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ABSTRACT

There is considerable interest in the investigation of the plasma column with an external cylindrical particle source in the core of the column. Examples for such devices are the contact-ionized cesium plasma and the hollow cathode discharge.[4,5] With reference to these experiments we here investigate theoretically the following model. An infinite cylindrical vessel of diameter 2R lies in a longitudinal magnetic field (B). A coaxial core of diameter $2r_0$, which contains electrons and ions of equal temperature T_0, is excluded from the plasma volume. This core is the external source of particle production, which provides an electron and ion current of density Γ_0 flowing into the plasma volume. These electrons and ions enter the discharge at r_0 and diffuse across the magnetic field towards the insulated wall of the container (R), where they recombine. In this part (I) of our investigation we assume that volume recombination and end effects are negligible. The effective mean free path of electron-ion collisions is small in comparision to 2R. We calculate density and temperature distributions from a simultaneous solution of the transport equations for mass, momentum, and energy. In general the results cannot be presented in analytic form but must be determined by machine solutions. Particularly noteworthy is a strong temperature variation across the discharge vessel. This variation has a remarkable influence on the density distributions and makes Schottky's approach inapplicable. The density at the edge of the core is determined from the eigenvalue problem as a function of the experimental parameters B, Γ_0, T_0, and R. A similarity law is given.

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INTRODUCTION

Discrepancies between experimental results and theoretical predictions have led to the conclusion that it is desirable to get more basic understanding of the properties of a plasma. It is believed that a fully ionized plasma in a magnetic field could be a rewarding subject for studying these basic properties. The most favorable condition would be to have such a plasma under symmetric geometrical conditions, as for instance in a cylindrical column.

The theory of the self-sustained positive column is well-known.[1,2] The particles in such a column are produced within the plasma volume by electron collisions with neutrals. This mechanism inherently requires high electron temperatures and, with that, an external electric field. The difficulties introduced by such an external field have been discussed elsewhere.[3,4]

In the recent past, therefore, experiments have been proposed and carried out which produce the charge carriers outside of the actual plasma column. This can be done, e.g., by contact ionization of atoms at the metal surface,[5,6,7] or by ionization in a hollow cathode.[8,9] The carriers are then introduced in the axial direction into the center of the column, where they form an electron-ion ensemble of given temperature and density. This core is the effective particle source for the rest of the plasma volume. From here particles diffuse across the magnetic field towards the wall, where they recombine.

It is the aim of this investigation to describe—within certain limits—the plasma volume between the core and the wall.

For this purpose we investigate the following model. The discharge volume is limited by two infinite coaxial cylinders of radius r_0 and R, respectively, lying in a longitudinal magnetic field (B). Within the smaller cylinder, of radius r_0, we have ensemble of electrons and ions of temperature T_0. This core is the external particle source of our discharge. It provides a radial-electron and ion-particle current of density Γ_0 which defines one of the boundary conditions of our problem. The electrons and

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ions entering from the core move across the magnetic field towards the wall (R) under the influence of mutual collisions and the radial electric field. As we neglect volume recombination, all particles recombine at the insulated wall of the container.[10] We assume that the effective mean free path of the electrons and ions is very much smaller than the extension of the discharge vessel (2R), and that the concept of quasi-neutrality is applicable.

Naturally these assumptions limit the applicability of our results to a certain range of the experimental parameters. In a second paper (II) the calculations will be extended to include end effects and volume recombination.

BASIC EQUATIONS

Undoubtedly the most detailed description of such a plasma would be given by the distribution function in phase space. We have attempted such a solution using an expansion in special Laguerre polynomials. One arrives at an infinite set of linear equations for the expansion coefficients. From this the coefficients may be evaluated by an approximation precedure, but are represented by determinants which include heavy integral expressions. This general solution has the decisive disadvantage of being practically unintelligible, and it appeared more appropriate to use the magneto-hydrodynamic approach.

Accordingly, we base our calculations on the following transport equations for the mass, momentum, and energy. These equations read[11]

$$\frac{\partial n_\pm}{\partial t} + \vec{\nabla} \cdot (\vec{v}_{d\pm} n_\pm) = \alpha_n, \qquad (1)$$

$$\pm \vec{B} \times \vec{\Gamma}_\pm + \frac{\Gamma_\pm}{\mu_\pm} + e\eta n_\pm (\vec{\Gamma}_\pm - \vec{\Gamma}_\mp) = \pm n_\pm \vec{X} - \vec{\nabla}\left(\frac{\overleftrightarrow{P}_\pm}{e}\right), \qquad (2)$$

$$\vec{\nabla} \cdot \overline{(n\vec{v}\, v^2)}_\pm \mp \frac{2n_\pm \cdot e \cdot \vec{v}_{d\pm}}{m_\pm} \cdot \vec{X} = \sum_i \frac{2n_\pm \nu_{\pm i}}{m_i + m_\pm} (m_i v_i^2 - m_\pm v_\pm^2)$$

$$- \sum_{i,x} \frac{2en_\pm \nu_{\pm x} V_{ix}}{m_\pm}, \qquad (3)$$

where $\vec{X}$ = electric field,
n = particle density,
$\vec{\Gamma}$ = particle current density,
$\vec{v}_d$ = drift velocity,
μ = mobility due to neutral particle collisions,
$\overleftrightarrow{P}$ = pressure tensor,
$\vec{v}$ = velocity,
$\vec{B}$ = magnetic field,
e = elementary charge,
m = particle mass,
α_n = net particle production per second and unit volume,
and η = electron-ion interaction parameter.

The bar indicates averages over the velocity space. The indexes +, - refer to ions and electrons, respectively.

Equations (1) to (3) represent six simultaneous differential equations. The problem is simplified considerably by the lack of a neutral gas component, and by the concept of quasi-neutrality, which means

$$\mu_+, \mu_- \rightarrow \infty, \tag{4}$$

$$n_+ \approx n_- = n. \tag{5}$$

THE SCHOTTKY APPROACH

Even with (4) and (5) the general problem is still complex. It is, therefore, instructive to consider the plasma first in the Schottky approximation, which has been successfully used in the case of the collision-dominated self-sustained positive column. This approximation is based on the assumption that the electron and ion temperature is constant across the discharge, thus it omits the energy balance, Eq. (3), altogether. So, with Eqs. (4) and (5), we then get the simple expression

$$\Gamma_r = \Gamma_{am} = -\frac{\eta n}{B^2}\frac{d}{dr}\left[nk(T_+ + T_-)\right] \tag{6}$$

for the current density, by using

$$\overleftrightarrow{P}_\pm = n_\pm kT_\pm \overleftrightarrow{\theta}, \tag{7}$$

where $\overleftrightarrow{\theta}$ is the identity tensor.

The mass continuity equation reads

$$\frac{r\eta nk}{B^2}\frac{d}{dr}\left[nk(T_+ + T_-)\right] = -\Gamma_0 r_0, \tag{8}$$

remembering that there is no particle production or destruction within the volume of the plasma.

Due to the assumption of constant temperature, it is possible to remove the term $(T_+ + T_-)$ from under the differentiation symbol and to treat the interaction term η as a constant. Under these circumstances integration of Eq. (8) is trivial and leads with the simplified boundary condition

$$n_\pm \approx 0, \tag{9}$$

used by Schottky, to

$$n \doteq n_0\left[\ln(R/r)/\ln(R/r_0)\right]^{1/2} \tag{10}$$

The eigenvalue

$$n_0 = [\ell n(R/r_0) \cdot \frac{B^2 \cdot \Gamma_0 \cdot r_0}{k \cdot T_0 \cdot \eta(T_0)}]^{\frac{1}{2}} \qquad (11)$$

gives the density at the edge of the core as a function of the discharge parameters.

The density distribution (10) is shown in Fig. 1 together with the Bessel distribution known for the self-sustained positive column. Fig. 2 gives the eigenvalue as a function of $(2\pi r_0 \Gamma_0/R)$.

GENERAL APPROACH

In the case of the self-sustained positive column the electron temperature is defined by the energy gain in the longitudinal electric field, and by the energy loss through collisions with neutral atoms. As both these quantities do not depend on the radial coordinate the assumption of constant electron temperature is reasonable.

In the fully ionized column with external particle production the particles enter the discharge volume with equal energy. Moving across the magnetic field, they interact directly or via the ambipolar electric field, exchanging energy in a rather complicated way. Here the assumption of constant temperature is not obvious. We therefore try to include the temperature variation.

Again we have Eq. (8). However, it is not possible now to remove the term $(T_+ + T_-)$ from under the differentiation symbol, and consequently the corresponding formula reads

$$n \frac{d}{dr} [n(T_+ + T_-)] = - \frac{\Gamma_0 r_0 B^2}{\eta_0 k} \left(\frac{T_-}{T_0}\right)^{\frac{3}{2}} \frac{1}{r}, \qquad (12)$$

where we have used the relation

$$\eta = \eta_0 \Big/ (T_-/T_0)^{\frac{3}{2}}. \qquad (13)$$

Substituting further in Eq. (3),

$$\nu_{+-} = \frac{e^2 \cdot n \cdot \eta}{m_+} ; \qquad \nu_{-+} = \frac{e^2 n \eta}{m_-}, \qquad (14)$$

and making use of the approximation

$$\vec{\nabla} \cdot \overline{(n\vec{v}_\pm v_\pm^2)} = \frac{3k}{m_\pm} \vec{\nabla} \cdot (\vec{\Gamma}_r T_\pm), \qquad (15)$$

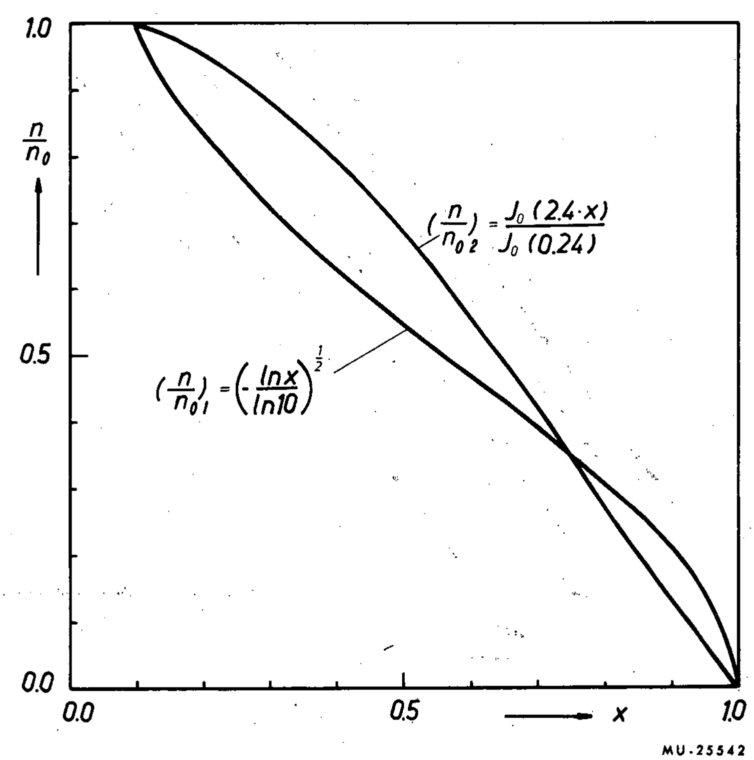

Fig. 1. Schottky approximation of the density distribution for the case of the fully ionized plasma column with external particle production $\left(\frac{n}{n_0}\right)_1$, and for the collision-dominated self-sustained positive column $\left(\frac{n}{n_0}\right)_2$.

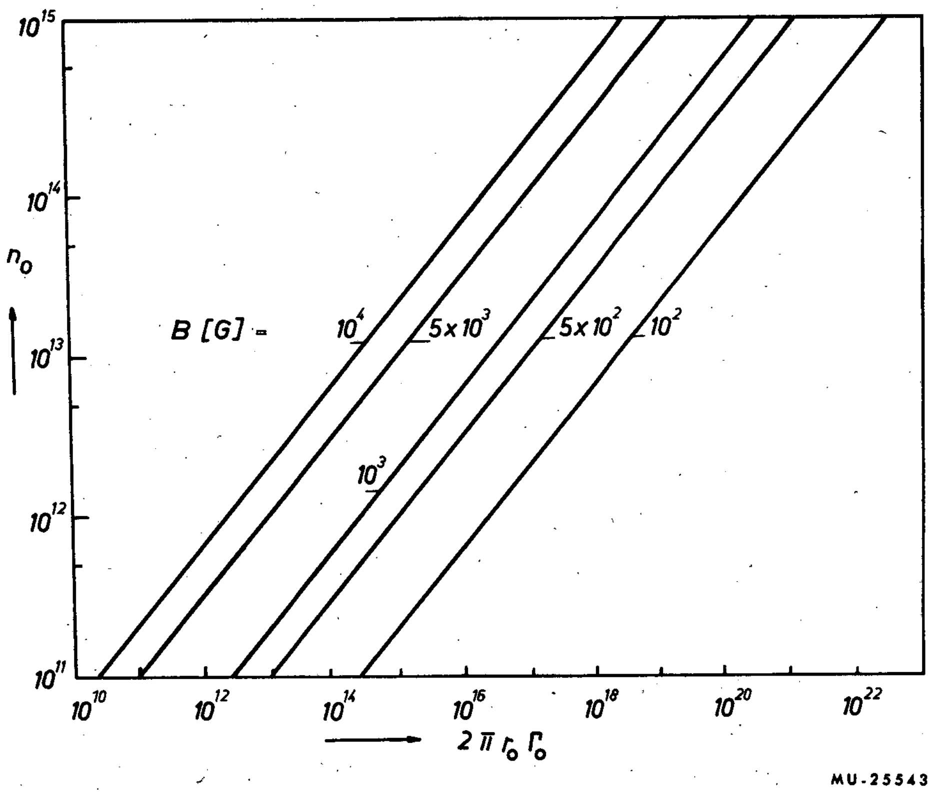

Fig. 2. Density n_0 at the edge of the particle source calculated as a function of $(2\pi\Gamma_0 r_0/R)$ by using Schottky's approach.

we find

$$\frac{2e^2\eta n^2}{m_+ + m_-}(T_- - T_+) = \vec{\nabla}\cdot(\vec{\Gamma}_r T_+) - \frac{2e}{3k}\vec{X}\cdot\vec{\Gamma}_r, \qquad (16a)$$

$$\frac{2e^2\eta n^2}{m_+ + m_-}(T_+ - T_-) = \vec{\nabla}\cdot(\vec{\Gamma}_r T_-) + \frac{2e}{3k}\vec{X}\cdot\vec{\Gamma}_r + \sum_x \frac{2e}{3k} n\ \nu_{-x} V_{+x}. \qquad (16b)$$

Adding these equations, we have

$$\vec{\nabla}\cdot[\vec{\Gamma}_r(T_+ + T_-)] = -\sum_x \frac{2e}{3k} n\nu_{-x} V_{+x}. \qquad (17)$$

As would be expected, this equation states that the divergence of the energy current is equal to the energy loss due to ion excitation and ionization. In those cases under consideration the temperature of the plasma is too low to cause such excitation to an appreciable extent. Consequently we have

$$(T_+ + T_-)\vec{\nabla}\cdot\vec{\Gamma}_r + \vec{\Gamma}_r\cdot \mathrm{grad}\,(T_+ + T_-) = 0, \qquad (18)$$

and therefore

$$T_+ + T_- = 2T_0. \qquad (19)$$

The sum of the average particle energies of the ions and electrons is constant under the conditions of our model since there is no net volume energy production.

Making use of Eqs. (19) and (12), we have

$$\frac{d}{dx}(z^2) = -\frac{C_1[1-(y/2)]^{3/2}}{x}, \qquad (20)$$

where

$$y = T_+/T_0;\ z = n/n_0;\ x = r/R;\ C_1 = [2^{3/2}\cdot R\cdot B^2\cdot \Gamma_0\cdot x_0]/[\eta(T_0)\cdot T_0\cdot k\cdot n_0^2]. \qquad (21)$$

The second relation for the variables y, z is given by Eq. (16a) which, after the introduction of the abbreviations (21), reads

$$\frac{4e^2R\eta n^2}{m_+ + m_-}(1-y) = \Gamma_r\left(\frac{dy}{dx} - \frac{2e X_r\cdot R}{3kT_0}\right). \qquad (22)$$

From Eq. (22) we eliminate the radial-field component with the help of the momentum Eqs. (2) which state

$$\Gamma_{\theta -} B - \Gamma_r/\mu_- = nX_r + \frac{d}{dr}(nkT_-/e), \qquad (23a)$$

$$\Gamma_{\theta -} = - \frac{B\mu_-}{1 + e\eta n(\mu_+ + \mu_-)} \Gamma_r ; \qquad (23b)$$

and eliminating $\Gamma_{\theta -}$ from (23) we have

$$-\Gamma_r \cdot \frac{1 + \mu_-^2 B^2 + e\eta n(\mu_+ + \mu_-)}{\mu_-[1 + e\eta n(\mu_+ + \mu_-)]} = nX_r + \frac{d}{dr}\left(\frac{nkT_-}{e}\right) . \qquad (24)$$

Further, by using Eqs. (4) and (6),

$$neX_r = \frac{1}{\mu_+ + \mu_-} \left[\mu_- \frac{d}{dr}(nkT_+) - \mu_+ \frac{d}{dr}(nkT_-)\right] \qquad (25)$$

or

$$\frac{eX_r R}{kT_0} = \left[\frac{dy}{dx} + \frac{1}{z}\frac{dz}{dx} g(y)\right] , \qquad (26)$$

with

$$g(y) = y \frac{1 - \kappa\left(\frac{2-y}{y}\right)^{3/2}}{1 + \kappa\left(\frac{2-y}{y}\right)^{1/2}} ; \qquad \kappa = \frac{\mu_{+0}}{\mu_{-0}} . \qquad (27)$$

Substituting (6) and (26) in Eq. (22), we finally arrive at the following two simultaneous differential equations:

$$\frac{d}{dx} z^2 = -C_1 \frac{[1 - (y/2)]^{3/2}}{x} \qquad (28a)$$

and

$$C_2(1-y) = \frac{1}{z}\frac{dz}{dx}\left[-\frac{dy}{dx} + 2g(y) \cdot \frac{1}{z}\frac{dz}{dx}\right] , \qquad (28b)$$

where C_1 and C_2 are defined by

$$C_1 = \frac{2^{3/2} \cdot \Gamma_0 \cdot r_0 \cdot B^2}{kT_0 \eta_0 \cdot n_0^2} ; \quad C_2 = \frac{6 \cdot e^2 \cdot B^2 \cdot R^2}{k \cdot T_0 \cdot (m_+ + m_-)} . \qquad (28c)$$

These equations (28) define the electron temperature T_-, the ion temperature T_+, and the particle number density n. Of course, in addition to the differential equations one needs boundary conditions. At the edge of the core (r_0) we have clearly

$$x_0 = r_0/R , \quad y = 1 , \quad z = 1 . \qquad (29a)$$

The situation at the wall of the discharge is more difficult. The relation (9) of Schottky's approximation is not sufficient here. We use the more refined boundary condition given elsewhere[10] for the case of the self-sustained collision-dominated positive column. This boundary condition is derived from the requirement of current continuity at the sheath edge of the plasma.

It is necessary to define what we mean by the edge of the sheath in our special case. The description of the plasma by a diffusion process is correct down to the point where we are about one effective mean free path of the ions away from the wall. From this point on, the motion of the ions should be described by the laws of free fall. Accordingly, the description of the plasma formulated in the preceding equations breaks down at about one ion mean free path away from the wall, and we will use this point to define the beginning of the sheath. It is designated by an index s. According to our general assumptions, the extension of the sheath is small in comparison to the radius R.

The wall boundary condition of our plasma can therefore be stated in the form[10]

$$\frac{r_0 \Gamma_0}{R} = \frac{(n \bar{v}_r)_s}{4}. \tag{29b}$$

Since we have no net volume particle production this formula is quite plausible.

As the sheath edge is practically at x = 1, the boundary conditions of our problem are

$$x = x_0 \rightarrow y = 1; \; z = 1, \tag{29c}$$

$$x = 1 \rightarrow z = \frac{4 x_0 \Gamma_0}{n_0} \left(\frac{m_+}{3kT_0 y} \right)^{1/2} \tag{29d}$$

SIMILARITY LAW

Without integrating Eqs. (28) and (29), we note an important feature. The coefficients of the differential equations and the boundary conditions include the parameters of our problem only in certain combinations. These are

$$\frac{\Gamma_0 r_0}{T_0^{5/2}}; \quad \frac{B^2}{T_0} \cdot \left(\frac{R^2}{m_+} \right); \quad \frac{n_0}{T_0^{\,2}} \left(\frac{R}{\sqrt{m_+}} \right). \tag{30}$$

Similar discharges have identical relative parameter values—density, temperature, etc.—at homolog points. This is true if the quantities (30) have the same values. From this we find the following rules for

T_0, B, R, r_0, Γ_0, m_+:

All other parameters being constant, we have similar discharges if

$$R \propto \sqrt{m_+}\ ; \qquad \Gamma_0 \propto 1/r_0 \tag{31}$$

If discharges are to be similar, then a variation of one of the parameters given above prescribes the necessary changes for all the other parameters. These relations are described by the following scheme:

		T_0	B	$R/\sqrt{m_+}$	$\Gamma_0 r_0$
T_0	→	$\propto T_0$	$\propto T_0^{-3/2}$	$\propto T_0^{2}$	$\propto T_0^{5/2}$
B	→	$\propto B^{-2/3}$	$\propto B$	$\propto B^{-4/3}$	$\propto B^{-5/3}$
$R/\sqrt{m_+}$	→	$\propto (R/\sqrt{m_+})^{1/2}$	$\propto (R/\sqrt{m_+})^{-3/4}$	$\propto (R/\sqrt{m_+})$	$\propto (R/\sqrt{m_+})^{5/4}$
$\Gamma_0 r_0$	→	$\propto (\Gamma_0 r_0)^{2/5}$	$\propto (\Gamma_0 r_0)^{-3/5}$	$\propto (\Gamma_0 r_0)^{-4/5}$	$\propto \Gamma_0 r_0$

INTEGRATION AND RESULTS

Equations (28) and (29) do not allow an analytic solution. Machine solutions are complicated by the fact that we are dealing with a boundary-value problem. However, as we have several parameters at our disposal, we can evade this difficulty by the following procedure.

The magnetic field B, the radius R, and the core temperature T_0 define the constant C_2. Choosing values of C_1, we integrate simultaneously Eqs. (29) starting from $z(x_0) = 1$ and $y(x_0) = 1$. At $x = 1$ we find values z_1 and y_1. By introducing these into Eqs. (29d), we have the two following relations:

$$C_1 = \frac{2^{3/2}\,\Gamma_0 r_0\, B^2}{k T_0 \eta_0 n_0^{\,2}} \tag{32a}$$

and

$$Z_1 = \frac{4\Gamma_0 r_0}{n_0 R} \cdot \left(\frac{m_+}{3kT_0 y_1}\right)^{1/2}, \tag{32b}$$

from which we calculate the parameter values $\Gamma_0 r_0$ and n_0 belonging to these density and temperature distributions. We also have the corresponding eigenvalue n_0. An example of results of such calculations is given in Figs. 3(a) to 5(b) for Cs ($m_+ = 133$).

In addition, Fig. 6 gives the eigenvalues n_0 as a function of the magnetic field B and the effective particle production $\Gamma_0 r_0$.

DISCUSSION

The discussion uses the two parameters

$$P_1 = \frac{\Gamma_0 r_0}{T_0^{5/2}} ; \qquad P_2 = \frac{(BR)^2}{T_0} . \tag{33}$$

The characteristic features of the relative density distributions z shown in Figs. 3(a), 4(a), and 5(a) may be summarized as follows:

All distributions decrease from the edge of the core towards the wall, the slope $|dz/dx|$ being stronger near the two limiting cylinders than in between. With increasing parameter value p_1 the relative density (and, according to Fig. 6, also the absolute density) increases in all cases. The influence of p_1 is stronger for small values of p_2. With increasing p_2 the relative density decreases. (However, this cannot be said of the absolute density, because—according to Fig. 6—n_0 increases with p_2.)

These features may be qualitatively understood simply from the mass conservation law, which requires that the radial particle current shall be constant across the plasma volume.

If the diffusion coefficient were constant, the slope $|dz/dx|$ would be required to decrease towards the wall in proportion to $1/r$. However, in the case of a fully ionized positive column in a longitudinal field the effective transverse diffusion coefficient is not only proportional to the particle density but also inversely proportional to the square of the magnetic field [see Eq. (6)]. This means that $|dz/dx|$ should increase again in regions of small particle density. Consequently, starting from the edge of the core, $|dz/dx|$ should first be expected to decrease because of the increase in r, but then, approaching regions of low particle density, should increase again due to the decrease in the effective diffusion coefficient. This agrees well with the analytic result. An increase in the magnetic field means an increase in the parameter p_2. It decreases the diffusion coefficient, and therefore requires in general a larger slope $|dz/dx|$ and, with that, a decrease in the relative density—again in agreement with the analytic results. From an increase of p_1 we would expect—and Fig. 6 confirms this—an increase in the absolute density across the plasma volume. An increase of p_1 would also increase the effective diffusion coefficient, which results in a decrease of $|dz/dx|$, as demonstrated by the curves of Figs. 3(a) to 5(a).

As T_0 is a constant experimental parameter, the relative temperature distributions y(x) shown in Figs. 3(b) to 5(b) are proportional to the absolute temperature distributions. We see that the temperature variation is in general not at all negligible. The ion temperature always decreases. In some cases it decreases monotonically towards the wall, but it can also show a minimum—or even a minimum and a maximum—as a function of x. As parameter p_1 increases, the temperature decrease is reduced. This influence becomes stronger as the parameter p_2 becomes smaller. As

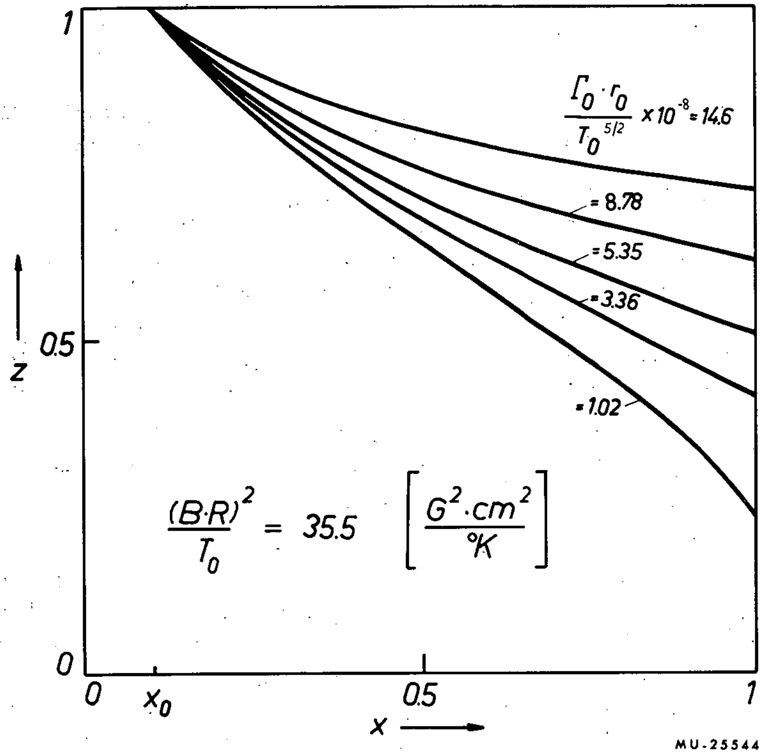

Fig. 3(a). Relative density distribution $z(x) = n(x)/n$ for $p_2 = 35.5$ and various parameter values of p_1 [see Eq. (33)], calculated from Eqs. (28) and (29) for the example of cesium.

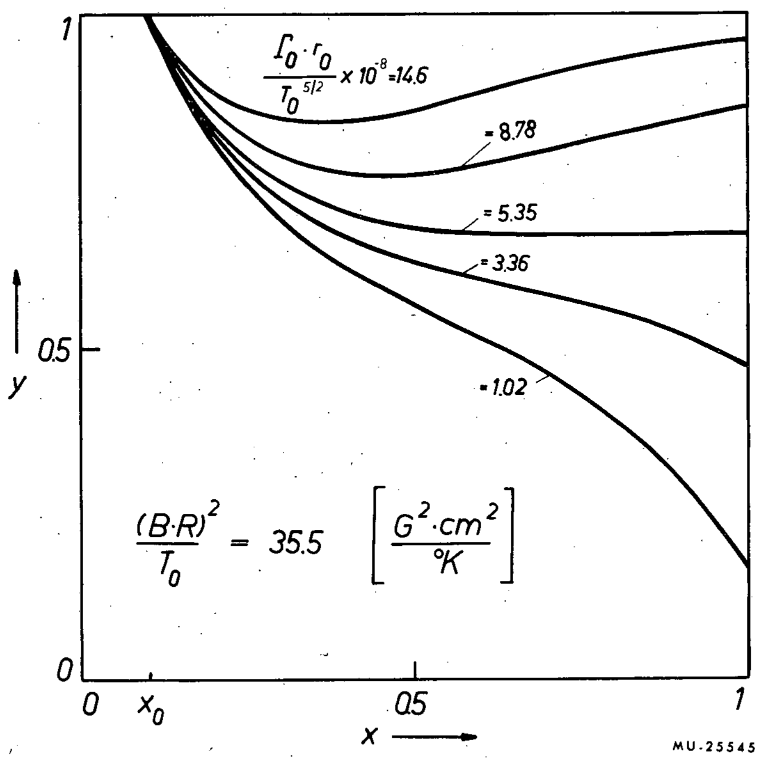

Fig. 3(b). Relative temperature distribution $y(x) = T_+(x)/T_0$ for $p_2 = 35.5$ and various parameter values of p_1 [see Eq. (33)], calculated from Eqs. (28) and (29) for the example of cesium.

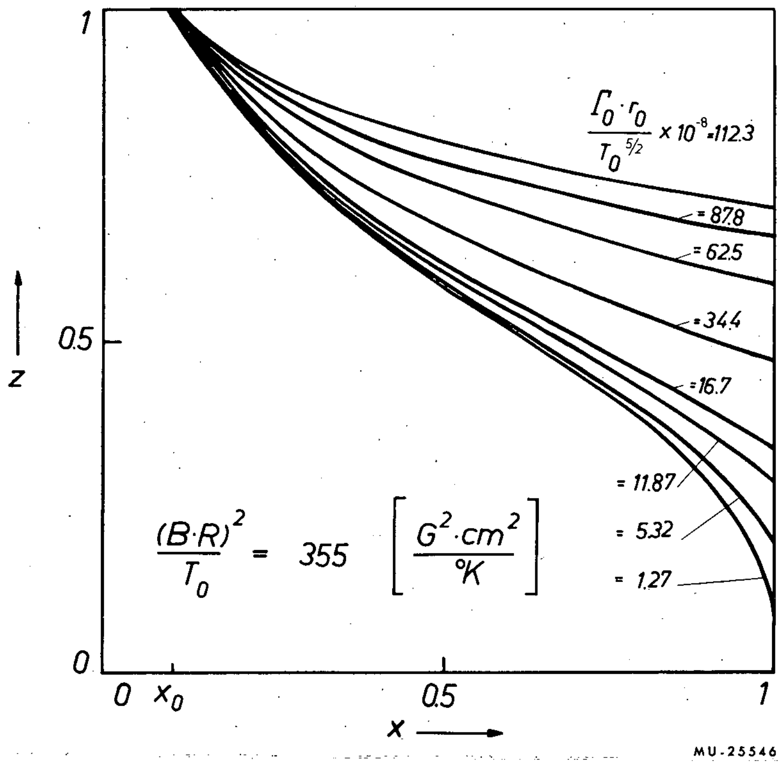

Fig. 4(a). Relative density distribution $z(x) = n(x)/n$ for $p_2 = 355$ and various parameter values of p_1 [see Eq. (33)], calculated from Eqs. (28) and (29) for the example of cesium.

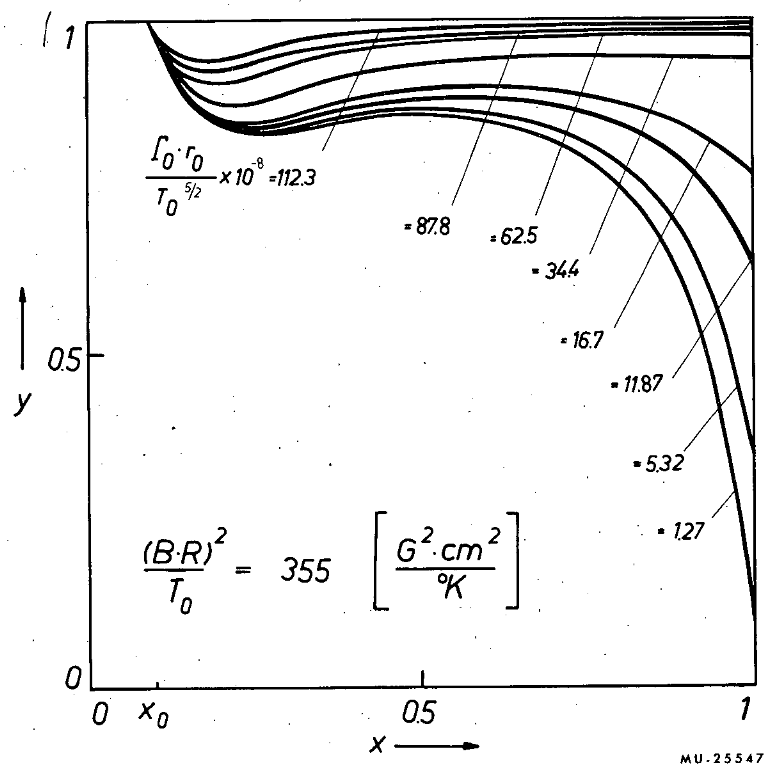

Fig. 4(b). Relative temperature distribution $y(x) = T_+(x)/T_0$ for $p_2 = 355$ and various parameter values of p_1 [see Eq. (33)], calculated from Eqs. (28) and (29) for the example of cesium.

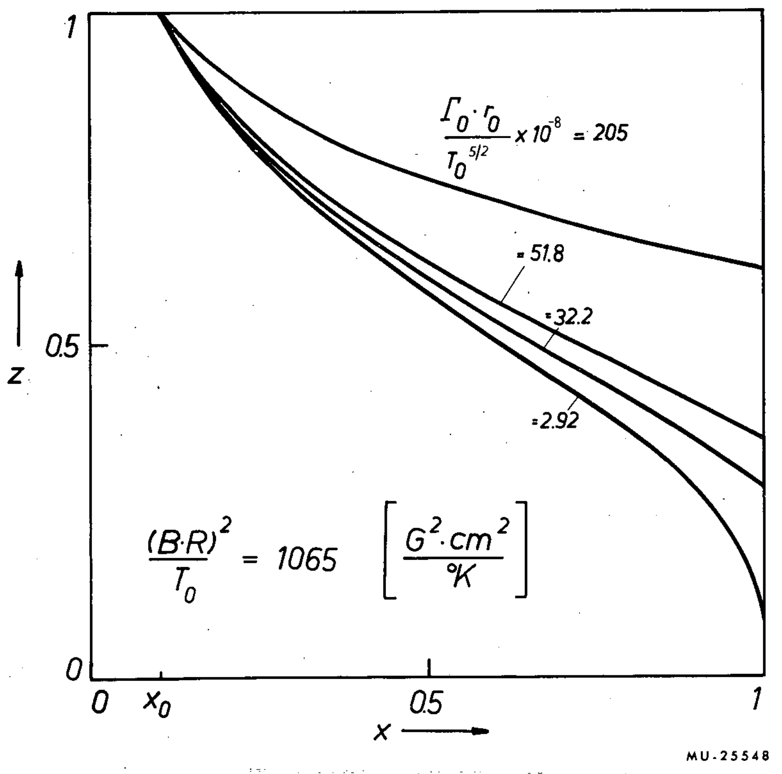

Fig. 5(a). Relative density distribution $z(x) = n(x)/n$ for $p_2 = 1065$ and various parameter values of p_1 [see Eq. (33)], calculated from Eqs. (28) and (29) for the example of cesium.

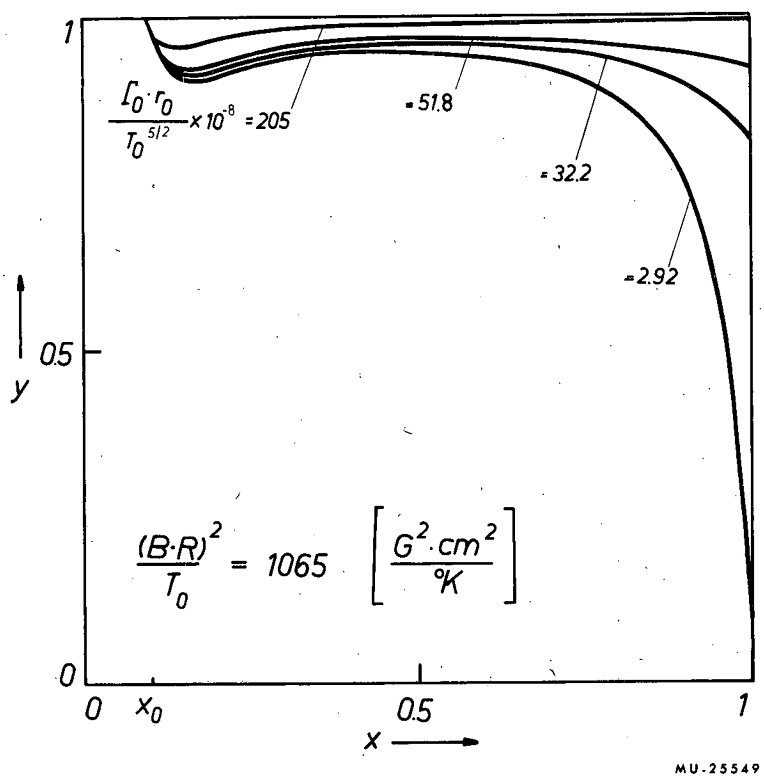

Fig. 5(b). Relative temperature distribution $y(x) = T_+(x)/T_0$ for $p_2 = 1065$ and various parameter values of p_1 [see Eq. (33)], calculated from Eqs. (28) and (29) for the example of cesium.

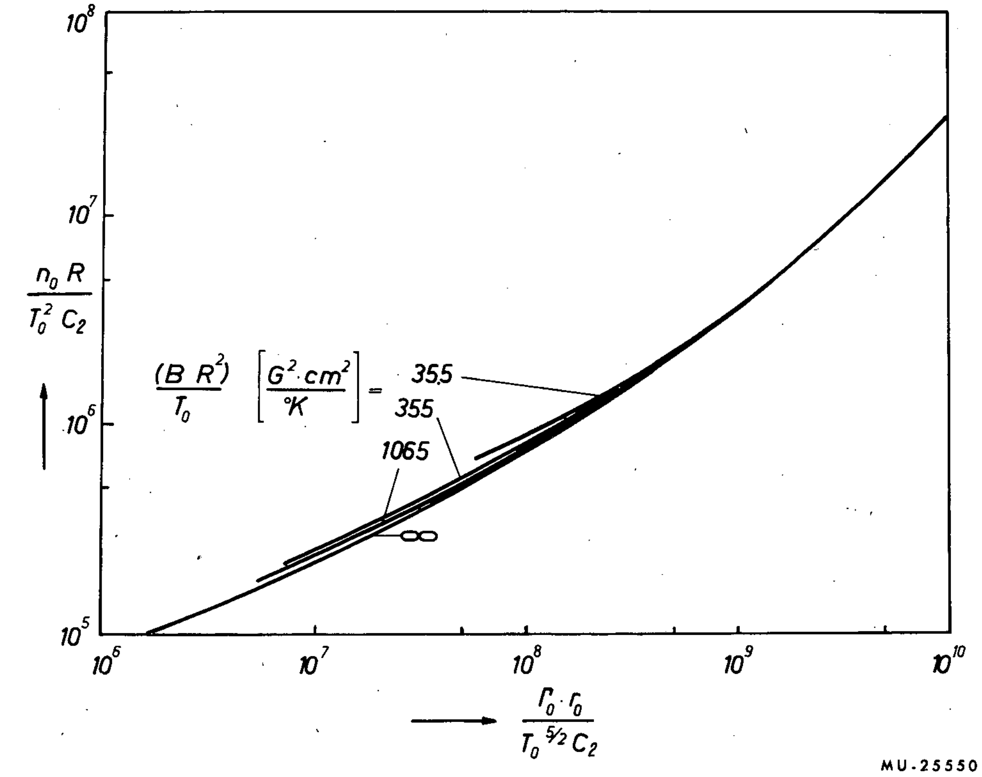

Fig. 6. Density n_0 at the edge of the particle source as a function of $[(\Gamma_0 r_0 / T_0^{5/2} C_2]$, calculated for various values of p_2 as the eigenvalue of Eqs. (28) and (29).

parameter p_2 increases, the temperature distribution y(x) approaches a constant value, except for a decrease near the core edge and near the wall of the vessel.

Again these features can be qualitatively understood, remembering that two processes govern the change in temperature. There is the collective interaction of the particles of the fully ionized column via the space and wall charge (ambipolar field). This interaction takes energy from the ions and gives it to the electrons. This effect increases with the magnetic field. The other process—the energy exchange due to individual particle interactions—tends to decrease the temperature difference between ions and electrons.

At the edge of the core, where the two temperatures are identical, only the ambipolar field is in action, which causes a decrease in the ion temperature (and, with that, an increase in the electron temperature) as shown in all of the Figs. 3(b) to 5(b). This increase in temperature difference brings the individual energy exchange of the unlike particles into play. It causes an increase in dy/dx. Remembering that this individual exchange varies in proportion to the particle density, we expect dy/dx to decrease again in the regions of low particle density, close to the wall. This is confirmed in Figs. 3(b) to 5(b), except in those cases where the particle density does not really decrease very much near the wall. An increase in p_1 causes an increase in the particle density, as described in the preceding paragraph. This favors the individual energy exchange, and consequently increases the ion temperature, in agreement with the calculated results shown in the Figures. An increase in the magnetic field p_2 reduces the influence of heat conduction and collective interaction, and so favors the individual energy exchange, which moves the temperature distributions y(x) closer to y = 1. This is also demonstrated in Figs. 3(b) to 5(b).

Finally, let us discuss Fig. 6. We note that in the appropriate units used in Fig. 6 the dependence of n_0 on $\Gamma_0 r_0$ is practically not influenced by the value of the magnetic field p_2. In a first approximation the relation may be represented in the double logarithmic plot by a straight line of slope 2/3, which justifies the analytical representation

$$n_0 = (6e^2/km_+)^{1/3} \quad (B \cdot \Gamma_0 r_0)^{2/3} \cdot R^{-1/3}. \qquad (34)$$

Equation (34) or Fig. 6 may be compared with Eq. (11) or Fig. 2. The inapplicability of Schottky's approach to the present problem becomes obvious at once.

SUMMARY

We have calculated the qualities of an infinite fully ionized cylindrical plasma with external particle production and without volume recombination. The results were derived from the magnetohydrodynamic relations for the conservation of mass, momentum, and energy. We find that the Schottky approach using a constant temperature and a simplified boundary condition is not applicable to this problem. The general calculation, which includes temperature variation and utilizes a more general boundary condition, produces results which cannot be represented in an analytical form but have been determined by machine solutions for the example of a cesium column. The results show density and temperature distributions which vary strongly with the discharge parameters (r_0, R, B, Γ_0, T_0). The general features of these distribution functions have been discussed and interpreted in physical terms. The calculations also produce the maximum density n_0 at the edge of the core as the eigenvalue of the problem. The dependence of n_0 on the experimental parameters can be reasonably approximated by a simple analytical relations. A general similarity law relating the parameters (r_0, R, B, Γ_0, T_0) has been given.

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