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Authors
Moretto, L.G.
Thompson, S.G.
Routti, J.
et al.

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INFLUENCE OF SHELLS AND PAIRING ON THE FISSION PROBABILITIES OF NUCLEI BELOW RADIUM*

L. G. Moretto, S. G. Thompson, J. Routti, and R. C. Gatti

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

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Realistic level densities have been employed in analyzing experimental fission probabilities. The inclusion of shell and pairing effects in Γ_N allows one to obtain reliable information concerning the fission barriers as well as the saddle point pairing and level densities.

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A large body of experimental information concerning the fission cross sections of nuclei below radium has been obtained, mainly through the work of the Berkeley group [1]. Attempts to fit the fission probabilities by means of simple expressions based on Fermi gas level densities indicated that reliable values for fission barriers could be obtained. These experimental values showed that the barriers could adequately be accounted for as the sum of ground state shell effect plus the liquid drop fission barrier as proposed by Myers and Swiatecki [2]. However, difficulties in fitting the fission probabilities over a large energy range were apparent for nuclei close to the shell region $Z = 82$, $N = 126$: at low excitation energy a large ratio of the level density parameters involved in Γ_F and in Γ_N seemed to be required ($\frac{a_f}{a_n} \sim 1.4$), while at higher energies a ratio more nearly unity seemed to be in order. This effect was suspected to be an influence in Γ_n of shell effects and their disappearance with excitation energy.

A further problem developed as a result of experimental fission angular distributions close to the barriers of ^{210}Po and ^{211}Po. These data seemed to indicate the existence of substantially large pairing effects at the saddle point [3]. Such large pairing effects might have an important influence on the correct interpretation of the experimental data. A more satisfactory interpretation of the experimental results depended on the development of improved theoretical expressions. Recently very effective formalisms have been developed for the calculation of the level densities on the basis of realistic single particle schemes [4]. Furthermore, the pairing effects and their energy dependence could be included in the calculation by means of the B.C.S. Hamiltonian [5,6,7]. These calculations have been shown to predict the disappearance of shell and pairing effects with increasing excitation energy [5,8].

In order to analyze the fission probabilities, a code based on the above formalism has been written to evaluate the first chance fission probability $\frac{\Gamma_F}{\Gamma_F + \Gamma_N}$. The neutron width Γ_N is obtained by computing the relevant level density on the basis of the Nilsson diagram [9]. Pairing is accounted for by the B.C.S. Hamiltonian used in the statistical formalism. No free parameters have been introduced in the calculation, aside from the oscillator shell spacing which has usually been kept constant and given the value $41/A^{1/3}$. Two sets of Nilsson levels have been used: a set for spherical nuclei for $195 \leqslant A \leqslant 213$ and another set for deformed nuclei for $A < 195$.

The fission width Γ_F is calculated on the basis of the uniform model with pairing again accounted for by means of the B.C.S. Hamiltonian. Four free parameters have been introduced: the fission barrier B_f, the density of the doubly degenerate levels g_f, the gap parameter Δ and the barrier penetrability.

The possibility of using shell model single particle levels at the saddle point, although it could be done easily, has been discarded for two reasons:

i) the single particle calculations available in this region have been carried out only for a very limited number of deformation coordinates. This tends to locate spuriously the saddle point in strong antishell regions;

ii) the acceptance of a given shell model scheme to describe the statistical behavior of the nucleus at the saddle point implies the acceptance of the fission barrier height which can be obtained by means of the Strutinski procedure. However these models do not yet predict the barriers within 1 MeV. A deviation larger than 1 MeV would not allow one to reproduce the experimental fission probabilities. Therefore it is not feasible to eliminate the barriers as free parameters without having available a much more realistic single particle model.

The angular momentum dependence of the fission probability is controlled by the spin cut-off parameters used in Γ_N and Γ_F. The spin cut-off parameter in Γ_N is calculated directly by the level density subroutine from the Nilsson levels. The spin cut-off parameter in Γ_F is calculated in such a way as to reproduce the average shell model value at sphericity and it is modified in order to account for the saddle point deformation. The mean square angular momentum of the compound nucleus is calculated by means of an optical model. The primary experimental data are the fission cross sections: the reactions and the source of the data considered in the present analysis are presented in Table 1. The total fission probabilities are then obtained by dividing the experimentally measured fission cross sections σ_F, by the reaction cross sections σ_R derived from optical model calculations. Two corrections are necessary to transform this quantity into the first chance fission probability: the higher order fission contribution and the non compound nucleus reaction cross section [10].

Although the corrections are negligible in our region of interest at energies below ∿ 40 MeV, they may become rather serious at high excitation energies; however they act in opposite directions and to some extent at least they are expected to cancel. At this time there is neither sufficient experimental information nor reliable theory available for making these corrections on a more than qualitative basis. Therefore we decided not to make any correction at all. The data fitting has been performed up to 70 MeV excitation energy below which the cancellation of the two corrections is expected to be more nearly complete. Fits were also made up to the highest available excitation energy to determine the extent to which the free parameters would vary; it was found that the final values of the parameters were very close. The fitting procedure was also performed in many modes:

i) a preliminary attempt to fit all the data with all the parameters free including the quantity $\hbar\omega_o$ (five parameters) did not indicate any preferred value of $\hbar\omega_o$. Equivalently good fits over a large range of $\hbar\omega_o$ ($30\ A^{-1/3}$ to $50\ A^{-1/3}$) could be obtained provided the product of $\hbar\omega_o$ and the saddle point level density parameter g_f was maintained essentially constant;

ii) the shell spacing was then fixed to $41\ A^{-1/3}$ as by Nilsson et al.[9] and the other four parameters were left free. The fits obtained in this fashion ranged from good to very good over both energy ranges. The average of the best fit values of the barrier penetrabilities for the cases in which data were obtained close to the barrier or below was ∿ 1.0 MeV. The values of the gap parameters at the saddle point turned out to be unexpectedly low, even lower than the known ground state values;

iii) the gap parameters at the saddle point were fixed at the average ground state values $\Delta = 11/A^{1/2}$. The relatively poor quality of the fits seems to indicate that the data are inconsistent with pairing effects of this magnitude;

iv) in order to obtain a consistent set of fission barrier heights, the barrier penetrabilities were fixed at 1 MeV which was the average value obtained in ii). The fits can be rated from good to very good with χ^2 values almost identical to those obtained in ii). We note also that the values of the barriers and other parameters are not much changed in the three and four parameter fits.

The results of the least squares fits to the data are reported in Table 1. Typical examples of fits can also be seen in fig. 1.

The fission barriers obtained as indicated in iv) are expected to be generally accurate within 1 MeV, aside from some possible systematic deviations associated with the fixed value of the barrier penetrability. If one assumes that no shell effects are present at the saddle point, the differences between the measured fission barriers and the ground state shell effects should be smooth and follow closely the liquid drop prediction. This is indeed the case as can be seen in fig. 2. The fluctuations about the liquid drop predictions are at most 1 MeV, of the order of the experimental uncertainty. Therefore the saddle point shell effects seem to be very small.

The single particle level densities g_f at the saddle point are plotted as a function of mass number A in fig. 2. The two lines bracketing the data correspond to the level density parameters $a_f = \frac{A}{9}$ and $a_f = \frac{A}{8}$ where $a_f = \frac{\pi^2}{3} g_f$. We note that a reasonable average line representing a smooth dependence on A passes through the data and corresponds to $a_f = \frac{A}{8.5}$. The fluctuations about the average are smaller than ∿ 5%. Thus it seems clear that the level densities based on the Nilsson diagram have accounted for the major part of the shell effects and their disappearance with energy. However, a closer examination of the deviations from the average indicates a systematic trend rather than statistical differences. There seems to be a correlation of the fluctuations and residual shell effects which are not accounted for by the Nilsson model (see for instance fig. 16 in ref. 9).

The average value $a_f = \frac{A}{8.5}$ indicated in fig. 2 may also have additional significance, if it is compared with the corresponding quantity in Γ_n obtained by smoothing the Nilsson spectrum. This smoothed result gives $a_n \sim \frac{A}{9.2}$. The ratio $a_f/a_n \sim 1.08$ agrees very well with a prediction by W. Myers which is based on the effect of increased surface area on single particle level densities at the saddle point.

The gap parameters obtained from the data fitting are surprisingly small: it seems that it is not possible to reproduce the remarkably steep rise of the fission probabilities at low energies without reducing pairing substantially. Perhaps this could be caused by the breakdown of pairing due to the influence of angular momentum. However, this is not clear and it is quite possible that pairing is reduced to compensate for other effects which are not accounted for by the model.

A rather disturbing fact is evident from the comparison of the proton induced fission and alpha induced fission. The former cases seem to require much smaller pairing than the latter. In particular the two reactions ^{209}Bi + p and ^{206}Pb + ^{4}He leading to the same compound nucleus ^{210}Po also yield somewhat different values both for the barrier and for pairing. The reason for the difference is not understood.

A conclusion of this work is that a major step forward has been made in the interpretation of the fission probabilities by using more realistic level densities. However, some of the results are still not completely understood.

FOOTNOTES AND REFERENCES

[*] Work performed under the auspices of the U. S. Atomic Energy Commission.

1. For the sources of experimental data, see references in fig. 1.
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Table 1. Parameters obtained from the analysis of fission probabilities*

Reaction	Ref.	B_f MeV	g_f MeV^{-1}	Δ MeV	χ^2_{LOG} †
${}_{83}\mathrm{Bi}^{209} + {}_{2}\mathrm{He}^{4} \rightarrow {}_{85}\mathrm{At}^{213}$	a	17.0	7.67	0.38	0.060
${}_{82}\mathrm{Pb}^{208} + {}_{2}\mathrm{He}^{4} \rightarrow {}_{84}\mathrm{Po}^{212}$	a	19.5	7.36	0.06	0.027
${}_{82}\mathrm{Pb}^{207} + {}_{2}\mathrm{He}^{4} \rightarrow {}_{84}\mathrm{Po}^{211}$	a	19.7	7.08	0.84	0.001
${}_{82}\mathrm{Pb}^{206} + {}_{2}\mathrm{He}^{4} \rightarrow {}_{84}\mathrm{Po}^{210}$	a	20.5	7.42	0.60	0.030
${}_{83}\mathrm{Bi}^{209} + {}_{1}\mathrm{H}^{1} \rightarrow {}_{84}\mathrm{Po}^{210}$	a	21.4	7.33	0.17	0.024
${}_{82}\mathrm{Pb}^{208} + {}_{1}\mathrm{H}^{1} \rightarrow {}_{83}\mathrm{Bi}^{209}$	a	23.3	7.55	0.22	0.020
${}_{82}\mathrm{Pb}^{206} + {}_{1}\mathrm{H}^{1} \rightarrow {}_{83}\mathrm{Bi}^{207}$	a	21.9	7.63	0.11	0.035
${}_{79}\mathrm{Au}^{197} + {}_{2}\mathrm{He}^{4} \rightarrow {}_{81}\mathrm{Tl}^{201}$	a	22.3	7.57	0.39	0.051
${}_{79}\mathrm{Au}^{197} + {}_{1}\mathrm{H}^{1} \rightarrow {}_{80}\mathrm{Hg}^{198}$	a	20.4	7.43	0.68	0.015
${}_{75}\mathrm{Re}^{187} + {}_{2}\mathrm{He}^{4} \rightarrow {}_{77}\mathrm{Ir}^{191}$	b	23.7	7.16	0.05	0.003
${}_{75}\mathrm{Re}^{185} + {}_{2}\mathrm{He}^{4} \rightarrow {}_{77}\mathrm{Ir}^{189}$	b	22.6	6.84	0.10	0.023
${}_{74}\mathrm{W}^{184} + {}_{2}\mathrm{He}^{4} \rightarrow {}_{76}\mathrm{Os}^{188}$	c	24.2	6.89	0.54	0.005
${}_{74}\mathrm{W}^{183} + {}_{2}\mathrm{He}^{4} \rightarrow {}_{76}\mathrm{Os}^{187}$	c	22.7	6.84	0.83	0.004
${}_{74}\mathrm{W}^{182} + {}_{2}\mathrm{He}^{4} \rightarrow {}_{76}\mathrm{Os}^{186}$	c	23.4	6.66	0.43	0.006
${}_{73}\mathrm{Ta}^{181} + {}_{2}\mathrm{He}^{4} \rightarrow {}_{75}\mathrm{Re}^{185}$	a	24.0	6.51	0.60	0.008
${}_{71}\mathrm{Lu}^{175} + {}_{2}\mathrm{He}^{4} \rightarrow {}_{73}\mathrm{Ta}^{179}$	b	26.1	6.53	0.99	0.002
${}_{69}\mathrm{Tm}^{169} + {}_{2}\mathrm{He}^{4} \rightarrow {}_{71}\mathrm{Lu}^{173}$	b	28.0	6.17	0.87	0.003

(continued)

Table 1. (continued)

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[a] Khodai-Joopary, A., Ph.D. Thesis, University of California, Lawrence Radiation Laboratory UCRL-16489, July 11, 1966.

[b] G. M. Raisbeck, J. W. Cobble, Phys. Rev. <u>153</u>, 1270 (1967).

[c] L. G. Moretto, R. C. Gatti, and S. G. Thompson, Lawrence Radiation Laboratory Report UCRL-17989, Nuclear Chemistry Division Annual Report, January 1968 (unpublished), p. 141.

[*] The Barrier Penetrations have been set equal to 1.0 MeV.

[†] $\chi^2_{\mathrm{LOG}} = \frac{1}{N} \sum_{K=1}^{N} [\mathrm{LOG}(\mathrm{Exp.}_K) - \mathrm{LOG}(\mathrm{Theor.}_K)]^2.$

FIGURE CAPTIONS

Fig. 1. Examples of theoretical fits to experimental fission probabilities.

Fig. 2. Measured fission barriers, corrected for the ground state shell effects, as a function of the fissility parameter x. The solid line represents the liquid drop prediction.

Fig. 3. Single particle level density at the saddle point (doubly degenerate levels). The three lines correspond to level density parameter a equal to $\frac{A}{8}$, $\frac{A}{8.5}$ and $\frac{A}{9}$.

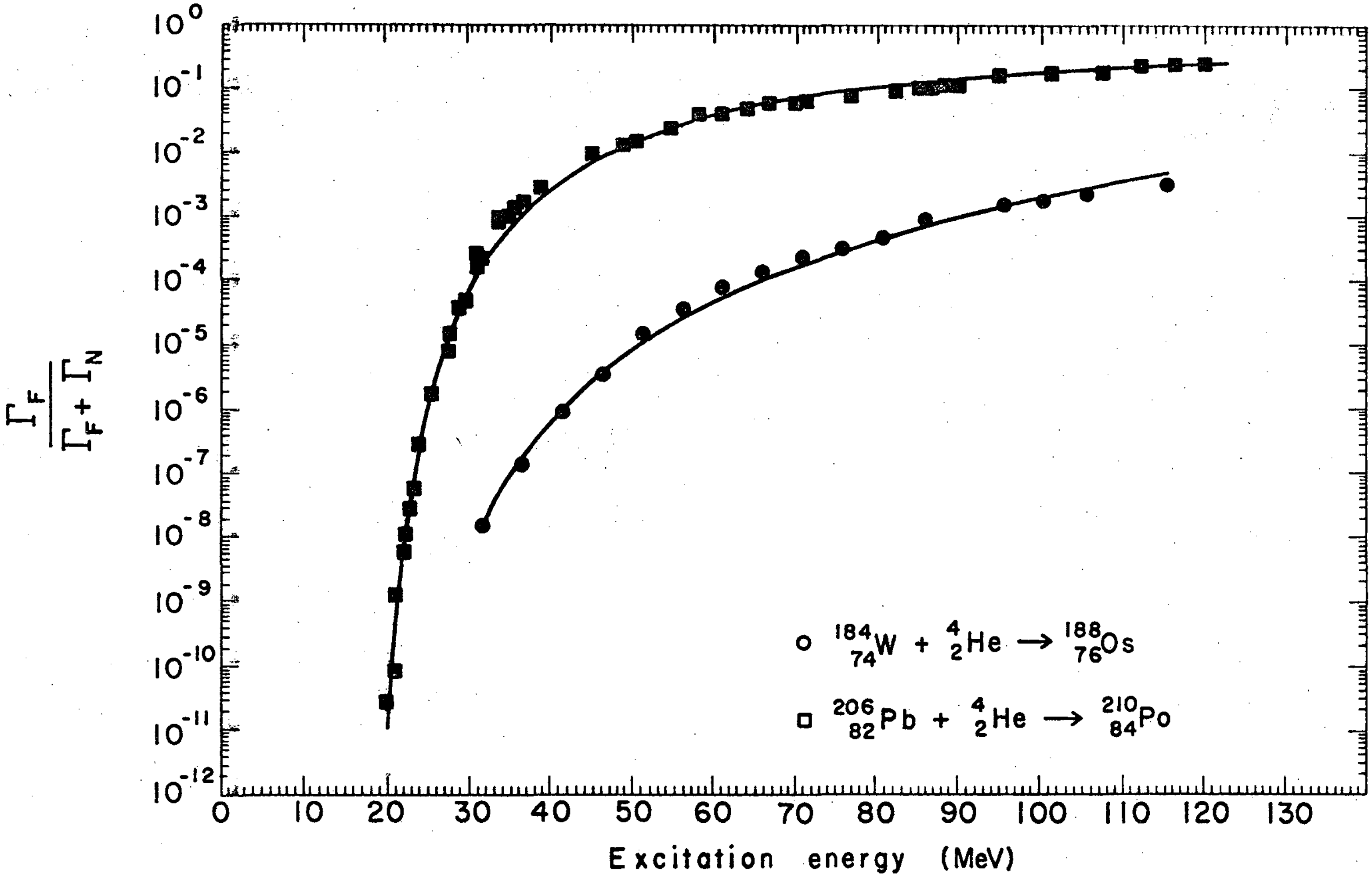

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Fig. 1

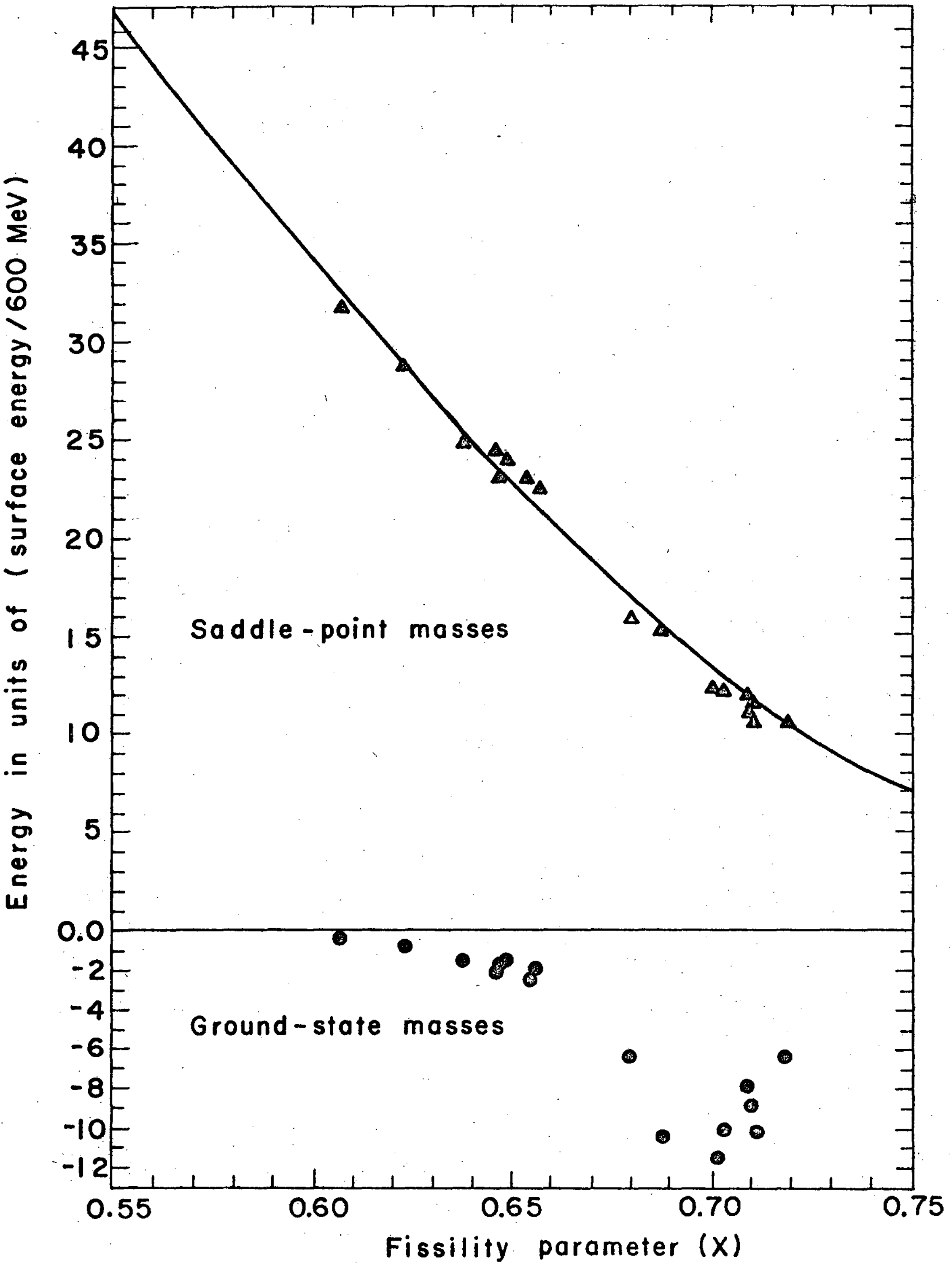

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Fig. 2

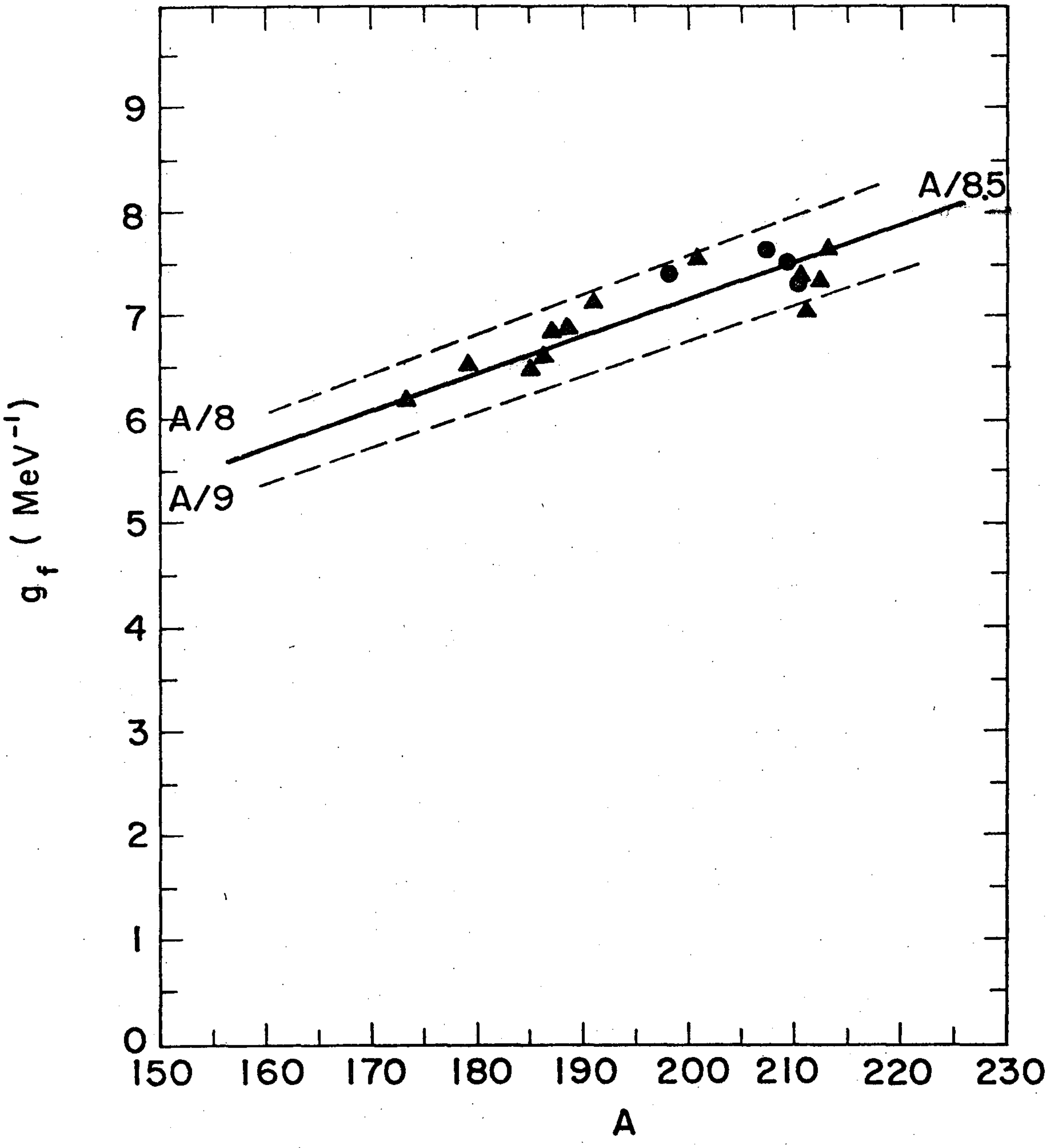

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Fig. 3

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LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720