Title

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π^-p ELASTIC SCATTERING IN THE CMS ENERGY RANGE 1400-2000 MEV*

A COMPARISON OF THE RESULTS OF ELASTIC SCATTERING PHASE SHIFT ANALYSES

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Introduction

In this supplement a comparison of the many different elastic phase shift analyses is made.

The dynamics of the interaction of a pion with a nucleon are contained in the partial wave amplitudes $T_L^{\pm}$ ($j = L \pm \frac{1}{2}$). These T-matrix elements are related to the centre-of-mass scattering amplitude through the following relations[1]

$$M = f(\theta) + g(\theta)\, \sigma \cdot n$$

where

$$f(\theta) = \frac{1}{k} \sum_L \left\{ (L+1)\, T_L^+ + L\, T_L^- \right\} P_L(\cos\theta)$$

and

$$g(\theta) = \frac{i}{k} \sum_L (T_L^+ - T_L^-)\, P_L^1(\cos\theta)$$

$f(\theta)$ and $g(\theta)$ are the spin non-flip and spin flip scattering amplitudes.

The differential cross-section and polarization are then given by:

$$\frac{d\sigma}{d\Omega} = \left|M(\theta)\right|^2 = \left|f(\theta)\right|^2 + \left|g(\theta)\right|^2$$

and

$$\frac{d\sigma}{d\Omega} \vec{P} = 2\, \mathrm{Re}(f^* g)\, \hat{n}$$

where

$$\hat{n} = \vec{k}_i \wedge \vec{k}_f / \left|\vec{k}_i \wedge \vec{k}_f\right|$$

The partial wave amplitude is then parameterized by a phase shift, δ, and an absorption parameter, η, according to

$$T_L^{\pm} = \frac{\eta_L^{\pm}\, e^{2i\delta_L^{\pm}} - 1}{2i}$$

The object of an elastic phase shift analysis is to determine the values of η and δ as a function of energy. This is accomplished by adjusting the values of η and δ until the predicted cross-sections and polarizations give a reasonable fit to the observed experimental distributions. There are two main types of phase shift analysis — energy-independent and energy-dependent, which are discussed in turn.

Energy Independent

In an energy-independent analysis fits to the experimental data are undertaken at each energy where measurements have been made. Unfortuantely, a large number of 'good' fits are often obtained at each energy and some selection has then to be made. This selection is made in a different manner in each of three major energy-independent analyses.

(i) Saclay[2]

Smooth variation of the η and δ are required together with smoothness in other functions of the partial wave amplitudes.

(ii) Berkeley[3]

In this case a distance function is defined for the change in the value of a partial wave amplitude from one energy to the adjacent energy point. Their final solution is then that set of solutions which has the smallest 'total distance' throughout the whole energy range.

(iii) Cern Experimental and Theory[4]

In this analysis continuity is applied through partial wave dispersion relations. After first making an energy-independent analysis at each energy the parameters are used in a partial wave dispersion relation to predict the partial wave amplitudes at all energies. These are then fed in as 'data' together with the experimental measurements and a new fit performed. Using the results of this fit new partial wave amplitudes can be determined from the dispersion relations and the whole process iterated until consistency is achieved. When the final step of the process is a fit to the experimental data the CERN Experimental Phase Shifts are obtained, and when the last step is a calculation of the partial wave amplitudes through dispersion relations CERN Theory Phase Shifts result.

Energy Dependent

In this type of analysis a smooth variation of the partial wave amplitudes is ensured by making a parameterization of the phase shift etc. All experimental data are then fitted at the same time, the parameters describing the phase shifts etc. being determined. The various analyses of this type differ in their parameterizations.

(i) Roper[5]

In this case polynomial expansions of the δ's and η's are used for the non-resonant contributions to each partial wave. The resonances are parameterized by a Breit-Wigner Amplitude. In all cases the correct threshold dependence is ensured. One drawback of this analysis is that it simply adds resonant and non-resonant amplitudes in a given partial wave, leading to the possibility of violating unitarity.

(ii) Chilton[6]

In this analysis a parameterization of the partial wave amplitudes is used which is consistent with a partial wave dispersion relation. The analysis is performed in two parts, the first for pion kinetic energies less than 700 MeV and the second for the range above 700 MeV, resulting in some discontinuity at the break.

(iii) Glasgow[7]

The analysis used here contains some features of the energy independent method. The phase shift δ and inelasticity parameter η are expanded as a series

$$\delta = \sum_{n=0}^{2} a_n \, (k^2 - k_o^2)^n$$

$$\eta = \sum_{n=0}^{2} b_n \, (k^2 - k_o^2)^n$$

over a limited range of the data eg. 100 MeV. The only restriction imposed is that the value of the phase shift and inelasticity should be continuous in moving from one energy region to the next. The requirement of a continuous first derivative is also made but not strictly enforced. In this manner a continuous set of phase shifts are obtained over the whole range. A fit is then made to this solution with partial wave amplitudes which contain both background and resonances, some partial waves containing more than one resonance. Finally a fit is then made to all of the data allowing the background and resonance parameters to vary. This method leads to the solution GLASGOW A. If the CERN Theory values for the partial wave amplitudes are used to allow an initial determination of the background and resonance parameters before the final fit is attempted, GLASGOW B results.

The properties of these different analyses are summarized in the figures, the elastic and inelast c partial cross-sections being calculated as

$$\sigma^{LIJ}_{\text{elastic}} = \pi\lambda\!\!\!^{-2}(j + \tfrac{1}{2}) \left[1 - 2\eta^{LIJ}\cos 2\delta^{LIJ} + (\eta^{LIJ})^2\right]$$

$$\sigma^{LIJ}_{\text{inelastic}} = \pi\lambda\!\!\!^{-} (j + \tfrac{1}{2}) \left[1 - (\eta^{LIJ})^2\right]$$

where L - orbital angular momentum

J - total angular momentum

I - total isospin

Comments

We would like to add the following remarks;

(i) All of the six methods are biassed against the existence of narrow resonances.

(ii) In many diagrams the small scale obscures the differences in values of η and particularly δ. However, although these differences are small they do correspond to the differences in the elastic and inelastic cross-sections indicated.

(iii) A comparison of these phase shifts results with experimental data can be found in the paper[8] to which this is a supplement.

(iv) In these graphs the solution described as CERN-EXP. corresponds to that described as CERN-KIRSOPP in another compilation[9].

References

1. M. L. Goldberger and K. M. Watson, Collision Theory (John Wiley and Sons, Inc., New York, 1964).

2. P. Bareyre, C. Bricman, A. V. Stirling and G. Villet, Phys. Letters 18, 342 (1965); P. Bareyre, C. Bricman and G. Villet, Phys. Rev. 165, 1730 (1968).

3. C. H. Johnson, UCRL - 17683 (1967).

4. A. Donnachie, R. G. Kirsopp, and C. Lovelace, Phys. Letters 26B, 161 (1968); C. Lovelace, Proceedings of the 1967 Heidelberg Conference (North Holland Publishing Company, Amsterdam, 1967), P.79; C. Lovelace, Invited paper at the Conference on πN Scattering, Irvine (1967); CERN Preprint TH-839 (1967).

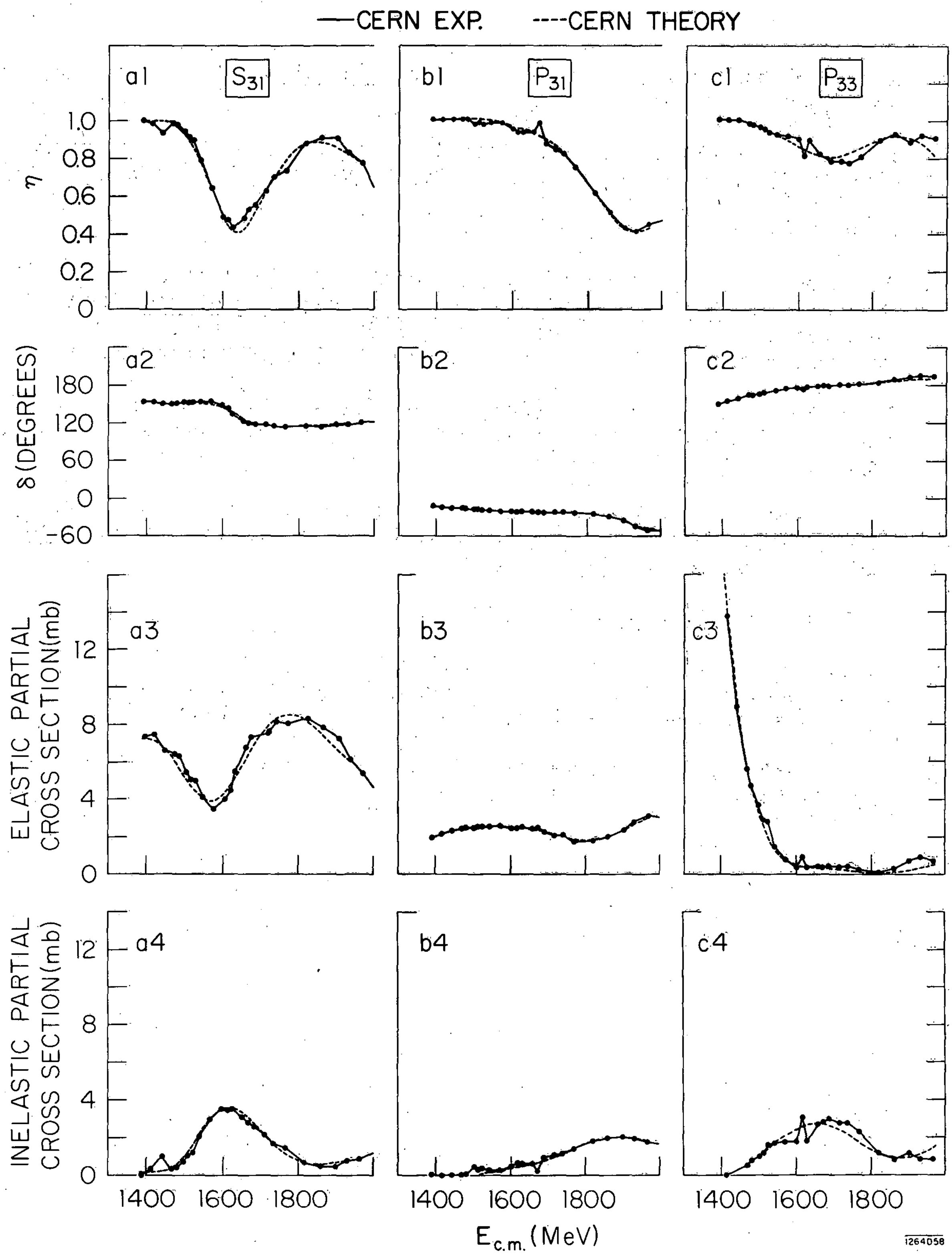
5. L. D. Roper, Phys. Rev. Letters 12, 340 (1964); L. D. Roper, R. M. Wright, and B. T. Feld, Phys. Rev. 138B, 190 (1965); L. D. Roper and R. M. Wright, Phys. Rev. 138B, 921 (1965).

6. B. H. Bransden, P. J. O'Donnell and R. G. Moorhouse, Phys. Letters 11, 339 (1964); Phys. Rev. 139B, 1566 (1965); Proc. Roy. Soc. (London) A289, 538 (1966); Phys. Letters 19, 420 (1965).

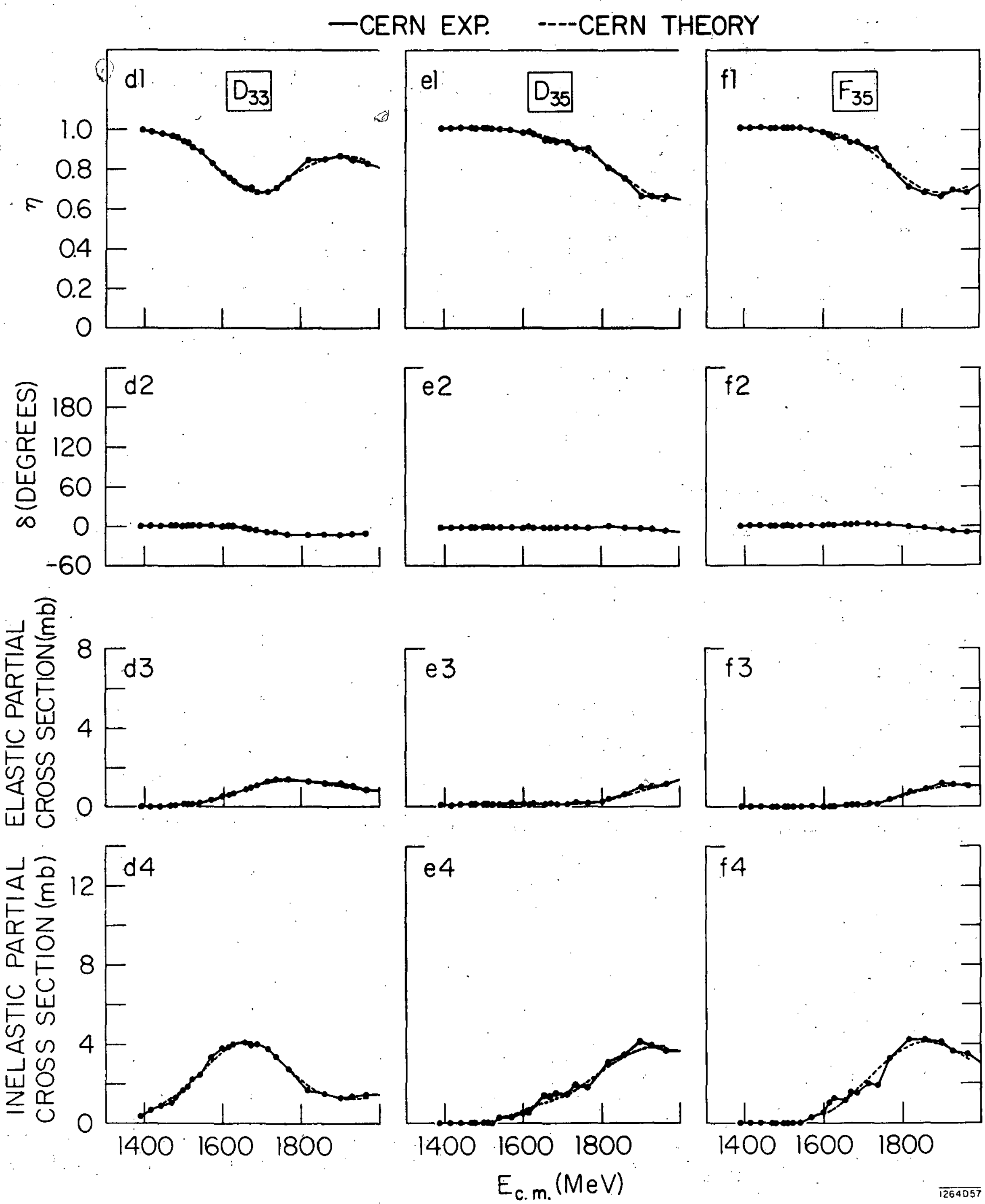
7. A. T. Davies, N.P. B21, 359 (1970).

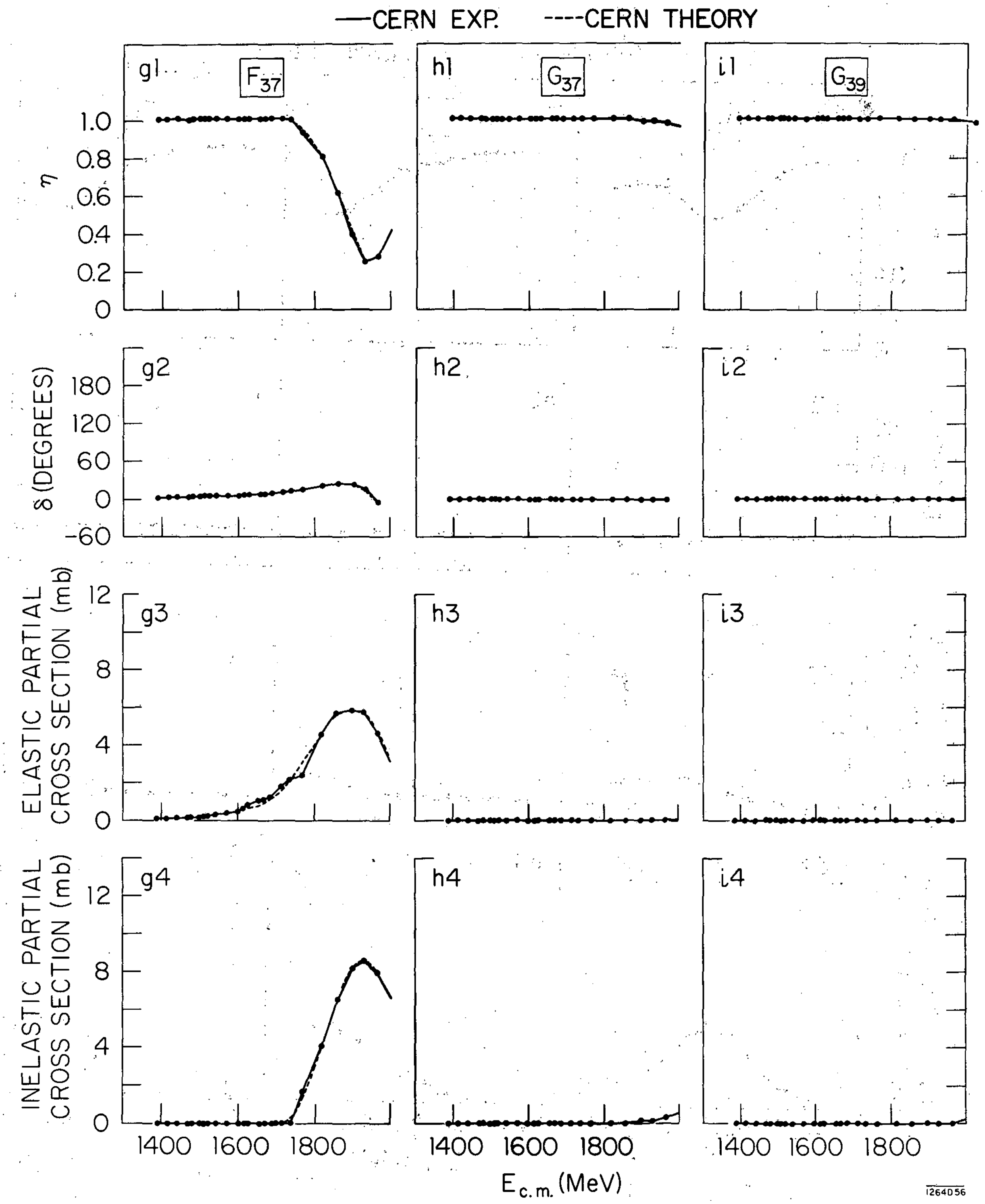
8. A. D. Brody, R. J. Cashmore, A. Kernan, D. W. G. S. Leith, B. S. Levi, B. C. Shen, J. P. Berge, D. J. Herndon, R. Longacre, L. R. Price, A. H. Rosenfeld and P. Söding, Phys. Rev. (to be published).

9. D. J. Herndon, A. Barbaro-Galtieri and A. H. Rosenfeld, UCRL-20030 πN (1970); unpublished.



—CERN EXP.
----CERN THEORY
a1
S_{31}
b1
P_{31}
c1
P_{33}
η
1.0
0.8
0.6
0.4
0.2
0
a2
b2
c2
δ (DEGREES)
180
120
60
0
-60
a3
b3
c3
ELASTIC PARTIAL CROSS SECTION (mb)
12
8
4
0
a4
b4
c4
INELASTIC PARTIAL CROSS SECTION (mb)
12
8
4
0
1400
1600
1800
$E_{c.m.}$ (MeV)
1264058


—CERN EXP. ----CERN THEORY
d1
D33
e1
D35
f1
F35
η
1.0
0.8
0.6
0.4
0.2
0
d2
e2
f2
δ (DEGREES)
180
120
60
0
-60
d3
e3
f3
ELASTIC PARTIAL CROSS SECTION (mb)
8
4
0
d4
e4
f4
INELASTIC PARTIAL CROSS SECTION (mb)
12
8
4
0
1400 1600 1800
E c.m. (MeV)
1264D57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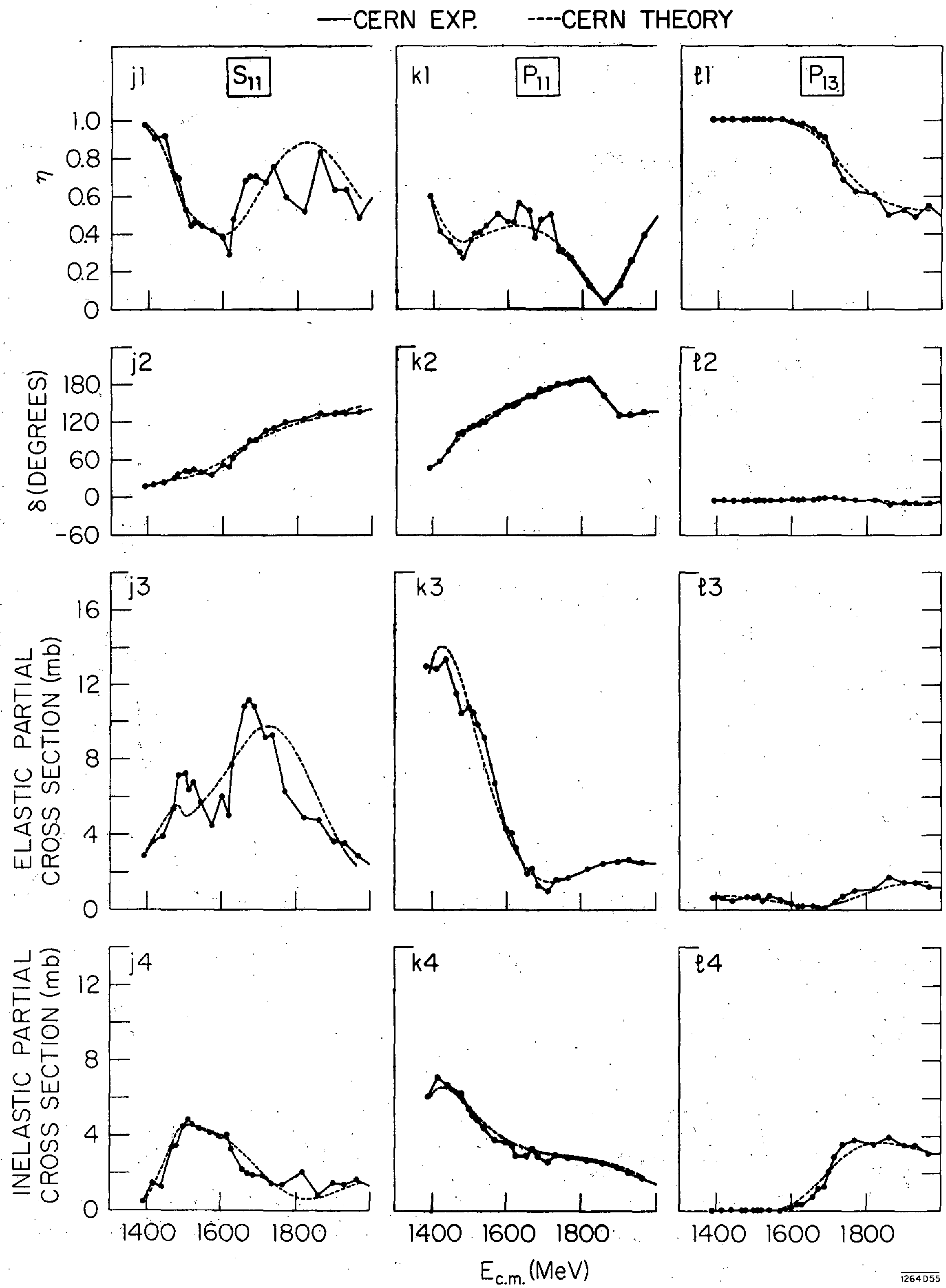
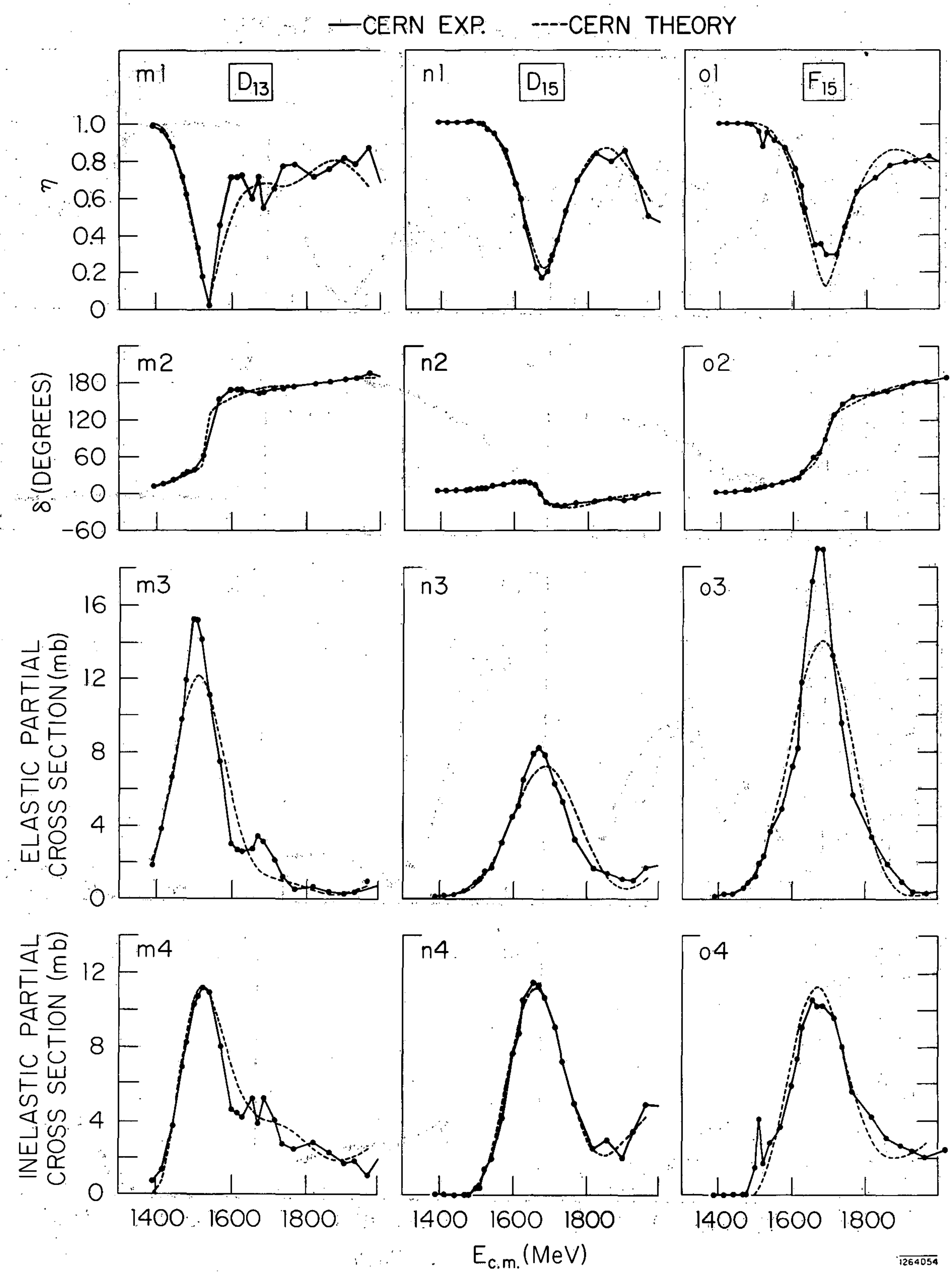
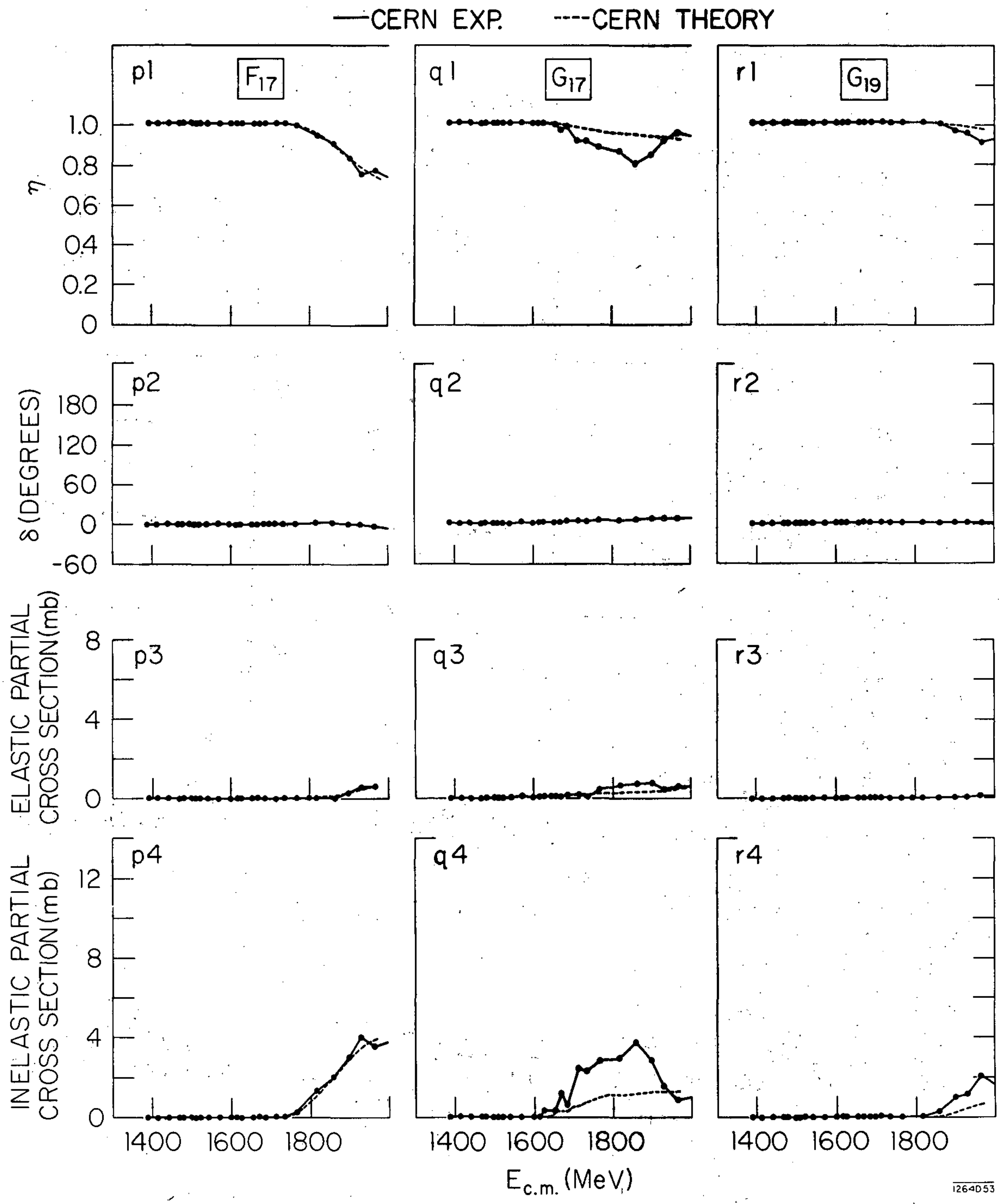
g1
F37
h1
G37
i1
G39
η
1.0
0.8
0.6
0.4
0.2
0
g2
h2
i2
δ (DEGREES)
180
120
60
0
-60
g3
h3
i3
ELASTIC PARTIAL CROSS SECTION (mb)
12
8
4
0
g4
h4
i4
INELASTIC PARTIAL CROSS SECTION (mb)
12
8
4
0
1400 1600 1800
1400 1600 1800
1400 1600 1800
$E_{c.m.}$ (MeV)
1264D56

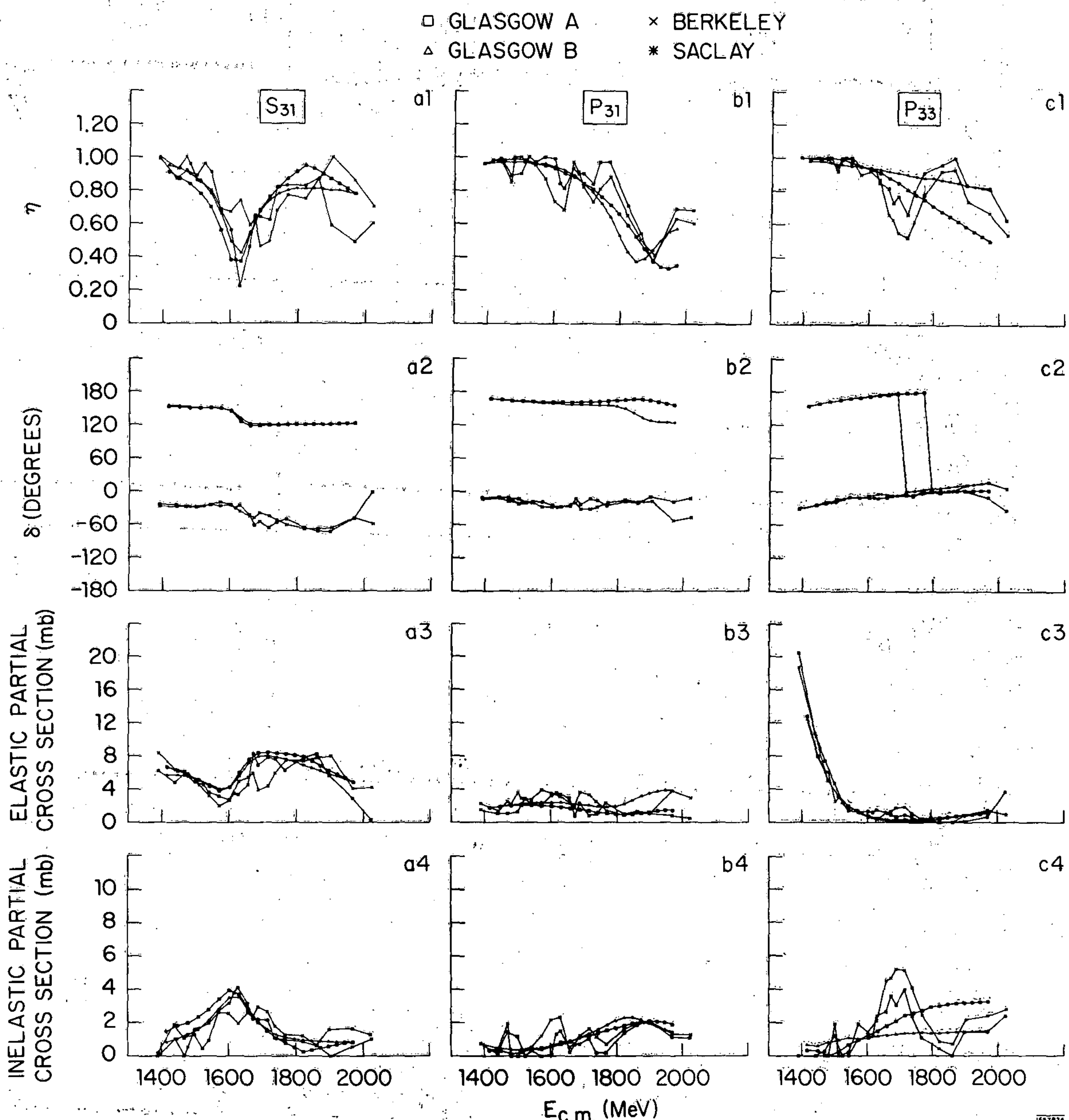


—CERN EXP.
----CERN THEORY
S11
P11
P13
j1
k1
ℓ1
j2
k2
ℓ2
j3
k3
ℓ3
j4
k4
ℓ4
η
δ (DEGREES)
ELASTIC PARTIAL CROSS SECTION (mb)
INELASTIC PARTIAL CROSS SECTION (mb)
E c.m. (MeV)
1400
1600
1800
1264D55

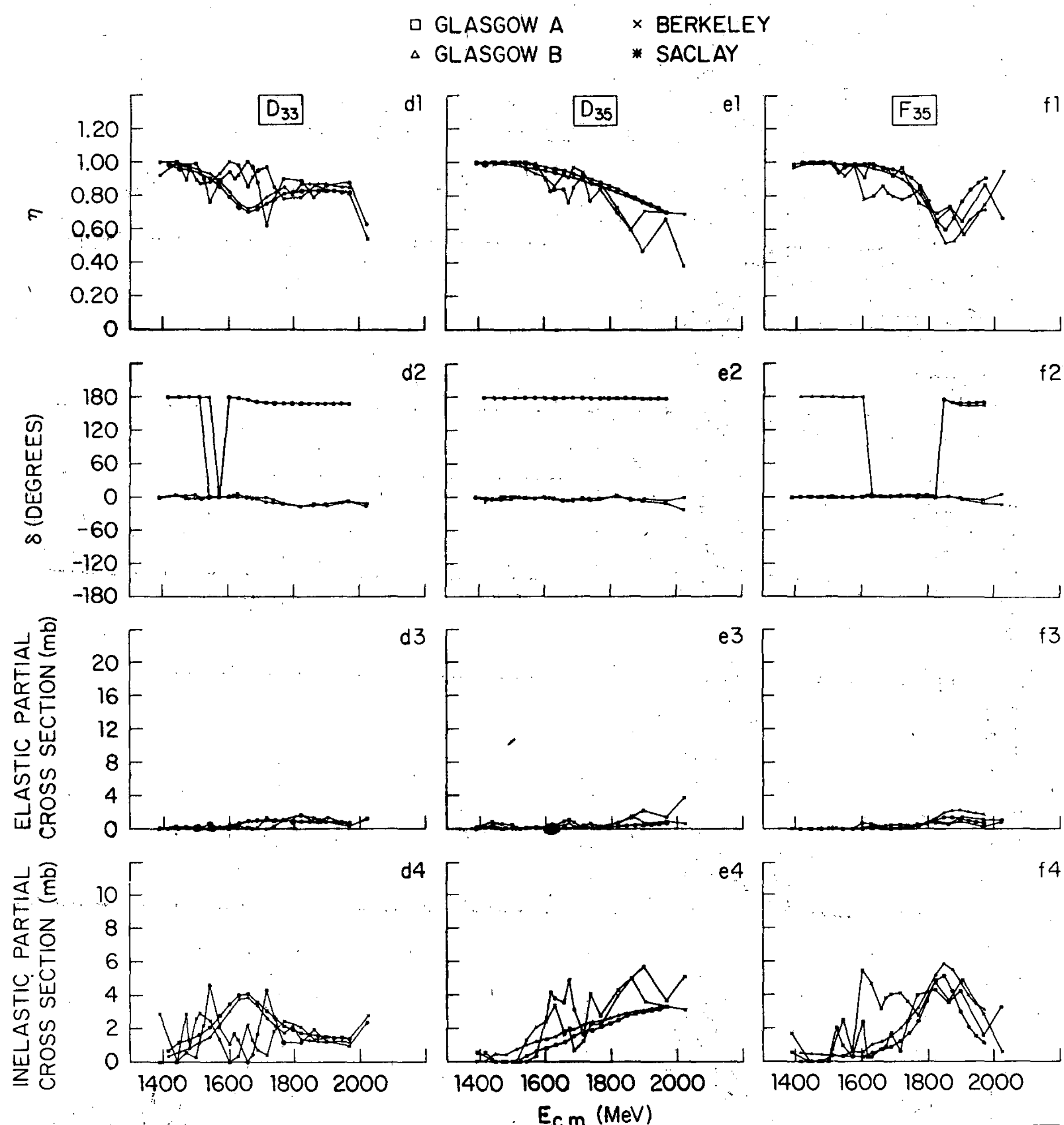

—CERN EXP.
----CERN THEORY
m1
D13
n1
D15
o1
F15
η
1.0
0.8
0.6
0.4
0.2
0
m2
n2
o2
δ (DEGREES)
180
120
60
0
-60
m3
n3
o3
ELASTIC PARTIAL CROSS SECTION (mb)
16
12
8
4
0
m4
n4
o4
INELASTIC PARTIAL CROSS SECTION (mb)
12
8
4
0
1400
1600
1800
E c.m. (MeV)
1264054



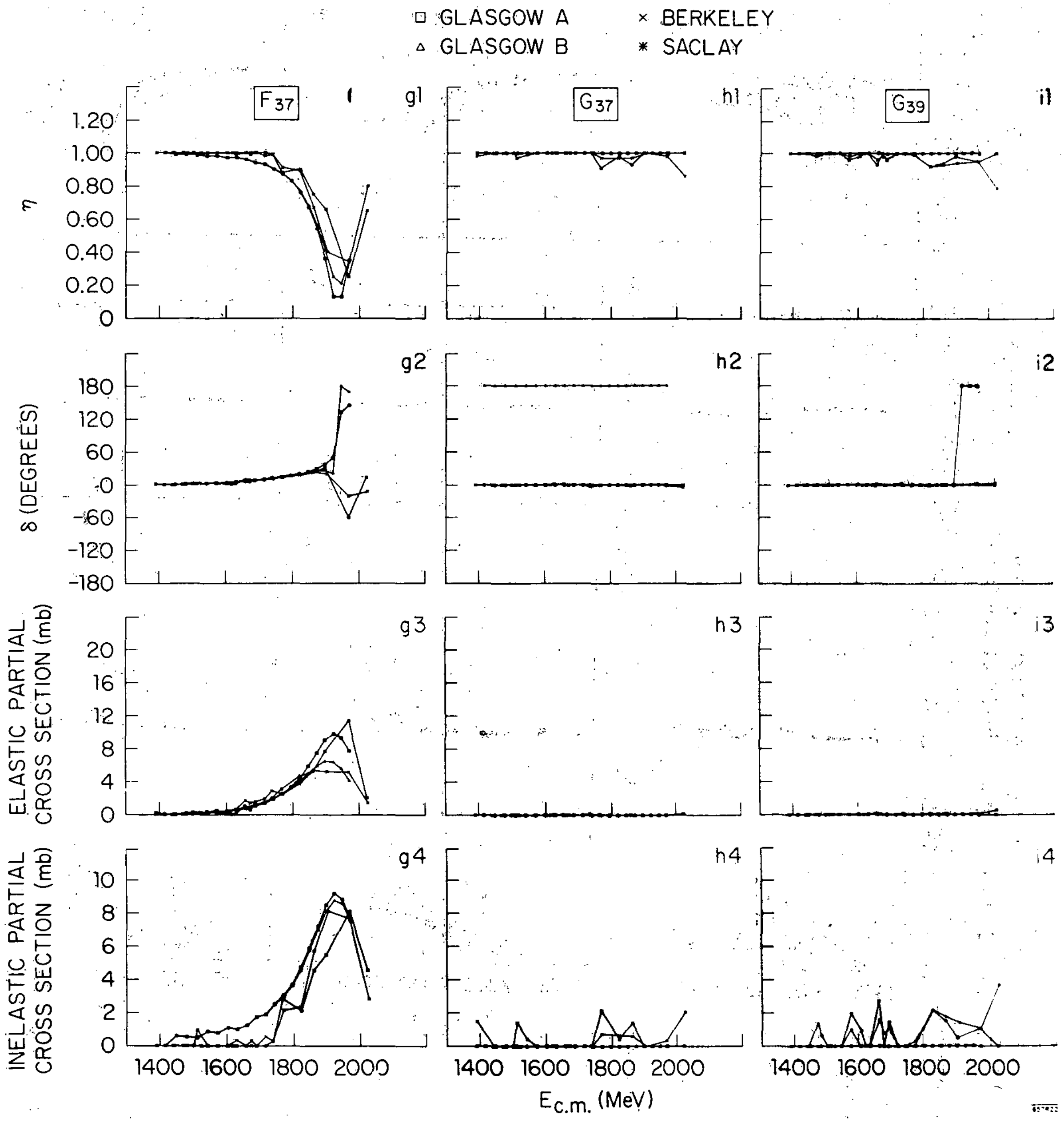
—CERN EXP.
----CERN THEORY
p1
F_{17}
q1
G_{17}
r1
G_{19}
η
1.0
0.8
0.6
0.4
0.2
0
p2
q2
r2
δ (DEGREES)
180
120
60
0
-60
p3
q3
r3
ELASTIC PARTIAL CROSS SECTION (mb)
8
4
0
p4
q4
r4
INELASTIC PARTIAL CROSS SECTION (mb)
12
8
4
0
1400
1600
1800
$E_{c.m.}$ (MeV)
1264D53



□ GLASGOW A
△ GLASGOW B
× BERKELEY
* SACLAY
S31
P31
P33
a1
b1
c1
a2
b2
c2
a3
b3
c3
a4
b4
c4
η
δ (DEGREES)
ELASTIC PARTIAL CROSS SECTION (mb)
INELASTIC PARTIAL CROSS SECTION (mb)
E c.m. (MeV)
1400
1600
1800
2000



□ GLASGOW A
△ GLASGOW B
× BERKELEY
✱ SACLAY
D33
D35
F35
d1
e1
f1
d2
e2
f2
d3
e3
f3
d4
e4
f4
η
1.20
1.00
0.80
0.60
0.40
0.20
0
δ (DEGREES)
180
120
60
0
-60
-120
-180
ELASTIC PARTIAL CROSS SECTION (mb)
20
16
12
8
4
0
INELASTIC PARTIAL CROSS SECTION (mb)
10
8
6
4
2
0
1400 1600 1800 2000
E c.m. (MeV)



□ GLASGOW A
△ GLASGOW B
× BERKELEY
* SACLAY
F37
G37
G39
g1
h1
i1
g2
h2
i2
g3
h3
i3
g4
h4
i4
η
1.20
1.00
0.80
0.60
0.40
0.20
0
δ (DEGREES)
180
120
60
0
-60
-120
-180
ELASTIC PARTIAL CROSS SECTION (mb)
20
16
12
8
4
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INELASTIC PARTIAL CROSS SECTION (mb)
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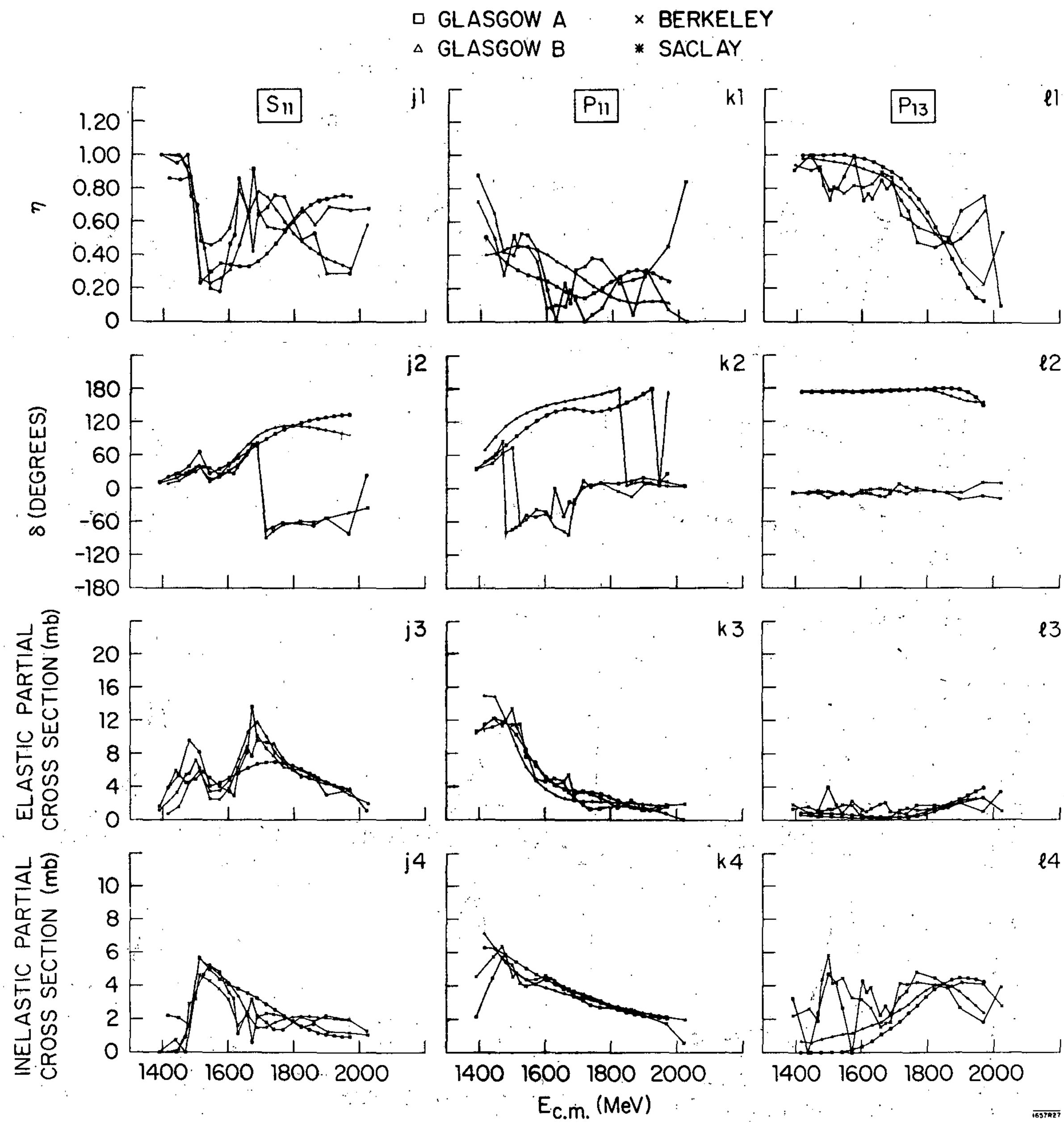
2000
E c.m. (MeV)



□ GLASGOW A
× BERKELEY
△ GLASGOW B
* SACLAY
S11
P11
P13
j1
k1
ℓ1
j2
k2
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k3
ℓ3
j4
k4
ℓ4
η
δ (DEGREES)
ELASTIC PARTIAL CROSS SECTION (mb)
INELASTIC PARTIAL CROSS SECTION (mb)
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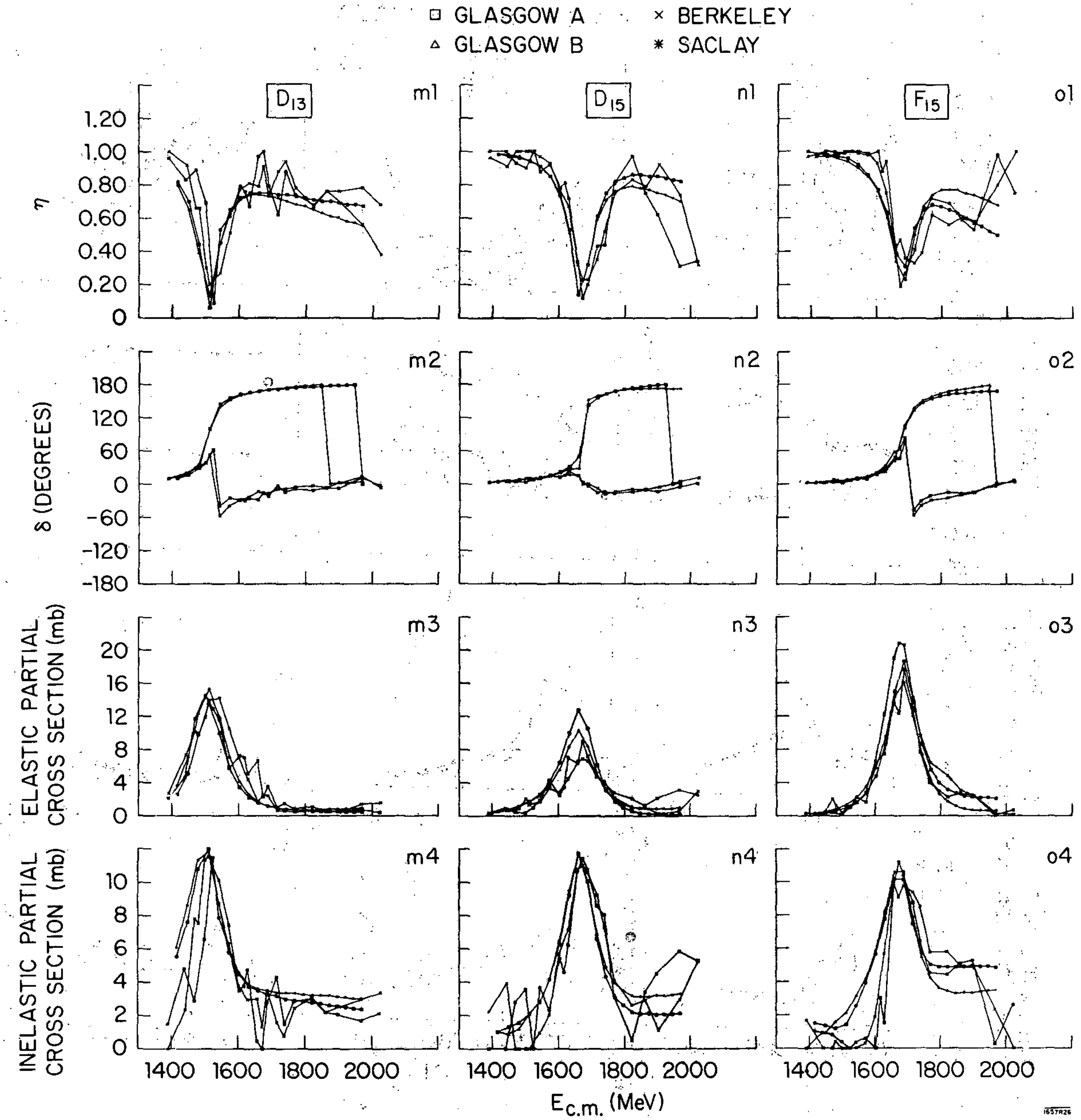
$E_{c.m.}$ (MeV)



GLASGOW A
GLASGOW B
BERKELEY
SACLAY
D13
D15
F15
m1
n1
o1
m2
n2
o2
m3
n3
o3
m4
n4
o4
η
δ (DEGREES)
ELASTIC PARTIAL CROSS SECTION (mb)
INELASTIC PARTIAL CROSS SECTION (mb)
1.20
1.00
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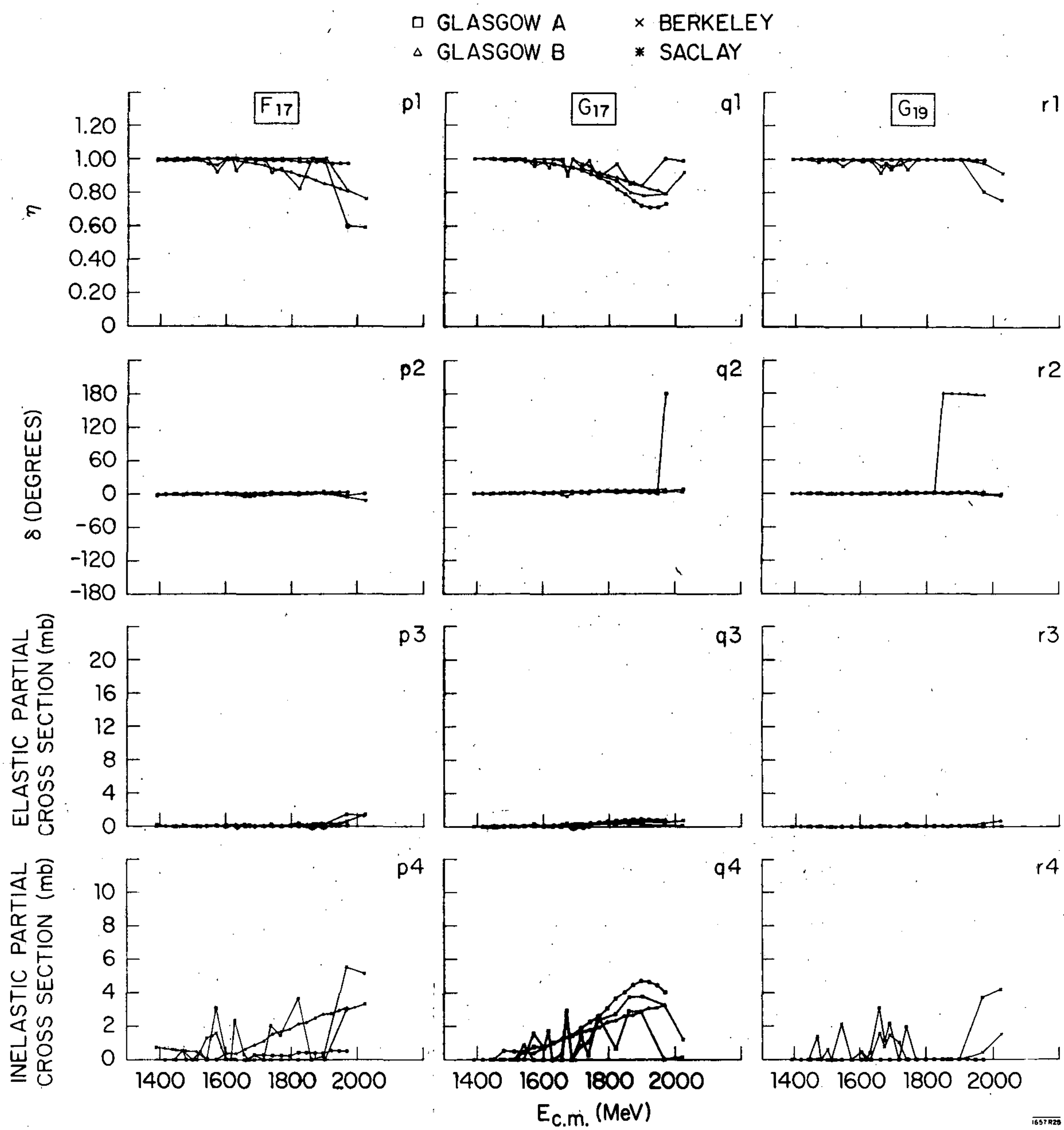
E c.m. (MeV)



□ GLASGOW A
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× BERKELEY
* SACLAY
F17
G17
G19
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q3
r3
p4
q4
r4
η
δ (DEGREES)
ELASTIC PARTIAL CROSS SECTION (mb)
INELASTIC PARTIAL CROSS SECTION (mb)
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E c.m. (MeV)