Topological Invariants and Apparent Motion

Lin Chen

China University of Science & Technology
Hofei, Anhui Province
The People's Republic of China

and

Program in Cognitive Science
University of California at Irvine
Irvine, CA 92717

INTRODUCTION

As many authors (Ullman, 1979; Anstis, 1979; Marr, 1982) have pointed out, at the core of how to understand apparent motion lies the correspondence problem: in the process of perceiving apparent motion, one has to establish at some level a correspondence identifying the parts of two succeeding stimulus frames which represent the same object.

One distinguishing fact of apparent motion is that when one perceives apparent motion, often he also perceives some sort of transformation from one stimulus pattern to another; for example, a square is moving and changing shape simultaneously to become a circle. Kolers & Pomerantz (1971) found that three kinds of transformations, not only translation and rotation of rigid shape but also intriguing plastic deformation, occured when the dissimilar pairs gave apparent motion. The interesting question is: What kinds of invariants under the transformation of "plastic deformation" does the visual system depend on to determine that two figures, however shape-changed they may be, nevertheless represent the same object?

As Chen (1980, 1981, 1982a,b,c,d) has argued, a primitive and general function of the visual system may be the perception of global topological invariants, such as connectivity, closure and holes, which are defined as invariants under topological transformations (plastic deformations without breaking and fusion). Many paradigms and approaches in visual perception, such as the object superiority effect (Weisstein & Harris, 1974; McClelland, 1978; Williams & Weisstein, 1978; Chen, 1982c), grouping (Olson and Attneave, 1970; Pomerantz, Sager & Stover, 1977; Chen, 1982a), card sorting (Palmer, 1978), effortless texture discrimination (Julesz, 1981), visual sensitivity to distinction made in topology (Pomerantz, 1980; Chen, 1982b) and competing organization with several simultaneous factors (Chen, 1982d) have provided some evidence for detection of topological properties in visual perception. The many facts mentioned above also lead us to consider topological invariants as candidates for the correspondence tokens in apparent motion.

METHOD

The general method for the following experiments was advanced by Ullman (1979) and is called "the competing motion technique". In this method two stimulus displays are successively presented. The first one contains a single figure in the center, while the second contains two figures located on either sides of, and at the same distance from, the center. The question asked is whether the figure in the first stimulus display is perceived to move to one or the other of the two figures in the second. In Ullman's demonstrations, each set of two stimulus displays were alternated repeatedly, and without providing precise data, he reported the subjects' motion preference. For collecting accurate data, in the following experiments each set of two stimulus displays was presented on just one cycle for each trial, but many trials were used as experimental presentations. This method provides us an experimental measure to compare and characterize the effect of various structural invariants.

A three-field tachistoscope was used for presenting stimuli. Subjects were asked to look at a fixation point at the center of the preexposure field then press a button which resulted in a presentation cycle. Each first stimulus display containing a middle figure of each pair was presented for 100-150 msec. and each second stimulus display, for 1000 msec., with a inter-stimulus interval (ISI) of 20-30 msec.. For each presentation subjects were required to choose in a forced choice procedure one of two responses: "right" (motion from a middle figure to a figure at right) or "left" (motion to a figure at left), guessing if necessary. For each subject the presentation durations were adjusted in order to produce strong effects of apparent motion. Each subject was initially familiarized with the phenomenon of apparent motion under the condition of single alternate exposure. At least three trials of each pair were used as a practice presentation. The order of presentations was randomized and counterbalanced across subjects. Four blocks of 21 trials per block, which contained three presentations of each pair, were used for test presentation.

Four subjects participated in all these experiments involving seven pairs of stimulus displays.

Pair 1 (adapted from Pomerantz et al, 1977) consists of the two stimulus displays shown in Fig.1. The first contains an arrow (stimulus a). The second contains two figures which are made up of exactly the same three line segments as the arrow. The difference between stimulus b and c is just that the one of the shorter line segments is located in two different positions, displaced by a constant distance from the same line segment in the arrow. But the closed nature of a triangle makes it topologically different from the other two figures.

Pair 2 is shown in Fig.2. Each of the three figures is made up of five line segments with two sorts of lengths. Among the three figures are stimulus a and b adapted from Julesz (1980). Although stimulus a has a different number of "terminators" from stimulus c and the same number of terminators as stimulus b, stimulus a possesses the same topological invariant, simple connectivity, as stimulus c and is different in topological invariants from stimulus b, which is disconnected with a closure.

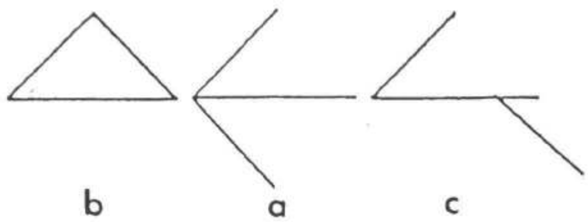

FIGURE 1

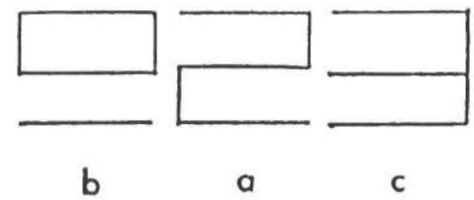

FIGURE 2

In Pair 3, stimulus a is a solid circle, stimulus b is a ring and stimulus c, a solid square. From our intuitive experiences, a solid circle seems to have more "similarity" to a ring than to a square; however, from topology, the difference between a disk and a ring is much deeper than that between a disk and a solid square. For the latter, the difference will dissolve under a topological transformation, a plastic deformation without breaking and fusion. On the other hand, stimulus b is characterized as a connected component with a hole in it, a typical topologically invariant description.

Fig.4 shows Pair 4, which is similar to Pair 3 except that a square with a square hole in it was used instead of the disk, the middle figure in Pair 3.

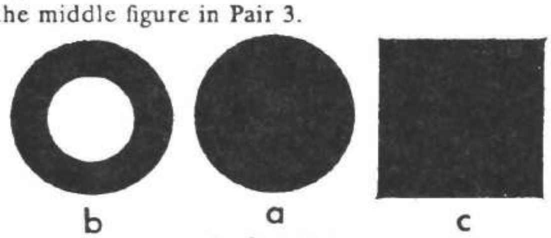

FIGURE 3

FIGURE 4

The three figures contained in Pair 5 have the common feature of possessing holes in a connected component (shown in Fig.5). However, stimulus b has two holes in it, while stimulus a and c, just one. The number of holes is a topological invariant, "the order of connectivity". So, stimulus a is topologically equivalent to stimulus c, but not to stimulus b. The inner diameter of stimulus c is 0.71 (2/2) times that of stimulus b, so the total area of the two inner circles of stimulus b is equal to that of the inner circle of stimulus c.

Pair 6 (shown in Fig.6) involves another kind of global topological invariant, whether a target line is within a closed curve or outside it. If the two stimulus displays were superimposed, both stimulus a (a line segment) and the target line in stimulus c would lie within the big circle of stimulus c, while stimulus b (another line segment) would be outside the circle.

Discrete dots were used to make up Pair 7 (shown in Fig.7). These figures are adapted from Zeeman (1965). Each figure is a set of discrete dots, but globally they look like Pair 3. Even though at the viewing distance in the experiment one can see that these stimuli are discrete dots, one still has a wholistic perception, that is, stimulus a looks like a solid circle; stimulus c, a solid square; stimulus b, a ring. For a clearly discrete set, why do we have continuous and wholistic perception? This fact suggests that the visual system can ignore details within a certain range for attaching importance to global structure (Chen, 1980, 1981).

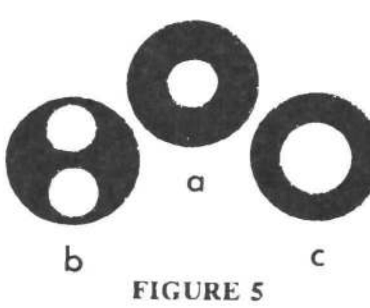

FIGURE 5

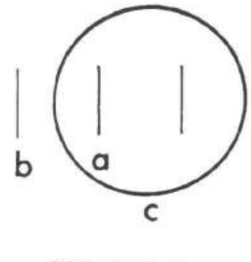

FIGURE 6

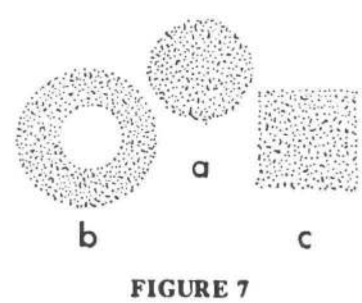

FIGURE 7

RESULTS AND DISCUSSION

	Subject 1	Subject 2	Subject 3	Subject 4	Average
Pair 1	92%	83%	75%	75%	81%
Pair 2	83%	92%	83%	83%	85%
Pair 3	83%	92%	92%	83%	88%
Pair 4	92%	83%	92%	92%	90%
Pair 5	83%	100%	83%	92%	90%
Pair 6	92%	92%	83%	83%	88%
Pair 7	83%	92%	83%	75%	83%

TABLE 1. The percentages of reporting motion from each middle figure to the figure with the same topological invariants. The difference in average percentages between figures with the same topological invariants and figures with different ones are all significant, $p < .01$.

The results with all seven pairs are shown in Table 1. They clearly show that subjects saw strong preference for motion from a middle figure to a figure that has the same topological invariants as the middle figure. The figures used in the first pair were made up of exactly the same line segments. According to the theory (Ullman, 1979) considering line segments as correspondence tokens, the only difference between stimulus b and c is that one component line segment was translated with a constant distance. It seems difficult for this kind of theory to interpret the preference for motion from stimulus a to stimulus c.

In Marr's "primal sketch" (1978) and Julesz' theory of effortless texture discrimination (1981), terminators are also considered as basic primitives. Along this line, somebody might like to argue that the preference for motion between stimulus a and c in Pair 1 was observed not because of closure but just because of terminators. However, the result with Pair 2 seems to rule out this objection. Stimulus a possesses the same number of terminators as stimulus b but not stimulus c. In some cases, terminators may reflect closure and connectivity properties, but not always.

Assuming that the visual response to closure resulted from topological structure in visual perception, it would be predicted that subjects would see motion preferentially between a disk and a solid square rather than between a disk and a ring. The result with Pair 3 supports this prediction.

It is interesting to note that for Pair 3 and 4 only the stimulus in the first display changed; yet the preferred direction of apparent motion strongly reversed from the solid square in Pair 3 to the ring in Pair 4. This pattern strongly suggests topological explanation. In fact, Ramachandran, Anstis and Ginsburg (1982) have already reported that subjects display preference for motion from a solid square to a solid circle rather than an outline square and from a cross to a rotated cross rather than a square made by the same line segments. The facts have been interpreted assuming that "low spatial frequency dominates apparent motion". Along this line of thinking, the preference for motion from a hollow square to a ring would have to be interpreted as high spatial frequency domination of apparent motion. These two observations, therefore, cast some doubt on the value of explanations of apparent motion using the notion of spatial frequency. Under plastic deformation of stimuli, like lines or blobs, spatial frequencies, whether low or high, would be difficult to imagine as invariants for the correspondence tokens.

Because the number of holes in a connected component represents a typical topological invariant and its intuitive meaning is not obvious, Pair 5 was designed to give more evidence for the topological hypothesis and also to further rule out some other factors that are often relevant to the study of brightness sensitivity or spatial frequency analysis. The fact that the total area of the two inner circles in stimulus b is equal to that of the bigger inner circle in stimulus c makes the explanation in terms of differences in brightness or spatial frequency difficult; however, the result is consistent with the topological explanation. One might argue that the motion from stimulus a to stimulus c arises from the "similarity" between them. However, in Pair 3, the motion from the disk to the square rather than to the ring already indicates that the correspondence tokens are not these kinds of similarity factors.

Some long-standing debates about the nature of apparent motion have often centred on the fundamental question of how to understand grouping (Anstis, 1979). Chen (1980, 1981) has considered grouping also as the extraction of global topological invariants. The very nature of visual perception is discrete, and the approach of perceptual organization, say, the Gestalt laws, is often aimed at some obviously discrete stimuli, such as dot arrays. Therefore, general topology cannot be directly used to describe perceptual organization. For a clearly discrete set, why do we have continuous and wholistic perceptions? In this sense, the Gestalt perceptual phenomena look puzzling. The mathematics of Tolerance Spaces (Zeeman, 1962) tells us how to formulate the global properties on a discrete set. Tolerance is an algebraic relation chosen not only to represent the concept of the least noticeable difference but also to represent a minimum measure within the range of which details will be ignored by the perceptual system for attaching importance to global properties.

In a tolerance space, we can build up a mathematical structure similar to topology. Grouping represents a visual function to ignore details within a certain tolerance and to extract global tolerance invariants, such as tolerance connectivity, closure and holes. Taking a tolerance of one centimeter, the two most noticeable global tolerance properties of the second stimulus of Pair 7 are that it has two pieces (two tolerance connected components) and one of them has a hole in it. So, if grouping can really be considered as the extraction of global tolerance invariants, then preference of apparent motion should be observed from the tolerance circle to the tolerance square rather than to the tolerance ring, the global tolerance property of which is different from that of the others. The result with Pair 7 helps us with the suggested theoretical basis for understanding grouping. Correspondence processes with either grouping or normal patterns can be described consistently in terms of topological invariants.

SUMMARY AND GENERAL DISCUSSION

Using the adapted "competing motion technique", seven pairs of stimulus figures showing topological variation were designed to reveal some evidence for that topological invariants play a role in correspondence processes of apparent motion. These experimental data, which show that subjects reported strong preference for motion from a central figure to a figure with the same topological invariants as its, came from various kinds of stimulus patterns that represent quite different structural forms and that control other explanatory factors, such as brightness, spatial frequency and terminators. Nonetheless all of them are consistent with the topological explanation and strongly suggest a topological structure in visual perception.

The key point is that the units of figure perceptual representation are invariants at different geometrical levels (Chen, 1981). Along this line we can deepen our comprehension of some of the long-standing debates about apparent motion. For example, now it is clear that the question of whether motion perception precedes form perception is not a good question. Form perception includes different levels and at the level of extraction of topological invariants it precedes motion perception; however, at the levels of more detailed properties, say, the difference between a square and a triangle, motion perception may precede form perception. So, we cannot simply claim that "the correspondence tokens are not structured forms" (Ullman, 1979). The right question is which kinds of structured forms should be considered as the correspondence tokens, and which kinds are not. In fact, topological invariants are a kind of important structured form.

The critical act in formulating Ullman's computational theory for apparent motion is of using the rigidity constraint on the way the world behaves (Marr, 1978). Ullman's theory is noted for the discovery of a valid constraint of rigidity, which "enables us to solve the structure-from-motion problem unambiguously"(Marr, 1982). But at the same time a certain limitation of the theory also comes from rigidity. Plastic deformation is a strong and common phenomenon in apparent motion. Marr (1982) pointed out that a new theory may be needed for when the object is not only moving but also changing. The experimental facts reported in the present paper and their topological explanation have suggested a new type of analysis of apparent motion, which has been motivated by the general transformation, plastic deformation.

The suggested topological approach, supported by empirical data, has also raised some interesting issues about Marr's "primal sketch". "The primal sketch" has emphasized "the local geometry of an image". How should we consider the relationship between topological properties and "the primal sketch"? It seems that it is implausible mathematically to compute out topological properties from local geometrical properties, such as oriented edges, lines, blobs. And the fact that perception of topological invariants precedes motion perception has shown the early extraction of topological properties. Many other experiments, for example, the configural superiority effect (Chen, 1981) and competing organization with several simultaneous factors (Chen, 1982d), have also provided some evidence for the extraction of global topological invariants earlier than that of local geometrical properties, such as orientations, positions. So, considering the time dependence of perceived properties, it seems difficult to assume that topological properties are derived from local geometrical properties. Minsky and Papert (1972) proved that for perceptrons, the topological predicate is not finite order; however, lower-order perceptrons can be used for computing geometrical properties. Thus from the perspective of computational theory, the global nature of topological properties makes them essentially different from local geometrical properties. It therefore seems difficult for "the primal sketch" to accommodate topological properties without changing its local nature. These questions are fundamental for understanding vision and deserve further study.

Note about figures

All stimulus figures used in these experiments are black on white paper. Line segments in Pair 1, 2 and 6 were drawn by a pen with line width in .50 mm.

Two of three line segments of each stimulus figure in Pair 1 have equal length in 24 mm, the other, 32 mm. The distance between stimulus b and c is 36 mm.

Three of five line segments of each stimulus figure in Pair 2 have equal length in 26 mm, the others, 11 mm. The distance between stimulus b and c is 34 mm.

In Pair 3, the diameter of the disk is the same 32 mm as the outer diameter of the ring and one side length of the square. The inner diameter of the ring is 18 mm. The distance between the ring and the square is 40 mm.

In Pair 4, one outer side of stimulus a is the same 27 mm in length as one side of the square and the outer diameter of the ring. The inner side of stimulus a is 14 mm in length. The inner diameter of the ring is 18 mm. The distance between the square and the ring is 40 mm.

The outer diameters of three stimuli in Pair 5 are all the same 40 mm. The diameter of the inner circle in stimulus a is the same 15 mm as that of one inner circle of stimulus b. The inner diameter of stimulus c is approximately 21 mm. The distances between stimulus a and b and stimulus a nd c are all 2 mm.

The lengths of three line segments in Pair 6 are all the same 21 mm. The diameter of the circle is 59 mm. The distance between stimulus a and b is the same 25 mm as that between stimulus a and the target line in stimulus c.

In Pair 7, the diameter of stimulus a is 33 mm, the outer diameter and the inner diameter of stimulus b are, respectively, 43 mm and 19 mm, and one side length of stimulus c is 30 mm. The distances between stimulus a and b and stimulus a and c all 3 mm.

REFERENCES

Anstis, S. M. Apparent movement. In R. Held, H.W. Leibowitz, H.-L. Teuber(Eds.), *Handbook of Sensory Physiology vol. 8, Perception*, New York: Spring-Verlag, 1979.

Chen, L. Tolerance spaces and Gestalt. paper presented at the 4th Biophysical Academic Conference of China, Beijing(1980).

Chen, L. Perceptual organization and tolerance spaces. paper presented at the 14th Annual Mathematical Psychology Meeting, Santa Barbara(1981).

Chen, L. What are the units of figure perceptual representatin? *Studies in Cognitive Science*, UCI, No. 22, 1982a.

Chen, L. Topological structure in visual perception. *Science*, 218, 669-700, 1982b.

Chen, L. Connectedness and the object superiority effect. *Investigative Ophthamology and Visual Science, 22, 124*, 1982c(Supplement).

Chen, L. Competing organization with several simultaneous factors. *Bulletin of the Psychonomic Society*, September 1982d, 145-146(abstract).

Julesz, B. Figure and ground perception in brief presented isodio-pole texture. In M. Kubovy & J.R. Pomerantz(Eds.), *Perceptual Organization*, Hillsdale, New Jersey: Lawrence Erlbaum Associates, Publisher, 1981.

Julesz, B. Textons, the elements of texture perception, and their interactions. *Nature*, 1981, 290, 91-97.

Koler, P.A. & Pomerantz, J.R. Figural change in apparent motion. *Journal of Experimental Psychology*, 1971, 87, 99-108.

Marr, D. Representing visual information. *Lectures on Mathenatics in the Life Science*, vol.10, 101-180. reprinted in Hanson, A.R. & Riesman, E.M.(Eds.) *Computer Vision Systems*. New York: Academi Press, 1979.

Marr, D. *Vision*. San Francisco: Freeman, 1982.

McClleland, J.L. Perception and masking of wholes and parts. *Journal of Experimental Psychology: Human Perception and Performance*, 978, 4, 210-229.

Minsky, M. & Papert, S. *Perceptrons*, MIT Press, 1969.

Olson, R.K. & Attneave, F. What variables produce similarity grouping? *American Journal of Psychology*, 1970, 83, 1-23.

Palmer, S.E. Structural aspects of visual similarity. *Memory and Cognition*, 1978, 6, 91-97.

Pomerantz, J.R., Sager, L.C. & Stoever, R.J. Perception of wholes and their component parts: some configural effects. *Journal of Experimental Psychology: Human Perception and Performance*, 1977, 3, 422-435.

Pomerantz, J.R. Are complex visual features derived from simple ones? in E.I.J. Leeuwenbert & H.F.M. Buffart(Eds.), *Formal Theory of Visual Perception*, Sussex, England: Wiley Ltd., 1980.

Ramachandran, V.S. Perception of apparent motion. *Studies in the cognitive Sciences*, No. 11, UCI, 1981.

Ullman, S. *the Interpretation of Visual Motion*. The MIT press, 1979.

Weisstein, N. & Harris, C.S. Visual detection of line segments: an object-superiority effect. *Science*, 1974, 186, 725-755.

Williams, A. & Weisstein, N. Line segments are perceived better in coherent contex than alone: an object-superiority effect in visual perception. *Memory & Cognition*, 1978, 6, 85-90.

Zeeman, E.C. The topology of the brain and visual perception. in Fort, K.(Ed.) *Topology of 3-manifolds*. Prentice Hall, Englewood Cliffs, N.J., 1962.

Zeeman, E.C. & Buneman, O.P. Tolerance spaces and the brain. In *Mathematics and Computer Science in Biology and Medicine*. (Proceedings of Conference held by the Medical Research Council in Association with the Health Departments, Oxford, 1964).

ACKNOWLEDGEMENTS. I am grateful to William Batchelder for valuable suggestion, helpful discussion and careful review of drafts of this paper. In particular, I would like to thank Paul Smolensky for stimulating comments and reviewing this paper and to thank Bing Chen for helpful suggestion and helping with preparing these experiments. Helpful comments came from Michael Braunstein, Donald Macleod, Donald Norman, V.S. Ramachandran, David Rumulhart and Jack Yellot. Finally, I would also like to thank Eileen Conway for preparing the manuscript. This research was supported by a grant from Sloan Foundation to Program in Cognitive Science at UCI.