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SINGLE REGGE POLE ANALYSIS OF $\pi^- + p \to \eta^0 + n$ †

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There is particular interest in high-energy reactions in which a single Regge pole in the crossed channel may be believed to dominate. At present accelerator energies, elastic scatterings are not in this category. One must include several Regge poles chosen from the presently well-established mesons which form a spectrum grouped into nonets with quantum numbers 2^+ and 1^-. For other reactions, however, the cross-channel quantum numbers are more restrictive, and in some cases only a single Regge pole is known with the appropriate quantum numbers. The first case of this kind to be analyzed was $\pi^- + p \to \pi^0 + n$, at small momentum transfers, for which only the ρ Regge pole is known to be relevant and for which a single-pole analysis is successful.[1,2]

This letter presents the analysis of a second case, $\pi^- + p \to \eta^0 + n$, at small momentum transfers, for which high-energy data have just become available.[3] Here only the R Regge pole[4,5,6] (associated with the A_2 meson) is known to have the correct cross-channel quantum numbers. We show that these data are consistent with a single Regge pole whose trajectory in turn is consistent with the A_2 meson mass.

We already had information about the R trajectory from KN and $\overline{K}N$ scattering.[1] Furthermore, the couplings of R to the $\overline{K}K$ and $\pi\eta$ systems are approximately related by SU_3 symmetry, so that we were able to predict the $\pi^- + p \rightarrow \eta^0 + n$ cross section before the data arrived.(7) This prediction was remarkably successful.[3] Nevertheless, it is desirable to reanalyze the KN and $\overline{K}N$ data simultaneously with the new information about $\pi^- + p \rightarrow \eta^0 + n$, without the use of SU_3 symmetry (which is not exact), to show that the same R trajectory is consistent with both sets of data. We also achieve thereby a precise test of the accuracy of SU_3 symmetry.

Our formalism follows that of Ref. 1, for pseudoscalar meson-nucleon scattering. At high energies the $\eta - \pi$ mass difference effects are negligible compared with experimental errors, and we simply use elastic kinematics. The contributions of R to the nonflip and helicity-flip amplitudes A and B (which correspond to A' and B in Singh's notation)[8] are parameterized as follows:

$$A = - C_0\, \alpha(2\alpha + 1) \exp(C_1 t)\, \frac{\exp(-i\pi\alpha) + 1}{\sin \pi\alpha} \left(\frac{E}{E_0}\right)^{\alpha}, \qquad (1)$$

$$B = - D_0\, \alpha \exp(D_1 t)\, \frac{\exp(-i\pi\alpha) + 1}{\sin \pi\alpha} \left(\frac{E}{E_0}\right)^{\alpha-1}. \qquad (2)$$

Here $\alpha(t)$ is the R trajectory, t is the squared momentum transfer, E is the total incident lab energy, and E_0 is an arbitrary scale parameter which we choose to be 1 GeV; C_0, C_1, D_0, and D_1 are real constants.

The trajectory $\alpha(t)$ is given the two-parameter (Pignotti) form

$$\alpha(t) = -1 + [1 + \alpha(0)]^2/[1 + \alpha(0) - \alpha'(0)t] , \qquad (3)$$

$\alpha(0)$ and $\alpha'(0)$ being the intercept and slope at $t = 0$.

The differential cross section, in terms of A and B, is

$$\frac{d\sigma}{dt} = \frac{1}{\pi s} \left(\frac{m_N}{4k} \right)^2 \left\{ \left(1 - \frac{t}{4m_N^2} \right) |A|^2 - \frac{t}{4m_N^2} \frac{st + 4m_N^2 p^2}{4m_N^2 - t} |B|^2 \right\} , \qquad (4)$$

where s is the total c.m. energy squared, m_N is the nucleon mass, p is the pion lab momentum, and k is the c.m. momentum.

We first fitted the six parameters of R to the $\pi^- + p \rightarrow \eta^0 + n$ data alone. The best fit, to 39 data points, has $\chi^2 = 27.9$, which is more than adequate. The corresponding parameters are shown in the first line of Table I (labeled solution 0). Note that a substantial slope, $\alpha_R'(0)$, is found, consistent with the position of the A_2 meson at $\alpha = 2$, which is 1.1 GeV from Eq. (3) compared with 1.32 GeV from experiment. The fit to data is illustrated in Fig. 1.

The best fit with no shrinkage $[\alpha_R'(0) = 0]$ has an intercept, $\alpha_R(0)$, which is 0.29 ± 0.03, and $\chi^2 = 37.4$, several standard deviations off from a good fit, and much worse than the case above wherein the single extra shrinking parameter $\alpha_R'(0)$, evaluated to be 0.65 ± 0.15, is used to bring down the χ^2 by 9.5.

We then reanalyzed these data together with the KN and $\overline{K}N$ data previously considered.[1] The new constraints were that the trajectory

$\alpha_R(t)$ and the ratio A_R/B_R should be the same when both sets of data are fitted (the A/B requirement comes from factorization). This reanalysis was made for solutions 1 and 2 of Ref. 1; the corresponding R parameters are shown on the second and third lines of Table I, and the corresponding values of χ^2 are 182 and 170 respectively, for 154 data points and a total of 18 parameters.

For completeness, the parameters for the KN and $\overline{K}N$ systems are shown in Tables II and III; these correspond to Tables IV and V of Ref. 1. The notation is fully explained in Ref. 1. Briefly, however, we may add that the amplitudes for P, P', and ρ Regge poles are expressed in terms of the πN amplitudes, if we use the factorization condition

$$A_i(KN)/A_i(\pi N) = B_i(KN)/B_i(\pi N) = F_0 \exp(F_1 t), \qquad (5)$$

the πN amplitudes being already fixed for each of the solutions. The ω Regge pole contribution to B is ignored: its contribution to A is parameterized by using a difference of two exponentials--hence four parameters instead of two. The ω trajectory, not shown in the Tables, was not re-search, and retained the same values as in Ref. 1.

In the limit of exact SU_3 symmetry, if P is a singlet and ρ belong to an octet, we expect to find in Table II

$$\left\{\begin{aligned} &F_0(P) = 1.0\ , \qquad F_1(P) = 0.0\ , \\ &F_0(\rho) = 0.5\ , \qquad F_1(\rho) = 0.0\ . \end{aligned}\right. \qquad (6)$$

The results confirm what was already noted in Ref. 1, namely, that the symmetry holds quite well for P and ρ, though P' behaves neither like pure singlet nor pure octet.

If R is a pure octet member, we expect to find

$$C_0(R:\pi^- + p \to \eta^0 + n) = (4/\sqrt{3})F_0\, C_0(R:KN) ,$$

$$C_1(R:\pi^- + p \to \eta^0 + n) = C_1(R:KN) + F_1 , \tag{7}$$

with similar relations for D_0 and D_1 :

$$D_0(R:\pi^- + p \to \eta^0 + n) = (4/\sqrt{3})F_0\, D_0(R:KN)$$

$$D_1(R:\pi^- + p \to \eta^0 + n) = D_1(R:KN) + F_1 , \tag{8}$$

and $F_0 = 1$ and $F_1 = 0$.[9]

In our analysis the F_0's were made the same in Eqs. (7) and (8), in order to satisfy the factorization principle [see, for instance, Eq. (5)]; likewise for the F_1's. Their values indicate the degree of breaking of SU_3 .

The measurements of Ref. 3 refer directly to the η-meson production followed by 2γ decay of η . To convert this to the complete η-production cross section, we have used the currently accepted branching ration $(\eta \to 2\gamma)/(\eta \to \text{all}) = 0.386$, [10] for case (a). However, a recent experiment[11] suggests this branching ratio is closer to 0.30; if this new value is used instead, the values of F_0 and also C_0 and D_0 in Table I are multiplied by 1.13, case (b). The results shown in the Table below.

Solution 1	Case a	Case b
F_0	0.66	0.75
F_1	-0.11	
Solution 2		
F_0	0.68	0.77
F_1	0.02	

At the resonance of $A_2(\alpha=2)$ the branching ration $A_2 \rightarrow \pi\eta/K\bar{K}$ requires F_0 to be $(0.56)^{1/2} = 0.75$ as given by Glashow and Socolow.[12] We note in Table I that all the parameters except C_1 for the three separate solutions show good agreement. The present data seem not to be accurate nor extensive enough to determine C_1 more precisely. That the least massive system having the quantum numbers of R is three pions suggests that C_1 and D_1 of Table I should be limited by $(3m_\pi)^{-2} \approx 5.6\,(\text{GeV})^{-2}$. We observe that our three solutions satisfy this condition.

To summarize, we find:

(a) The $\pi^- + p \rightarrow \eta^0 + n$ data are consistent with a single R trajectory with "substantial shrinkage".

(b) The R parameters are also consistent with KN and $\bar{K}N$ data.

(c) The R trajectory is consistent with the A_2 meson position.

(d) The R couplings to $\bar{K}K$ and $\pi\eta$ differ by 33% from the ratio predicted by SU_3 symmetry, if R is pure octet and the currently accepted $\eta \rightarrow 2\gamma$ branching ratio[10] is used. However, a recent experiment suggests this branching ratio may be different and the agreement with exact SU_3 symmetry may be even better.

(e) The factorization principle is a useful constraint in establishing the R parameters. We expect it will prove a powerful tool in explaining related reactions.

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Table I. R parameters for $\pi^- + p \to \eta^0 + n$.

Solution	$\alpha(0)$	$\alpha'(0)$ [(GeV)$^{-2}$]	C_0 (mb × GeV)	C_1 [(GeV)$^{-2}$]	D_0 (mb)	D_1 [(GeV)$^{-2}$]
0	0.40±0.03	0.65±0.15	(a) 2.91 (b) 3.30	1.06	(a) -48 (b) -54	1.97
1	0.41±0.02	0.8 ±0.1	(a) 2.90 (b) 3.29	4.64	(a) -53 (b) -60	1.86
2	0.37±0.01	0.60±0.05	(a) 3.76 (b) 4.27	4.77	(a) -55 (b) -62	2.04

(a) 0.386 used as branching ratio

(b) 0.30 used as branching ratio

Table II. Parameters relating P, P', and ρ contributions to πN and KN.

Solution	P		P'		ρ	
	F_0	F_1 [(GeV)$^{-2}$]	F_0	F_1 [(GeV)$^{-2}$]	F_0	F_1 [(GeV)$^{-2}$]
1	0.90	-0.21	0.29	-1.84	0.51	0.51
2	0.90	-0.22	0.29	-1.22	0.50	0.47

Table III. KN amplitude coefficients for R and ω .

Solution	R				ω			
	C_0 (mb × GeV)	C_1 $[(GeV)^{-2}]$	D_0 mb	D_1 $[(GeV)^{-2}]$	C_0 (mb × GeV)	C_1 $[(GeV)^{-2}]$	C_3 $[(GeV)^{-2}]$	G
1	1.91	4.75	-35	1.98	6.03	11.0	0.09	0.84
2	2.38	4.75	-35	2.02	6.69	11.0	0.002	0.65

FOOTNOTES AND REFERENCES

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FIGURE CAPTION

Fig. 1. $\pi^- + p \rightarrow \eta^0 + n$ differential cross sections at 5.9, 9.8, 13.3, and 18.2 GeV/c, from Ref. 3 converted to complete η^0 production by using the currently accepted branching ratio of Ref. 10, that is, 0.386. The full lines are the results of Solution 0. The sets of data are spaced by a decade. The dots are the Group I and the squares are the Group II data of Ref. 3.

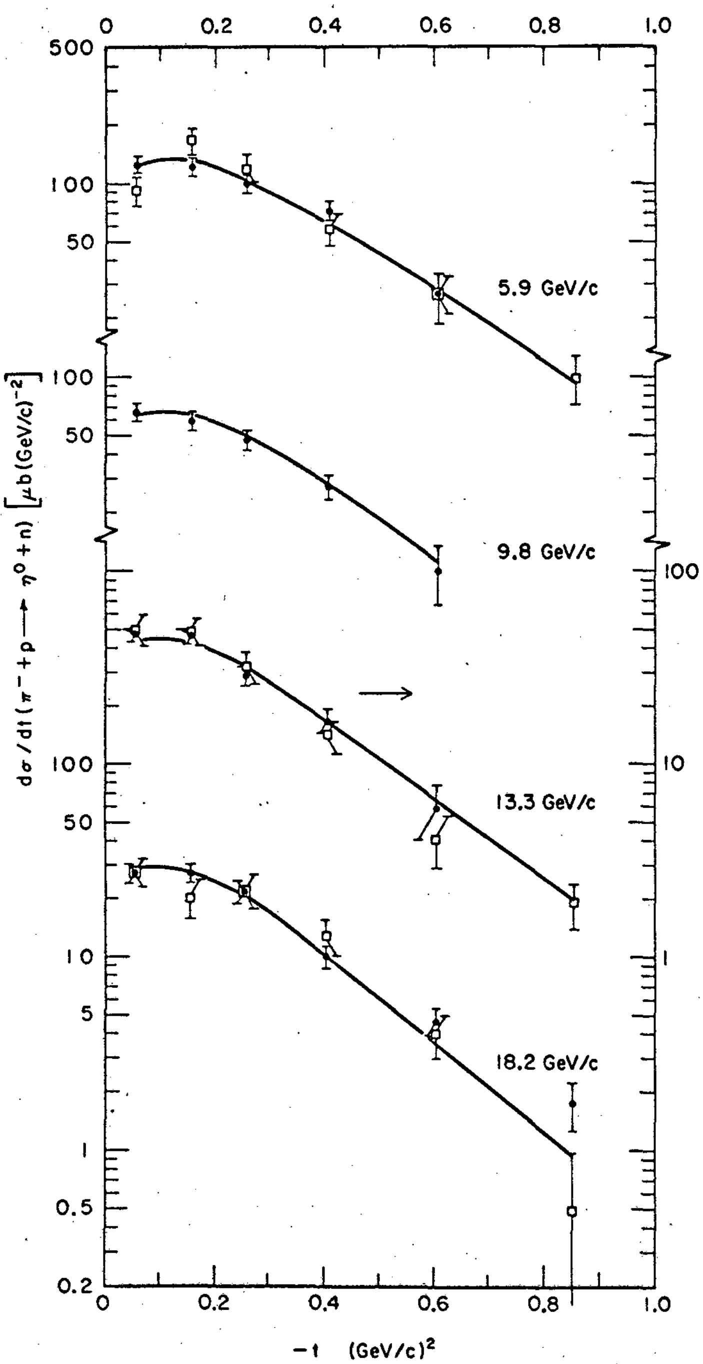
0
0.2
0.4
0.6
0.8
1.0
500
100
50
5.9 GeV/c
100
50
9.8 GeV/c
100
13.3 GeV/c
10
100
50
10
5
1
18.2 GeV/c
1
0.5
0.2
dσ / dt (π⁻ + p ⟶ η⁰ + n) [μb (GeV/c)⁻²]
−t (GeV/c)²
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Fig. 1