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Publication Date 2016

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA

Los Angeles

Essays on International Finance

A dissertation submitted in partial satisfaction of the requirements for the degree Doctor of Philosophy in Economics

by

Hyo Sang Kim

2016

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ABSTRACT OF THE DISSERTATION

Essays on International Finance

by

Hyo Sang Kim

Doctor of Philosophy in Economics University of California, Los Angeles, 2016 Professor Aaron Tornell, Chair

This dissertation studies two issues on international finance: predictability of foreign exchange market and distributional approach to economic distress from small from small economic disturbances to catastrophic crises. Chapter 1, which is coauthored with Aaron Tornell and Zhipeng Liao, investigates whether social learning can help to account for the existence of predictability in the foreign exchange market. We present an heterogeneous-agent asset pricing model where fundamental shocks lead to amplification cycles (bubbles), and the principle of contrarian opinion holds: in equilibrium, less-informed speculators become overly optimistic (pessimistic) when prices diverge enough from fundamentals and the bubble is likely to burst. Informed forward-looking speculators find it optimal to ride the bubble until a time when they switch the sign of their positions. At this switching time, the bubble continues to grow as less-informed speculators become more optimistic (pessimistic). Based on the implications of the model, we propose a forecasting strategy that estimates structural breaks in the bivariate process followed by exchange rates and speculators' positions. Across the six major currencies, our forecasts outperform the random walk over forecasting horizons from 1 months to 12 months. Chapter 2, which is co-authored with Aaron Tornell and KeyYong Park, looks at the size distribution of economic distress events over the recent period of globalization (1970 - 2014) and the long historical period (1830 - 2013). We find that there exists a remarkable relation between the magnitude of economic distress events and the frequency with which they occur. We document that there is a threshold below which the size of ED events follows an exponential distribution while a Pareto distribution (a power-law) applies for ED events larger than the threshold. To explain the empirical results, we present a wildfire model in which the dynamics of an individual ED event is determined by the interaction of two opposing forces: (i) the natural stochastic growth of the ED, which is proportional to the size of the damage that has already occurred; and (ii) a policy that attempts to extinguish the economic distress. We then derive the steady-state cross-sectional distribution of the final size of the ED events. Chapter 3 analyzes the forward premium puzzle both on developed and emerging economies. The forward premium puzzle tends to exist on developed economies, but not on emerging economies. From the theoretical model of Gourinchas and Tornell (2004), the forward premium puzzle can be explained when investors have a biased belief which overestimates transitory shocks to persistent shocks about the interest rate process. I decompose interest rate differential process with transient and persistent components by using the state space model. Both developed and emerging countries have persistent interest rate differential processes, but developed countries tend to have relatively larger shocks that connect to the persistent component.

The dissertation of Hyo Sang Kim is approved.

Romain T. Wacziarg Zhipeng Liao Roger E. Farmer Aaron Tornell, Committee Chair

University of California, Los Angeles 2016

To my family,

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ACKNOWLEDGEMENTS

I am extremely grateful to my academic advisor, Professor Aaron Tornell, for all his invaluable guidance. I would also like to thank my other committee members, Professor Roger E. Farmer, Professor Zhipeng Liao, and Professor Romain T. Wacziarg for their insightful advice. I have also benefited from conversations with participants of the International Proseminars for their helpful comments. All remaining errors are my own.

I have met many colleagues who have supported and encouraged me throughout the Ph.D. Especially; I am very grateful to Young Ju Kim, Sangyup Choi, Carlos Cantú, and KeyYong Park for their encouragement.

Finally, I thank my wife, Jihye Kwon, and my parents for their unconditional love and endless support. Without them, I would not have been able to study and finish the Ph.D.

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1 The Principle of Contrarian Opinion and Its Implications for Forecasting Exchange Rates

1.1 Introduction

We find that publicly available speculator position data contains useful information to forecast exchange rates. In contrast to much of the literature, our forecasts are nonlinear as they are based on structural breaks in the exchange rates and speculators positions series. Our forecast strategy is based on what may be termed the principle of contrarian opinion, which states that when a group of speculators tends to agree that the price will go up(down), it is very likely they are wrong, and the price will soon start a declining(increasing) path. The contribution of this paper is twofold. First, we present a minimal model where prices are endogenous and where this puzzling pattern occurs in equilibrium. Second, we propose a forecasting strategy that estimates the times when informed forward-looking speculators switch their positions, signaling the imminent end of the bubble. We have shown that our forecast strategy beats random-walk forecasts.

In our model economy, there are three types of traders: hedgers, informed speculators, and uninformed trend-chasing traders. In equilibrium, a transitory shock to fundamentals may give rise to a bubble. Informed speculators find it optimal to ride the bubble for several periods. However, when the price is far enough from the fundamentals so that the probability of a bubble burst becomes high enough, informed speculators switch their positions. A key property of the equilibrium is that at this switching time, the price continues to move in the same direction. Thus, backward induction does not unravel back to the initial time, and so informed speculators ride the bubble initially. Our empirical strategy estimates these switching times by testing for structural breaks in the bivariate process followed by exchange rates and the net positions of speculators. Based on these structural break estimates, we then construct out-ofsample directional forecasts and test two null hypothesis against the random walk.

We use the net speculators position data of the Commitments-of-traders (COT) report from the Commodity Futures Trading Commission (CFTC) for the six major currencies vis-a-vis the US dollar over the 1992-2015 (August) period. Over horizons ranging from 1 to 12 months, our out-of-sample directional forecasts have a 59.5% average success ratio across the six most traded currency pairs vis-a-vis the US Dollar and are greater than 50% in 29 out of the 30 currency-horizon pairs we consider.¹ The forecast success ratios are particularly accurate at the 6-to-12 months horizons, reaching 83% for the Yen, 68% for the Australian Dollar, and 66% for the Euro.

To evaluate whether our directional forecasts succeed in predicting *big* swings in exchange rates, we use the directional forecast test proposed by Kim et al. (2014). Unlike the traditional directional test, the KLT test weights each directional forecast by the realized exchange rate change, and evaluates whether the weighted directional forecasts outperform the driftless random walk forecasts. At the 6-month horizon, the weighted directional test rejects the random-walk null in favour of our model across all currencies, except the Canadian Dollar, controlling for auto-correlation using the long-run variance estimators proposed in Newey and West (1987). The same holds true at the 9-month horizon (except for the British Pound) and at the 12-month horizon (except for the British Pound and the Australian Dollar). At the 1-month and 3-month horizons, the null is rejected in 7 out of 12 currency-horizon pairs.

The rest of the paper is organized as follows. In section 2, we present the model

¹We consider the Euro, Japanese Yen, British Pound, Australian Dollar, Canadian Dollar and Swiss Franc at 1m, 3m, 6m, 9m, and 12m forecasting horizons.

and derive the bubbly equilibrium that links the informed speculators' positions with the dynamics of the exchange rates. Section 3 discusses the empirical implications of the model, constructs the forecasts and tests their accuracy vis-a-vis the random walk. Section 4 discusses the related literature. Finally, Section 5 concludes. Proofs and technical results are located in the Appendices.

1.2 Model

We present a minimal dynamic asset pricing model with heterogeneous agents. We consider an economy in which there is one-good which is the numeraire, a safe asset and a risky asset. The safe asset pays zero interest and its price is one. The risky asset is in zero net supply and is traded in a futures market. We will denote the price of the risky asset at time t by P_t .

There are three types of overlapping generations of traders who live 2 periods: hedgers, momentum speculators, and informed speculators. Hedgers trade for insurance purposes, not speculation. Their demand for the risky asset is

$$q_t^h(P_t, Z_t) = \phi(Z_t - P_t), \quad \phi > 0.$$
 (1)

We will refer to Z_t as the "fundamental." It is given by

$$Z_t = \mu_z + \zeta_t,\tag{2}$$

where μ_z is a real constant, and ζ_t has mean zero and can take three possible values: -z, 0 and z with probabilities r/2, 1 - r, and r/2 ($r \in (0, 1)$) respectively. In the absence of speculators, the equilibrium price equals Z_t .

There is a set of a measure one of informed speculators who are risk neutral and

optimally choose q_t^I to maximize their expected profits

$$\pi_t^I = q_t^I \cdot \left(\mathbf{E}_t [P_{t+1}] - P_t \right),$$

where $E_t[\cdot] = E[\cdot | \mathcal{I}_t]$ denotes the conditional expectation operator given the information I_t available to them at time t. Their choice set is $\{-c, 0, c\}$ and hence,

$$q_t^I(P_t, \mathcal{E}_t[P_{t+1}]) = C_t(\mathcal{E}_t[\Delta P_{t+1}]) = \begin{cases} c & \text{if } \mathcal{E}_t[\Delta P_{t+1}] > 0 \\ 0 & \text{if } \mathcal{E}_t[\Delta P_{t+1}] = 0 \\ -c & \text{if } \mathcal{E}_t[\Delta P_{t+1}] < 0 \end{cases}$$
(3)

where $\Delta P_{t+1} = P_{t+1} - P_t$ and c is a real positive number.

There is a set of measure one of momentum speculators' whose demand for the risky asset is proportional to the price change. Momentum speculators need to borrow to finance their trades, and there may be some situations in which they are not able to borrow. When such event occurs we say that there is *saturation*. We define a binary state variable S_t which takes the value 0 if they can borrow to finance their trades and the value 1 if they cannot get credit at time t. Thus, their demand for the risky asset is

$$q_t^m(P_t, P_{t-1}) = \begin{cases} \theta(P_t - P_{t-1}) & \text{if } S_t = 0\\ 0 & \text{if } S_t = 1 \end{cases}, \quad \theta > 0.$$
(4)

Equation (4) captures the essence of trend-following observed in several asset markets. If momentum speculators can find financing and the price has been increasing, then their demand increases over time. As the asset price diverges from the mean of the fundamental, it becomes less likely that momentum speculators will find financing. That is, the probability of saturation next period σ_{t+1} is increasing in the distance between the mean of the fundamental and the price. To concentrate on the gist of the mechanism, we consider the following simple process for the probability of saturation next period. The binary variable S_t^* has the following law of motion:

$$\sigma_{t+1} \equiv \Pr(S_t^* = 1 | S_{t-1}^* = 0, P_{t-1}^*) = \begin{cases} 0, & |P_{t-1}^* - \mu_z| \le \vartheta \text{ and } P_{t-1}^* > \underline{P} \\ \overline{\sigma}, & |P_{t-1}^* - \mu_z| > \vartheta \text{ and } P_{t-1}^* > \underline{P} \\ 1, & P_{t-1}^* \le \underline{P}, \end{cases}$$
(5)

where $\vartheta > 0$ and $\overline{\sigma} \in (0, 1)$ are some fixed constants. <u>P</u> is a small positive lower bound that ensures prices are positive along a negative bubbly path.

To close the model we assume that the economy starts at time 0 with $S_0 = 1$. We further assume that if there is saturation at time t, the momentum speculators cannot find financing at times t and t + 1.

The equilibrium concept we use is standard in the literature. During every period, each agent submits a demand schedule, taking prices as given. The equilibrium price is then determined by the market clearing condition:

$$q_t^h(P_t, Z_t) + q_t^m(P_t, P_{t-1}) + q_t^I(P_t, \mathcal{E}_t[P_{t+1}]) = 0.$$
(6)

Informed speculators know the process followed by σ_t and know that prices are determined by market clearing condition (6). Thus, they can correctly forecast the distribution of future equilibrium prices.

1.2.1 Discussion of the Setup

We consider a minimal model that allows us to construct an internally consistent mechanism for the endogenous amplification of price bubbles, where positions of informed forward looking speculators have predictive power over prices. The three types of agents are typically present in asset markets: informed forward looking speculators; momentum driven speculators; and those who do not trade for speculative purposes and whose excess demands arguably reflect fundamentals.

As we shall show, if a bubbly equilibrium exists, then a key property of the equilibrium is that along a positive bubbly path, the price goes up at the time informed forward looking speculators start to exit the market (or go short). This result implies that: (i) informed speculators find it profitable to ride the bubble initially (backwards induction does not unravel to initial time); and (ii) they start to exit their long positions when the price is very likely to change direction and revert back to its fundamental value. The analogous pattern arises along a negative bubbly path.

The empirical implication of this bubbly equilibrium is that if one has time series data which contains information about the positions of the informed speculators, then he/she can estimate the switching times when the price is very likely to change direction by identifying times when such time series has structural breaks. In other words, the objective is to test whether there is a structural break within a given window of the time series, and estimate the break date when a break is detected.

1.2.2 Bubbly Equilibrium

Here, we characterize bubbly equilibrium paths along which the price of the risky asset follows either a strictly increasing or a strictly decreasing path during a period over which there are no shocks.

Definition 1.1 Suppose that in the time interval $\{t, ..., t'\}$, no saturation happens and $Z_j = \mu_Z$ for any $j \in \{t, ..., t'\}$. Then there is a positive bubbly path on $\{t, ..., t'\}$ if prices satisfy:

- 1. $P_j^* > P_{j'}^*$ for any $j, j' \in \{t, \dots, t'\}$ with j > j';
- 2. There exists at least one $j \in \{t + 1, \dots, t'\}$ such that $\Delta P_j^* > \Delta P_{j-1}^*$.

Similarly, there is a negative bubbly path on $\{t, \ldots, t'\}$ if prices satisfy

- 3. $P_j^* < P_{j'}^*$ for any $j, j' \in \{t, \dots, t'\}$ with j > j';
- 4. There exists at least one $j \in \{t + 1, \dots, t'\}$ such that $\Delta P_j^* < \Delta P_{j-1}^*$.

We say that there exists a bubbly path if there is either a positive bubbly path or a negative bubbly path.

The uncertainty in this economy comes from the random shocks ζ_t to the fundamentals, and the possibility of saturation for momentum speculators. The key endogenous variables are the equilibrium price P_t^* , and the binary variable S_t^* indicating whether there is saturation at time t. The demands of the risky asset can then be calculated using these two variables together with equations (1), (3) and (4).

To derive the equilibrium price, notice that the market-clearing condition in nonsaturation periods is

$$0 = \theta(P_t - P_{t-1}) + \phi(Z_t - P_t) + C_t(\mathbf{E}_t[\Delta P_{t+1}]).$$
(7)

It follows that the equilibrium price P_t^* satisfies:

$$P_t^* = \frac{\theta P_{t-1}^* - \phi Z_t - C_t(\mathbf{E}_t[\Delta P_{t+1}^*])}{\theta - \phi} I\{S_t^* = 0\} + Z_t I\{S_t^* = 1\},\tag{8}$$

where $I\{\cdot\}$ is the indicator function.

The following Lemma says that in equilibrium, a bubbly path exists only if the demand of speculators is strong enough.

Lemma 1.1 An equilibrium bubbly path exists only if the momentum speculator's demand is more sensitive to current prices than the hedgers' demand. That is:

$$\theta > \phi. \tag{9}$$

Proof. By definition, along a positive bubbly path, $\Delta P_t^* > 0$. As a result, price dynamics are given by:

$$\Delta P_t^* = \frac{\phi \cdot (P_{t-1}^* - \mu_z) - c}{\theta - \phi} = \frac{\phi}{\theta - \phi} P_{t-1}^* - \frac{\phi \mu_Z + c}{\theta - \phi},$$
(10)

which further implies that

$$\Delta P_t^* - \Delta P_{t-1}^* = \frac{\phi}{\theta - \phi} \Delta P_{t-1}^*.$$
(11)

Suppose that t is a period such that $\Delta P_t^* > \Delta P_{t-1}^*$. Then equation (11) implies that $\frac{\phi}{\theta-\phi}\Delta P_{t-1}^* > 0$, which together with $\Delta P_{t-1}^* > 0$ and $\phi > 0$ immediately proves the claim. The proof for the negative bubbly path is the same and hence omitted.

Restriction (9) will be key to characterizing the equilibrium paths, and so we will impose this condition throughout the rest of the paper. It is clear that under condition (9), $\Delta P_j^* > \Delta P_{j-1}^*$ along the positive bubbly path and $\Delta P_j^* < \Delta P_{j-1}^*$ along the negative bubbly path, which means that P_t^* is an explosive process along the bubbly path.

To see why (9) is necessary for a bubbly path consider first a positive bubbly path. It follows from (10) that along a bubbly path.

If
$$\theta > \phi$$
, then $\Delta P_t > 0 \iff P_{t-1} > \mu_z + \frac{c}{\phi}$. (12)

Clearly, if $\Delta P_t > 0$ and $E_t (\Delta P_{t+1}) > 0$, then (12) implies that ΔP_{t+1} is positive for any $E_{t+1} (\Delta P_{t+2})$. A positive ΔP_{t+1} in turn implies a positive ΔP_{t+2} and so on. In contrast, if $\theta < \phi$, a bubbly path cannot exist

If
$$\theta < \phi$$
, then $\Delta P_t > 0 \iff P_{t-1} < \mu_z + \frac{c}{\phi}$. (13)

If $\Delta P_t > 0$, there is a time such that the last inequality in (13) is violated. The same argument shows that a negative bubbly path exists only if $\theta > \phi$.

Next, we derive an equilibrium positive bubbly path. That is, we need to verify that along the path where Z_t is constant, there is a sequence of increasing market clearing prices and sequences of market participants' positions that are in turn best responses to past and future returns generated by those price sequences. In particular, we characterize the conditions under which starting with $P_0 = \mu_z$, a transitory fundamental shock can induce a bubbly path. Concretely, $Z_1 = \mu_z + z$, and $Z_t = \mu_z$ for t > 1.

The next Proposition characterizes such a bubbly path where informed speculators find it optimal to ride the bubble for a period of time: at t they choose $C_t = +c$ expecting $E_t(\Delta P_{t+1}) > 0$, at t + 1 they set $C_{t+1} = +c$ expecting $E_{t+1}(\Delta P_{t+2}) > 0$, and so on until a time τ^* , which we call switching time. This switching time will typically occur before saturation time.

Proposition 1.2 (Bubbly Equilibrium) Suppose that

$$z > \frac{2c}{\phi} \quad and \quad \overline{\sigma} > \frac{\phi}{\theta}$$
 (14)

and moreover

$$\frac{z}{(1-\phi/\theta)^{\tau_*}} + \frac{c}{\phi} > \vartheta \tag{15}$$

where is defined below in (16). Then a positive bubbly equilibrium path exists.

- At time 0, the equilibrium price P^{*}₀ = μ_z and the informed speculators' demand is C^{*}₀ = 0.
- A transitory shock $\zeta_1 = z$, induces a positive bubbly path starting at t = 1.
- Informed speculators choose $C_t^* = c$ up to a switching time τ^* , where $\tau^* = \tau_* + 1$ and

$$\tau_* = \max\left\{t \ge 2: \left|\frac{z}{(1-\phi/\theta)^{t-1}} + \frac{1+\phi/\theta}{1-\phi/\theta}\frac{c}{\phi}\right| < \vartheta\right\}.$$
(16)

Moreover, the price P_t^* satisfies

$$P_t^* = \mu_z + \frac{c}{\phi} + \frac{z}{(1 - \phi/\theta)^{t-1}}, \text{ for } 1 \le t < \tau^*.$$
(17)

• At switching time τ^* , informed speculators revert their positions and choose $C^*_{\tau^*} = -c$. The price is

$$P_{\tau^*}^* = \mu_z + \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta} + \frac{z}{(1 - \phi/\theta)^{\tau^* - 1}}.$$
(18)

• After τ^* , informed speculators choose $C_t^* = -c$ and the price P_t^* satisfies

$$P_t^* = \mu_z + \frac{c}{\phi} \left[\frac{2}{(1 - \phi/\theta)^{t - \tau^* + 1}} - 1 \right] + \frac{z}{(1 - \phi/\theta)^{t - 1}}, \text{ for } \tau^* < t < s^*.$$
(19)

- At s^{*}, the equilibrium price P^{*}_{s^{*}} = μ_z and the informed speculators' demand is
 C^{*}_{s^{*}} = 0.
- Momentum speculators choose long positions from period t = 2 up to saturation time s*.

The proof of Proposition 1.2 is in the Appendix.

Remark 2.1. This proposition makes two key points that will be exploited by our estimation strategy. First, initially informed speculators find it optimal to choose long positions and ride the positive bubble. Second, the equilibrium price necessarily increases at the time when saturation becomes imminent and informed speculators close their long positions. Thus, backwards induction does not unravel all the way to initial time. The intuition is the following. Even though the price is above its fundamental value, initially informed speculators find it optimal to ride the positive bubble and choose long positions. As the price gets farther away from the mean of the fundamental value μ_z , there is an increase in the probability of saturation $\overline{\sigma}$, as well as the size of the potential price fall $\mu_z - P_t$. Because informed speculators observe these events, there is a "switching time" when they rationally close their long positions and establish short positions. At this switching time, the price increases despite the fact that informed speculators go short. In response to the price increase, momentum speculators choose long positions up to saturation time, when the price falls to μ_z . This result is generated by the requirement that the sensitivity of momentum specs demand be large $\theta > \phi$, which is a necessary condition for the existence of a bubbly equilibrium.

Remark 2.2. The first restriction $z > 2c/\phi$ in (14) requires that the magnitude of the shock to fundamentals be large. The second restriction $\overline{\sigma} > \phi \theta^{-1}$ in (14) requires that the probability of saturation be large enough. The restriction in (15) ensures that the switching time τ_* is unique. When this condition is not satisfied, we can show that τ_* lies in some finite interval and the dynamics of the equilibrium price P_t^* still satisfy (17), (18) and (19).

Remark 2.3. Because $\theta > \phi$, from (17) we see that the equilibrium price P_t^* for

 $t < \tau^*$ is strictly increasing because

$$\Delta P_t^* = \frac{z}{(1 - \phi/\theta)^{t-1}} \frac{\phi}{\theta} > 0.$$

$$\tag{20}$$

Moreover, we have

$$\frac{\Delta P_t^*}{\Delta P_{t-1}^*} = \frac{1}{1 - \phi/\theta} > 1, \tag{21}$$

which implies that the increment of the equilibrium price along the bubbly path is explosive.

Remark 2.4. At switching time, we have

$$\Delta P_{\tau^*}^* = \frac{2c}{\phi} \frac{\phi/\theta}{1 - \phi/\theta} + \frac{z}{(1 - \phi/\theta)^{\tau^* - 1}} \frac{\phi}{\theta}$$
(22)

and

$$\frac{\Delta P_{\tau^*}^*}{\Delta P_{\tau^*-1}^*} = \frac{1}{1 - \phi/\theta} + \frac{c\theta}{\phi^2 z} \frac{\phi/\theta}{(1 - \phi/\theta)^{\tau^*-1}}.$$
(23)

Comparing equations (20) with (22), we can see that there is a jump in the equilibrium price change at switching time τ^* . Similarly, a jump happens on the ratio of the changes of equilibrium price at time τ^* .

Remark 2.5. After the switching time, we have

$$\Delta P_t^* = \frac{2c}{\phi} \frac{\phi/\theta}{(1 - \phi/\theta)^{t - \tau^* + 1}} + \frac{z\phi/\theta}{(1 - \phi/\theta)^{t - 1}}$$
(24)

and

$$\frac{\Delta P_t^*}{\Delta P_{t-1}^*} = \frac{1}{1 - \phi/\theta}.$$
(25)

It is interesting to see that the jump in the ratio of the changes of equilibrium price is temporary, because it comes back to the level before the switching time τ^* .

1.2.3 Negative Bubbly Path

An analogous result to that in the Proposition 1.2 applies to a negative bubbly path. In particular, we characterize the conditions under which starting with $P_0 = \mu_z$, a transitory fundamental shock can start a negative bubbly path. Concretely, $Z_1 = \mu_z - z$, and $Z_t = \mu_z$ for t > 1.

Along the negative bubbly path, informed speculators anticipate continuation of negative bubble and find it optimal to ride the bubble for a period of time: they choose $C_t = -c$ expecting $E_t (\Delta P_{t+1}) < 0$, at t + 1, and informed speculators set $C_{t+1} = -c$ expecting $E_{t+1} (\Delta P_{t+2}) < 0$, and so on until a time (which we call switching time). To ensure the negative bubbly path may arise, we impose

$$\mu_z > z + \underline{P} + \frac{c}{\theta - \phi},\tag{26}$$

so that $P_1 = \mu_Z - z - \frac{c}{\theta - \phi} > \underline{P} > 0.$

Corollary 1.3 (Negative Bubbly Equilibrium) Suppose that

$$z > \frac{2c}{\phi}, \quad \overline{\sigma} > \frac{\phi}{\theta} \quad and \quad \left| \frac{z}{(1 - \phi/\theta)^{\tau_*}} + \frac{c}{\phi} \right| > \vartheta$$
 (27)

where τ_* is defined in (29). Then:

- At time 0, the equilibrium price P^{*}₀ = μ_z and the informed speculators' demand is C^{*}₀ = 0.
- A transitory shock $\zeta_1 = -z$, induces a positive bubbly path starting at t = 1.
- Informed speculators choose $C_t^* = -c$ up to a switching time τ^* , where

$$\tau^* = \min\left\{\tau_*, \underline{\tau}\right\} + 1 \tag{28}$$

The switching time τ_* is related to the postive probability of saturation,

$$\tau_* = \max\left\{t \ge 2: \left|\frac{z}{(1-\phi/\theta)^{t-1}} + \frac{1+\phi/\theta}{1-\phi/\theta}\frac{c}{\phi}\right| < \vartheta\right\},\tag{29}$$

The switching time $\underline{\tau}$ is when informed speculators close their position because the price P_t^* is close enough to the positive lower bound \underline{P} .

$$\underline{\tau} = \max\left\{t \ge 2: \mu_z - \frac{z}{(1 - \phi/\theta)^{t-1}} - \frac{1 + \phi/\theta}{1 - \phi/\theta}\frac{c}{\phi} > \underline{P}\right\},\tag{30}$$

The price P_t^* satisfies

$$P_t^* = \mu_z - \frac{c}{\phi} - \frac{z}{(1 - \phi/\theta)^{t-1}}, \text{ for } 1 \le t < \tau^*.$$
(31)

• At time τ^* , informed speculators choose either $C^*_{\tau^*} = c$ and the price $P^*_{\tau^*}$ satisfies

$$P_{\tau^*}^* = \mu_z - \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta} - \frac{z}{(1 - \phi/\theta)^{\tau^* - 1}}.$$
(32)

- If the price reaches the lower bound (P^{*}_{τ*} ≤ <u>P</u>), then saturation happens for sure. Thus At time s^{*} = τ^{*} + 1, the equilibrium price P^{*}_{s*} = μ_z and the informed speculators' demand is C^{*}_{s*} = 0.
- After time τ^* , informed speculators choose $C_t^* = c$ and the price P_t^* satisfies

$$P_t^* = \mu_z - \frac{c}{\phi} \left[\frac{2}{(1 - \phi/\theta)^{t - \tau^* + 1}} - 1 \right] - \frac{z}{(1 - \phi/\theta)^{t - 1}}, \text{ for } \tau^* < t < s^*.$$
(33)

• Time s^* can be either the realization of saturation or the price reaches the lower bound in the previous period $P^*_{s^*-1} \leq \underline{P}$.

- At s^{*}, the equilibrium price P^{*}_{s^{*}} = μ_z and the informed speculators' demand is
 C^{*}_{s^{*}} = 0.
- Momentum speculators choose short positions from period t = 2 up to saturation time s*.

The proof of Corollary 1.3 is almost identical to the proof of Proposition 1.2. The only difference is the possibility that the price may be lower than the lower bound \underline{P} . If this were to occur, the equilibrium price would jump immediately to the fundamental value. We can choose the mean of the fundamental μ_z high enough so that we rarely encounter this situation.

1.3 Empirical Implications of the Model

Recall that informed speculators observe the probability of saturation σ_t and the size of the potential price fall $\mu_z - P_t$. Based on this information they choose their position q_t^I . Along a positive equilibrium bubbly path, when the price gap $P_t - \mu_z$ and the probability of saturation σ_t become large enough, informed speculators close their long positions and go short.

Therefore, if our data contains information about the informed speculators' positions, the equilibrium of the model implies that the econometrician can backup useful information about an imminent expected price reversal by estimating dates when the time series data has structural changes. Based on these estimates one can make forecasts of future price changes.

Suppose that we observe the following demand for the risky asset Y_t :

$$Y_t = q_t^I + q_t^m + e_t,$$

where q_t^I , q_t^m and e_t are the demands of informed speculators, momentum speculators and some noisy traders respectively. From the theoretical model, we can further write

$$Y_t = c_t + \theta_t \Delta P_t + e_t,$$

where $c_t = -c$, 0 or c, $\theta_t = 0$ or θ , where c and θ are unknown parameters. We have data $\{Y_t, \Delta P_t\}_{t=1}^T$ and we would like to test whether c_t and/or θ_t are constant from 1 to T. If our test shows that there is a break in c_t or θ_t , we would also like to estimate the break date.

Let T_0 be a conjectured break date. Then we can divide the full sample into two subsamples: $\{Y_t, \Delta P_t\}_{t=1}^{T_0}$ and $\{Y_t, \Delta P_t\}_{t=T_0+1}^{T}$, which are from two possibly different models.

$$Y_t = \begin{cases} c_1 + \theta_1 \Delta P_t + e_t, & 1 \le t \le T_0 \\ c_2 + \theta_2 \Delta P_t + e_t, & T_0 + 1 \le t \le T \end{cases}$$

Our null hypothesis is

$$H_0: c_1 = c_2 \text{ and } \theta_1 = \theta_2. \tag{34}$$

Under the null, the two models are the same. So we can use the two subsamples to estimate (c_1, θ_1) and (c_2, θ_2) respectively and then construct the Wald statistic for testing the null. Define

$$\begin{pmatrix} \widehat{c}_{1,T_a} \\ \widehat{\theta}_{1,T_b} \end{pmatrix} = \begin{pmatrix} T_0 & \sum_{t=1}^{T_0} \Delta P_t \\ \sum_{t=1}^{T_0} \Delta P_t & \sum_{t=1}^{T_0} (\Delta P_t)^2 \end{pmatrix}^{-1} \begin{pmatrix} \sum_{t=1}^{T_0} Y_t \\ \sum_{t=1}^{T_0} \Delta P_t Y_t \end{pmatrix}$$

and

$$\begin{pmatrix} \widehat{c}_{2,T_b} \\ \widehat{\theta}_{2,T_b} \end{pmatrix} = \begin{pmatrix} T - T_0 & \sum_{t=T_0+1}^T \Delta P_t \\ \sum_{t=T_0+1}^T \Delta P_t & \sum_{t=T_0+1}^T (\Delta P_t)^2 \end{pmatrix}^{-1} \begin{pmatrix} \sum_{t=T_0+1}^T Y_t \\ \sum_{t=T_0+1}^T \Delta P_t Y_t \end{pmatrix}.$$

Then the Wald statistics for testing H_0 against $H_1(T_0)$ is given by

$$L_T(T_0) = T \left(\begin{array}{c} \widehat{c}_{1,T_a} - \widehat{c}_{2,T_b} \\ \widehat{\theta}_{1,T_b} - \widehat{\theta}_{2,T_b} \end{array} \right)' \left(\widehat{W}_1 \frac{T}{T_0} + \widehat{W}_2 \frac{T}{T - T_0} \right)^{-1} \left(\begin{array}{c} \widehat{c}_{1,T_a} - \widehat{c}_{2,T_b} \\ \widehat{\theta}_{1,T_b} - \widehat{\theta}_{2,T_b} \end{array} \right),$$

where \widehat{W}_1 and \widehat{W}_2 are the estimators of the variance-covariance matrixes of $\begin{pmatrix} \widehat{c}_{1,T_a} \\ \widehat{\theta}_{1,T_b} \end{pmatrix}$

and
$$\begin{pmatrix} \widehat{c}_{2,T_b} \\ \widehat{\theta}_{2,T_b} \end{pmatrix}$$
, respectively. Based on $L_T(T_0)$, the following test statistic can be used

$$\sup_{T_0\in\left[\underline{T_0},\overline{T_0}\right]}L_T(T_0),$$

where $\underline{T}_0 < \overline{T}_0$ are between 1 and T. As a rule of thumb, we set $\underline{T}_0 = 0.15T$ and $\overline{T}_0 = 0.85T$. Andrews (1993) reports the asymptotic critical values c_{α} at several confidence levels for the test statistics. While exchange rate changes can be non-stationary along the bubbly path from the model, Andrews (1993) assume that the regressors are stationary. To remedy that, we use a 'fixed regressor bootstrap' method proposed by Hansen (2000). Thus, our estimator of the break date is, if exists,

$$\widehat{T}_0 = \underset{T_0 \in [\underline{T}_0, \overline{T}_0]}{\operatorname{argsup}} L_T(T_0) \quad \text{s.t.} \quad L_T(T_0) > c_\alpha$$

Given the estimated break date (if one exists) and the positions of informed spec-

ulators, we can form our directional forecast using the following rule:

$$sgn(\widehat{P}_{T+h} - P_T) = \begin{cases} +, & \text{if } P_{\widehat{T}_0} - P_1 < 0 & \text{and} & \widehat{c}_{2,T_b} > 0 \\ -, & \text{if } P_{\widehat{T}_0} - P_1 > 0 & \text{and} & \widehat{c}_{2,T_b} < 0, \end{cases}$$

where the difference $P_{\widehat{T}_0} - P_1$ is used to determine if we are on the positive bubbly path or the negative bubbly path and \widehat{c}_{2,T_b} determines the position of informed speculators after the estimated break date \widehat{T}_0 .

1.3.1 Data

We use the net speculators position data of the Commitments-of-traders (COT) report from the Commodity Futures Trading Commission (CFTC). The CFTC requires all large traders to identify themselves as commercial or non-commercial. In the literature, commercial and non-commercial traders are considered hedgers and speculators, respectively. We construct our variable Y_t by considering the net (long minus short) futures position of non-commercial traders in a foreign currency, expressed as a fraction of open interest (the total number of open future contracts). The CFTC typically reports the positions as of the close of each Tuesday and releases them three days later, on Friday. To be consistent with the released date, we use weekly Friday spot exchange rates, released by the Federal Reserved Board. We collect weekly data for six major foreign currencies, the Australian dollar (AUD), the Canadian Dollar (CAD), the euro (EUR), the Japanese yen (JPY), the British pound (GBP), and the Swiss franc (CHF) from October 1992 (January 1999 for Euro) to August 2015.

1.3.2 Estimation of the Structural Break Model

For each of the six currencies, we use a rolling window regression with a window size T = 120 weeks² to estimate structural break on COT data after controlling the effect of the past change in exchange rate. As we use a rolling window, each week we add a new observation and drop the first observation in the previous sample.

Each vertical black line on Figure 1 is where a structural break is estimated. We consider exchange rate will appreciate (depreciate) if the COT is positive (negative) when a structural break is detected. Across the six currencies, structural breaks are estimated about 75-95 percent of periods by the 'fixed regressor bootstrap' method proposed by Hansen (2000). We can commonly observe that the same break dates are estimated over the following different rolling windows. Figure 2 shows the profitability of the directional forecasts based on estimated structural breaks. If there is a signal which implies that a structural break is detected, and the sign of the COT is considered, we take a long or short position on the foreign currency until another signal is detected or no structural break is estimated. The average path of return on the directional forecasts is positive across all the six currencies.

1.3.3 Point Forecasts

From the model, estimated $c_{2,t}$ is current position of informed speculators, so it contains useful information of the future exchange rate changes. Figure 3 in the appendix plot the estimated coefficients $c_{1,t}$ and $c_{2,t}$. We evaluate the out-of-sample predictive ability of the exchange rate models. One standard way to do so is to compare the mean squared prediction errors (henceforth, MSPE) of the exchange rate models with the benchmark random walk model of no predictability. The model under null is a

 $^{^{2}}$ Estimation results of structural breaks are quite robust with respect to the size of the rolling window ranging from 80 to 150 observations

zero mean martingale difference process.

Random walk: $P_{t+h} - P_t = e_{t+h}$ Fundamental Model: $P_{t+h} - P_t = \beta f_t + e_{t+h}$

Here, we use estimated informed position as the fundamental f_t . Thus,

$$f_t = \hat{c}_{2,t}$$
 or $f_t = \{\hat{c}_{1,t}, \hat{c}_{2,t}\}$

The hypothesis that we test is that the MSPEs are equal under the null against the alternative that the MSPE of the fundamental model is lower than the MSPE of the random walk. The random walk model is nested in the model under alternative, so we use the Clark and West test statistic (henceforth CW) as proposed by Clark and West (2006) for evaluating the predictability of the exchange rate models. The CW statistic adjusts for the bias in MSPE comparison for the Diebold and Mariano (1995) and West (1996) (DMW) test statistic when the model under alternative nests the model under null.

Table 3 contains the CW test statistics, and their p-values for the 6 currencies and the 5 horizons. The null is rejected if the CW test statistic is significantly greater than zero. For the one-sided test we consider, a t-value greater than 1.282 implies a 0.1 significance level. After controlling for auto-correlation using the Newey-West LRV estimator, we can see in Table 3 that the null is strongly rejected in all currency pairs, over all forecasting horizons.

1.3.4 Directional Forecasts

We construct the exchange rate forecasts in the principle contrary opinion manner. We forecast appreciation at time t if we observe exchange rate depreciates before the structural break and informed speculators take long positions after the break. Conversely, We forecast depreciation at time t if we observe exchange rate appreciates before the structural break and informed speculators take short positions after the break. Thus, our directional exchange rate forecasts are:

$$D_{t,h} = \begin{cases} 1, & P_{\hat{t}_0} - P_{t-T} < 0 \text{ and } \hat{c}_{2,t_b} > 0 \\ -1, & P_{\hat{t}_0} - P_{t-T} > 0 \text{ and } \hat{c}_{2,t_b} < 0 \\ 0 & \text{Otherwise} \end{cases}$$
(35)

For each week, we generate out-of-sample exchange rate directional forecasts. Table 1 shows the forecast success ratio of our out-of-sample directional forecasts at the five forecasting horizon; 1m, 3m, 6m, 9m and 12m. The forecast success ratio is the number of successful appreciation or depreciation forecasts divided by the total number of appreciation or depreciation forecasts have made. The overall forecast success ratio is 59.5 percent between October 1992 and August 2015.

1.3.5 Weighted Directional Test

Kim et al. (2014) propose the weighted directional test, which consider the profitability of trading strategies. If the exchange rate follows the random walk, then no market participants can make expected profit, so it must be zero. We consider the following test statistic.

$$T_{a,n} = \frac{1}{n_1} \sum_{t=n_0}^{n-h} D_{t,h} (P_{t+h} - P_t), \text{ where } n_1 \equiv n - n_0 - h + 1.$$
(36)

where n is the total number of sample. As we need n_0 observations to construct the initial window, n_1 is the number of periods making forecasts. Under the random walk model, the optimal forecast is a zero exchange rate change. The null hypothesis underlying the test statistic $T_{a,n}$ can be specified as

$$H_0: \quad E[D_{t,h}(P_{t+h} - P_t)] = 0 \quad \text{for any } i \text{ and } t.$$
(37)

That is, under the null our directional forecasts are uncorrelated with future realized exchange rate changes. In order to test null hypothesis (37), let $V_{T_{a,n}}$ denote the consistent estimator of the asymptotic variance of $T_{a,n}$. Then by Slutsky's theorem and the martingale central limit theorem, we deduce that

$$\sqrt{n_1} V_{T_{a,n}}^{-1/2} T_{a,n} \to_d N(0,1).$$
(38)

Table 4 present the values of the $T_{a,n}$ statistic, and its t-values (test statistics), for the 6 currencies and the 5 horizons we consider. The null (37) tested is that our directional forecasts are uncorrelated with future realized exchange rate changes. The null is rejected if the test statistic $T_{a,n}$ is significantly larger than zero. For the one-sided test, a *t*-value greater than 1.282 implies a 0.1 significance level.

As you can see in panel B of Table 4, using the Newey-West LRV estimator, 22 out of 30 currency-horizon pairs are statistically significant. At the 6-month horizon, the weighted directional test rejects the random-walk null in favour of our model across all currencies, except the Canadian Dollar. The same holds true at the 9-month horizon (except for the British Pound) and at the 12-month horizon (except for the British Pound and the Australian Dollar). At the 1-month and 3-month horizons, the null is rejected in 7 out of 12 currency-horizon pairs. The high forecast success ratios in Table 2 translate into strong predictability of future exchange rate changes using our directional forecasts. However, they are not the same. Even though the success ratio of the Swiss Franc at 12 month horizon is slightly above 50 percents, it reject the null of the weighted directional test at 1 percent level even after controlling the autocorrelation.

1.3.6 Binomial Test

Here, we test the significance of our model in forecasting the sign of $P_{t+h} - P_t$. So this test is linked to the forecast success ratio. When the exchange rate is a driftless random walk, we define a new dummy variable $R_{t,h}$ that captures the direction of the realized exchange rate change over horizon h:

$$R_{t,h} = \begin{cases} 1, & \text{if } P_{t+h} - P_t \ge 0\\ -1, & \text{if } P_{t+h} - P_t < 0 \end{cases}$$
(39)

The null hypothesis we test is that our directional forecasts $D_{t,h}$ are uncorrelated with the future direction of the exchange rate $R_{t,h}$.

$$H_0: \quad Cov \left(D_{t,h}, R_{t,h} \right) = 0, \tag{40}$$

Consider then the following test statistics.

$$T_{b,n} = \frac{1}{n_1} \sum_{t=n_0}^{n-h} D_{t,h} R_{t,h} - \frac{1}{n_1} \sum_{t=n_0}^{n-h} D_{t,h} \frac{1}{n_1} \sum_{t=n_0}^{n-h} R_{t,h},$$

which is the sample covariance of the two random variables: $D_{t,h}$ and $R_{t,h}$. In order to test null hypothesis (40), let $V_{T_{b,n}}$ denote the consistent estimator of the asymptotic
variance of $T_{b,n}$. Then we have

$$\sqrt{n_1} V_{T_{b,n}}^{-\frac{1}{2}} T_{b,n} \to_d N(0,1).$$

Table 5 reports the test results using the sample variance (Panel A) and Newey-West LRV estimator (Panel B) to control for auto-correlation. The null is rejected if the t-value of the $T_{b,n}$ statistic is positive and statistically significant. For the one-sided test, a *t*-value greater than 1.282 implies a 0.1 significance level.

As you can see in panel B of Table 5, using the Newey-West LRV estimator, 18 out of 30 currency-horizon pairs are statistically significant. At the 9-month horizon, the binomial test rejects the random-walk null in favour of our model across all currencies. The results are exceptionally good for Australian Dollar, Euro and Japanese Yen. The null is rejected at all 5 forecasting horizons.

1.4 Related Literature

There is a large literature on exchange rate forecasting. Recent surveys include Cheung et al. (2005), Rogoff and Stavrakeva (2008) and Rossi (2013). Since ?Meese and Rogoff (1983b), it has been found that fundamentals-based forecasting models do not have a better ability to forecast exchange rates than the random walk over periods of less than 12 months. Mark (1995) reports success over longer horizons. Recently, several papers have reported somewhat positive short-term forecasting results: Engel et al. (2007), Gourinchas and Rey (2007), and Molodtsova and Papell (2009). In contrast to most of the literature this paper looks at speculators position data rather than fundamentals and uses a non-linear forecasting method. In this sense, this paper is linked to Kim et al. (2014) (hereafter KLT). KLT assume that the exchange rate follows a Markov-switching process, and derive an AR(1) Markov-switching model to extract the information in the speculators' positions data. In contrast, here the exchange rate is endogenously determined. Our goal is to write down a micro-founded model that generates the patterns of speculators' positions and exchange rates observed in the data. Furthermore, our forecasting method is based on identifying structural breaks in the joint process followed by speculators' positions and exchange rates.

The tests proposed by Diebold and Mariano (1995) and West (1996) as well as Clark and West (2006) tests are commonly used to test the accuracy of point forecasts and the linear relationship between exchange rate changes and macro fundamentals. In this paper, our focus is on directional forecasts, and so we conduct the familiar binomial test based on the forecast success ratio, as well as the KLT test that weights each directional forecast by the subsequent exchange rate change. The latter test captures the profitability of our directional forecasts and in this sense has the same spirit as the Diebold-Mariano test, which is based on the mean-squared errors of the predictions.

The pioneering paper by Evans and Lyons (2002) finds that the daily differences between buyer- and seller-initiated order flow capture roughly 45 to 65 percent of contemporaneous daily exchange rate movements for the Deutsche Mark and the Japanese Yen. Evans and Lyons (2005) compare forecasting performance of a microbased model against a standard macro model and a random walk and find that the micro-based model using order flow has significant forecasting power at up to 1-month horizon. While the order flow data they consider is private information, the COT data we use is public information.

There is vast empirical literature that documents the short-run positive and longrun negative autocorrelation of returns observed across different asset classes. A contribution of this paper is to identify switching times when such correlations are likely to change sign. Jegadeesh and Titman (1993), Badrinath and Wahal (2002), Brunnermeier and Nagel (2004), Greenwood and Shleifer (2014), Dasgupta et al. (2011b) focus on stock market returns. Cutler et al. (1990), Brunnermeier et al. (2008), Menkhoff et al. (2012), Moskowitz et al. (2012), Asness et al. (2013) find on currency market returns.

In proposing a model where rational investors can reap gains from riding a bubble at the expense of less informed investors, our paper is related to Cutler et al. (1980), De Long et al. (1990), Abreu and Brunnermeier (2003), Brunnermeier and Pedersen (2009), Dasgupta et al. (2011a), Barberis et al. (2013) and Barberis et al. (2015).

1.5 Conclusion

We have found that publicly available speculator position data contains useful information to forecast exchange rates. In contrast to much of the literature, our forecasts are non-linear as they are based on structural breaks in the exchange rates and speculators positions series.

Our forecast strategy is based on what may be termed "the principle of contrarian opinion," which states that when a group of speculators tend to agree that the price will go up(down), it is very likely they are wrong, and the price will soon start a declining(increasing) path.

The contribution of this paper is twofold. First, we present a minimal model where prices are endogenous and where this puzzling pattern occurs in equilibrium. Second, we propose a forecasting strategy that estimation the switching times when a group of informed forward-looking speculators switch their positions, signalling the imminent end of the bubble. We have shown that our forecast strategy beats random-walk forecasts.

1.6 Appendix

1.6.1 Figures



Figure 1: Estimated Structural Breaks

Notes: This figure plots the estimated structural breaks based on the model, exchange rates, and directional forecasts between October 1992 and August 2015. When the directional forecast is 1 (-1), we can expect exchange rate will appreciate (depreciate) in near future.



Figure 2: Profitability of the Directional Forecasts Based on Estimated Structural Breaks

Notes: Each red lines depict return of directional forecasts based on estimated structural breaks. We start to make forecasts when there are structural breaks and close the position when there is another estimated structural break or no break. The black line with circles is the average path of return, and the black dashed lines are one standard deviation band.



Figure 3: Estimated $c_{1,t}$ and $c_{2,t}$ with the Rolling Windows

Notes: Blue lines depict estimated $\hat{c}_{2,t}$ and red dashed lines depict estimated $\hat{c}_{1,t}$ from the structural break model from October 1992 to August 2015. Rolling window require initial 120 observations to start estimation.



Figure 4: Out-of-sample Directional Forecasts

Notes: This figure plots the exchange rates and directional forecasts between October, 1992 and August, 2015.



Figure 5: The Performance of Our 1 Month Ahead Directional Forecasts

Notes: The dark grey line depicts the exchange rates. Blue circle means success of appreciation forecast and light blue triangle means failure of appreciation forecast. Red circle means success of depreciation forecast and orange triangle means failure of depreciation forecast.



Figure 6: The Performance of Our 9 Months Ahead Directional Forecasts

Notes: The dark grey line depicts the exchange rates. Blue circle means success of appreciation forecast and light blue triangle means failure of appreciation forecast. Red circle means success of depreciation forecast and orange triangle means failure of depreciation forecast.



Figure 7: Cumulative Forecast Success Ratio (h = 1m)

Notes: This figure plots the cumulative forecasting success ratio (black) and 5-year rolling window forecasting success ratio (blue dot). The forecast success ratio is defined the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts.



Figure 8: Cumulative Forecast Success Ratio (h = 3m)

Notes: This figure plots the cumulative forecasting success ratio (black) and 5-year rolling window forecasting success ratio (blue dot). The forecast success ratio is defined the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts.



Figure 9: Cumulative Forecast Success Ratio (h = 6m)

Notes: This figure plots the cumulative forecasting success ratio (black) and 5-year rolling window forecasting success ratio (blue dot). The forecast success ratio is defined the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts.



Figure 10: Cumulative Forecast Success Ratio (h = 9m)

Notes: This figure plots the cumulative forecasting success ratio (black) and 5-year rolling window forecasting success ratio (blue dot). The forecast success ratio is defined the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts.



Figure 11: Cumulative Forecast Success Ratio (h = 12m)

Notes: This figure plots the cumulative forecasting success ratio (black) and 5-year rolling window forecasting success ratio (blue dot). The forecast success ratio is defined the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts.

1.6.2 Tables

	Forecasting Horizon (h)					
Currency	1m	3m	6m	9m	12m	
AUD	0.599	0.603	0.664	0.663	0.628	
	(451)	(453)	(453)	(451)	(441)	
CAD	0.537	0.518	0.547	0.593	0.529	
	(475)	(477)	(477)	(477)	(467)	
EUR	0.546	0.537	0.596	0.591	0.595	
	(414)	(404)	(394)	(381)	(368)	
JPY	0.584	0.681	0.753	0.831	0.815	
	(433)	(433)	(433)	(433)	(432)	
GBP	0.551	0.523	0.511	0.527	0.504	
	(481)	(474)	(462)	(455)	(454)	
CHF	0.534	0.513	0.544	0.529	0.526	
	(584)	(575)	(563)	(550)	(549)	

Table 2: Success Ratio of the Directional Forecasts

Notes: 1. The forecast success ratio is the ratio of the number of successful appreciation and depreciation forecasts divided by the total number of appreciation and depreciation forecasts. 2. The total number of appreciation and depreciation forecasts is in parentheses. 3. Our sample starts on 10/02/1992 for all currencies except the Euro, which starts on 01/08/1999. Our sample ends on 08/14/2015 for all currencies.

	Forecasting Horizon (h)					
Currency	1m	3m	6m	9m	12m	
	Panel A: $\hat{c}_{2,t}$ only					
AUD	1.593^{*}	2.551^{***}	3.299***	4.112***	5.190***	
	(0.056)	(0.005)	(0.000)	(0.000)	(0.000)	
CAD	3.090^{***}	4.002^{***}	4.612^{***}	4.923^{***}	5.487^{***}	
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	
EUR	1.318^{*}	2.729^{***}	3.849^{***}	4.533^{***}	4.108^{***}	
	(0.094)	(0.003)	(0.000)	(0.000)	(0.000)	
JPY	1.882^{**}	2.853^{***}	4.067^{***}	5.328^{***}	5.890^{***}	
	(0.030)	(0.002)	(0.000)	(0.000)	(0.000)	
GBP	0.771	0.958	1.756^{**}	4.598^{***}	5.384^{***}	
	(0.220)	(0.169)	(0.040)	(0.000)	(0.000)	
CHF	1.511^{*}	2.709^{***}	4.231***	5.272^{***}	4.965^{***}	
	(0.065)	(0.003)	(0.000)	(0.000)	(0.000)	
		Par	nel B: $\hat{c}_{1,t}$ and	$\hat{c}_{2,t}$		
AUD	2.211**	3.352^{***}	3.975^{***}	4.416^{***}	5.563^{***}	
	(0.014)	(0.000)	(0.000)	(0.000)	(0.000)	
CAD	3.243^{***}	4.035^{***}	4.474^{***}	4.641^{***}	5.314^{***}	
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	
EUR	1.286^{*}	2.461^{***}	3.662^{***}	5.708^{***}	6.373^{***}	
	(0.099)	(0.007)	(0.000)	(0.000)	(0.000)	
JPY	3.227^{***}	4.081^{***}	4.866^{***}	6.979^{***}	8.672^{***}	
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	
GBP	1.792^{**}	1.469^{*}	2.162^{**}	4.221^{***}	3.984^{***}	
	(0.037)	(0.071)	(0.015)	(0.000)	(0.000)	
CHF	2.382^{***}	3.979^{***}	5.201^{***}	6.524^{***}	6.245^{***}	
	(0.009)	(0.000)	(0.000)	(0.000)	(0.000)	

Table 3: Clark and West (2006) Test

Note: 1. Panel A reports the CW test results using estimated $c_{2,t}$ as an independent variable. Panel B reports the CW test results using estimated $c_{1,t}$ and $c_{2,t}$. 2. All test results use Newey-West LRV estimators to control for auto-correlation. 3. p-values are in parentheses. We use the test as an one-sided test. Critical values from a standard normal distribution are used for inference. *, ** and *** indicate significance at 10, 5 and 1 percent, respectively.

	Forecasting Horizon (h)				
Currency	1m	3m	6m	9m	12m
	Panel A: No autocorrelation adjusted				
AUD	0.161***	0.511^{***}	0.996***	1.154^{***}	0.966***
	(2.823)	(4.841)	(6.010)	(5.147)	(3.561)
CAD	0.039	0.110^{*}	0.277^{***}	0.460***	0.522^{***}
	(1.097)	(1.542)	(2.654)	(3.532)	(3.788)
EUR	0.161^{***}	0.404^{***}	0.860***	1.166***	1.408***
	(2.445)	(3.441)	(4.954)	(5.099)	(5.129)
JPY	0.221***	0.832***	1.864***	2.414^{***}	2.596^{***}
	(4.124)	(7.779)	(12.986)	(15.108)	(14.132)
GBP	0.144^{***}	0.247^{***}	0.216^{*}	0.063	-0.011
	(2.858)	(2.620)	(1.617)	(0.448)	(-0.073)
CHF	0.111^{**}	0.365^{***}	0.711^{***}	1.134^{***}	1.342^{***}
	(1.661)	(3.556)	(5.026)	(6.420)	(6.330)
	Panel B: Newey-West				
AUD	0.161*	0.511^{**}	0.996***	1.154^{**}	0.966^{*}
	(1.484)	(1.987)	(2.333)	(2.007)	(1.372)
CAD	0.039	0.110	0.277	0.460^{*}	0.522^{*}
	(0.643)	(0.653)	(1.056)	(1.387)	(1.500)
EUR	0.161^{*}	0.404^{*}	0.860^{**}	1.166^{**}	1.408^{**}
	(1.390)	(1.474)	(2.044)	(2.084)	(2.088)
JPY	0.221^{**}	0.832^{***}	1.864^{***}	2.414^{***}	2.596^{***}
	(2.078)	(3.140)	(5.160)	(5.898)	(5.522)
GBP	0.144^{*}	0.247	0.216	0.063	-0.011
	(1.534)	(1.047)	(0.635)	(0.179)	(-0.030)
CHF	0.111	0.365^{*}	0.711^{**}	1.134^{***}	1.342^{***}
	(0.993)	(1.509)	(1.989)	(2.440)	(2.395)

Table 4: Weighted Directional Forecasts Test

Note: 1. Panel A reports the test results using sample variance, and B report the test results using Newey-West LRV estimators to control for auto-correlation. 2. t-values are in parentheses. We use the test as an one-sided test. Critical values from a standard normal distribution are used for inference. *, ** and *** indicate significance at 10, 5 and 1 percent, respectively. 3. The information on our directional forecasts is described in the notes to Table 1.

	Forecasting Horizon (h)					
Currency	1m	3m	6m	9m	12m	
	Panel A: No autocorrelation adjusted					
AUD	0.073***	0.069***	0.120^{***}	0.118^{***}	0.083***	
	(4.193)	(3.907)	(6.867)	(6.674)	(4.727)	
CAD	0.030^{**}	0.019	0.043^{***}	0.080^{***}	0.029^{*}	
	(1.649)	(1.017)	(2.327)	(4.264)	(1.529)	
EUR	0.044^{*}	0.031^{*}	0.083^{***}	0.071^{***}	0.071^{***}	
	(1.862)	(1.334)	(3.667)	(3.218)	(3.275)	
JPY	0.054^{***}	0.121^{***}	0.180^{***}	0.248^{***}	0.246^{***}	
	(3.322)	(7.521)	(11.397)	(16.232)	(15.780)	
GBP	0.042^{**}	0.020	0.013	0.025^{*}	0.005	
	(2.269)	(1.068)	(0.724)	(1.352)	(0.293)	
CHF	0.034^{*}	0.027^{*}	0.060^{***}	0.063^{***}	0.063^{***}	
	(1.786)	(1.391)	(3.162)	(3.315)	(3.308)	
		Par	nel B: Newey-	West		
AUD	0.073***	0.069^{*}	0.120***	0.118^{***}	0.083^{*}	
	(2.393)	(1.793)	(2.882)	(2.746)	(1.870)	
CAD	0.030	0.019	0.043	0.080^{**}	0.029	
	(1.075)	(0.473)	(1.016)	(1.847)	(0.640)	
EUR	0.044	0.031	0.083^{*}	0.071^{*}	0.071^{*}	
	(1.124)	(0.631)	(1.624)	(1.403)	(1.372)	
JPY	0.054^{**}	0.121^{***}	0.180^{***}	0.248^{***}	0.246^{***}	
	(2.012)	(3.727)	(5.451)	(7.193)	(6.786)	
GBP	0.042^{*}	0.020	0.013	0.025	0.005	
	(1.557)	(0.525)	(0.334)	(0.589)	(0.126)	
CHF	0.034	0.027	0.060^{*}	0.063^{*}	0.063^{*}	
	(1.101)	(0.653)	(1.393)	(1.396)	(1.378)	

Table 5: Binomial Directional Forecasts Test

Note: 1. Panel A report the test results using sample variance, and B report the test results using Newey-West LRV estimators to control for auto-correlation. 2. t-values are in parentheses. We use the test as an one-sided test. Critical values from a standard normal distribution are used for inference. *, ** and *** indicate significance at 10, 5 and 1 percent, respectively. 3. The information on our directional forecasts is described in the notes to Table 1.

1.6.3 Proof of Main Results

Proof of Proposition 1.2. In any period t > 1, when there is no saturation, the price P_t^* and the demand of the informed speculators C_t^* satisfy

$$\phi \left(Z_t - P_t^* \right) + \theta \left(P_t^* - P_{t-1}^* \right) + C_t^* = 0, \tag{41}$$

which implies that

$$P_{t}^{*} = \frac{\theta P_{t-1}^{*} - \phi Z_{t} - C_{t}^{*}}{\theta - \phi}$$
(42)

and hence

$$\Delta P_t^* = \frac{\phi(P_{t-1}^* - Z_t) - C_t^*}{\theta - \phi} = \frac{\phi(P_{t-1}^* - \mu_z) - C_t^*}{\theta - \phi} - \frac{\phi\zeta_t}{\theta - \phi}.$$
(43)

When there is saturation at period t, he price P_t^* and the demand of the informed speculators C_t^* satisfy

$$\phi \left(Z_t - P_t^* \right) + C_t^* = 0, \tag{44}$$

which implies that

$$P_t^* = Z_t + \frac{C_t^*}{\phi}.$$
 (45)

The above equations will be used extensively in the proof.

Step 0. We show that at period 0,

$$P_0^* = \mu_z \text{ and } C_0^* = 0.$$
 (46)

At period 0, the momentum speculator's demand is zero. From (44), we have

$$P_0^* = \mu_z + \frac{C_0^*}{\phi}.$$
 (47)

By the definition of saturation, the momentum speculator's demand at period 1 is also zero. So the market clearing condition at period 1 implies

$$P_1^* = Z_1 + \frac{C_1^*}{\phi} \tag{48}$$

which combined with (47) yields

$$\Delta P_1^* = \zeta_1 + \frac{C_1^* - C_0^*}{\phi}.$$
(49)

Suppose that $C_0^* = c$, then by (49) we know that

$$\zeta_1 + \frac{C_1^* - c}{\phi} \le \zeta_1 \text{ for any } C_1^* \in \{-c, 0, c\},\$$

which implies that $E_0[\Delta P_1^*] \leq 0$. This contradicts the demand rule of the informed speculators. Next, suppose that $C_0^* = -c$, then by (49) we know that

$$\zeta_1 + \frac{C_1^* + c}{\phi} \ge \zeta_1 \text{ for any } C_1^* \in \{-c, 0, c\},\$$

which implies that $E_0[\Delta P_1^*] \ge 0$. This also contradicts the demand rule of the informed speculators. Hence we must have $C_0^* = 0$ and $P_0^* = \mu_z$.

Step 1. We show that at period 1 when there is a positive shock to the fundamental, we have

$$P_1^* = \mu_z + z + \frac{c}{\phi} \text{ and } C_1^* = c.$$
 (50)

At time t = 1, the informed speculators observe $Z_1 = \mu_z + z$ and choose a demand C_1 contingent on the expected future price change $E_1[\Delta P_2]$; momentum speculator's demand is zero; hedgers demand is $\phi(Z_1 - P_1)$. The market clearing condition at

period 1 is

$$\phi \left(Z_1 - P_1^* \right) + C_1^* = 0 \tag{51}$$

which implies that

$$P_1^* = \mu_z + z + \frac{C_1^*}{\phi}.$$
 (52)

Because $\vartheta > z + c/\phi$, the informed speculators know that the probability of saturation at period 2 is zero. Using (43) with t = 2 and (52), we have

$$\Delta P_2^* = \frac{\phi(P_1^* - \mu_z) - C_2^*}{\theta - \phi} - \frac{\phi \zeta_2}{\theta - \phi}$$
$$= \frac{\phi z + C_1^* - C_2^*}{\theta - \phi} - \frac{\phi \zeta_2}{\theta - \phi} > -\frac{\phi \zeta_2}{\theta - \phi}$$
(53)

for any $C_1^*, C_2^* \in \{-c, 0, c\}$, where the last inequality is by $\theta > \phi$ and $z > 2c/\phi$. As $E_1[\zeta_2] = 0$, by the inequality in (53) we immediately get $E_1[\Delta P_2^*] > 0$, which together with the demand function of the informed speculators and (52) implies that $C_1^* = c$ and $P_1^* = \mu_z + z + c/\phi$. This proves (50).

Step 2. Define

$$\tau_* = \max\left\{t \ge 2: \left|\frac{z}{(1-\phi/\theta)^{t-1}} + \frac{1+\phi/\theta}{1-\phi/\theta}\frac{c}{\phi}\right| < \vartheta\right\}$$

We show that for any t with $1 \le t \le \tau_*$, we have

$$P_t^* = \mu_z + \frac{c}{\phi} + \frac{z}{(1 - \phi/\theta)^{t-1}} \text{ and } C_t^* = c.$$
 (54)

As ϑ is sufficiently large constant, we know that τ_* is well defined. By (50), we know that (54) holds for t = 1. We prove that (54) for t with $2 \le t < \tau_*$ by mathematical induction. Suppose that (54) holds for t-1. Then at period t, we have $Z_t = \mu_z$ and

$$P_t^* = \mu_z + \frac{z}{(1 - \phi/\theta)^{t-1}} + \frac{\theta c/\phi - C_t^*}{\theta - \phi}.$$
(55)

Because $t < \tau_*$, we know that

$$|P_t^* - \mu_z| = \left| \frac{z}{(1 - \phi/\theta)^{t-1}} + \frac{\theta c/\phi - C_t^*}{\theta - \phi} \right|$$
$$\leq \left| \frac{z}{(1 - \phi/\theta)^{t-1}} + \frac{1 + \phi/\theta}{1 - \phi/\theta} \frac{c}{\phi} \right| < \vartheta,$$
(56)

which implies that the informed speculators know that the probability of saturation at period t + 1 is zero. Hence, by (43) and (55), we have

$$\Delta P_{t+1}^* = \frac{\phi(P_t^* - \mu_z) - C_{t+1}^*}{\theta - \phi} - \frac{\phi\zeta_{t+1}}{\theta - \phi} = \frac{\phi z - (1 - \phi/\theta)^{t-1}C_{t+1}^*}{(\theta - \phi)(1 - \phi/\theta)^{t-1}} + \frac{\theta c - \phi C_t^*}{(\theta - \phi)^2} - \frac{\phi\zeta_{t+1}}{\theta - \phi}.$$
(57)

Under the restrictions $\theta > \phi$ and $z > 2c/\phi$, we have

$$\frac{\phi z - (1 - \phi/\theta)^{t-1} C_{t+1}^*}{(\theta - \phi)(1 - \phi/\theta)^{t-1}} > 0 \text{ and } \frac{\theta c - \phi C_t^*}{(\theta - \phi)^2} > 0$$
(58)

for any $C_t^*, C_{t+1}^* \in \{-c, 0, c\}$, which together with (57) and $E_t[\zeta_{t+1}] = 0$ implies that

$$E_t[\Delta P_{t+1}^*] = \frac{\phi z - (1 - \phi/\theta) E_t[C_{t+1}^*]}{(\theta - \phi)(1 - \phi/\theta)} + \frac{\theta c - \phi C_t^*}{(\theta - \phi)^2} > 0.$$
(59)

Combing (59) with the demand function of the informed speculators we get $C_t^* = c$, which together with (55) shows that (54) holds at t. **Step 3.** We show at time $\tau^* = \tau_* + 1$, we have

$$P_{\tau^*}^* = \mu_z + \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta} + \frac{z}{(1 - \phi/\theta)^{\tau^* - 1}} \text{ and } C_{\tau^*}^* = -c.$$
(60)

By definition, at τ_* we have $|P_{\tau_*}^* - \mu_z| < \vartheta$, which means that the probability of saturation at period τ^* is zero. Hence, by (43) and (54),

$$P_{\tau^*}^* = \mu_z + \frac{z}{(1 - \phi/\theta)^{\tau^* - 1}} + \frac{\theta c/\phi - C_{\tau^*}^*}{\theta - \phi}.$$
(61)

Because

$$\frac{z}{(1-\phi/\theta)^{\tau^*-1}} + \frac{c}{\phi} > \vartheta, \tag{62}$$

we know that

$$|P_{\tau^*}^* - \mu_z| \ge \vartheta. \tag{63}$$

This means that at period $\tau^* + 1$, regardless the demand of the informed speculators, the probability of saturation becomes $\overline{\sigma}$. When there is no saturation at period $\tau^* + 1$, (43) and (61) imply that

$$\Delta P_{\tau^*+1}^* = \frac{\phi z - (1 - \phi/\theta)^{\tau^* - 1} C_{\tau^*+1}^*}{(\theta - \phi)(1 - \phi/\theta)^{\tau^* - 1}} + \frac{\theta c - \phi C_{\tau^*}^*}{(\theta - \phi)^2} - \frac{\phi \zeta_{\tau^*+1}}{\theta - \phi} \\ = \frac{\phi z/\theta}{(1 - \phi/\theta)^{\tau^*}} - \frac{C_{\tau^*+1}^*}{\theta - \phi} + \frac{\theta c - \phi C_{\tau^*}^*}{(\theta - \phi)^2} - \frac{\phi \zeta_{\tau^*+1}}{\theta - \phi}.$$
(64)

On the other hand, when there is saturation, the market clearing condition implies that

$$\Delta P_{\tau^*+1}^* = \zeta_{\tau^*+1} + \frac{C_{\tau^*+1}^*}{\phi} - \frac{z}{(1-\phi/\theta)^{\tau^*-1}} - \frac{\theta c/\phi - C_{\tau^*}^*}{\theta - \phi}.$$
(65)

Combining the results in (64) and (65), and applying $E_{\tau^*}[\zeta_{\tau^*+1}] = 0$, we get

$$E_{\tau^{*}}[\Delta P_{\tau^{*}+1}^{*}] = (1 - \overline{\sigma}) \left[\frac{\phi z/\theta}{(1 - \phi/\theta)^{\tau^{*}}} - \frac{E_{\tau^{*}}[C_{\tau^{*}+1}^{*}]}{\theta - \phi} + \frac{\theta c - \phi C_{\tau^{*}}^{*}}{(\theta - \phi)^{2}} \right] + \overline{\sigma} \left[\frac{E_{\tau^{*}}[C_{\tau^{*}+1}^{*}]}{\phi} - \frac{z}{(1 - \phi/\theta)^{\tau^{*}-1}} - \frac{\theta c/\phi - C_{\tau^{*}}^{*}}{\theta - \phi} \right] = \frac{z}{(1 - \phi/\theta)^{\tau^{*}-1}} \frac{\phi/\theta - \overline{\sigma}}{1 - \phi/\theta} + \frac{\theta^{2}(\phi/\theta - \overline{\sigma})c}{\phi(\theta - \phi)^{2}} + \frac{\theta(\overline{\sigma} - \phi/\theta)}{\phi(\theta - \phi)} E_{\tau^{*}}[C_{\tau^{*}+1}^{*}] + \frac{\theta(\overline{\sigma} - \phi/\theta)}{(\theta - \phi)^{2}} C_{\tau^{*}}^{*}.$$
(66)

Define

$$Q(c_1, c_2) = \frac{\theta^2(\phi/\theta - \overline{\sigma})c}{\phi(\theta - \phi)^2} + \frac{\theta(\overline{\sigma} - \phi/\theta)}{(\theta - \phi)^2}c_1 + \frac{\theta(\overline{\sigma} - \phi/\theta)}{\phi(\theta - \phi)}c_2.$$
 (67)

Note that

$$Q(c,c) = \frac{\theta^2(\phi/\theta - \overline{\sigma})c}{\phi(\theta - \phi)^2} + \frac{\theta(\overline{\sigma} - \phi/\theta)}{(\theta - \phi)^2}c + \frac{\theta(\overline{\sigma} - \phi/\theta)}{\phi(\theta - \phi)}c$$
$$= \frac{(\phi/\theta - \overline{\sigma})\theta c}{(\theta - \phi)} \left[\frac{\theta/\phi}{\theta - \phi} - \frac{1}{\theta - \phi} - \frac{1}{\phi}\right] = 0.$$
(68)

As $\overline{\sigma} > \phi/\theta$, we know that $Q(c_1, c_2) \leq Q(c, c)$ or any $c_1, c_2 \in \{-c, 0, c\}$, which together with (68) implies that

$$Q(c_1, c_2) \le 0 \text{ for any } c_1, c_2 \in \{-c, 0, c\}.$$
(69)

Collecting the results in (66), (69) and using the restriction $\overline{\sigma} > \phi/\theta$, we deduce that $E_{\tau^*}[\Delta P^*_{\tau^*+1}] < 0$. By the demand function of the informed speculators, we get $C^*_{\tau^*} = -c$. The market clearing condition at period τ^* is

$$\phi \left(Z_{\tau^*} - P_{\tau^*}^* \right) + \theta \left(P_{\tau^*}^* - P_{\tau^*-1}^* \right) - c = 0 \tag{70}$$

which together with $Z_{\tau^*} = \mu_z$ and

$$P_{\tau^*-1}^* = \mu_z + \frac{c}{\phi} + \frac{z}{(1 - \phi/\theta)^{\tau^*-2}}$$
(71)

implies that

$$P_{\tau^*}^* = \mu_z + \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta} + \frac{z}{(1 - \phi/\theta)^{\tau^* - 1}}.$$
(72)

Step 4. We show that after the switching time τ^* , we have

$$P_t^* = \mu_z + \frac{z}{(1 - \phi/\theta)^{t-1}} + \frac{c}{\phi} \left[\frac{2}{(1 - \phi/\theta)^{t-\tau^*+1}} - 1 \right] \text{ and } C_t^* = -c$$
(73)

for all $\tau^* \leq t < s^*$.

By (60), we know that (73) holds for $t = \tau^*$. We prove that (73) holds for t with $\tau^* + 1 \leq t < s^*$ using mathematical induction. Suppose that (73) holds for t - 1. Then at period t, we have $Z_t = \mu_z$ and

$$P_{t}^{*} = \frac{\theta P_{t-1}^{*} - C_{t}^{*} - \phi Z_{t}}{\theta - \phi}$$

$$= \frac{\theta \left[\mu_{z} + \frac{z}{(1 - \phi/\theta)^{t-2}} + \frac{c}{\phi} \left[\frac{2}{(1 - \phi/\theta)^{t-\tau^{*}}} - 1 \right] \right] - C_{t}^{*} - \phi \mu_{z}}{\theta - \phi}$$

$$= \mu_{z} + \frac{z}{(1 - \phi/\theta)^{t-1}} + \left[\frac{2}{(1 - \phi/\theta)^{t-\tau^{*}+1}} - \frac{1}{1 - \phi/\theta} \right] \frac{c}{\phi} - \frac{C_{t}^{*}}{\theta - \phi}.$$
(74)

When there is no saturation at period t + 1, the market clearing condition implies

that

$$\Delta P_{t+1}^{*} = \frac{\phi P_{t}^{*} - \phi \mu_{z}}{\theta - \phi} - \frac{C_{t+1}^{*} + \phi \zeta_{\tau^{*}+1}}{\theta - \phi}$$

$$= \frac{\frac{\phi z}{(1 - \phi/\theta)^{t-1}} + \phi \left[\frac{2}{(1 - \phi/\theta)^{t-\tau^{*}+1}} - \frac{1}{1 - \phi/\theta}\right] \frac{c}{\phi} - \frac{\phi C_{t}^{*}}{\theta - \phi}}{\theta - \phi} - \frac{C_{t+1}^{*} + \phi \zeta_{\tau^{*}+1}}{\theta - \phi}$$

$$= \frac{\phi z}{(\theta - \phi)(1 - \phi/\theta)^{t-1}} + \left[\frac{2}{(1 - \phi/\theta)^{t-\tau^{*}+2}} - \frac{1}{(1 - \phi/\theta)^{2}}\right] \frac{c}{\theta}$$

$$- \frac{\phi C_{t}^{*}}{(\theta - \phi)^{2}} - \frac{C_{t+1}^{*} + \phi \zeta_{\tau^{*}+1}}{\theta - \phi}.$$
(75)

On the other hand, when there is saturation, the market clearing condition implies that

$$\Delta P_{t+1}^* = \zeta_{t+1} + \frac{C_{t+1}^*}{\phi} - \frac{z}{(1 - \phi/\theta)^{t-1}} - \left[\frac{2}{(1 - \phi/\theta)^{t-\tau^*+1}} - \frac{1}{1 - \phi/\theta}\right] \frac{c}{\phi} + \frac{C_t^*}{\theta - \phi} = -\frac{z}{(1 - \phi/\theta)^{t-1}} - \left[\frac{2}{(1 - \phi/\theta)^{t-\tau^*+1}} - \frac{1}{1 - \phi/\theta}\right] \frac{c}{\phi} + \frac{C_{t+1}^*}{\phi} + \frac{C_t^*}{\theta - \phi} + \zeta_{t+1}.$$
(76)

Note that

$$\frac{(1-\overline{\sigma})\phi z}{(\theta-\phi)(1-\phi/\theta)^{t-1}} + \left[\frac{2}{(1-\phi/\theta)^{t-\tau^{*}+2}} - \frac{1}{(1-\phi/\theta)^{2}}\right]\frac{(1-\overline{\sigma})c}{\theta} \\
- \frac{\overline{\sigma}z}{(1-\phi/\theta)^{t-1}} - \left[\frac{2}{(1-\phi/\theta)^{t-\tau^{*}+1}} - \frac{1}{1-\phi/\theta}\right]\frac{\overline{\sigma}c}{\phi} \\
= \frac{\phi/\theta-\overline{\sigma}}{1-\phi/\theta}\frac{z}{(1-\phi/\theta)^{t-1}} \\
+ \left[\frac{2}{(1-\phi/\theta)^{t-\tau^{*}+1}} - \frac{1}{1-\phi/\theta}\right]\left[\frac{(1-\overline{\sigma})}{(1-\phi/\theta)\theta} - \frac{\overline{\sigma}}{\phi}\right]c \\
= \frac{\phi/\theta-\overline{\sigma}}{1-\phi/\theta}\frac{z}{(1-\phi/\theta)^{t-1}} + \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{t-\tau^{*}+1}} - \frac{1}{1-\phi/\theta}\right]\frac{\phi/\theta-\overline{\sigma}}{1-\phi/\theta}.$$
(77)

Note that

$$\frac{C_{t+1}^*}{\phi}\overline{\sigma} + \frac{C_t^*}{\theta - \phi}\overline{\sigma} - \frac{(1 - \overline{\sigma})C_{t+1}^*}{\theta - \phi} - \frac{(1 - \overline{\sigma})\phi C_t^*}{(\theta - \phi)^2} \\
= \frac{\overline{\sigma} - \phi/\theta}{1 - \phi/\theta}\frac{C_{t+1}^*}{\phi} + \frac{\overline{\sigma} - \phi/\theta}{(1 - \phi/\theta)^2}\frac{C_t^*}{\theta} \\
\leq \frac{\overline{\sigma} - \phi/\theta}{1 - \phi/\theta}\frac{c}{\phi} + \frac{\overline{\sigma} - \phi/\theta}{(1 - \phi/\theta)^2}\frac{c}{\theta} = \frac{\overline{\sigma} - \phi/\theta}{(1 - \phi/\theta)^2}\frac{c}{\phi}.$$
(78)

Under the restrictions that $\theta > \phi$ and $\overline{\sigma} > \phi/\theta$, we have

$$\frac{c}{\phi} \left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+1}} - \frac{1}{1-\phi/\theta} \right] \frac{\phi/\theta - \overline{\sigma}}{1-\phi/\theta} + \frac{\overline{\sigma} - \phi/\theta}{(1-\phi/\theta)^2} \frac{c}{\phi} \\
= \frac{2c}{\phi} \frac{\phi/\theta - \overline{\sigma}}{(1-\phi/\theta)^2} \left[\frac{1}{(1-\phi/\theta)^{t-\tau^*}} - 1 \right] < 0.$$
(79)

Collecting the results in (75), (76), (77), (78) and (79), we deduce that $E_t \left[\Delta P_{t+1}^*\right] < 0$ which combined with the demand function of the informed speculators imply that $C_t^* = -c$. Plugging $C_t^* = -c$ into equation (74), we get

$$P_t^* = \mu_z + \frac{z}{(1 - \phi/\theta)^{t-1}} + \left[\frac{2}{(1 - \phi/\theta)^{t-\tau^*+1}} - \frac{1}{1 - \phi/\theta}\right]\frac{c}{\phi} + \frac{c}{\theta - \phi}$$
$$= \mu_z + \frac{z}{(1 - \phi/\theta)^{t-1}} + \left[\frac{2}{(1 - \phi/\theta)^{t-\tau^*+1}} - 1\right]\frac{c}{\phi}$$

which finishes the proof.

Step 5. We show that at saturation s^* ,

$$P_{s^*}^* = \mu_z \text{ and } C_{s^*}^* = 0.$$
 (80)

The proof of this claim is the same as that of (46) and hence is omitted. **Proof of Corollary 1.3.** Along the negative bubble, the equilibrium price P_t^* and the demand of the informed speculators C_t^* also satisfy (41), (42), (43), (44) and (45).

Step 0. At period 0,

$$P_0^* = \mu_z \text{ and } C_0^* = 0.$$
 (81)

The proof will be exactly same with the step 0 in the proof of the Proposition 1.2.

Step 1. We show that at period 1 when there is a negative shock to the fundamental, we have

$$P_1^* = \mu_z - z - \frac{c}{\phi} \text{ and } C_1^* = -c.$$
 (82)

At time t = 1, the informed speculators observe $Z_1 = \mu_z - z$ and choose a demand C_1 contingent on the expected future price change $E_1[\Delta P_2]$; momentum speculator's demand is zero; hedgers demand is $\phi(Z_1 - P_1)$. The market clearing condition at period 1 is

$$\phi \left(Z_1 - P_1^* \right) + C_1^* = 0 \tag{83}$$

which implies that

$$P_1^* = \mu_z - z + \frac{C_1^*}{\phi}.$$
(84)

Because $\vartheta > |z + c/\phi|$, the informed speculators know that the probability of saturation at period 2 is zero. Using (43) with t = 2 and (84), we have

$$\Delta P_2^* = \frac{\phi(P_1^* - \mu_z) - C_2^*}{\theta - \phi} - \frac{\phi \zeta_2}{\theta - \phi}$$
$$= \frac{-\phi z + C_1^* - C_2^*}{\theta - \phi} - \frac{\phi \zeta_2}{\theta - \phi} < -\frac{\phi \zeta_2}{\theta - \phi}$$
(85)

for any $C_1^*, C_2^* \in \{-c, 0, c\}$, where the last inequality is by $\theta > \phi$ and $z > 2c/\phi$. As

 $E_1[\zeta_2] = 0$, by the inequality in (85) we immediately get $E_1[\Delta P_2^*] < 0$, which together with the demand function of the informed speculators and (84) implies that $C_1^* = -c$ and $P_1^* = \mu_z - z - c/\phi$. This proves (82).

Step 2. Define

$$\tau_* = \max\left\{t \ge 2: \left|\frac{z}{(1-\phi/\theta)^{t-1}} + \frac{1+\phi/\theta}{1-\phi/\theta}\frac{c}{\phi}\right| < \vartheta\right\},\$$

and

$$\underline{\tau} = \max\left\{t \ge 2: P_t = \mu_z - \frac{z}{(1 - \phi/\theta)^{t-1}} - \frac{1 + \phi/\theta}{1 - \phi/\theta}\frac{c}{\phi} > \underline{P}\right\}.$$

We show that for any t with $1 \le t \le \min{\{\tau_*, \underline{\tau}\}}$, we have

$$P_t^* = \mu_z - \frac{c}{\phi} - \frac{z}{(1 - \phi/\theta)^{t-1}} \text{ and } C_t^* = -c.$$
 (86)

Suppose $\tau_* < \underline{\tau}$ holds which implies that saturation can happen before price reaches the lower bound. As ϑ is sufficiently large constant, we know that τ_* is well defined. By (82), we know that (86) holds for t = 1. We prove that (86) for t with $2 \le t < \tau_*$ by mathematical induction. Suppose that (86) holds for t - 1. Then at period t, we have $Z_t = \mu_z$ and

$$P_t^* = \mu_z - \frac{z}{(1 - \phi/\theta)^{t-1}} - \frac{\theta c/\phi + C_t^*}{\theta - \phi}.$$
(87)

Because $t < \tau_*$, we know that

$$|P_t^* - \mu_z| = \left| \frac{z}{(1 - \phi/\theta)^{t-1}} + \frac{\theta c/\phi - C_t^*}{\theta - \phi} \right|$$
$$\leq \left| \frac{z}{(1 - \phi/\theta)^{t-1}} + \frac{1 + \phi/\theta}{1 - \phi/\theta} \frac{c}{\phi} \right| < \vartheta,$$
(88)

which implies that the informed speculators know that the probability of saturation at period t + 1 is zero. Hence, by (43) and (87), we have

$$\Delta P_{t+1}^* = \frac{\phi(P_t^* - \mu_z) - C_{t+1}^*}{\theta - \phi} - \frac{\phi \zeta_{t+1}}{\theta - \phi} = -\frac{\phi z + (1 - \phi/\theta)^{t-1} C_{t+1}^*}{(\theta - \phi)(1 - \phi/\theta)^{t-1}} - \frac{\theta c + \phi C_t^*}{(\theta - \phi)^2} - \frac{\phi \zeta_{t+1}}{\theta - \phi}.$$
(89)

Under the restrictions $\theta > \phi$ and $z > 2c/\phi$, we have

$$-\frac{\phi z + (1 - \phi/\theta)^{t-1} C_{t+1}^*}{(\theta - \phi)(1 - \phi/\theta)^{t-1}} < 0 \text{ and } -\frac{\theta c + \phi C_t^*}{(\theta - \phi)^2} < 0$$
(90)

for any $C_t^*, C_{t+1}^* \in \{-c, 0, c\}$, which together with (89) and $E_t[\zeta_{t+1}] = 0$ implies that

$$E_t[\Delta P_{t+1}^*] = -\frac{\phi z + (1 - \phi/\theta)^{t-1} C_{t+1}^*}{(\theta - \phi)(1 - \phi/\theta)^{t-1}} - \frac{\theta c + \phi C_t^*}{(\theta - \phi)^2} < 0.$$
(91)

Combing (91) with the demand function of the informed speculators we get $C_t^* = c$, which together with (55) shows that (54) holds at t. Along the negative bubbly path, we also need to consider the possibility that price can be lower than the lower bound \underline{P} . If $\tau_* > \underline{\tau}$, then the informed speculators know that the probability of saturation next period is zero, as we derived above, $E_t[\Delta P_{t+1}^*] < 0$. Thus, the demand of informed speculators is $C_t^* = -c$, the equilibrium price is (87).

Step 3. We show at the time of saturation $\tau^* = \min \{\tau_*, \underline{\tau}\} + 1$, we have

$$P_{\tau^*}^* = \mu_z - \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta} - \frac{z}{(1 - \phi/\theta)^{\tau^* - 1}} \text{ and } C_{\tau^*}^* = c.$$
(92)

Suppose $\tau_* < \underline{\tau}$. Then by definition, at τ_* we have $|P_{\tau_*}^* - \mu_z| < \vartheta$, which means

that the probability of saturation at period τ^* is zero. Hence, by (43) and (86),

$$P_{\tau^*}^* = \mu_z - \frac{z}{(1 - \phi/\theta)^{\tau^* - 1}} - \frac{\theta c/\phi + C_{\tau^*}^*}{\theta - \phi}.$$
(93)

Because

$$\left|\frac{z}{(1-\phi/\theta)^{\tau^*-1}} + \frac{c}{\phi}\right| > \vartheta,\tag{94}$$

we know that

$$|P_{\tau^*}^* - \mu_z| \ge \vartheta. \tag{95}$$

This means that at period $\tau^* + 1$, regardless the demand of the informed speculators, the probability of saturation becomes $\overline{\sigma}$. When there is no saturation at period $\tau^* + 1$, (43) and (93) imply that

$$\Delta P_{\tau^*+1}^* = -\frac{\phi z + (1 - \phi/\theta)^{\tau^* - 1} C_{\tau^*+1}^*}{(\theta - \phi)(1 - \phi/\theta)^{\tau^* - 1}} - \frac{\theta c + \phi C_{\tau^*}^*}{(\theta - \phi)^2} - \frac{\phi \zeta_{\tau^*+1}}{\theta - \phi}$$
$$= -\frac{\phi z/\theta}{(1 - \phi/\theta)^{\tau^*}} - \frac{C_{\tau^*+1}^*}{\theta - \phi} - \frac{\theta c + \phi C_{\tau^*}^*}{(\theta - \phi)^2} - \frac{\phi \zeta_{\tau^*+1}}{\theta - \phi}.$$
(96)

On the other hand, when there is saturation, the market clearing condition implies that

$$\Delta P_{\tau^*+1}^* = \zeta_{\tau^*+1} + \frac{C_{\tau^*+1}^*}{\phi} + \frac{z}{(1-\phi/\theta)^{\tau^*-1}} + \frac{\theta c/\phi + C_{\tau^*}^*}{\theta - \phi}.$$
(97)

Combining the results in (96) and (97), and applying $E_{\tau^*}[\zeta_{\tau^*+1}] = 0$, we get

$$E_{\tau^*}[\Delta P_{\tau^*+1}^*] = (1 - \overline{\sigma}) \left[-\frac{\phi z/\theta}{(1 - \phi/\theta)^{\tau^*}} - \frac{E_{\tau^*}[C_{\tau^*+1}^*]}{\theta - \phi} - \frac{\theta c + \phi C_{\tau^*}}{(\theta - \phi)^2} \right] + \overline{\sigma} \left[\frac{E_{\tau^*}[C_{\tau^*+1}^*]}{\phi} + \frac{z}{(1 - \phi/\theta)^{\tau^*-1}} + \frac{\theta c/\phi + C_{\tau^*}}{\theta - \phi} \right] = -\frac{z}{(1 - \phi/\theta)^{\tau^*-1}} \frac{\phi/\theta - \overline{\sigma}}{1 - \phi/\theta} - \frac{\theta^2(\phi/\theta - \overline{\sigma})c}{\phi(\theta - \phi)^2} + \frac{\theta(\overline{\sigma} - \phi/\theta)}{\phi(\theta - \phi)} E_{\tau^*}[C_{\tau^*+1}^*] + \frac{\theta(\overline{\sigma} - \phi/\theta)}{(\theta - \phi)^2} C_{\tau^*}^*.$$
(98)

Define

$$Q(c_1, c_2) = -\frac{\theta^2(\phi/\theta - \overline{\sigma})c}{\phi(\theta - \phi)^2} + \frac{\theta(\overline{\sigma} - \phi/\theta)}{(\theta - \phi)^2}c_1 + \frac{\theta(\overline{\sigma} - \phi/\theta)}{\phi(\theta - \phi)}c_2.$$
 (99)

Note that

$$Q(-c,-c) = -\frac{\theta^2(\phi/\theta - \overline{\sigma})c}{\phi(\theta - \phi)^2} - \frac{\theta(\overline{\sigma} - \phi/\theta)}{(\theta - \phi)^2}c - \frac{\theta(\overline{\sigma} - \phi/\theta)}{\phi(\theta - \phi)}c$$
$$= \frac{(\phi/\theta - \overline{\sigma})\theta c}{(\theta - \phi)} \left[-\frac{\theta/\phi}{\theta - \phi} + \frac{1}{\theta - \phi} + \frac{1}{\phi} \right] = 0.$$
(100)

As $\overline{\sigma} > \phi/\theta$, we know that $Q(c_1, c_2) \ge Q(-c, -c)$ or any $c_1, c_2 \in \{-c, 0, c\}$, which together with (100) implies that

$$Q(c_1, c_2) \ge 0 \text{ for any } c_1, c_2 \in \{-c, 0, c\}.$$
(101)

Collecting the results in (98), (101) and using the restriction $\overline{\sigma} > \phi/\theta$, we deduce that $E_{\tau^*}[\Delta P^*_{\tau^*+1}] > 0$. By the demand function of the informed speculators, we get $C^*_{\tau^*} = c$. The market clearing condition at period τ^* is

$$\phi \left(Z_{\tau^*} - P_{\tau^*}^* \right) + \theta \left(P_{\tau^*}^* - P_{\tau^*-1}^* \right) - c = 0 \tag{102}$$

which together with $Z_{\tau^*} = \mu_z$ and

$$P_{\tau^*-1}^* = \mu_z - \frac{c}{\phi} - \frac{z}{(1 - \phi/\theta)^{\tau^*-2}}$$
(103)

implies that

$$P_{\tau^*}^* = \mu_z - \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta} - \frac{z}{(1 - \phi/\theta)^{\tau^* - 1}}.$$
(104)

Suppose $\tau_* < \underline{\tau}$. Then at time $\tau^* + 1$, regardless the demand of the informed speculators, saturation happens for sure. Thus,

$$E_{\tau^*}[\Delta P^*_{\tau^*+1}] = \frac{E_{\tau^*}[C^*_{\tau^*+1}]}{\phi} + \frac{z}{(1-\phi/\theta)^{\tau^*-1}} + \frac{\theta c/\phi + C^*_{\tau^*}}{\theta - \phi} > 0$$
(105)

Hence, by the demand function of the informed speculators, we get $C^*_{\tau^*} = c$. The equilibrium price at time τ^* is

$$P_{\tau^*}^* = \mu_z - \frac{c}{\phi} \frac{1 + \phi/\theta}{1 - \phi/\theta} - \frac{z}{(1 - \phi/\theta)^{\tau^* - 1}}.$$
(106)

Saturation happens for sure when the price reaches the lower bound. Thus as you will see at Step 5, at time $s^* (= \tau^* + 1)$,

$$P_{s^*}^* = \mu_z \text{ and } C_{s^*}^* = 0.$$
 (107)

Step 4. Suppose $\tau^* < \underline{\tau}$. We show that after the switching time τ^* , we have

$$P_t^* = \mu_z - \frac{z}{(1 - \phi/\theta)^{t-1}} - \frac{c}{\phi} \left[\frac{2}{(1 - \phi/\theta)^{t-\tau^* + 1}} - 1 \right] \text{ and } C_t^* = c$$
(108)

for all $\tau^* \leq t < s^*$, where s^* is the time of saturation.

By (92), we know that (108) holds for $t = \tau^*$. We prove that (108) holds for twith $\tau^* + 1 \le t < s^*$ using mathematical induction. Suppose that (108) holds for t-1. Then at period t, we have $Z_t = \mu_z$ and

$$P_t^* = \frac{\theta P_{t-1}^* - C_t^* - \phi Z_t}{\theta - \phi}$$

$$= \frac{\theta \left[\mu_z - \frac{z}{(1 - \phi/\theta)^{t-2}} - \frac{c}{\phi} \left[\frac{2}{(1 - \phi/\theta)^{t-\tau^*}} - 1 \right] \right] - C_t^* - \phi \mu_z}{\theta - \phi}$$

$$= \mu_z - \frac{z}{(1 - \phi/\theta)^{t-1}} - \left[\frac{2}{(1 - \phi/\theta)^{t-\tau^*+1}} - \frac{1}{1 - \phi/\theta} \right] \frac{c}{\phi} - \frac{C_t^*}{\theta - \phi}.$$
(109)

When there is no saturation at period t + 1, the market clearing condition implies that

$$\Delta P_{t+1}^{*} = \frac{\phi P_{t}^{*} - \phi \mu_{z}}{\theta - \phi} - \frac{C_{t+1}^{*} + \phi \zeta_{\tau^{*}+1}}{\theta - \phi}$$

$$= \frac{-\frac{\phi z}{(1 - \phi/\theta)^{t-1}} - \phi \left[\frac{2}{(1 - \phi/\theta)^{t-\tau^{*}+1}} - \frac{1}{1 - \phi/\theta}\right] \frac{c}{\phi} - \frac{\phi C_{t}^{*}}{\theta - \phi}}{\theta - \phi} - \frac{C_{t+1}^{*} + \phi \zeta_{\tau^{*}+1}}{\theta - \phi}$$

$$= -\frac{\phi z}{(\theta - \phi)(1 - \phi/\theta)^{t-1}} - \left[\frac{2}{(1 - \phi/\theta)^{t-\tau^{*}+2}} - \frac{1}{(1 - \phi/\theta)^{2}}\right] \frac{c}{\theta}$$

$$- \frac{\phi C_{t}^{*}}{(\theta - \phi)^{2}} - \frac{C_{t+1}^{*} + \phi \zeta_{\tau^{*}+1}}{\theta - \phi}.$$
(110)

On the other hand, when there is saturation, the market clearing condition implies that

$$\Delta P_{t+1}^{*} = \zeta_{t+1} + \frac{C_{t+1}^{*}}{\phi} + \frac{z}{(1-\phi/\theta)^{t-1}} + \left[\frac{2}{(1-\phi/\theta)^{t-\tau^{*}+1}} - \frac{1}{1-\phi/\theta}\right]\frac{c}{\phi} + \frac{C_{t}^{*}}{\theta-\phi} = \frac{z}{(1-\phi/\theta)^{t-1}} + \left[\frac{2}{(1-\phi/\theta)^{t-\tau^{*}+1}} - \frac{1}{1-\phi/\theta}\right]\frac{c}{\phi} + \frac{C_{t+1}^{*}}{\phi} + \frac{C_{t}^{*}}{\theta-\phi} + \zeta_{t+1}.$$
(111)

Note that

$$-\frac{(1-\overline{\sigma})\phi z}{(\theta-\phi)(1-\phi/\theta)^{t-1}} - \left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+2}} - \frac{1}{(1-\phi/\theta)^2}\right]\frac{(1-\overline{\sigma})c}{\theta}$$

$$+\frac{\overline{\sigma}z}{(1-\phi/\theta)^{t-1}} + \left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+1}} - \frac{1}{1-\phi/\theta}\right]\frac{\overline{\sigma}c}{\phi}$$

$$= -\frac{\phi/\theta-\overline{\sigma}}{1-\phi/\theta}\frac{z}{(1-\phi/\theta)^{t-1}} - \left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+1}} - \frac{1}{1-\phi/\theta}\right]\left[\frac{(1-\overline{\sigma})}{(1-\phi/\theta)\theta} - \frac{\overline{\sigma}}{\phi}\right]c$$

$$= -\frac{\phi/\theta-\overline{\sigma}}{1-\phi/\theta}\frac{z}{(1-\phi/\theta)^{t-1}} - \frac{c}{\phi}\left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+1}} - \frac{1}{1-\phi/\theta}\right]\frac{\phi/\theta-\overline{\sigma}}{1-\phi/\theta}.$$
 (112)

Note that

$$\frac{C_{t+1}^{*}}{\phi}\overline{\sigma} + \frac{C_{t}^{*}}{\theta - \phi}\overline{\sigma} - \frac{(1 - \overline{\sigma})C_{t+1}^{*}}{\theta - \phi} - \frac{(1 - \overline{\sigma})\phi C_{t}^{*}}{(\theta - \phi)^{2}} \\
= \frac{\overline{\sigma} - \phi/\theta}{1 - \phi/\theta}\frac{C_{t+1}^{*}}{\phi} + \frac{\overline{\sigma} - \phi/\theta}{(1 - \phi/\theta)^{2}}\frac{C_{t}^{*}}{\theta} \\
\ge -\frac{\overline{\sigma} - \phi/\theta}{1 - \phi/\theta}\frac{c}{\phi} - \frac{\overline{\sigma} - \phi/\theta}{(1 - \phi/\theta)^{2}}\frac{c}{\theta} = -\frac{\overline{\sigma} - \phi/\theta}{(1 - \phi/\theta)^{2}}\frac{c}{\phi}.$$
(113)

Under the restrictions that $\theta > \phi$ and $\overline{\sigma} > \phi/\theta$, we have

$$-\frac{c}{\phi} \left[\frac{2}{(1-\phi/\theta)^{t-\tau^*+1}} - \frac{1}{1-\phi/\theta} \right] \frac{\phi/\theta - \overline{\sigma}}{1-\phi/\theta} - \frac{\overline{\sigma} - \phi/\theta}{(1-\phi/\theta)^2} \frac{c}{\phi}$$
$$= -\frac{2c}{\phi} \frac{\phi/\theta - \overline{\sigma}}{(1-\phi/\theta)^2} \left[\frac{1}{(1-\phi/\theta)^{t-\tau^*}} - 1 \right] > 0.$$
(114)

Collecting the results in (110), (111), (112), (113) and (114), we deduce that $E_t \left[\Delta P_{t+1}^*\right] > 0$ which combined with the demand function of the informed speculators imply that $C_t^* = c$. Plugging $C_t^* = -c$ into equation (109), we get

$$P_t^* = \mu_z - \frac{z}{(1 - \phi/\theta)^{t-1}} - \left[\frac{2}{(1 - \phi/\theta)^{t-\tau^*+1}} - \frac{1}{1 - \phi/\theta}\right]\frac{c}{\phi} - \frac{c}{\theta - \phi}$$
$$= \mu_z - \frac{z}{(1 - \phi/\theta)^{t-1}} - \left[\frac{2}{(1 - \phi/\theta)^{t-\tau^*+1}} - 1\right]\frac{c}{\phi}$$

which finishes the proof.

Step 5. We show that at saturation s^* ,

$$P_{s^*}^* = \mu_z \text{ and } C_{s^*}^* = 0.$$
 (115)

The proof of this claim is the same as that of (81) and hence is omitted. \blacksquare
2 Economic Recessions & Wildfires

2.1 Introduction

Economic crises, while rare and catastrophic, are not outliers. Our findings suggest that the size distribution of economic crises are a smooth extrapolation of smaller economic distress events, as it is often the case with extreme natural disasters.

We consider all economic distress (ED) events over the period 1970-2014. These ED events range from small deviations from trend-growth to large recessions, to wellknown catastrophic crises, such as the 2008 crash, and they cover different economic systems and time periods.

Our contribution is to unearth a remarkable relation between the magnitude of economic distress events and the frequency with which they occur. Figure 13 plots the size of ED events in the abscissa against the (logarithm) of the complementary CDF, i.e., the probability that the distress is larger than a given size. As we can see, from a birds-eye's perspective, a linear regression fits quite well the ED magnitude-frequency data ranging from small economic disturbances to catastrophic crises ($R^2 = 0.995$).

Using the more rigorous statistical techniques of Clauset et al. (2009), we find that the ED size distribution follows a power law with an exponential cutoff distribution. In other words, there is a threshold \underline{x} below which the size of ED events follows an exponential distribution, while a Pareto distribution (a power-law) applies for ED events larger than \underline{x} , as shown in Figure 15. As we can see in the bottom panel, there is a linear relation between log(frequency) and log(magnitude) for ED events greater than \underline{x} . Meanwhile, in the top panel, we see a linear relation between log(frequency) and magnitude for ED events smaller than \underline{x} .

To understand the economic mechanism that may give rise to a power law with an exponential cutoff distribution of ED events, we model an ED event as a wildfire. We present a model in which the dynamics of an individual ED event is determined by the interaction of two opposing forces: (i) the natural stochastic growth of the ED, which is proportional to the size of the damage that has already occurred; and (ii) a policy that attempts to extinguish the economic distress. We then derive the steady-state cross-sectional distribution of the final size of the ED events. We show that the size distribution is exponential for $x < \underline{x}$ and Pareto for $x \ge \underline{x}$ whenever the extinguishment policy is irresponsive to the spread of the fire up to a distress size \underline{x} , but for $x \ge \underline{x}$ it becomes increasingly responsive to the size of the fire.

Our findings are linked to the log(magnitude)-log(frequency) linear relation that characterizes many natural catastrophes: Earthquakes (Gutenberg-Richter relation); wildfires, landslides, hurricanes, epidemics, social upheavals, stock-market crashes, etc. Gabaix (2009, 2016) surveys the evidence for power law distributions in Economics and Finance. In the context of the equity-premium puzzle, Barro (2006) and Barro and Jin (2011) document the existence of such a power-law relation for economic catastrophes.

An implication of our findings is that policymakers' attempts to stop an ED-event may simply result in a larger future ED-event and eventually in a catastrophic crisis. This possibility stands in contrast to the centuries-old accepted wisdom that economic crises are the result of misguided macroeconomic and regulatory policies, and that they are avoidable with the appropriate policy menu.

2.2 Data and Methodology

We base our analysis on annual real GDP growth rates and level of GDP per capita. Two datasets are considered. The first dataset which comes from the World Bank Development Indicators (WDI, Code: NY.GDP.MKTP.KD.ZG) covers annual real GDP growth rates over the period 1960-2014. The second data cover the level of GDP per capita over the period 1830-2014 and were obtained from Maddison Project Database (MPD).

2.2.1 Countries

We consider all countries with well-functioning financial systems, satisfying either of the following criteria.

- 1. High-income OECD members (with a GNI per capita of \$12,736 or more)
- 2. Countries with a GNI per capita of more than \$4,125 (high-income economies and upper-middle-income economies) that the World Bank classifies as financial creditworthy so as to be eligible to borrow from the International Bank for Reconstruction and Development (IBRD).

This generates us a set of 60 countries with data availability in both datasets. We classify an economy as "advanced" if they fall under the MSCI maket classification of developed markets. All others countries are classified as "emerging". The list of countries are summarized in Table 12 in the appendix.

2.2.2 Time Periods

We consider two time periods: the recent period of globalization (1970 - 2014) and the long historical period (1830 - 2013). For the recent period of globalization (1970 -2014), both datasets (WDI and MPD) are available for all of the 60 countries on the list. In case of the long historical period (1830 - 2013), we only consider the advanced countries in MPD because most of the emerging market economies have short time series of data available. Table 12 in the appendix.

2.2.3 Identifying recessions

There exists an episode of "Economic Distress" (ED) if $g_{it} = y_{it} - \mu_{it} < 0$ holds where y_{it} is the growth rate of real GDP for country *i* in year *t* and μ_{it} is a filter that captures the potential GDP growth rate. Duration of "Economic Distress" (ED) is $t_1 - t_0$ where t_0 is the year when real output growth falls below the trend and t_1 is the first year after t_0 when it recovers the trend. This method is successful in identifying financial crises, which is validated by comparison with other crises database in Reinhart and Rogoff (2009), Laeven and Valencia (2013) and Ranciere and Tornell (2015).³ Moreover, this method is successful in identifying the relatively small ED episodes. The ED episodes captured by this identifying method (when 10-yr MA is used for the filter) show significant overlap with the official recession dates announced by NBER. It is well visualized in Figure 18.

2.2.4 Measuring the Degree of Economic Distress

The degree of Economic Distress, X, is measured as the cumulative sum of standardized deviations from trend growth

$$r_{i,t} = \frac{y_{it} - \mu_{it}}{\sigma_{it}}$$

over "Economic Distress" years. It can be written as:

$$X_{i,t_1-t_0} = \sum_{t=t_0}^{t_1} r_{it} \cdot I(r_{it} < 0)$$

We consider two specific ED measures:

³This method has achieved 100 percent accuracy in identifying all types of financial crises indicated by the papers.

- Measure 1 (Standardized Growth Gap): We set μ_{it} to be 10-year moving average of y_{it} and σ_{it} to be 10-year moving standard deviation of y_{it} . This normalized growth gap in real GDP growth is similar to that used by Bordo et al. (2001) and Hoggarth et al. (2002).
- Measure 2 (Proportional Contraction): We set μ_{it} to be 0 and σ_{it} to be 1. This corresponds to the proportional contraction in the level of GDP per capita between t_0 and t_1 (Barro (2006) and Barro and Jin (2011)).

'Standardized growth gap' has a comparative advantage over 'proportional contraction' in capturing a economic stagnation such as "Japan's Lost Decade" because an event of sluggish economic growth is not identified as an ED episode by 'proportional contraction'.

In the following section we perform a power law test on various distributions:

- Size distribution of ED episodes experienced by all 60 countries during the recent period of globalization (1970 - 2014). Standardized growth gap is used to measure the degree of ED. See Figure 13 in the Appendix.
- Size distribution of ED episodes experienced by all 60 countries during the recent period of globalization (1970 - 2014). Proportional contraction is used to measure the degree of ED. See Figure 14 in the Appendix.
- 3. Size distribution of ED episodes experienced by 23 advanced countries and 37 emerging market economies respectively during the recent period of globalization (1970 2014). Standardized growth gap is used to measure the degree of ED. See Figure16 in the Appendix.
- 4. Size distribution of ED episodes experienced by 23 advanced countries during

the long historical period (1830 - 2013). Standardized growth gap is used to measure the degree of ED. See Figure 17 in the Appendix.

2.3 Test for Power Law and Exponentiality

2.3.1 Test for Power Law

To test whether the empirical distribution of our data follows a power law in the upper tail, we use an empirical methodology introduced by Clauset et al. (2009). A powerlaw distribution is described by a probability density p(x) such that $p(x)dx = Pr(x \le X < x + dx) = Cx^{-\alpha}dx$, where X is the observed value and C is a normalization constant. Clearly, this density diverges as $x \to 0$ so it cannot hold for all $x \ge 0$; there must be some lower bound to the power-law behavior. We will denote this bound by \underline{x} . Then a density of continuous power law distribution is given by:

$$p(x) = \frac{\alpha - 1}{\underline{x}} \left(\frac{x}{\underline{x}}\right)^{-\alpha} \tag{116}$$

The maximum likelihood estimator (MLE) of the power law exponent, α , is

$$\hat{\alpha} = 1 + n \left(\sum_{i=1}^{n} ln \frac{x_i}{\underline{x}} \right) \tag{117}$$

where x_i , i = 1, 2, ..., n are independent observations such that $x_i > \underline{x}$. The lower bound on the power law distribution, \underline{x} , will be estimated using the following procedure. For each $x_i > \underline{x}$, we estimate α using the MLE and then compute the KolmogorovSmirnov (KS) statistic which is the maximum distance between the CDFs of the data and the fitted model. \underline{x} is then selected as a value of x_i minimizing the KS statistic. That is to say, our estimate $\underline{\hat{x}}$ is the value of \underline{x} that minimizes D which is

$$D = \max_{x \ge \underline{x}} |S(x) - F(x)| \tag{118}$$

where S(x) is the CDF of the data for the observations with value at least \underline{x} , and F(x) is the CDF of the best fitted power law model in the region $x \ge \underline{x}$.

With the estimated \hat{x} , the scaling parameter of the power law model (α) is estimated using maximum likelihood estimators (MLE) as in the equation (2). Next we test the goodness of fit of the power law model based on a semi-parametric bootstrap approach following Clauset et al. (2009). We generate a large number of power-law distributed synthetic data sets with the estimated scaling parameters, \hat{x} and $\hat{\alpha}$. Then, power law models are fitted to each of the synthetic data sets individually using the same method as for the original data set and the KS statistics are calculated. P-value is defined to be the fraction of the synthetic distances that are larger than the empirical distance. According to Clauset et al. (2009), the rule-of-thumb P-value is 0.1 which means that if the resulting p-value is greater than 0.1 the power law is a plausible hypothesis for the data, otherwise it is rejected.

Table 6: Power Law Estimation when Measure 1 used (1970 - 2014)

Estimation of	Power Law (MLE)	& Goo	dness of fit test using KS	S stati	stic
Measure 1	# of observations	\underline{x}	# of observations $> \underline{x}$	α	P-value
All Countries (60)	496	6.69	66	3.92	0.46
ADV (23)	184	5.42	43	3.76	0.74
EME (37)	312	6.69	41	3.97	0.29

Table 6 summarizes the result of the power law test when standardized growth gap is used to measure the degree of Economic Distress for the recent period of globalization (1970 - 2014). It shows that a power law behavior is observable for the whole sample (all 60 countries) and for the subsets of the sample (advanced countries and emerging market economies). The estimated scaling parameter (α) is

Estimation of	Power Law (MLE)	& Goo	dness of fit test using KS	S statis	stic
Measure 2	# of observations	\underline{x}	# of observations $> \underline{x}$	α	P-value
All Countries (60)	328	7.06	82	2.56	0.3
ADV (23)	114	3.79	33	3.17	0.34
EME (37)	214	7.64	67	2.52	0.17

Table 7: Power Law Estimation when Measure 2 used (1970 - 2014)

Table 8: Power Law Estimation when Measure 1 used (1830 - 2013)

Estimation	n of Power Law (MI	LE) &	Goodness of fit test usin	g KS s	statistic
Measure 1 ADV (23)	# of observations 735	$\frac{x}{5.65}$	# of observations $> \underline{x}$ 87	lpha 3.34	P-value 0.24

Table 9: Power Law Estimation when Measure 2 used (1830 - 2013)

Estimation	n of Power Law (MI	LE) &	Goodness of fit test usin	g KS s	statistic
Measure 1 ADV (23)	# of observations 543	$\frac{\underline{x}}{5.51}$	# of observations $> \underline{x}$ 158	lpha 2.44	P-value 0.27

found to be stable ranging from 3.76 to 3.97. The estimated lower bound of the power-law behavior \underline{x} for the advanced countries is 5.42 which is lower than that for the emerging market economies (6.69). The upper tail of the datasets (in the region $x \geq \underline{x}$) include the large economic distress episodes such as the Mexican 1982 debt crisis (10.18), 1997 Asian financial crisis (Korea: 12.82, Indonesia: 18.32, Malaysia: 22.23), "Japan's Lost Decade" from 1991 to 1999 (11.71), the U.S. subprime mortgage crisis (8.02), Greece's debt crisis (14.5), etc. See table 11 in the Appendix. According to our estimates, about 13% of all available observations are above \underline{x} in the case of datasets covering all countries. In the case of dataset covering the advanced countries and the emerging market economies, 23% and 13% of all available observations follow the power law behavior.

Table 7 summarizes the result of the power law test when proportional contraction

is used to measure the degree of ED for the recent period of globalization (1970 - 2014). It shows that a power law pattern in the upper tail of the datasets is still valid for all the cases. The estimated scaling parameter (α) ranges from 2.52 to 3.17. The estimated lower bound of the power-law behavior \underline{x} for the advanced countries is 3.79 which is much lower than that for the emerging market economies (7.64). 25%, 29%, and 31% of all available observations are above \underline{x} in the case of datasets covering the all countries, the advanced countries, and the emerging market economies, respectively.

Table 8 and Table 9 summarize the result of the power law test when standardized growth gap and proportional contraction are used to measure the degree of Economic Distress, respectively for the long historical period (1830 - 2013). It shows that a power law pattern in the upper tail (12% and 29% of the observations) of the datasets is still observable (p-value is 0.24 and 0.27). Furthermore, it is worth noting that the estimated parameters (α and \underline{x}) are almost identical even though we now have much more observations in the long historical period than in the recent period of globalization (Table 8). Unlike the result of Table 8, the estimated \underline{x} are quite different in Table 9. This is because proportional contraction, in comparison to standardized growth gap, is not a STD-adjusted measure and hence the estimated \underline{x} is larger for the longer time period because we are adding more drastic events such as the great depression and the world wars.

2.3.2 Test for Exponentiality

Now, we test exponentiality of the empirical distributions using a nonparametric goodness-of-fit test, Kolmogorov-Smirnov test. Let $x_1,...,x_n$ be an ordered sample

with $x_1 \leq \ldots \leq x_n$ and define $S_n(x)$ as follows:

$$S_n(x) = \begin{cases} 0 & x \le x_1 \\ k/n & x_k \le x \le x_{k+1} \\ 1 & x_n \le x \end{cases}$$

Now suppose that the sample comes from a population with CDF, F(x) and define the KolmogorovSmirnov statistic, D_n as follows:

$$D_n = \max_{x} |F(x) - S_n(x)|$$
(119)

If F is continuous then under the null hypothesis $\sqrt{n}D_n$ converges to the Kolmogorov distribution for n sufficiently large. The goodness-of-fit test or the KolmogorovSmirnov test is constructed by using the critical values of the Kolmogorov distribution. The null hypothesis is rejected at level α if

$$\sqrt{n}D_n > K_a,\tag{120}$$

where K_a is found from

$$Pr(K \le K_a) = 1 - \alpha \tag{121}$$

If $D_{n,a}$ is the critical value from the table, where n is the number of observations and a is the significance level. Then $P(D_n \leq D_{n,a}) = 1 - a$. D_n can be used to test the hypothesis that the data came from a population with a specific distribution function F(x).

Table 10 summarizes the result of KS-test for exponentiality. For both measures, when we test the whole range, we reject the null hypothesis that the data is exponen-

Table 10: KS test

Kolmogorov-Smirnov	Test for	Exponentiality
	D_n	$D_{n,a}, a = 0.05$
Measure 1	0.088	0.061
Measure 1 (w/o tail)	0.045	0.066
Measure 2	0.130	0.075
Measure 2 (w/o tail)	0.059	0.085

tially distributed. It is straightforward in the sense that the upper tail of the data exhibits a power law behavior as seen in section 3.1 and this heavy upper tail generates some large deviations from the theoretical distribution which are picked up by the KS-test. If we truncate the sample to $x < \underline{x}$, however, then we do not reject the null at 5 % significance level. This implies that the data is exponentially distributed up to \underline{x} and follows a power law pattern thereafter. This is called a exponential with a power law cutoff.

In summary, we used several empirical tests and found that the size-frequency distribution of ED episodes follows a mixture of a power law and an exponential distribution. In the following section, we introduce a stochastic model that explains the empirical distribution.

2.4 Model

We may think of an episode of economic distress (ED) as a wildfire. In a wildfire, the mass of trees burned determines the share of a forest that is destroyed. Intuitively, we may think of a ED episode as one where a mass of firms goes bankrupt, which reduces the rate of economic growth below its trend or may even lead to a recession. In both situations, the dynamics may be modelled as the interaction between two forces:

- 1. A recessionary stochastic process that spreads the distress over a larger share of firms in the economy and
- 2. An stochastic extinguishment policy that attempts to stop the economic distress.

In each ED episode, the interaction of these two forces determines the size of the economic loss (i.e., the area of the forest that is destroyed by the wildfire) as well as the duration of the episode. We have data on the cross-sectional distribution of final economic distress. Our objective is to establish a closed-form link between the dynamics of individual ED episodes and the cross-sectional distribution of final sizes. To such end we consider a specific recessionary process followed by a representative ED episode and an extinguishment policy.

Let t denote the time since the onset of distress: $t \in [0, T_i]$ and $X_i(t)$ be the cumulative share of output that has been lost since the onset of the ED event in economy i. Like in wildfire models, we can think of the rate at which distress progresses throughout the economy as a function of the share of the economy that has been distressed since the onset of the ED episode. In particular, we consider a monotone stochastic process that gives rise to a mean rate of destroyed output which is proportional to $X_i(t)$

$$E[dX_i|X_i(t)] = \mu(X_i(t))dt \ge 0 \quad \mu(X_i(t)) \equiv X_i(t).$$
(122)

To ensure that the sample paths of an individual fire are increasing, we consider the following "pure birth" continuous time setup with discrete states, labelled 1, 2, 3, \dots .etc.⁴ These states capture how wide has the fire spread.

The process is in state j at time t if the area $X_i(t)$ burned by time t exceeds ⁴See Berman and Halina (1996) and Reed and McKelvey (2002). marker size x_j , but not marker size x_{j+1} . That is,

$$x_j < X_i(t) < x_{j+1}$$
, with $x_{j+1} - x_j \equiv \Delta > 0$ for all j.

where the assumption that states are equally spaced (i.e., $x_{j+1} - x_j \equiv \Delta$ for all j) is made for simplicity. If the process is in state j at time t, then the probability that it will be in state j + 1 at time t + dt, is

$$P(X_i(t+1) = x_{j+1}|x_j) = \lambda_j dt + o(dt), \quad \text{with } \lambda_j = \frac{\mu(x_j)}{\Delta}.$$
 (123)

Similarly, the probability that it will remain in state j is

$$P\left(X_i(t+1) = x_j | x_j\right) = 1 - \lambda_j dt + o(dt).$$

It follows that the expected growth in the size of the area burned in the infinitesimal interval (t, t + dt), given that $X_i(t) = x_j$, is

$$E(X_{i}(t+dt) - X_{i}(t)|X_{i}(t) = x_{j}) = \lambda_{j}\Delta dt + o(dt) = \mu(x_{j})dt + o(dt)$$
(124)

Next, we model the extinguishment rate stochastically using a so-called killing rate function

$$k(t) = \lim_{dt \to 0} \frac{1}{dt} P(T_i < t + dt | T_i \ge t)$$
(125)

As we shall see below, the shape of the extinguishment policy is a key determinant of the cross-sectional distribution of final distress sizes. We assume that the extinguishment policy is a state-dependent step function:

$$k(t) = \nu(X_i(t)) = \begin{cases} C_0 & \text{if } X_i(t) < \underline{x} \\ C_1 X_i(t) & \text{otherwise} \end{cases}$$
(126)

This policy captures the notion that when an ED episode is mild (i.e., $X_i(t) < \underline{x}$) the government (or international organizations like the IMF or the EMS) do not face pressure to implement emergency interventions beyond the existing authomatic stabilizers, and so the ED episode is left to extinguish itself. However, if the ED episode morphes into a crisis (i.e., a threshold is crossed ($X_i(t) \ge \underline{x}$) then government policies to stop the crisis are implemented. The intensity of these policies grows proportionally to the size of the ED.

Let $\nu_j = \nu(x_j), j = 1, 2, ...$ so that the probability of the fire (ED episode) ending in the infinitesimal interval (t, t+dt), given that it was in state j at time t is $\nu_j dt + o(dt)$.

We next derive the final size of the burned area (i.e., the final size of the ED) when extinguishment occurs. Let \bar{X} denote the state when the ED process is killed. Then the discrete PDF is:

$$f_j \equiv P(\bar{X} = j) = \frac{\nu_j}{\nu_j + \lambda_j} \prod_{n=1}^{j-1} \frac{\lambda_n}{\lambda_n + \nu_n}$$
(127)

$$= \frac{\rho_j \Delta}{1 + \rho_j \Delta} \prod_{n=1}^{j-1} \frac{1}{1 + \rho_n \Delta}, \quad \text{where } \rho_n = \frac{\nu_n}{\lambda_n \Delta} = \frac{\nu_n}{\mu(x_n)}.$$
(128)

We can rewrite this expression in terms of the the discrete hazard function $\theta_n = \frac{\nu_n}{\nu_n + \lambda_n}$

$$f_j = \theta_j \prod_{n=1}^{j-1} (1 - \theta_n), \quad \theta_n = \frac{\nu_n}{\nu_n + \lambda_n} = \frac{\rho_n \Delta}{1 + \rho_n \Delta}$$
(129)

To derive this result consider the transition diagram of a birth-killing process in

Figure 12. There are two types of transition: (i) a "birth" that moves the system from state n to state n+1, with a birth rateat λ_n ; and (ii) a killing that moves system from state n to state 0, with a killing rate ν_n . As the transition diagram indicates, if the system moves to state 0, the process ends and the final size of economic distress is given by \overline{x}_n . To derive equation (127) notice that the likelihood that the ED episode ends after reaching state n is simply the likelihood that is not killed in any state lower than $j\left(\text{i.e., } S(j) = \prod_{n=1}^{j-1} \frac{\lambda_n}{\lambda_n + \nu_n}\right)$ times the likelihood that it is killed in state j+1 (i.e., $\theta_j = \frac{\nu_j}{\nu_j + \lambda_j}$). In other words, with discrete states, the likelihood that the ED episode's final size equals x_n is given by the product of the discrete survival function S(j) times the discrete hazard function θ_j .

To obtain the continuous limit we first obtain the continuous hazard function and then use it to derive the continuous survival function. The continuous hazard function $\rho(x)$ is defined as

$$\rho(x) = \lim_{dx \to 0} \frac{1}{dx} P(X < x + dx | X \ge x) = \frac{f(x)}{S(x)}.$$
(130)

To obtain the continuous hazard function $\rho(x)$ we divide the discrete hazard function $\theta_j = \frac{\rho_j \Delta}{1+\rho_j \Delta}$ by Δ and let $\Delta \to 0$. We get

$$\rho(x) = \lim_{\Delta \to 0} \frac{\rho_j}{1 + \rho_j \Delta} = \rho_j \equiv \frac{\nu(x_j)}{\mu(x_j)}.$$
(131)

Let the cumulative hazard rate function be

$$P(x) = \int_{x_0}^x \rho(u) du = \int_{x_0}^x \frac{f(u)}{S(u)} du = \int_{x_0}^x \frac{-\frac{dS(u)}{du}}{S(u)} du = -\log S(x)$$
(132)

Thus, the continuous survival function $S_{\bar{X}}(x) \equiv P(\bar{X} > x)$ is

$$S_{\bar{X}}(x) = exp(-P(x)) = exp\left(-\int_{x_0}^x \rho(x')dx'\right), \quad \text{where } \rho(x) = \frac{\nu(x)}{\mu(x)}.$$
 (133)

Taking the derivative of $S_{\bar{X}}(x)$, yields the following density for \bar{X}

$$f_{\bar{X}}(x) = \rho(x) \exp\left(-\int_{x_0}^x \rho(x')dx'\right)$$
(134)

From equation (133) and (134), it follows that

$$\rho(x) = -\frac{d}{dx} \log S_{\bar{X}}(x) = -\frac{S'_{\bar{X}}(x)}{S_{\bar{X}}(x)}$$
(135)

Then it follows that $x\rho(x)$ is constant if and only if the cross-section of final sizes of ED events \bar{X} follows a power-law distribution:

$$\log S_{\bar{X}}(x) = b - a \log x$$

Meanwhile, $\rho(x)$ is constant if and only if \overline{X} follows an exponential distribution:

$$\log S_{\bar{X}}(x) = b - ax$$

We are now equiped to interpret our findings in the Empirical section since the empirical counterpart of $\log S_{\bar{X}}(x)$ is is the ordinate in Figure 13 through Figure 17. Since we have assumed that the growth rate of an individual ED episode $\mu(X)$ is X and the extinguishing policy $\nu(X)$ follows (126), we have that in the cross section

$$\rho(x) = \frac{\nu(X)}{\mu(X)}$$
 follows

$$\rho(x) = \frac{C_0}{X} \text{for } x \le \underline{x} \text{ and}$$

$$\rho(x) = \frac{C_1 X}{X} = C_1 \text{ for } x > \underline{x}$$

This is consistent with the PL with an exponential cutoff we characterized in the empirical section.

2.5 Literature Review

This paper is linked to a vast literature both theoretical and empirical on economic downturns. Most of the studies concentrate on the catastrophic events such as financial crises and wars, so they focus only on the tail distribution of economic distress episodes. Barro (2006) and Barro and Jin (2011) document a power-law distribution of rare disasters, which they define as a decline in per-capita GDP of more than 15 percent. While they only consider rare disasters whose probability is quite slim (1.5-2 percent per year), our study covers all economic downturns from small economic disturbances to catastrophic crises. Laeven and Valencia (2013) identify the starting date of systemic financial crises by policy indices. Bordo et al. (2001), Hoggarth et al. (2002) and Reinhart and Rogoff (2009) focus on frequency and severity of several different types of financial crises based on internal propagation mechanism of several kinds of economic crises.

While there is huge literature on economic disturbances and financial crises, a distributional analysis of the economic distress has rarely been performed. A distributional approach to other economic issues is growing and is comprehensively surveyed in Gabaix (1999, 2009, 2016)). He documents that there is much empirical evidence for the existence of power-laws in Economics.

However, Clauset et al. (2009) show that in most cases, the hypothesized power law distribution is not tested rigorously against the data, and hence the power law appears to be not convincing. They argue that the standard practice of identifying and quantifying power-law distributions by the approximately straight-line behavior of a histogram on a doubly logarithmic plot should not be trusted. Clauset et al. (2009) present a statistically principled set of techniques that test a power law along with the likelihood ratio tests for model selection based on Vuong (1989). Pisarenko and Sornette (2006) provide a statistical tool to compare the behavior of tail distributions with power-law and exponential distributions.

In the context of measuring the size of economic distress, Ormerod and Mounfield (2001) analyze the duration of the recession of 17 capitalist economies and reports that the duration follows a power-law distribution. Similarly, Redelico et al. (2008) collect data from 19 additional Latin America countries and reinforce the results of Ormerod and Mounfield (2001). Duration of recessions is closely related to economic distress, however, the dispersion in the size of economic distress for the same length of duration is very huge. See Figure 19. Moreover, it is hard to analyze the duration of recession statistically as it is categorical data. Wright (2005) concludes that the duration of recessions follows an exponential distribution using the same dataset of Ormerod and Mounfield (2001).

2.6 Conclusion

Power laws appear widely in both the natural and social sciences. In this paper, we use the analytical tools in Clauset et al. (2009) and the nonparametric goodnessof-fit test to characterize the frequency and size distribution of the past economic distress episodes in history. Our empirical results demonstrate that power law distributions provide a clear explanation to the upper tail of the frequency and size distributions. It has been also found that the power law pattern is valid for different measurements of economic distress, different set of countries, and for different time periods. Furthermore, We document that there is a threshold below which the size of ED events follows an exponential distribution. After characterizing the empirical distribution, we provide a stochastic wildfire model explaining how the distribution could be generated.

2.7 Appendix

2.7.1 Birth and Killing Process

The transition diagram of a birth/killing process looks like the following:

Figure 12: Transition Diagram



There are two transition types: λ_n (birth rate) moves system from n to n + 1. ν_n (killing rate) moves system from n to 0 (extinguishment stage). As the transition diagram indicates, if the system is moved to "stage 0", the process is terminated and the final size of economic distress is determined. With discrete states, $\theta_n = \frac{\nu_n}{\nu_n + \lambda_n} = \frac{\rho_n \Delta}{1 + \rho_n \Delta}$ is the discrete hazard function. This gives that the discrete survival function is $S(j) = \prod_{n=1}^{j-1} \frac{\lambda_n}{\lambda_n + \nu_n}$.

In a continuous setting, dividing $\frac{\rho_n \Delta}{1+\rho_n \Delta}$ by Δ and then letting $\Delta \to 0$, yields the continuous hazard function, $\rho(x)$.

$$\rho(x) = \lim_{dx \to 0} \frac{1}{dx} P(X < x + dx | X \ge x) = \frac{f(x)}{S(x)}$$
(136)

Let the cumulative hazard rate function be

$$P(x) = \int_{x_0}^x \rho(u) du = \int_{x_0}^x \frac{f(u)}{S(u)} du = \int_{x_0}^x \frac{-\frac{dS(u)}{du}}{S(u)} du = -\log S(x)$$
(137)

Thus

$$S(x) = exp(-P(x)) = exp\left(-\int_{x_0}^x \rho(x')dx'\right)$$
(138)

2.7.2 Figures







Figure 14: Size-frequency Distribution of Economic Distress Events (Measure 2)



Figure 15: Exponential with a Power Law Cutoff

Figure 16: Size-frequency Distribution of Economic Distress Events (Advanced Economies vs. Emerging Market Economies)





Figure 17: Size-frequency Distribution of Economic Distress Events (1830 - 2014)



Figure 18: Official recession dates announced by NBER



Figure 19: Duration vs. Size of Economic Distress

3 The Forward Premium Puzzle: Developed vs. Emerging Countries

3.1 Introduction

According to the uncovered interest rate parity (UIP), expected changes in the nominal exchange rate should have a positive relation to the difference in the nominal interest rate across countries. More specifically, under the UIP condition, the slope coefficient from the regression of the exchange rate change on the interest rate differential should be equal to one. Empirical evidence, however, suggests the opposite. The slope coefficient from this regression is close to zero or sometimes significantly negative. This observed pattern, known as the forward premium puzzle, is widely documented in the international finance literature.

Most studies, starting from Fama (1984), focus on developed countries, and there is only few research analyzing the forward premium puzzle on emerging economies Bansal and Dahlquist (2000) and Frankel and Poonawala (2010) report that the puzzle is less severe in emerging countries. This empirical evidence makes the puzzle more difficult because the risk premia of foreign exchange in emerging economies must be greater than that in developed countries. Thus, I consider an alternative approach to understanding the puzzle: relaxing rationality.

In this paper, I replicate the Fama regression both on developing and emerging economies. I have similar results to Bansal and Dahlquist (2000) and Frankel and Poonawala (2010). The forward premium puzzle exists on developed economies, but the UIP condition tends to hold on emerging economies. I also conduct the Fama

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Table

	Ŋ	Major Financial Crises in recent decades		
	Name of Crisis	Size of Economic Distress (Measure 1 used)	Duration	Ending Year of ED
Mexico	Latin American debt crisis	10.18	7 years	1988
Finland	Scandinavian banking crisis	11.19	4 years	1993
\mathbf{S} weden	Scandinavian banking crisis	8.62	$4 \mathrm{years}$	1993
$\operatorname{Argentina}$	Argentine economic crisis (1999-2002)	6.19	4 years	2002
Malaysia	1997 Asian Financial Crisis	22.23	$3 \mathrm{years}$	1999
Indonesia	1997 Asian Financial Crisis	18.32	3 years	1999
Greece	European sovereign debt crisis	14.5	7 years	2013
Spain	European sovereign debt crisis	15.61	7 years	2013
Japan	Collapse of Japanese asset price bubble	11.71	$9 \mathrm{years}$	1999
United States	Global financial crisis	8.02	$5 \mathrm{years}$	2009

Emerging Economies	Data Availability (WDI)	Data Availability (MPD)	Advanced Economies	Data Availability (WDI)	Data Availability (MPD)
Algeria (DZA)	1961 - 2014	1956-2014	Austrailia (AUS)	1961 - 2014	1831-2014
Argentina (ARG)	1961 - 2014	1886-2014	Austria (AUT)	1961 - 2014	1881 - 2014
Belize (BLZ)	1961 - 2014	1962-2014	$\operatorname{Belgium}\left(\operatorname{BEL} ight)$	1961 - 2014	1856-2014
Botswana (BWA)	1961 - 2014	1956-2014	Canada (CAN)	1961 - 2014	1881-2014
Brazil (BRA)	1961 - 2014	1861-2014	Denmark (DNK)	1961 - 2014	1831-2014
Chile (CHL)	1961 - 2014	1831-2014	Finland (FIN)	1961 - 2014	1871-2014
China (CHN)	1961 - 2014	1851-2014	France (FRA)	1961 - 2014	1831-2014
Colombia (COL)	1961 - 2014	1916-2014	Germany (DEU)	1971 - 2014	1861-2014
Costa Rica (CRI)	1961 - 2014	1931-2014	Greece (GRC)	1961 - 2014	1931-2014
Dominican Rep. (DOM)	1961 - 2014	1961-2014	Ireland (IRL)	1961 - 2014	1931-2014
Ecuador (ECU)	1961 - 2014	1951-2014	Israel (ISR)	1961 - 2014	1931-2014
Egypt (EGY)	1966 - 2014	1916-2014	Italy (ITA)	1971 - 2014	1831-2014
El Salvador (SLV)	1966 - 2014	1931-2014	Japan (JPN)	1961 - 2014	1881-2014
Gabon (GAB)	1961 - 2014	1956-2014	Luxembourg (LUX)	1961 - 2014	1960-2014
Georgia (GEO)	1966 - 2014	1901-2014	Netherlands (NLD)	1961 - 2014	1831-2014
Guatemala (GTM)	1961 - 2014	1931-2014	New Zealand (NZL)	1978 - 2014	1881-2014
Honduras (HND)	1961 - 2014	1931-2014	Norway (NOR)	1961 - 2014	1841-2014
Iceland (ISL)	1961 - 2014	1881-2014	Portugal (PRT)	1961 - 2014	1876-2014
India (IND)	1961 - 2014	1895-2014	Spain (ESP)	1961 - 2014	1861-2014
Indonesia (IDN)	1961 - 2014	1881-2014	Sweden (SWE)	1961 - 2014	1831-2014
Iran, Islamic Rep. (IRN)	1961 - 2014	1956-2014	Swiss (CHE)	1981 - 2014	1861-2014
Iraq (IRQ)	1969 - 2014	1961-2014	United Kingdom (GBR)	1961 - 2014	1831-2014
Jamaica (JAM)	1967 - 2014	1951-2014	United States (USA)	1961 - 2014	1831-2014
Korea (KOR)	1961 - 2014	1926-2014			
Malaysia (MYS)	1961 - 2014	1926-2014			
Mexico (MEX)	1961 - 2014	1911-2014			
Morocco (MAR)	1961 - 2014	1961-2014			
Panama (PAN)	1961 - 2014	1956-2016			
Paraguay (PRY)	1961 - 2014	1951-2014			
Peru (PER)	1961 - 2014	1926-2014			
Phillippines (PHL)	1961 - 2014	1926-2014			
South Africa (ZAF)	1961 - 2014	1831-2014			
Thailand (THA)	1961 - 2014	1961-2014			
Trinidad and Tobago (TTO)	1961 - 2014	1961-2014			
Turkey (TUR)	1961 - 2014	1936-2014			
Uruguay (URY)	1961 - 2014	1882-2014			
Venezuela (VEN)	1961 - 2014	1896-2014			

Table 12: Classifications of countries based on their level of development

regression with dummy variables to eliminate the periods that interest rate differential is unusually high or low. If interest rate spread is in the middle range, the result is robust to the Fama regression. However, when interest rate spread is exceptionally high or low compared to historical level, the results are different. When interest rate in the U.S. is high compared to that in a foreign country, the US dollar tends to appreciate both on developed and emerging countries. Gourinchas and Tornell (2004) show that the puzzle can be explained if investors have a biased belief which overestimate transitory shocks to persistent shocks about the interest rate process. I decompose interest rate differential with transient and persistent components by using the state space model. Developed countries tend to have larger persistent shocks, so investors in developed economies can overdraw the likelihood that current shock he observes is persistent.

The rest of the paper is organized as follows. Section 2 presents the background of the forward premium puzzle. Section 3 discusses the empirical results: the Fama regression and the state space model. Section 4 discusses the related literature. Finally, Section 5 concludes.

3.2 Background

Uncovered interest parity (UIP) represents the relationship between the log of spot exchange rate in next period s_{t+1} to the log of spot exchange rate today s_t , and interest rates in each country i_t (Home), and i_t^* (Foreign).

$$E_t s_{t+1} = s_t + i_t - i_t^* \tag{139}$$

If a foreign country has a higher interest rate than home country, then this should be compensated by the expected depreciation of its currency. The following equation is the Fama regression,

$$s_{t+1} - s_t = \alpha + \beta(i_t - i_t^*) + \varepsilon_{t+1} \tag{140}$$

Under the UIP condition, α and β must be equal to zero and one, respective. Empirically, however, β tend to be around zero or even negative. The UIP condition lies on several strong assumptions: (1) investors are risk-neutral, (2) investors have a rational expectation, (3) transaction cost is zero, (4) default risk is equally spread over Home and Foreign countries. Many studies focus on the risk premiums.

The excess return on the Home deposit held from period t to period t+1, inclusive of currency return is given by

$$\rho_t = i_t - i_t^* - (E_t s_{t+1} - s_t) \tag{141}$$

The variance of the risk premium can be decomposed as:

$$var(\rho_t) = var(i_t - i_t^*) - 2cov(i_t - i_t^*, E_t s_{t+1} - s_t) + var(E_t s_{t+1} - s_t)$$
(142)

According to the result of the Fama regression, $\beta = \frac{cov(i_t - i_t^*, s_{t+1} - s_t)}{var(i_t - i_t^*)} < \frac{1}{2}$ implies:

$$var(\rho_t) > var(E_t s_{t+1} - s_t) \tag{143}$$

The variance of risk premium is greater than the variance of the change of exchange rate. However, it is empirically known that exchange rates are more volatile than the predictions of monetary models based on interest rate parity or no foreign exchange risk premium. Engel (2016) analyzes the forward premium puzzle in real terms and found that the puzzle also hold in real term

$$cov(E_t q_{t+1} - q_t, r_t - r_t^*) < 0 \tag{144}$$

where q_t and q_{t+1} are real exchange rates at time t and t+1. r_t and r_t^* are real interest rates in Home and Foreign countries repectively. This implies that

$$cov(\lambda_t, r_t - r_t^*) < -var(r_t - r_t^*) < 0$$
 (145)

where $\lambda_t \equiv E_t q_{t+1} - q_t - (r_t - r_t^*)$. If real exchange rate (q_t) is stationary, then Engel (2016) empirically find that

$$cov(\Lambda_t, r_t - r_t^*) > 0$$
 where $\Lambda_t = \sum_{j=0}^{\infty} E_t \lambda_{t+j}.$ (146)

Engel (2016) conclude that representative agent models of the risk premium are difficult to explain this empirical findings (145) and (146).

Gourinchas and Tornell (2004) explains the forward premium puzzle by assumption that investors misperceive the persistence of interest rate shocks and they do not learn the true interest process over time. In the model, the true interest rate process follows an AR process with autocorrelation θ :

$$x_t = \theta x_{t-1} + \varepsilon_t \quad \text{where} \quad \varepsilon_t \sim N(0, \sigma_{\varepsilon}^2)$$

$$(147)$$

However, the interest rate process that investors believe is:

$$x_t = z_t + v_t \quad \text{where} \quad v_t \sim N(0, \sigma_v^2)$$
$$z_t = \theta z_{t-1} + \varepsilon_t \quad \text{where} \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

Depending on parameters $\{\lambda, \eta(=\sigma_v^2/\sigma_\varepsilon^2)\}$, this model can generate the puzzle.

3.3 Data and Empirical Results

For this analysis, I investigate exchange rates and interest rates for the U.S. about 38 countries, 1978:1-2012:5 (for most emerging countries, 1992:1-2012:5). According to the *International Finance Corporation* (IFC) of the *World Bank* in 1998, countries are classified as 18 developed countries.⁵ and 20 emerging countries.⁶

I obtain daily foreign exchange rate series from federal reserve historical data. Countries with a fixed exchange rate system (Hong Kong), and with capital controls (India) are included. I construct the monthly data by using the last business day of each month. I use monthly data of one-month annual interbank interest rate from the *Datastream* database. For some countries, available data period is relatively short, and then a different one-month interest rate is used. I transform annual rate into monthly rate.

$$i_M = 100 \left\{ \left(1 + \frac{i_Y}{100}\right)^{\frac{1}{12}} - 1 \right\}$$
(148)

3.3.1 The Fama Regressions

The Fama regression (Fama (1984)) is the bases for the forward premium puzzle. It is usually reported as a regression of the change in the log of the exchange rate between time t and t + 1 on the time t interest differential:

$$s_{t+1} - s_t = \alpha + \beta(i_t - i_t^*) + \varepsilon_{t+1} \tag{149}$$

⁵Developed countries: Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Italy, Japan, Netherland, Norway, Singapore, Spain, Sweden, Swiss and the United Kingdom

⁶Emerging countries: Argentina, Brazil, Chile, Colombia, Czech Republic, Greece, Hungary, India, Indonesia, Korea, Kuwait, Malaysia, Mexico, Philippines, Poland, Portugal, Saudi Arabia, Taiwan, Thailand, and Turkey

where dependent variable $s_{t+1} - s_t$ denotes the log difference of the exchange rate, expressed as the U.S. dollar price of foreign country currency, independent variable $i_t - i_t^*$ denotes the monthly nominal interest rate differential between the U.S. and a foreign country. Under the uncovered interest parity, $\beta_0 = 0$, and $\beta_1 = 1$.

Table 13 reports the point estimate and standard deviation of α and β . The estimated coefficient β is negative in most developed countries, whereas the estimated coefficient β is positive in most emerging countries. The forward premium puzzle is less severe in emerging countries. The estimated coefficient β is negative for 14 out of 18 cases in developed countries and only 3 out of 20 cases in emerging countries. Interestingly, The coefficients of major currencies such as Canadian Dollar, Japanese Yen, Swiss Franc and Britsh Pound are significantly negative. This result is consistent with Bansal and Dahlquist (2000) and Frankel and Poonawala (2010).

3.3.2 The Fama Regressions with Dummy Variables

To further characterize when the forward premium puzzle is present, I consider the following regression.

$$s_{t+1} - s_t = \alpha + (\beta_1 - \beta_2)(i_t - i_t^*)I^- + \beta_2(i_t - i_t^*) + (\beta_3 - \beta_2)(i_t - i_t^*))I^+ + \varepsilon_{t+1}$$
(150)

where I^- is 10 percent left tail of the interest rate differential and I^+ is 10 percent right tail of the interest rate differential. Thus, β_1 , β_2 , and β_3 are the coefficients of left tail, middle range, right tail interest rate differential, respectively.

Table 14 reports the point estimate and standard deviation of α , β_1 , β_2 and β_3 for developed countries. The result of the Fama regression is robust, as the coefficient of the middle range interest differential shows negative for 16 out of 18 cases. There are asymmetry on the left tail and the right tail interest rate differentials. The left tail means that interest rate in the foreign country is high compared to that in the US. It seems that the forward premium puzzle disappears as only 3 out of 18 cases has negative coefficients. When interest rate in the US is relatively high to the foreign country (interest rate spread is positive (or close to zero), the forward premium is strong as 12 out of 18 cases has negative coefficients.

Table 15 reports the point estimate and standard deviation of α , β_1 , β_2 and β_3 for emerging countries. The coefficient of the middle range interest differential shows negative for 6 out of 20 cases. It seems that the UIP condition does not work well on the both left and right tail interest rate differentials. For the left tail coefficient, 10 out of 20 cases hold the forward premium puzzle. For the right tail coefficient, 14 out of 20 cases hold the puzzle.

The result of Fama regression is robust, as the coefficient of the middle range interest spread shows similar results. Interestingly, there exists the forward premium puzzle in both developing and emerging countries on the right tail.

3.3.3 Persistence of Interest Spread

Gourinchas and Tornell (2004) propose an explanation for the forward premium puzzles based on a distortion in beliefs about future interest rates. The persistence of interest rate differential is important, as agents continually predict interest rate differentials. If the prediction of investors is biased, this can lead to negative coefficients in the Fama regression. As the forward premium puzzle appears in developed countries and it does not appear in emerging countries, it may be useful to compare interest rate differential processes of both developed and emerging countries.

Consider the following interest rate differentials (x_t) , which consists of a persistent
component (z_t) and a transitory component (v_t) .

$$x_t = z_t + v_t \tag{151}$$

$$z_t = \lambda z_{t-1} + \varepsilon_t \tag{152}$$

where $v_t \sim N(0, \sigma_v^2)$ and $\varepsilon_t \sim N(0, \sigma_{\varepsilon}^2)$ and both are independent. This is a simple form of the state space model. I try to estimate $\{\lambda, \eta(=\sigma_v^2/\sigma_{\varepsilon}^2)\}$ by Maximum Likelihood.

Figure 20 and Figure 21 shows interest rate differential process of developed and emerging countries. Table 16 reports the estimated AR(1) coefficient λ and the estimated ratio of the variance of transitory shocks and persistent shocks η . Both developed and emerging countries have a very persistent component in the interest rate differential process. The variance of the persistent component is larger in developed countries compared to that in emerging countries.

3.4 Literature Review

There is a vast literature on the forward premium puzzle. Hansen and Hodrick (1980), Fama (1984), Engel (1996) Bansal and Dahlquist (2000), Frankel and Poonawala (2010), Lothian and Wu (2011) and Engel (2016) document predictability of exchange rate movements and the violation of the uncovered interest rate parity.

There have been many attempts to solve this puzzle through the risk premium. Chari et al. (2002) explain volatility and persistence of exchange rate by Sticky prices and monetary shock. Bacchetta and Van Wincoop (2010) adopt infrequent portfolio decisions. Colacito and Croce (2011) and Bansal and Shaliastovich (2013) develop a long-run risks model in two country setting. Verdelhan (2010) uses Habit model, Farhi and Gabaix (2015) consider the probability of rare disasters to explain the risk premium on foreign exchange markets.

There is another strand of literature that tries to explain the forward premium puzzle by relaxing rationality assumption. Froot and Frankel (1989) empirically decompose the bias in foreign exchange market into risk premium and expectation errors. Froot and Frankel (1989) conclude that the systemic expectational errors are more significant than the risk premium. Gourinchas and Tornell (2004) and Burnside et al. (2011) provide theoretical models that can generate the forward premium puzzle in foreign exchange market based on investors' misbelief and overconfidence.

3.5 Conclusion

In the paper, I investigate the forward premium puzzle both developed and emerging market currencies. The UIP condition is more violated in developed countries than in emerging countries, which is consistent with Bansal and Dahlquist (2000) and Frankel and Poonawala (2010). Unlike developed countries which show the slope coefficient significantly less than zero in the Fama regression, the coefficient for emerging market is close to zero. Thus, the puzzle is less severe in emerging countries. Two categorized countries have different interest rate differential processes. Both developed and emerging countries have a persistent process of interest rate differential, however, developed countries tend to larger shocks that are linked to persistent component compared to emerging countries. This difference may affect to agents' beliefs, and from the model of Gourinchas and Tornell (2004), the distortion can generate the puzzle.

3.6 Appendix

3.6.1 Figures and Tables



Figure 20: Interest Rate Differentials: Developed Countries



Figure 21: Interest Rate Differentials: Emerging Countries

Panel A: Developec	l countries				Panel B: Emergin	g countries			
Country	σ	$SE(\alpha)$	β	$\operatorname{se}(\beta)$	Country	σ	$se(\alpha)$	β	$se(\beta)$
Austria	0.045	0.194	-1.128	1.182	Argentina	-0.517	0.481	0.372	0.632
Belgium	0.002	0.179	-0.305	0.642	Brazil	-0.163	0.567	0.173	0.169
Canada	-0.051	0.115	-1.329	0.651	Chile	0.460	0.402	-2.290	1.337
Denmark	0.136	0.186	-0.580	0.758	Colombia	0.311	0.394	0.815	0.267
Finland	0.060	0.193	0.878	1.198	Czech Republic	0.293	0.279	0.837	0.572
France	-0.054	0.177	-0.372	0.833	Greece	0.078	0.252	0.610	0.566
Germany	0.113	0.186	-0.920	0.876	Hungary	0.350	0.522	1.027	0.499
Hong Kong	0.000	0.006	-0.029	0.049	India	0.762	0.354	2.645	1.105
Ireland	0.142	0.336	-0.159	1.283	Indonesia	-1.102	0.963	-0.460	1.560
Italy	-0.032	0.201	0.518	0.611	Korea	-0.039	0.366	0.450	1.279
Japan	0.841	0.221	-2.512	0.742	Kuwait	0.101	0.061	0.954	0.529
Netherland	0.178	0.187	-1.622	0.945	Malaysia	-0.061	0.182	1.473	1.145
Norway	0.156	0.233	0.481	1.030	Mexico	-0.569	0.401	0.143	0.347
Singapore	0.249	0.156	-1.330	0.872	Philippines	0.316	0.429	1.370	1.118
Spain	-0.019	0.211	1.266	0.893	Poland	0.344	0.497	0.928	0.394
Sweden	-0.034	0.227	-0.538	1.169	Portugal	-0.024	0.210	0.324	0.792
Swiss	0.474	0.245	-1.289	0.742	Saudi Arabia	-0.005	0.009	-0.216	0.318
United Kingdom	-0.390	0.196	-2.350	1.206	Taiwan	-0.076	0.113	0.108	0.659
					Thailand	0.139	0.220	1.213	1.726
					Turkey	0.359	0.530	0.783	0.260
<i>Notes:</i> This table restimated coefficient.	ports the e ß is negati	stimated conversion of the second sec	oefficients o he forward	f α , β and	their standard de transference	viation. Tl	he gray bo	oxes represe	nt that the
		· (mm - · · · ·							

Table 13: Fama Regressions: $s_{t+1} - s_t = \alpha + \beta(i_t - i_t^*) + \varepsilon_{t+1}$ for 1978:1-2012:5

Austria 0.17	$se(\alpha)$	β_1	$\operatorname{se}(eta_1)$	β_2	$\operatorname{se}(\beta_2)$	β_3	$\operatorname{se}(\beta_3)$
	1 0.224	1.501	2.738	-0.793	1.791	-4.221	2.930
Belgium 0.01 ⁴	4 0.186	4.078	2.055	-1.879	1.606	2.749	1.992
Canada 0.03	1 0.133	0.444	1.245	-0.592	1.357	-1.781	2.137
Denmark 0.089	9 0.207	8.187	1.880	-3.834	1.753	5.675	3.344
Finland 0.23	0 0.207	4.232	1.812	-0.868	1.097	-3.907	2.636
France 0.07:	9 0.182	1.834	1.626	-0.529	1.425	-3.245	2.364
Germany 0.31	0 0.215	4.051	2.338	-1.213	1.366	-0.471	1.672
Hong Kong 0.00	5 0.006	0.389	0.325	-0.178	0.324	-0.007	0.415
Ireland 0.48;	3 0.572	-9.541	3.990	4.526	4.391	-13.263	7.937
Itary 0.11	5 0.261	-0.815	1.172	0.870	1.000	-5.126	3.452
Japan 1.03	5 0.297	11.066	4.000	-2.977	1.268	2.698	1.295
Netherland 0.38	0.214	5.339	2.435	-1.722	1.410	-1.088	1.596
Norway 0.20;	3 0.245	1.742	1.657	-0.153	1.259	-3.615	3.898
Singapore 0.20	2 0.228	-4.316	22.193	-0.889	1.552	0.427	1.151
SPA 0.04	0.233	3.893	1.553	-0.835	1.311	-1.067	2.985
SWE -0.10	1 0.253	6.556	2.344	-2.899	1.593	7.557	3.222
SWI 0.65a	8 0.307	5.845	4.374	-1.960	1.214	1.593	1.107
UK -0.28	9 0.230	4.539	1.840	-2.527	1.645	-1.745	3.122
<i>Notes:</i> This table reports t estimated coefficient β is ne	he estimated of station of the states of the	coefficients of α the forward pre-	, β and their mium puzzle e	standard devi exists.	ation. The g	tay boxes repr	resent that the

 Table 14: Fama Regressions with Dummy Variables (Developed Countries)

Argentina -0.239 Brazil -0.565 Chile 0.908		۲		1		C	
Brazil -0.565 Chile 0.908	0.415	-2.383	0.737	1.340	0.564	-7.824	4.458
Chile 0.908	0.925	0.157	0.585	-0.045	0.753	-2.327	1.404
	0.421	91.817	38.718	-4.051	1.502	5.868	1.578
Colombia 0.232	0.470	-0.973	0.284	0.818	0.361	-4.158	4.434
Czech Republic 0.257	0.369	-2.426	1.353	1.377	1.483	0.528	3.008
Greece 0.101	0.331	-1.976	1.119	1.124	1.061	-0.376	3.470
Hungary -0.174	0.826	1.051	0.653	0.009	1.086	-1.900	6.153
India -2.218	1.661	3.594	2.069	-2.167	2.469	-1.460	3.621
Indonesia 0.808	0.556	-1.613	1.396	2.430	1.932	3.254	4.477
Korea 0.142	0.435	-3.633	1.383	1.710	1.310	5.137	12.988
Kuwait 0.128	0.091	-3.050	1.215	1.834	1.327	-1.322	2.629
Malaysia 0.031	0.179	-2.045	4.867	2.099	2.055	-4.318	1.804
Mexico -0.631	0.547	-0.241	0.742	0.160	0.832	-3.739	3.449
Philippines -0.231	0.426	1.233	1.123	-0.036	0.887	-11.500	7.542
Poland 0.174	0.704	-0.776	0.457	0.777	0.680	8.281	9.337
Portugal 0.058	0.233	1.358	1.795	-0.291	1.613	-2.525	3.214
Saudi Arabia 0.011	0.013	-0.178	0.218	0.142	0.252	-2.607	2.721
Taiwan 0.252	0.190	1.468	2.498	0.402	0.570	-5.529	2.113
Thailand -0.143	0.755	0.108	0.313	0.411	0.434	-1.015	1.364
Turkey -0.039	0.139	3.542	2.518	-1.277	1.816	3.007	1.951

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Table

Country	λ	$\operatorname{se}(\lambda)$	$\eta(=\sigma_v^2/\sigma_\varepsilon^2)$	$se(\eta)$
Panel A: Developed	countries			
Canada	0.961	0.021	0.220	0.143
France	0.973	0.008	0.108	0.213
Germany	0.995	0.005	0.092	0.132
Italy	0.931	0.079	0.232	0.351
Japan	0.992	0.005	0.167	0.228
United Kingdom	0.991	0.016	0.251	0.183
Panel B: Emerging	countries			
Brazil	0.992	0.021	1.097	0.980
Czech Republic	0.926	0.049	0.667	0.302
Korea	0.994	0.007	0.229	0.284
Mexico	0.961	0.003	0.421	0.310
Saudi Arabia	0.759	0.104	0.399	0.596
Taiwan	0.980	0.012	0.299	0.412

Table 16: Estimated coefficients of the State Space Model)

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