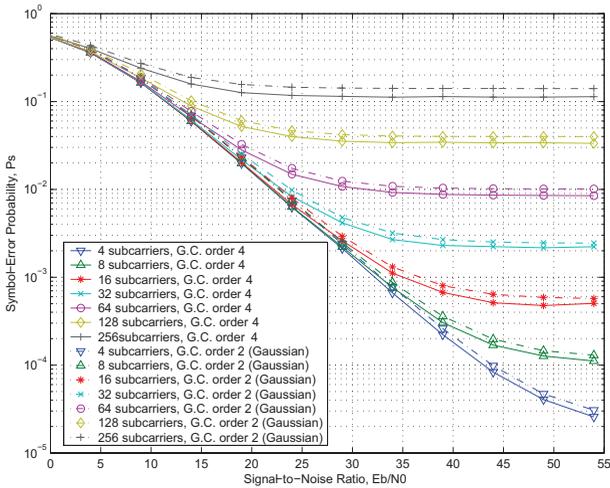


Fig. 9. Performance for 16-QAM OFDM system with $f_c = 5$ GHz, $T = 1\mu s$ ($W \approx 1MHz$), and speed of 100km/hr.

Gaussian approximation (Gram-Charlier expansion of order 4). We note that the Gaussian model ($K = 2$) give results that differ from the simulation results illustrated in Fig. 8 and the non-Gaussian approximation (G.C. order 4) in Fig. 9 by one or more dB depending on the SNR. The mismatch between the Gaussian approximation and the actual system performance is due to the fact that the ICI interference is not Gaussian distributed, as is illustrated in Section V. The most important feature of the Gram-Charlier approximation lies in the property that it makes use of the higher order statistics (moments) of the ICI, by employing a higher order Gram-Charlier approximation. Thus, the Gram-Charlier expansion provides a better match to the pdf of the ICI than the Gaussian approximation (which is based on second order statistics).

There is also a major difference between Gram-Charlier approximation of higher order $K \geq 4$ and a Gaussian approximation for the ICI. In a Gaussian approximation, the system performance only depends on the number of OFDM subcarriers and the normalized Doppler spread, due to the fact

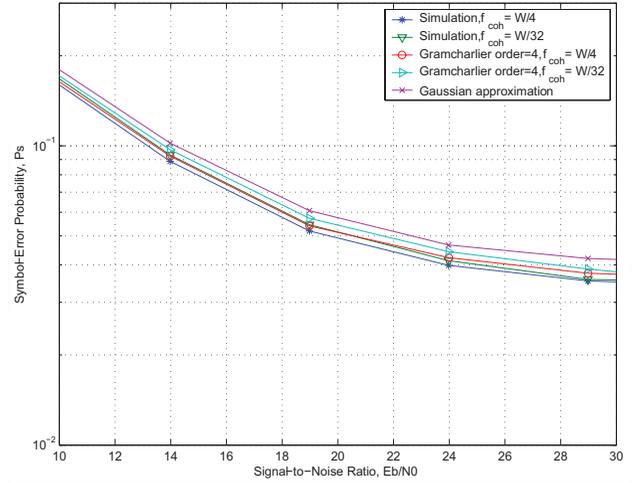


Fig. 10. Performance for 16-QAM OFDM system with 128 subcarriers, $f_c = 5$ GHz, $T = 1\mu s$ ($W \approx 1MHz$), speed of 100km/hr, and transmitting over different frequency-selective fading channels.

the second order moment of the ICI only depends on these two parameters. Therefore, the performance approximation obtained with the Gaussian model for the ICI can not represent the effect of channel frequency selectivity of the overall OFDM system. On the other hand, by using Gram-Charlier approximation, the ICI joint moments of order $K \geq 4$ do not only depend on the normalized Doppler frequency and the number of subcarriers, but also depend on the correlation structure among the subcarriers (hence the power delay profile of the frequency selective fading channel), and the baseband modulation constellations. Therefore, the Gram-Charlier approximation provides us with a much more accurate approach in evaluating the performance of an OFDM system under different frequency selective fading channel conditions. As an example, we demonstrate in Fig. 10 the system performance curves as well as their Gram-Charlier approximations (of order 4) of the same OFDM system transmitting over different frequency-selective fading channels with coherence bandwidth $W/4$ and $W/32$. We observe from the plot that the proposed Gram-Charlier approximation (with order $K = 4$) accounts for the effects of frequency-selectivity on ICI interference whereas the simple Gaussian approximation does not.

VII. CONCLUDING REMARKS

In this paper, the performance of an OFDM system was analyzed with respect to its sensitivity to Doppler related inter-carrier interference. We obtained explicit formulas for the bivariate probability density of the ICI, its joint moments, as well as two-dimensional Gram-Charlier representation and approximation. The performance of the OFDM system was obtained based on a truncated Gram-Charlier expansion of the ICI bivariate density. The effect of the Doppler spread in a time variant mobile radio channel on the performance of QAM and BPSK OFDM systems was evaluated. The performance of OFDM systems for M-ary PSK can be evaluated in a similar manner using the methods described in this paper.

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