Title
PRODUCTION CROSS SECTIONS IN THE MULTIPERIPHERAL MODEL

Authors
Ball, James S.
Marchesini, G.

Publication Date
1970-05-21

Submitted to Physical Review

UCRL-19816
Preprint

PRODUCTION CROSS SECTIONS IN THE MULTIPERIPHERAL MODEL

James S. Ball and G. Marchesini

May 21, 1970

AEC Contract No. W-7405-eng-48

LAWRENCE RADIATION LABORATORY
UNIVERSITY of CALIFORNIA BERKELEY

DISCLAIMER

PRODUCTION CROSS SECTIONS IN THE MULTIPERIPHERAL MODEL*

James S. Ball† and G. Marchesini††

Lawrence Radiation Laboratory
University of California
Berkeley, California

May 21, 1970

ABSTRACT

The total cross section for the multiperipheral model is obtained by solving a Bethe-Salpeter equation. We show that the N-particle production cross sections are given directly in terms of the eigenvalues and eigenfunctions of this Bethe-Salpeter equation, and hence are directly related to the Regge pole parameters. In simple models of Regge poles based on the multiperipheral model the production cross sections are completely determined. Results are given for several simple cases including the Chew and Snider model of the Pomeranchuk.

---

* This work was supported in part by the U.S. Atomic Energy Commission.

† Permanent address: Physics Department, University of Utah, Salt Lake City, Utah 84112.

†† Address after September 1970: Istituto di Fisica, Università di Parma, 43100 Parma, Italy.

I. INTRODUCTION

It has been proposed by Fubini and co-workers[1] that a multiperipheral model of high-energy scattering will provide a dynamical description of Regge poles and hence a dynamical model of the low-mass particles that lie on these trajectories. The basic form of this model is a ladder structure of the type shown in Fig. 1, where the multiparticle production contribution to the total cross section is constructed by the iteration of an elastic cross section. In our previous work[2] and in the work of Chew, Rogers, and Snider[3] the ladder graphs were summed by means of the Bethe-Salpeter equation. While the values of the various Regge pole parameters were obtained in the neighborhood of $t = 0$ little was said about the N-particle production cross sections which are the quantities most directly predicted by the model and are also directly measurable experimentally.

In this paper we would like to approach the same problem but beginning with a different point of view. We will assume that both the total cross section and the elastic cross section are known from experiment or theory and use the multiperipheral model to predict the N-particle production cross sections. It is hoped that this will allow the direct comparison of experimental data to check the validity of this model without depending on a specific calculation, and at the same time clarify the effect of certain assumptions made in previous calculations in regard to the parameterization of the elastic cross section and the assumed off-mass-shell dependence of the scattering amplitude.

Our starting point will be the assumption that the total cross section has a ladder structure of the type shown in Fig. 1 and that it can be calculated from a Bethe-Salpeter-type equation in which the inhomogeneous term and the kernel are related to the elastic cross section. It is further assumed that this equation is partially diagonalized when one invokes either $O(4)$ or $O(3)$ symmetry.

In the following section we will obtain a formal expression for the N-particle production cross sections in terms of the elastic and total cross sections. In Sec. 3 we investigate several applications to simple models, and we obtain the production cross sections as a function of energy and the number of pairs produced.

II. FORMULATION OF THE MULTIPERIPHERAL MODEL

The basic equation of the multiperipheral model can be written as the following Bethe-Salpeter equation:

$$T = T_o + T_o \, GT \,, \tag{2.1}$$

which corresponds formally to the diagram in Fig. 2. The kernel T_o, when evaluated on the mass shell, is related to the elastic cross section by

$$\sigma_{e\ell}(s) = [s(s - 4\mu^2)]^{-\frac{1}{2}} \quad \text{Im}\, T_o(s,t = 0) \,. \tag{2.2}$$

The solution $T(s,t)$ bares the same relation to the total cross section, and G is the two-particle propagator.

If we now expand both T and T_o in partial waves appropriate to whichever symmetry is assumed, i.e., $O(4)$ at $t = 0$ and $O(3)$ elsewhere, the Eq. (2.1) is diagonal with respect to this quantum number λ. The continuation of these quantities into the complex λ plane is accomplished in the standard manner. For simplicity we will assume that T_o has only a branch cut for positive s and hence no signature need be introduced. The resulting continuation is

$$T_\lambda(t) = \int_{4\mu^2}^{\infty} ds \quad s^{-\lambda-1} \, \text{Im}\, T(s,t) \quad , \tag{2.3}$$

where λ stands for either ℓ or n and we have used the asymptotic form of Q_ℓ or f_n to simplify the discussion. The inverse of this relation is:

$$\text{Im } T(s,t) = \frac{1}{2\pi i} \int_c d\lambda\, s^\lambda\, T_\lambda(t) , \tag{2.4}$$

where c is the usual contour to the right of all singularities of T_λ. The solution of the partial-wave Bethe-Salpeter equation can be expressed in terms of the eigenfunctions and eigenvalue of the homogeneous form of the equation:

$$\psi_\lambda^{\,i} = E_\lambda^{\,i}\, T_{o\lambda}\, G\, \psi_\lambda^{\,i} . \tag{2.5}$$

Then T_λ and $T_{o\lambda}$ have the following expansions:

$$T_\lambda(t) = \sum_i \frac{[\psi_\lambda^{\,i}(t)]^2}{E_\lambda^{\,i}(t) - 1} \tag{2.6}$$

and

$$T_{o\lambda}(t) = \sum_i \frac{[\psi_\lambda^{\,i}(t)]^2}{E_\lambda^{\,i}(t)} . \tag{2.7}$$

Note here that the off-mass shell dependence is contained in the functions ψ. It is clear from Eq. (2.6) that λ-plane poles of the amplitude $T_\lambda(t)$ arise from the denominator vanishing and hence the Regge trajectories $\alpha_i(t)$ and residues $\beta_i(t)$ are determined by the following equations:

$$E_{\alpha_i}^{\,i}(t) = 1,$$

$$\beta_i(t) = \psi_{\alpha_i}^{\,i}(t)^2 \Big/ \left[\frac{\partial E_\lambda^{\,i}(t)}{\partial \lambda}\right]_{\lambda=\alpha_i} . \tag{2.8}$$

In order to determine the cross section for production of N-particle pairs we define the amplitude T_N such that the imaginary part of T_N when evaluated on the mass shell is proportional to σ_N. It is clear that the T_N's satisfy the following recursion relation:

$$T_{N+1} = T_o \, G \, T_N \quad . \tag{2.9}$$

We can now define a generating function for the T_N's as follows:

$$T_h = \sum_{N=0}^{\infty} h^N \, T_N \, , \tag{2.10}$$

which will be the solution of the Bethe-Salpeter equation

$$T_h = T_o + hT_o \, G \, T_h \, , \tag{2.11}$$

and the production cross section can be extracted from T_h by projecting out the coefficient of h^N. The projection $T_{h,\lambda}$ has the same type of continuation into the λ plane as T_λ and $T_{o,\lambda}$ and hence satisfy the same partial-wave Bethe-Salpeter equation. The solution to Eq. (2.11) can again be expressed in terms of the eigenfunction solutions of the homogeneous Bethe-Salpeter equation:

$$T_{h,\lambda} = \sum_i \frac{(\psi_\lambda^{\,i})^2}{E_\lambda^{\,i} - h} \quad . \tag{2.12}$$

Since this expression is readily expandable in powers of h we obtain the following result for T_N:

$$T_{N,\lambda} = \sum_i \left(\frac{1}{E_\lambda^i}\right)^{N+1} (\psi_\lambda^i)^2 , \tag{2.13}$$

which, when evaluated on the mass shell and substituted into Eq. (2.4), yields the N-particle production cross section

$$\sigma_N(s) = \frac{1}{[s(s - 4\mu^2)]^{\frac{1}{2}}} \frac{1}{2\pi i} \int_c d\lambda\, s^\lambda\, T_{N,\lambda} . \tag{2.14}$$

The average multiplicity for the production process can easily be expressed directly in terms of T_h:

$$\langle N \rangle = \sum_N (N + 1)\, \sigma_N/\sigma_T = 1 + \frac{\partial}{\partial h} \mathrm{Im}\, T_{h=1}(s,0)/\mathrm{Im}\, T_{h=1}(s,0) . \tag{2.15}$$

Finally, we note that if the sum in Eq. (2.12) is well represented by the first term, i.e., the eigenvalues are widely separated, σ_N can be expressed directly in terms of integrals over $\sigma_{e\ell}$ and σ_T as follows:

$$\sigma_N(s) = \frac{1}{2\pi i} \int_c d\lambda\, s^{\lambda-1}\, T_{o\lambda} \left(\frac{T_\lambda - T_{o\lambda}}{T_\lambda}\right)^N , \tag{2.16}$$

where

$$T_\lambda = \int_{s_o}^\infty ds\, s^{-\lambda}\, \sigma_T(s)$$

and

$$T_{o\lambda} = \int_{s_o}^\infty ds\, s^{-\lambda}\, \sigma_{e\ell}(s) . \tag{2.17}$$

III. SPECIFIC MODELS

To illustrate the method presented in the previous sections, we will consider several specific examples. Our general procedure will be to assume certain types of λ-plane singularities and determine the resulting relation between the elastic, total, and production cross sections together with the multiplicity implied by these singularities. Since these relations are determined by the imaginary part of the amplitude at $t = 0$ we will simply parameterize the amplitude at this point, but the reader should keep in mind that in general all of the parameters introduced in the following discussion are functions of t. We study examples where a single term in the expansion given in Eq. (2.12) dominates the amplitude.

A. Single Pole

The simplest case we can consider is that T_λ has only a simple pole at $\lambda = \alpha_o$. Then E_λ is a linear function of λ:

$$E_\lambda = 1 + (\lambda - \alpha_o)/f \tag{3.1}$$

and the asymptotic form of σ_T is

$$\sigma_T(s) = \beta\, s^{\alpha_o - 1} , \tag{3.2}$$

where β is the residue of the pole. The σ_N's have the following Poisson distribution:

$$\sigma_N = \beta \frac{(f\ \ell n\ s)^N}{N!} s^{\alpha_o - f - 1} . \tag{3.3}$$

The average multiplicity for pairs is then the following:

$$\langle N \rangle = 1 + f \ln s \quad . \tag{3.4}$$

B. Two Poles

The simplest generalization of the single-pole model is to consider an eigenvalue which is a quadratic function of λ with the resulting amplitude containing two poles at α_+ and α_- with residues r_+ and r_-, respectively. If we assume

$$E_\lambda = (\lambda - \alpha_+)(\lambda - \alpha_-)/f + 1 \quad , \tag{3.5}$$

the amplitude $T_{h,\lambda}$ is

$$T_{h,\lambda} = f \frac{r_+(\lambda - \alpha_-) + r_-(\lambda - \alpha_+)}{(\lambda - \alpha_+)(\lambda - \alpha_-) + f(1 - h)} \cdot \tag{3.6}$$

Here f is a parameter which will be related to the average multiplicity. The Mellin transform of Eq. (3.6) is:

$$\mathrm{Im}\, T_h(s) = f s^{\alpha_R} \left\{ (r_+ + r_-) \cosh(R \ln s) + \alpha_D (r_+ - r_-) \frac{\sinh(R \ln s)}{R} \right\}, \tag{3.7}$$

where $\alpha_R = (\alpha_+ + \alpha_-)/2$, $\alpha_D = (\alpha_+ - \alpha_-)/2$, and $R = [\alpha_D^{\,2} + f(h - 1)]^{\frac{1}{2}}$.

By differentiation with respect to h we obtain the cross section for producing N pairs of particles σ_N:

$$\sigma_N(s) = f\, s^{\alpha_R - 1}\left\{R(r_+ + r_-)\, i_{N-1}(R\, \ell n\, s) + \alpha_D(r_+ - r_-)\, i_N(R\, \ell n\, s)\right\} \times \frac{\ell n\, s}{N!}\left(\frac{f}{2R}\, \ell n\, s\right)^N , \quad (3.8)$$

where $i_N(z) = (\frac{\pi}{2z})^{\frac{1}{2}}\, I_{N+\frac{1}{2}}(z)$ and R is evaluated at $h = 0$. Note that this is not a Poisson distribution except for large $R\, \ell n\, s$ where the asymptotic behavior of i_N is independent of N. The average multiplicity of pairs in this limit is

$$\langle N \rangle = 1 + \frac{f}{2R(h = 1)}\, \ell n\, s \ . \quad (3.9)$$

If the poles are a complex conjugate pair the i_N Bessel functions go into j_N's which are oscillating functions and σ_N is not positive definite. Therefore the complex poles must either be the nonleading singularities or must be an approximation to a cut of the type suggested by Ball, Marchesini, and Zachariasen[4] which is valid only over a limited energy region.

C. Approximate Solutions to the Multiperipheral Bootstrap

In our previous work on a realistic treatment of the multiperipheral model the Bethe-Salpeter equation was solved numerically. From these calculations we obtain a reasonable Pomeranchuk and ρ Regge poles. What this calculation predicts for σ_N is directly relevant to experiment, however, the fact that the eigenvalues were obtained numerically makes the required integration in Eq. (2.14) difficult. Recently, Chew and Snider,[5] investigating the same type of model, have employed a factorable kernel and have obtained results quite similar to our earlier result. Because of their approximation they have only one eigenvalue with the explicit λ dependence known. It is of interest to obtain the σ_N predicted by their model, and one would expect these results to be similar to those predicted by a realistic model. In their model the eigenvalue and eigenfunction are the following:

$$E_\lambda = \frac{(\lambda - \beta_\pi)(\lambda - \beta_p)}{g_p^4 + g_R^4(\lambda - \beta_p)} \tag{3.10}$$

and $\psi_\lambda^2 = (\lambda - \beta_\pi)$, where in the interest of simplicity we have set the internal and external couplings of the CS model equal. Here β_p is the position of the effective singularity due to the high-energy component of the kernel and β_π is the position of the singularity associated with the pion propagator. The parameters g_p and g_R control the strength of the high-energy and resonance components of the kernel, respectively.

Using Eq. (2.14), we obtain

$$\sigma_N(s) = \frac{1}{2\pi i} \int_c d\lambda\, s^{\lambda-1} \left(\frac{1}{\lambda - \beta_\pi}\right)^N \left(g_R^4 + \frac{g_p^4}{\lambda - \beta_p}\right)^{N+1} . \tag{3.11}$$

The evaluation of these integrals is straightforward but lengthy. In Fig. 3 are shown the total cross section and some typical production cross sections, for the value of the parameters determined by Chew and Snider[5] which are the following: $g_R^4 = 1.0$, $g_p^4 = 0.03$, $\beta_\pi = -0.3$, and $\beta_p = 0.9$. In Fig. 4 we compare the N dependence of production cross sections with the Poisson distribution for several values of the energy. The average multiplicity predicted by this model is shown in Fig. 5 together with two straight lines which agree with this curve in the accelerator energy region and in the asymptotic region. Note that the effective coefficient of $\ell n\ s$ changes from 2 to 0.65 at ultrahigh energy. This effect is similar to that we suggested in our previous paper,[2] in which the low-energy multiplicity is controlled by an "effective" Regge-pole that is actually the contribution of a cut, while at sufficiently high energy the "true" Pomeranchuk pole dominates and produces a much smaller average multiplicity.

CONCLUSION

The multiperipheral model provides a mathematical framework in which to study dynamical Regge trajectories and their associated cuts. In application of this model, previous work has generally concentrated on the elastic cross section which is directly related to the kernel of the integral equation and the total cross section which allows one to identify various Regge poles. These calculations also contain implicitly all the production cross sections which are also experimentally measured quantities. We have computed the production cross sections for a few simple models including the approximate multiperipheral model of Chew and Snider,[5] where the assumption of factorizability of the kernel leads to very simple predictions. While the production cross sections in this model deviate from a Poisson distribution, this deviation is not in disagreement with current experimental data.

FOOTNOTES AND REFERENCES

1. D. Amati, A. Stanghellini, and S. Fubini, Nuovo Cimento 26, 896 (1962), and references therein.

2. J. S. Ball and G. Marchesini, Phys. Rev. 188, 2508 (1969).

3. G. F. Chew, T. Rogers, and D. R. Snider, Relation Between the Multi-Regge Model and the ABFST Pion-Exchange Multiperipheral Model, Lawrence Radiation Laboratory Report UCRL-19457, Jan. 1970.

4. J. S. Ball, G. Marchesini, and F. Zachariasen, Phenomenological Implication of Colliding Regge Poles and Cuts, to be published in Phys. Letters B.

5. G. F. Chew and D. R. Snider, Multiperipheral Mechanism for a Schizophrenic Pomeron, Lawrence Radiation Laboratory Report UCRL-19455, Jan. 1970; hereafter referred to as CS.

FIGURE CAPTIONS

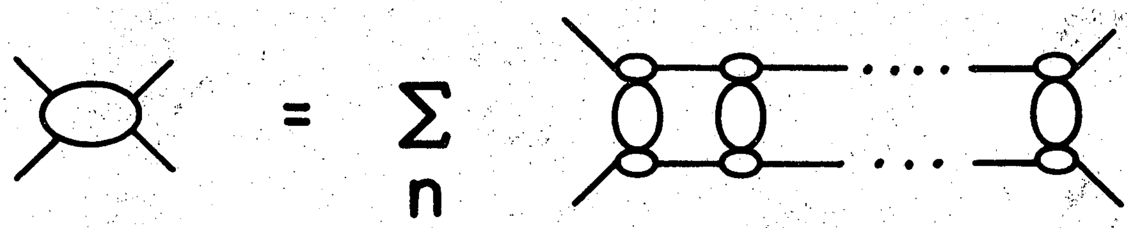
Fig. 1. Diagram representing the multiperipheral contribution to the total cross section.

Fig. 2. Diagram representing the multiperipheral Bethe-Salpeter equation.

Fig. 3. Production cross section as function of energy from CS model. Energy units: 1 GeV^2.

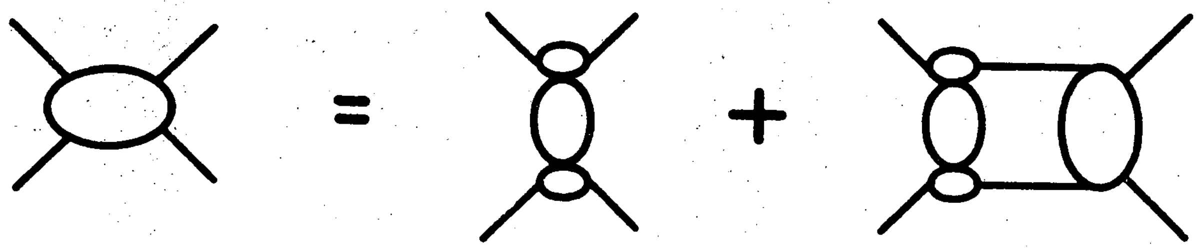
Fig. 4. Production cross section from CS model as function of number of pair (solid curves) and corresponding Poisson distribution (dashed curves) for several energies.

Fig. 5. Multiplicity of particles as function of energy from CS model.



XBL705-3006

Fig. 1



XBL705-3014

Fig. 2

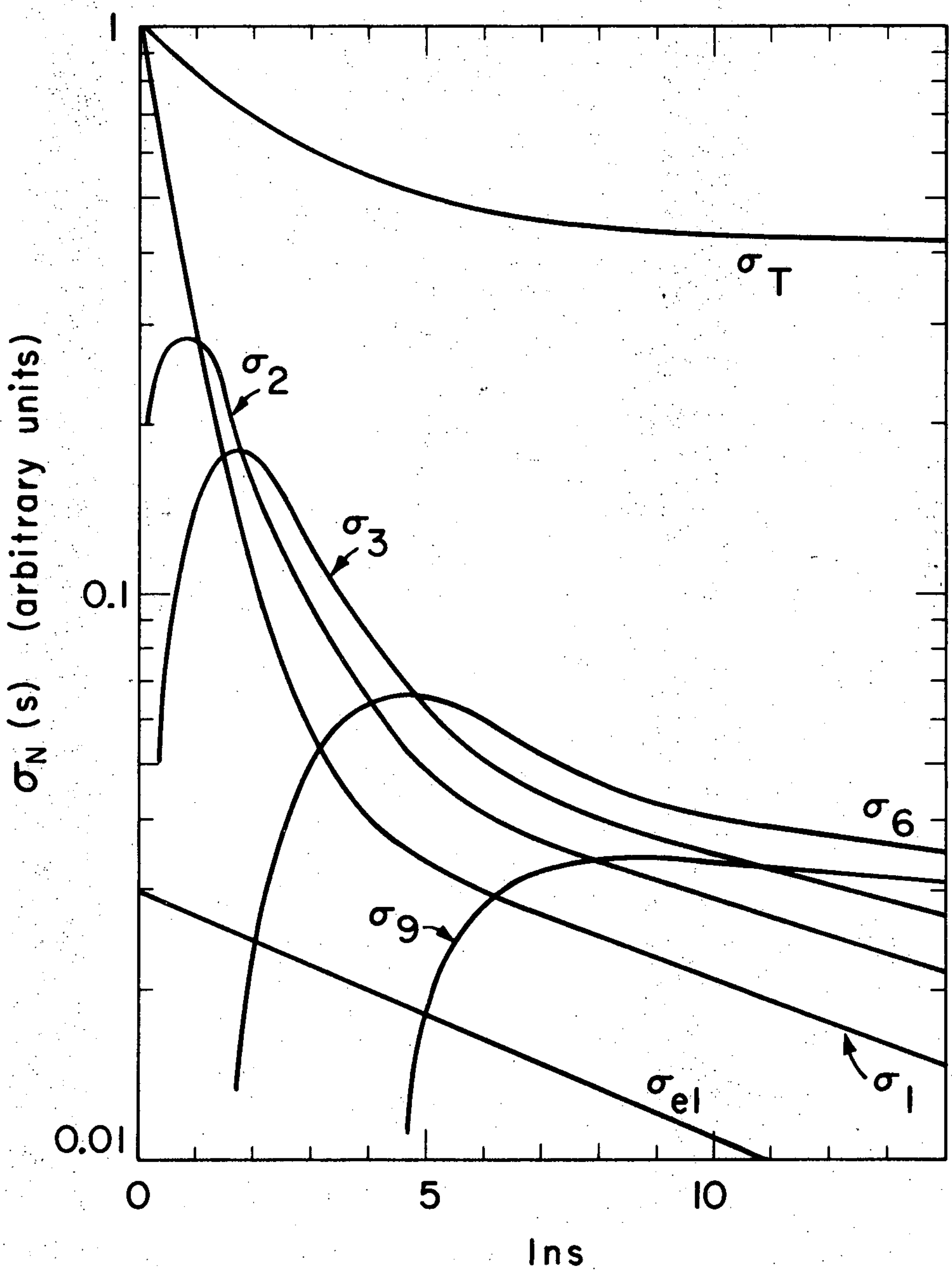

1
0.1
0.01
0
5
10
lns
σ_N (s) (arbitrary units)
σ_T
σ_2
σ_3
σ_6
σ_9
σ_1
σ_{el}
XBL705-3009

Fig. 3

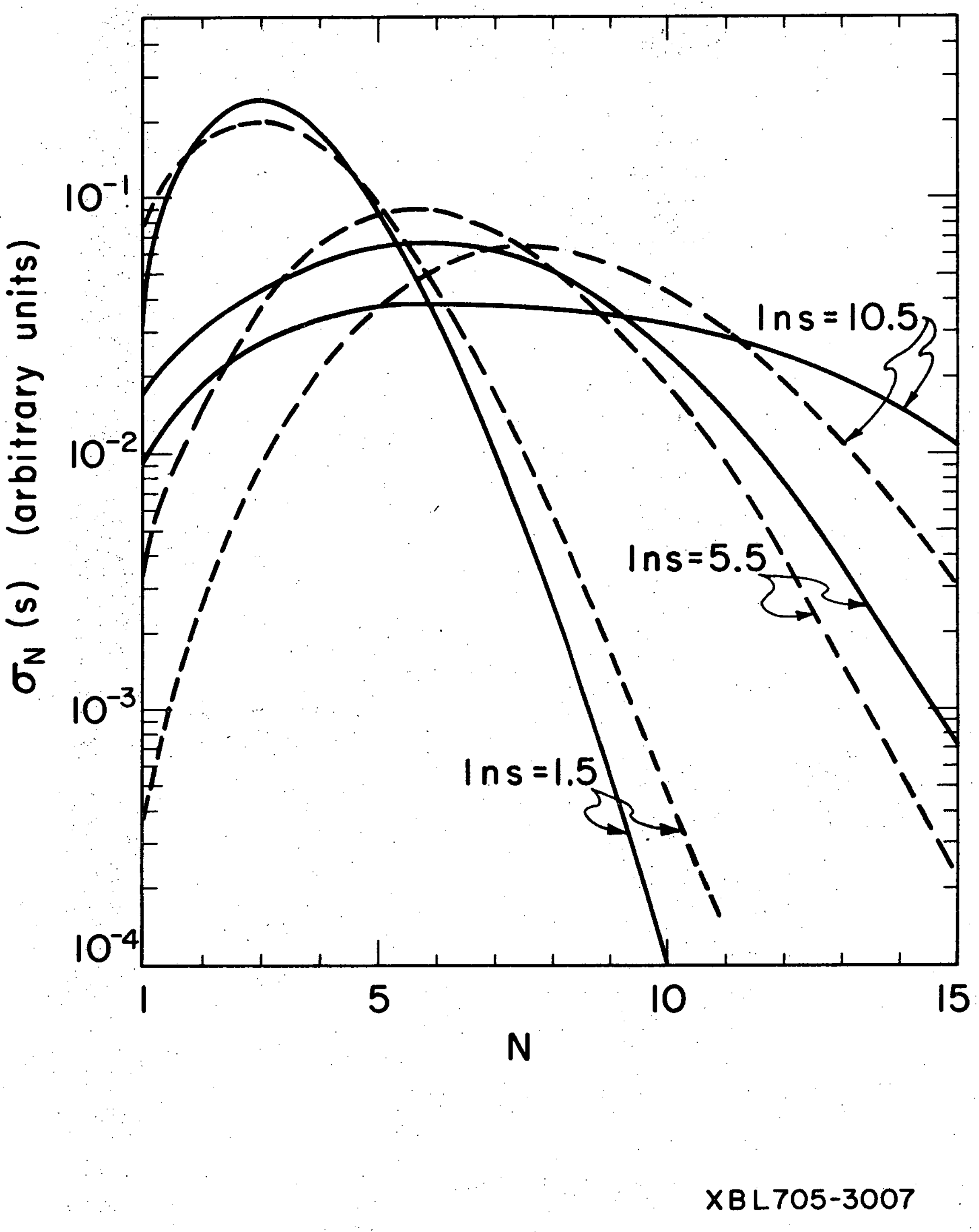
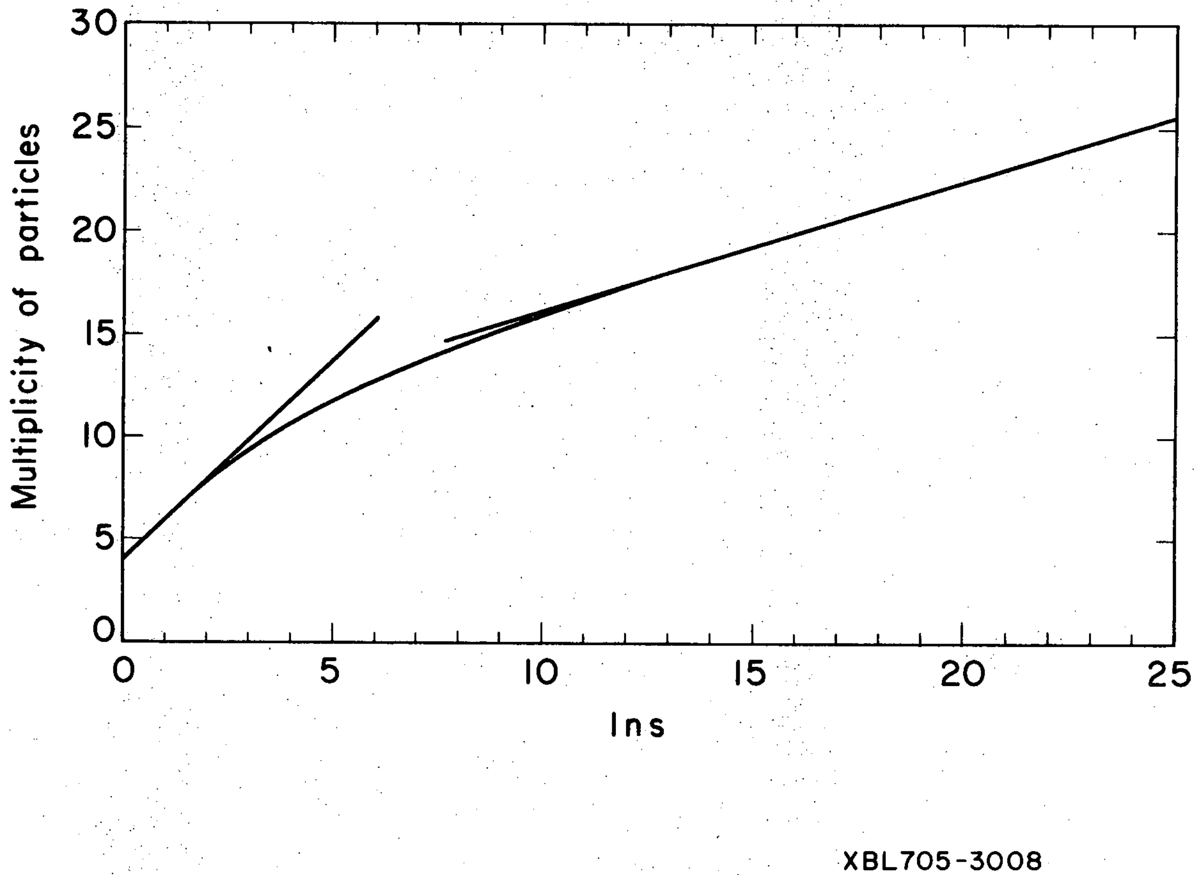

10^-1
10^-2
10^-3
10^-4
σN (s) (arbitrary units)
1
5
10
15
N
lns=10.5
lns=5.5
lns=1.5
XBL705-3007

Fig. 4



30
25
20
15
10
5
0
Multiplicity of particles
0
5
10
15
20
25
lns
XBL705-3008

Fig. 5

TECHNICAL INFORMATION DIVISION
LAWRENCE RADIATION LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720