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Performance Prediction of Nonlinear Degrading Systems

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Abstract—The lack of a fundamental theory of hysteresis is a major barrier to successful design of structures against deterioration. Development of a practical method for identification and prediction of degradation is an important task. This paper has a two-fold objective. First, a robust identification algorithm will be devised to generate models of degradation of a structure from its experimental load-displacement traces. This algorithm will be based upon the generalized differential model of hysteresis and the theory of genetic evolution, streamlined through sensitivity analysis. Second, it will be validated by experimentation that a model of degradation obtained by identification can be used to predict the future performance of a structure. Through brute-force identification of hysteretic evolution or degradation, it becomes possible to assess, for the first time in analysis, the performance of a real-life structure that has previously been damaged.

Keywords—System identification; hysteresis; degrading structures; nonlinear response

I. INTRODUCTION

All structures degrade when acted upon by cyclic forces such as those associated with earthquakes, high winds, and sea waves. Development of a practical method for the identification and prediction of degradation of structures is a problem of considerable practical significance. Under cyclic excitation, degradation manifests itself in the evolution of the associated hysteresis loops. Theoretical research in internal friction in the last few decades has noticeably increased the conceptual understanding of hysteresis [1, 2]. Practical issues related to internal friction, however, have not been adequately addressed.

In the past thirty years, cyclic performance testing of structural joints and subassemblies around the world has generated a substantial amount of experimental data on load-displacement traces. In the same period, generalization of the

Bouc-Wen differential model of hysteresis permits curve-fitting of practically any hysteretic trace with a suitable choice of its thirteen control parameters [3, 4]. Using system identification techniques, it appears highly feasible to utilize the generalized differential model of hysteresis and the extensive database of experimental hysteretic traces to deduce a working model for degrading structures. A fundamental objective of this paper is to do just that.

Two principal tasks in connection with hysteretic evolution will be addressed. First, a robust identification algorithm will be used to generate hysteretic models of a deteriorating structure from its experimental load-displacement traces. Second, it will be shown that a hysteretic model obtained by system identification can be used to predict the future performance of the same deteriorating structure. The organization of this article is as follows. In Section II, the smoothly-varying differential model of hysteresis in both its classical (non-degrading) and generalized forms are described. A robust identification algorithm is constructed in Section III to generate hysteretic models of a deteriorating structure from its experimental load-displacement traces. This algorithm is based upon the generalized Bouc-Wen model and differential evolution, streamlined through global sensitivity analysis. The model thus generated can account for degradation and pinching effects, which are prominent features of real-life structural deformation. To obtain experimental data for model validation, cyclic performance tests of simple joints and subassemblies are reported in Section IV. Using an experimental load-displacement trace, a working hysteretic model is identified. It will be shown that the hysteretic model obtained by identification may be used for predicting the nonlinear response of the same structure when driven by other cyclic loads. The requirements for accurate prediction of system response will be discussed.

II. DIFFERENTIAL MODEL OF HYSTERESIS

When a structure is subjected to severe cyclic loading, the hysteresis loops associated with the structural response are memory-dependent. That means the evolution of hysteresis loops depends not only on the instantaneous deformation but also on the history of deformation. The generalized Bouc-Wen model is one of the widely used empirical models capable of modeling the memory effects.

Suppose the equation of motion of a multi-degree-of-freedom system can be decoupled and, along the direction of the generalized coordinate x , the system is governed by

$$m\ddot{x} + c\dot{x} + r(x, z) = f(t) \quad (1)$$

where m , c are respectively the mass and damping coefficients, z is an imaginary hysteretic displacement, and $r(x, z)$ is the total restoring force. It is assumed that the excitation $f(t)$ is cyclic. In the development of differential model, the restoring force $r(x, z)$ is separated into an elastic (linear) component and a hysteretic (non-linear) component by

$$r(x, z) = akx + (1 - \alpha)kz \quad (2)$$

where k is the stiffness coefficient and $0 \leq \alpha \leq 1$ is a weighting parameter. Obviously, the restoring force is purely hysteretic if $\alpha = 0$; it is purely elastic if $\alpha = 1$. Hysteresis loops may be generated if the hysteretic displacement z and the total displacement x are connected by the nonlinear differential equation [3, 4]

$$\dot{z} = A\dot{x} - \beta|\dot{x}|z^{n-1}z - \gamma\dot{x}|z|^n \quad (3)$$

There are five unspecified loop parameters A , α , β , γ , n in Eqs. (2) and (3), which together represent the classical Bouc-Wen model. Over the years, the original Bouc-Wen model has been extended and new parameters have been added to fit hysteretic shapes arising from deteriorating systems. The result is a contemporary model with thirteen control parameters given by

$$\dot{z} = h(z) \left\{ \frac{A\dot{x} - v(\beta|\dot{x}|z^{n-1}z + \gamma\dot{x}|z|^n)}{\eta} \right\} \quad (4)$$

In the above expression, v and η are degradation shape functions [5] and $h(z)$ is a pinching shape function. In general, degradation depends on the response duration and severity. A convenient measure of such combined effect is the energy

$$E(t) = \int_0^t (1 - \alpha)kz\dot{x}dt \quad (5)$$

dissipated through hysteresis from initial time $t = 0$ to present time t . Since

$$\varepsilon(t) = \int_0^t z\dot{x}dt \quad (6)$$

is proportional to $E(t)$, it may also be used as a measure of response duration and severity. Both degradation shape functions v and η are assumed to depend linearly on ε as the system evolves:

$$v(\varepsilon) = 1 + \delta_v \varepsilon \quad (7)$$

$$\eta(\varepsilon) = 1 + \delta_\eta \varepsilon \quad (8)$$

Two unspecified degradation parameters δ_v and δ_η are thus introduced. Under cyclic excitation, the pinching of hysteresis loops is often observed. For example, pinching may be associated with slippage of longitudinal reinforcement in reinforced concrete or with X -braced steel frames driven by high-shear loads. Baber and Noori [6] introduced a slip-lock element behaving quite similarly to a hardening nonlinear spring, with the special characteristics that the "slip" zone stiffness is nearly zero while the "lock" zone stiffness is infinite. The "slip" zone stiffness is initially zero and increases continuously as the system degrades with time. Thus, the pinching shape function $h(z)$ takes the form [7]

$$h(z) = 1 - \zeta_1 e^{-[z \operatorname{sgn}(\dot{x}) - qz_u]^2 / \zeta_2^2} \quad (9)$$

where sgn is the signum function and z_u is the ultimate value of z given by

$$z_u = \left(\frac{A}{v(\beta + \gamma)} \right)^{1/n} \quad (10)$$

The two functions $\zeta_1(\varepsilon)$ and $\zeta_2(\varepsilon)$ control the progress of pinching and are written as

$$\zeta_1(\varepsilon) = \zeta_s [1 - e^{(-p\varepsilon)}] \quad (11)$$

$$\zeta_2(\varepsilon) = (\psi + \delta_\psi \varepsilon)(\lambda + \zeta_1) \quad (12)$$

Six pinching parameters ζ_s , q , p , ψ , δ_ψ , and λ are therefore present. Altogether there are thirteen loop parameters of hysteresis: A , α , β , γ , n , δ_v , δ_η , ζ_s , q , p , ψ , δ_ψ , and λ . This generalized model of hysteresis possesses all the important features observed in real structures, which include strength degradation, stiffness degradations, and pinching of the successive hysteresis loops.

Compared with other parametric models, a differential model of hysteresis has many advantages [7, 8]. The primary one is its ability to generate a large variety of realistic hysteresis loops. Another advantage is the coupling of the equation of motion (1) to either Eq. (3) or (4) to form an overall differential system. This greatly facilitates any theoretical and numerical manipulations [9]. It must be emphasized that the extended Bouc-Wen model of hysteresis is an empirical model. As such, it is not derivable from the fundamental postulates of mechanics and the exact physical meanings of its thirteen parameters are not fully understood. The probable role played by each parameter is summarized in Table I. Also contained in Table I are the sensitivity rankings [10] of the control parameters. As will be explained in the next section, in system

identification it is required to estimate only a subset of the thirteen unspecified parameters.

TABLE I. PARAMETERS OF THE GENERALIZED DIFFERENTIAL MODEL OF HYSTERESIS.

Parameter	Description	Local sensitivity ranking (highest = 1)	Global sensitivity ranking (highest = 1)
α	Ratio of linear to nonlinear response	1	2
A	Basic hysteresis shape control	Not varied	Not varied
β	Basic hysteresis shape control	5	4
γ	Basic hysteresis shape control	6	5
n	Sharpness of yield	8	7
δ_v	Strength degradation	12	9
δ_η	Stiffness degradation	4	8
ζ_s	Measure of total slip	2	1
q	Pinching initiation	9	6
p	Pinching slope	3	10
ψ	Pinching magnitude	7	3
δ_ψ	Pinching rate	11	12
λ	Pinching severity	10	11

III. SYSTEM IDENTIFICATION

In analysis, the response of a system is sought if the system model and excitation are known. This is sometimes termed a forward problem. In this interpretation, the inverse problem of finding a system model given the excitation and response is called system identification. System identification in the time domain involves the determination of unspecified parameters of an assumed system model. This can always be formulated as an optimization problem. In the present context, suppose the differential model of hysteresis is adopted and a set of measured excitation-response data from cyclic performance tests of an inelastic structure is given. How can the loop parameters of hysteresis be estimated from the measured data? For each choice of the parameters, the response of the degrading structure subjected to the given excitation can be obtained by numerical simulation. The calculated response data can then be compared to the measured data to see if there are large errors. Obviously, the assumed loop parameters provide a good fit if the errors are small. Thus, this amounts to estimating the thirteen control parameters of the differential model of hysteresis when a load-displacement trace is given. The

optimization problem can be stated as the determination of the parameter vector

$$\mathbf{p} = (A, \alpha, \beta, \gamma, n, \delta_v, \delta_\eta, \zeta_s, q, p, \psi, \delta_\psi, \lambda) \quad (13)$$

such that the objective function

$$g(\mathbf{p}) = \frac{1}{N} \sum_{j=1}^N [x(t_j) - \hat{x}(t_j | \mathbf{p})]^2 \quad (14)$$

is minimized. In the above expression, t_j is a sequence of time instants and $x(t_j)$ is the given system displacement at t_j , where $j = 1, 2, \dots, N$. On the other hand, $\hat{x}(t_j | \mathbf{p})$ is the system displacement at t_j calculated from Eqs. (1) and (4) when the parameter vector is equal to \mathbf{p} . The chosen objective function is simply the mean-square error in the displacement. Minimization of the objective function is subjected to the constraint that all parameters in \mathbf{p} with the exception of γ are positive [10].

Early studies of parametric identification of hysteresis typically employed the non-degrading classical differential model containing only five parameters [11, 12]. In the few studies that involved degradation [13-15], either a restricted Bouc-Wen model containing less than thirteen parameters or a restricted identification algorithm was used. For example, Zhang et al. [15] used gradient-based local-search algorithms to estimate some of the hysteretic control parameters for degrading structures. However, the generalized differential model of hysteresis is highly nonlinear and any gradient-based method tends to be trapped near local minima and therefore fails to converge. A robust and efficient identification algorithm is needed to estimate all thirteen unspecified parameters of the differential model of hysteresis [16, 17].

A. Reduction of Parameters

In order to streamline the identification of the control parameters of differential hysteresis, the generalized Bouc-Wen model has been re-examined [10]. Two significant issues have been uncovered. First, it was discovered that the unspecified parameters of the differential model are functionally dependent. One of the thirteen control parameters can be eliminated through suitable transformations in the parameter space. The number of unspecified parameters can thus be reduced from thirteen to twelve without any loss of generality. Elimination of a parameter will appreciably accelerate the convergence of any identification algorithm. As explained in [10], a convenient way to eliminate one parameter is to map the parameter A into 1. Henceforth $A=1$ will be assumed and there remain twelve loop parameters in the differential model of hysteresis.

The second issue uncovered is the existence of insensitive parameters in the generalized Bouc-Wen differential model: the variations of three or four control parameters in the generalized model do not appreciably alter the computed hysteretic evolution of the system. Through local and global sensitivity analyses, the twelve remaining parameters are ranked in order of decreasing sensitivity, as shown in Table I. The method of global analysis employed is a probabilistic method recently

expounded by Sobol [18]. It can account for the mutual interactions of the twelve parameters. On an overall basis, δ_v , δ_ψ , and λ are the least sensitive parameters. In identification of hysteresis, these parameters are likely to slow down numerical convergence. One extreme measure to deal with this problem is to set the insensitive parameters to constants in the beginning; at any rate they need not be estimated with high precision. It is decided in this investigation that a two-stage procedure will be adopted to streamline the identification process. In the first stage, the three least sensitive parameters δ_v , δ_ψ , and λ are fixed and a crude value of the nine-parameter vector

$$\mathbf{p}_1 = (\alpha, \beta, \gamma, n, \delta_\eta, \zeta_s, q, p, \psi) \quad (15)$$

is first estimated by minimization of the objective function in Eq. (14). This crude value of \mathbf{p}_1 , together with the fixed values of δ_v , δ_ψ , and λ , are then used as seeds in a second-stage identification to estimate the optimal value of the twelve-parameter vector

$$\mathbf{p}_2 = (\alpha, \beta, \gamma, n, \delta_v, \delta_\eta, \zeta_s, q, p, \psi, \delta_\psi, \lambda) \quad (16)$$

In the identification of hysteresis, this two-stage procedure economizes on both computer core memory as well as computing time.

B. Nonlinear Optimization Algorithms

The goal of an optimization problem is to find a vector \mathbf{p}^* in the search space S so that certain quality criterion is satisfied, namely the error norm $g(\mathbf{p})$ in Eq. (14) is minimized. The vector \mathbf{p}^* is a solution to the minimization problem if $g(\mathbf{p}^*)$ is a global minimum in S . For the constrained nonlinear optimization problem associated with the identification of differential hysteresis, the error surface defined by the objective function and constraints can exhibit many local minima or can even be multimodal. For this reason, different solution techniques will have dramatically different performance. The primary consideration in evaluating an optimization algorithm may be convergence speed or the minimum error achieved. Secondary consideration may be consistency, robustness, computational efficiency, or tracking capabilities.

Solution techniques are classified into local or population search. Basic local search methods rely on iterative descent towards a minimum in which the direction of descent depends either on the gradient $\nabla g(\mathbf{p})$ or the Hessian matrix $\nabla^2 g(\mathbf{p})$ of the objective function. There are drawbacks of most gradient-based techniques, which include difficulties in calculating the gradient and frequent trapping of the iterates near a local minimum [19]. In view of that, Zhang et al. [15] used a non-gradient method based upon the Nelder and Mead simplex method to minimize the objective function $g(\mathbf{p})$ in Eq. (14). The simplex method explores the search space S either by reflecting, contracting or expanding away from the worst

vertex, or shrinking towards the best vertex. An appropriate sequence of such movements converges to the nearest local minimum. This downhill simplex method requires only function evaluations and not derivatives. However, even this method does not have desirable convergence properties.

Population-based methods, also known as evolutionary computations, search the entire solution space S by maintaining a group of candidate vectors. Evolutionary techniques are inspired by natural evolution and adaptation with the essence of survival of the fittest. During the iterative process, new candidate vectors are generated from existing ones by means of variation and selection such that the set of candidates would move towards increasingly favorable regions of the search space. Variation is achieved by first combining several candidate vectors to form a new one and then performing a random modification of the components of the resulting vector. Selection is introduced by comparing the fitness of the candidate vectors; the smaller the objective function $g(\mathbf{p})$ the fitter the vector \mathbf{p} . The fitter vectors will be selected to survive and become members of a new generation. Popular evolutionary techniques include genetic algorithms [20], evolution strategies [21], and differential evolution [11, 22]. Instead of performing sequential search as in the von Neumann architecture, evolutionary techniques employ massively parallel schemes with many computational elements connected by links of various weights. As a consequence, these techniques tend to be relatively robust because the risk of being trapped near a local minimum is significantly reduced. The tradeoff is that evolutionary techniques are relatively slow compared with gradient methods. Genetic algorithms require conversion of floating-point values into bit strings and are less efficient for problems in which the parameters are real-valued. For most floating-point problems, evolution strategies are more complicated and computationally involved than differential evolution. Differential evolution emerges as the best evolutionary method for use in the identification of hysteretic parameters of the generalized Bouc-Wen model.

C. Differential Evolution

Differential evolution is a relatively new and efficient approach for minimizing real-valued nonlinear and non-differentiable continuous-space functions [23]. Let K be the dimension of the parameter vector \mathbf{p} in Eq. (13). Suppose each generation in differential evolution consists of a population of P vectors. New vectors are generated in successive generations through certain mechanisms such that the vectors are getting closer and closer to the optimal value. Basic mechanisms of differential evolution can be illustrated in Fig. 1.

In the differential evolution algorithm, three control parameters P , F , and CR must be pre-selected. Lampinen and Storn [22] indicated that convergence of differential evolution is not particularly dependent upon the choice of these control parameters. It suffices to choose F , CR as multiples of 0.1 and P as a multiple of 10. Finally, a convergence criterion is needed to determine if the evolution process has converged. A common procedure is to set a desired objective value or to establish a threshold based on the relative fitness

between the best and the worst vectors in each generation. Another method is to require a minimum percentage of improvement in fitness in order to continue the iteration process. One may also limit the number of generations or the computing time or both. Each convergence criterion has its advantages and drawbacks. Thus, a combination of convergence criteria is often used.

IV. PREDICTION OF PERFORMANCE

With a robust and relatively efficient algorithm devised in Section III, a differential model of hysteresis of any degrading system may be identified from a given load-displacement trace. Suppose a hysteretic model is generated with a given load-displacement trace. It will be shown that, under fairly broad conditions, the model predicts the response of the same structure when driven by other cyclic loads. This can be carried out experimentally by building a number of identically configured structural joints and subjecting them to different cyclic loads. One set of load-displacement trace is chosen to generate a hysteretic model of the joint by system identification. Subsequently, this identified model is used to compute the displacements of the joint under other cyclic loads. The computed displacements will be compared with experimental measurements. A reasonable match will validate the identified model and will demonstrate its prediction capability. In order to reduce the front-end costs of this investigation, only wood joints are used in the experiments. It must be emphasized that steel and concrete structures may be used as easily. Although a limited set of experimental data will be presented in this article, many different structural joints have been built and tested to support any observations made herein.

A. Test Set-Up

Cyclic tests of degrading structures are performed in the Forest Products Laboratory located at the Richmond Field Station of the University of California, Berkeley. Twenty identical specimens of a simple *T*-connection consisting of two wood members joined by plywood gusset-plates, as shown in Fig. 2, are constructed. The two wood members are made out of laminated veneer lumber (LVL), which is a highly predictable engineered wood product with relatively small material variability. Each LVL member is a uniform beam with a square cross-section of 3.5 in. x 3.5 in. and an elastic modulus of 2.0×10^6 psi. The LVL lumber is sawn to consistent sizes and is virtually free from warping and splitting. The two LVL members are connected on both sides by plywood gusset-plates of 0.5 in. thickness. Six 2-in. metal screws are arranged in three rows to fasten the plates to the vertical beam. The two plates are also connected securely to the horizontal beam with four metal screws and two retrofit bolts. Cyclic loads are applied parallel to the lower beam at a height 36 in. above the lower beam. The arrangement is such that the *T*-connection will fail through detachment of vertical LVL member from the gusset plates.

Common seismic testing protocols are employed to drive the *T*-connections. It is claimed that such protocols would simulate damage accumulation and degradation of the structural joint under real-life loads. One of the cyclic excitations used in the experiments are shown in Fig. 3. Excitation *C* was developed at the Forest Products Laboratory for the experiments reported herein. This protocol features trailing cycles that are decreasing in amplitude, which are different from the equal-amplitude trailing cycles in other excitations.



Fig.1. Basic mechanisms of differential evolution.



Fig. 2. Reaction frame test system with a specimen.

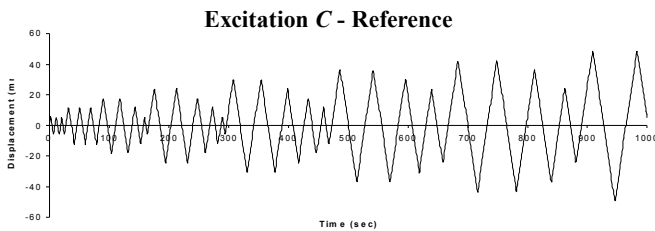


Fig. 3. Excitation of Forest Products Laboratory.

B. Model Identification and Validation

Each specimen of the identically configured T -connections is driven by a cyclic excitation and 1000 seconds of load-displacement data are recorded. Using any of these load-displacement traces, a model of hysteretic evolution can be identified. Two issues will now be examined. (1) Can a hysteretic model identified with a given cyclic load predict the future structural response if the same cyclic load continues beyond the duration used for identification? (2) Can a hysteretic model identified with a given cyclic load predict the structural response due to a different cyclic load?

In Fig. 4, the load-displacement trace associated with excitation C for the first 500 seconds is shown. As reflected in the hysteresis trace, the gusset joint is degrading in both strength and stiffness. There is also some pinching of the hysteretic loops, which is probably caused by slipping of the connecting screws. The load-displacement trace of Fig. 4 is used as input to the identification algorithm devised in the last section. Recall that $A=1$, and there remain twelve hysteretic parameters to be estimated of the gusset-plate connection. Once these twelve parameters are identified, a hysteretic model of the T -connection is obtained. This hysteretic model can now be used to compute the displacement history of the gusset joint associated with excitation C . As illustrated in Fig. 5, the computed displacement history closely matches the experimental measurements. This validates the identified hysteretic model over the first 500 seconds.

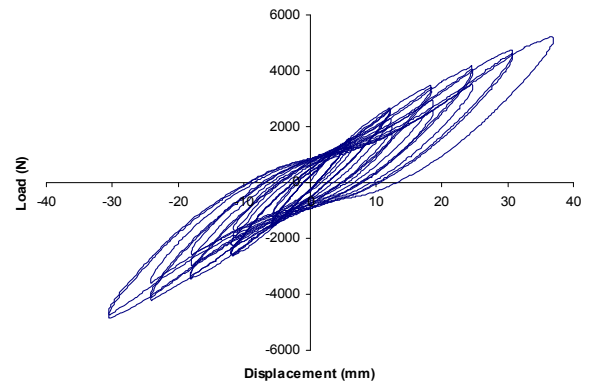


Fig. 4. Hysteresis loops generated by excitation C in 500 seconds.

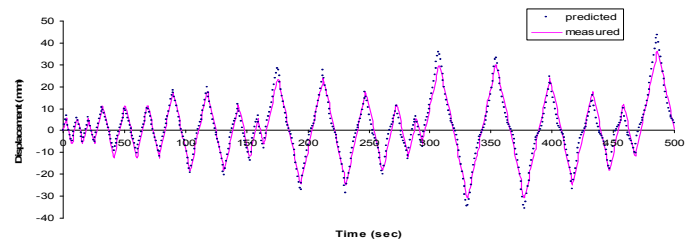


Fig. 5. Comparison of computed and measured displacements under excitation C .

In identifying a hysteretic model of the T -connection, the experimental load-displacement trace associated with excitation C has been used directly as input for identification. It can be argued that such input data are corrupted by Gaussian noise, and a more accurate model can be identified if noise is suppressed with a filter. To this end, the input was first taken through a median filter and then a least-squares low-pass filter. Such a filtering process is known to be effective in smoothing out random fluctuations caused by Gaussian noise [15]. However, it has been found that a better match than what is shown in Fig. 5 is not achieved with the noise-filtered data. It appears that differential evolution is not sensitive to a moderate level of Gaussian noise. Subsequently, noise-filtering is not used on any of the load-displacement traces.

If the duration of input used for identification already contains all features of hysteretic evolution, then a hysteretic model identified with a given excitation can accurately predict the future response when the same excitation continues beyond the duration used for identification. This basically answers the first question brought up at the beginning of this section. Can a hysteretic model obtained by identification using a given excitation predict the structural response due to a different cyclic load? This is the second question brought up earlier. Based upon experimentation and simulation, it is found that a hysteretic model identified with a given excitation may be used for response prediction under different excitations. However, the accuracy of prediction tends to diminish as time increases.

C. Requirements for Accurate Prediction

If structural degradation can be controlled so as to result in a failure configuration identical to what is contained in the input data used for identification, then nonlinear degrading response of a structure can be predicted accurately. Otherwise, prediction is only reliable up to a point at which the input data and the existing structure progress into different failure configurations. A study of the influence of failure configurations on the reliability of response prediction will be worthwhile in a subsequent course of investigation.

V. SUMMARY

A basic objective of this paper is to advance the methodology for predicting the performance of nonlinear deteriorating structures. Using the generalized Bouc-Wen differential model, it is shown how system identification can be utilized to predict the response of a degrading structure well beyond its linear range. Important observations reported in the paper are summarized in the following statements.

1. A robust identification algorithm based upon the generalized Bouc-Wen model and the theory of differential evolution can be used to generate practical models of hysteresis of degrading structures. Differential evolution is not sensitive to a moderate level of input noise.
2. If the duration of input used for identification already contains all features of hysteretic evolution, then a hysteretic model identified with a given cyclic load can accurately predict the future structural response when the same cyclic load continues beyond the duration used for identification.
3. A hysteretic model identified with a given cyclic load may be used to predict the nonlinear response under different cyclic loads. As damage accumulates, the precision of predicted response decreases and the reliability of prediction ultimately depends on the degree of similarity between failure configurations under different cyclic loads.

In the absence of a fundamental theory of degradation, the response prediction of degrading structures is indeed a challenging task. System identification of hysteretic evolution provides a brute-force procedure for prediction that has the potential of allowing a closer representation of reality. The research reported herein addresses various aspects of an identification methodology for predicting the performance of real-life deteriorating structures well beyond their linear ranges. Among other things, it is hoped that this paper would point to directions along which further research efforts should be made.

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