Market Entry and Structure under Uncertain and Disparate Market Expectations.

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or

Fools Rush In

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Abstract

An emerging market or market segment provides firms both the opportunity to enter early and capture market share and also the risk that the market will turn out to be less fruitful than expected. We formulate and analyze a game-theoretic model in which multiple firms with uncertain and/or disparate beliefs about the eventual market size decide whether to enter such a market. For increasingly general models, we show that the structure (i.e. the number and identity of participating firms) and profitability of equilibrium oligopolies can be determined by a classification scheme based on the firms' beliefs about the viable level of market concentration. This scheme is adapted to random forecasts (i.e. forecasts expressed as probability distributions) as well as point forecasts. This study was motivated by managerial issues encountered by a client firm engaged in semiconductor design.

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1 Introduction

A new or emerging market or market segment provides both the opportunity to enter early and capture market share and also the risk that the market will turn out to be less fruitful than expected. This leads to a basic question of when/whether it is in a firm's best interest to invest in the development of products or facilities to serve a new market. The nature of a new market makes this an inherently difficult decision in that a firm may need to commit its resources with limited information about future demand, and the issue is further complicated by the likelihood that other firms will also have the capability and the interest in entering this market. Furthermore, these other firms may have different beliefs as to the eventual size of the market perhaps as a result of different information, past experiences, or forecasting techniques. How does the presence of these competitor firms and their differing opinions of future market size change the firm's investment decision? How do these different opinions influence the market's structure (i.e. the number and identity of participating firms) and its profitability? These are the questions addressed in this paper.

This study was motivated by our work with a client firm that designs, develops, and markets semiconductor chipsets used in various types of digital communication. In this and other segments of the semiconductor industry, an important managerial question is when to take an existing chipset and adapt it to a new type of application. Such adaptation requires investing a substantial sum in the design and development of an "Application Specific Integrated Circuit," or "ASIC" in industry parlance. For example, a chip originally designed for use in digital wireless telephones might be adapted for use in digital wireless headsets; this is a new and potentially profitable application that requires smaller size, lower power consumption, and reduced transmission energy.

Once an ASIC is developed, the firm markets it to designers and assemblers of the communications products; thus, the decision to invest in ASIC development occurs well in advance of complete demand information. Indeed, the decision occurs in advance of even the design of the products from which the ASIC demand will eventually derive. Consequently, the firm must make this investment decision with very limited information about the eventual demand for its chips, and this is additionally confounded by the fact that other firms are likely to be coincidentally developing ASICs for the same market.

A key feature of this situation is that the ASIC development cost is an exogenous sunk cost in the terminology of Sutton [1996] and others, and there is a substantial literature (see Sutton [1996], Hay and Morris [1991], and the references therein) on the manner in which sunk costs are a determinant of industry concentration. A basic result in that literature is that large exogenous sunk costs act as a barrier to entry that induces market concentration (i.e. a small number of producers). This paper extends that literature in that it introduces the idea that the firms' forecasts or beliefs of market conditions may be an additional determinate of industry concentration. If our models are reduced to cases in which all firms are able to accurately predict the market conditions, the models would reduce to those already in the literature (again see Sutton, p. 27-45).

The sunk costs in our client's setting is a product development cost, but the analysis herein is also applicable to other forms of sunk costs. For example, an investment in production facilities and equipment is a sunk cost that is of much interest to the Operations-Management/Industrial-Engineering community. Most models in that literature, such as the "newsvendor model" and its many extensions (see Nahmias [1993] and references therein, Carr and Lovejoy [2000], Cachon [1997]) consider the perspective of a single firm in isolation. That is, the models do not recognize that multiple firms may be in competition and that the interaction between these firms may have important consequences to any individual firm (e.g. an important input into a firm's decision of whether to build capacity to serve a new market is a prediction of the number of competitors who will also enter that market). Recognizing that single firm models do not capture the important element of inter-firm competition, there is recent trend towards applying game theory to OM/IE problems in competitive settings; for instance, Carr et. al. [2000] analyzes the interplay between production variability and profits, van Mieghem and Dada [1999] considers the value of postponement strategies, and Cachon and Lariviere [1999] evaluate various inventory allocation schemes. This paper is that vein in that it applies game theory modelling techniques and results (see Friedman [1990]) to the issues of uncertainty, forecasting, and capacity investment that are common in the OM/IE literature.

Conversely, there are also papers that incorporate uncertainty and forecasting into economic models of industry concentration and market entry. For example, Maskin [1999] considers how uncertainty about demand and costs impacts an incumbent's decision of how much production capacity to install and the ability of this capacity to deter others from entering the industry, Dixit [1989] models an individual firm's entry and exit decisions when price follows a random walk, and De

Wolf and Smeers [1997] analyze a Stackelberg game in which the leader makes its production decision under demand uncertainty and the followers make their production decisions after the uncertainty is resolved. In our paper, entry decisions are made by multiple firms while demand is uncertain, and production decisions are made after the uncertainty is resolved.

In the next section we specify a model with two decision stages in which multiple firms first simultaneously decide whether to invest the sunk cost (e.g. the ASIC development cost) and enter a new market based on forecasted market conditions. Sometime after committing themselves to entry, the firms learn the actual market conditions, and the firms then use this new information to determine their production quantities. Finally, these production quantities together with the actual market conditions determine the selling price and firms' profits. Sections 3 through 5 investigate the above-described issues under increasingly general conditions about the nature of the firms' forecasts. Section 6 relaxes the assumption that entry decisions are made simultaneously by all the firms. Section 7 provides very general model under which all of our primary results hold; this model includes different modes of competition, Bertrand or price-setting competition for instance. Below are some of the insights derived from these models:

1. The equilibrium market structure (i.e. the level of industry concentration and the identity of the market entrants) is determined by the beliefs or forecasts that firms have of the actual conditions that will be experienced post-entry. For example, there may be a large number of potential entrants, but these firms may have low expectations for the market; this will result in a small number of entrants.

2. The market structure also depends upon the manner in which beliefs are "distributed" among potential participants. That is, a case in which firms have very similar forecasts of market conditions often results in a very different structure than a case in which the firms' forecasts are very dispersed or heterogeneous.

3. The market structure can be determined by a simple classification scheme based on firms' pre-entry beliefs, and this scheme is very robust to changes in the underlying assumptions.

4. The profits earned by the market participants are determined by both the actual market conditions and also by the market structure. Since the potential participants' forecasts determine market structure, they are an important albeit indirect determinant of market profitability.

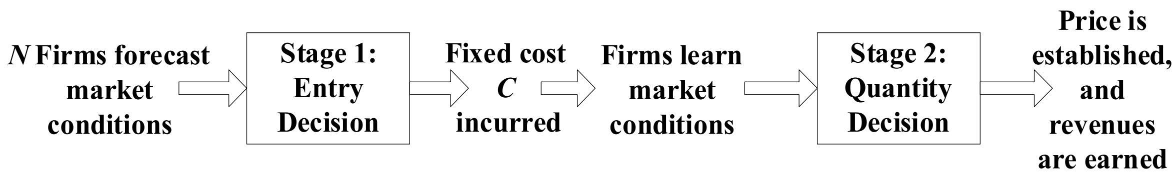

Figure 1: Sequence of Events

5. Firms may represent their forecasts as probability distributions to recognize their uncertainty about the market conditions. Increased variance in such a distribution makes it more likely that the firm will enter the market and less likely that the firm will be profitable.

6. Firms who forecast a large market are more likely participants than firms that forecast a small market, but this is only partially true. To a limited extent, participating firms may forecast a smaller market than other non-participating firms.

7. It is intuitively obvious and analytically true that the firms most likely to enter a new market are those who grossly overforecast the market's potential. If there are only a small number of these firms, then the overall market profitability will not suffer. This situation changes if there are a large number of these firms. If a sufficient number of firms share a sufficiently rosy view of the market's potential, then so many firms will rush into the market that it is unprofitable for all; hence this paper's subtitle.

2 Preliminaries

First, a brief statement of the model as illustrated by figure 1. There are initially N firms who simultaneously choose whether to invest a fixed and sunk cost C to enter a new market; this cost represents the ASIC development cost in our industrial setting. The firms may also costlessly decide not to enter. Importantly, the firms decide whether to enter the market before they know the actual market conditions, so they are unable to accurately predict the price that they will ultimately receive for their goods and must base their entry decision on a forecast of market conditions. Once all firms have made an entry/no-entry decision, they receive more accurate market information and then choose a quantity to produce. The actual price is ultimately determined by the quantity that the entrants produce in aggregate. The model is thus a two-stage game; firms make the entry decision in the first stage and the production decision in the second stage, and they receive additional market information between the stages. We employ the sub-game perfect Nash equilibrium for this competitive model, and we restrict attention to pure strategies. The three following sections describe models that differ in the forecasting assumptions; the remainder of this section describes modeling elements that are common to those models.

Market Conditions: We assume that the common post-production price p that is received by the firms is related to the aggregate quantity Q that is produced by all firms through the linear relation

$$p = a - bQ \ \text{ with } \quad a, b > 0 \tag{1}$$

The intercept a represents the market's *reservation price* (the supremum price at which demand is positive), and the slope b is the elasticity of price with respect to Q. An individual firm i's production decision is denoted q_i, so $Q := \sum_i q_i$.

Stage 1, Market entry: All N potential entrants simultaneously decide whether to invest C and enter the market. We assume, and this is assumption is central to our model, that the firms do not know the market's reservation price at the time of the entry decision. Rather, they calculate their anticipated post-entry margins based on a forecasted reservation price; specific forecasting assumptions will be discussed in the sections that follow.

Each possible (but perhaps not equilibrium) set of entry decisions is represented by an "entry vector." This is an N-vector in which the i-th element represents firm i's entry/no-entry decision, and we say that firms are "members," "entrants," or "participants" if they have selected entry in that vector. It may be that membership in an entry vector is undesirable to one or more firms; we say that these firms are "willing to leave" the vector. This occurs when a member anticipates that post-production revenues will be less than C. In other words, member firms are willing to leave if they believe that a non-entry decision, which guarantees zero profits, would be preferable to an entry decision given that the second-stage oligopoly will consist of the other members of that entry vector.

Similarly, it may be that non-members of an entry vector would prefer membership. In this case we say that these non-members are "willing to enter" the entry vector, and this occurs when they believe that an entry decision would be preferable to their current non-entry decision. Of course, it must be that no firms are willing to enter or to leave an equilibrium entry vector.

Our intention is not to analyze the manner in which different forecasting techniques lead to different forecasts, but rather to understand how the error and uncertainty that is inherent in any forecasting method affects the structure and profitability of a new market. We thus assume that each firm's entry decision is made on the basis of the forecasted reservation price, the required investment, the entry decisions of its competitor firms, and the anticipated post-entry production decisions of those competitors, and we exclude the possibility that a firm may, indeed perhaps should, actually change or update its own forecast upon receiving information about which other firms are electing to enter the market. This is a case of bounded rationality as discussed by Conlisk [1996].

Stage 2, Production: At the beginning of stage 2, the firms learn the actual reservation price. Each firm then selects a production quantity, and the market price p is determined by (1). A firm i who has chosen to enter the market thus receives revenue of $q_i \cdot p$ for a net profit of $q_i \cdot p - C$. This implicitly assumes the variable production costs are zero; relaxing this is a straightforward step, which we do not take, that adds notational complexity without providing additional insight. The production decision of non-members is simply handled by forcing their production quantities to be zero.

Before introducing specific forecasting assumptions, it may be noted that this model is a form of the "market-entry games" described by Selten and Güth [1982] and Gary-Bobo [1990]. The primary difference between those games and our models is that we introduce forecasting and focus upon its consequences. Nonetheless, it is possible to prove equilibrium existence in our models from Gary-Bobo's primary result.

3 Entry and Structure with Three Forecasting Classes

The model of this section is the simplest and illustrates several results that are shared by the more complicated models that follow. Here we assume that each firm falls into one of three forecasting classes, high, accurate, or low (abbreviated by subscripted "hi", "ac", "lo"), and that the number of firms in each class is some large integer. Also, we assume here and throughout that firms only desire membership in a market when their anticipated profits are **strictly** positive; this assumption eliminates some trivial cases and is otherwise benign.

In stage-1, high-forecasters anticipate a reservation price $f_{hi} > a$, accurate forecasters correctly forecast a reservation price $f_{ac} = a$, and low-forecasters anticipate a reservation price $f_{lo} < a$. Again, the actual reservation price a is learned at the beginning of stage 2.

Equilibrium: As is typical, subgame-perfect equilibrium conditions are identified by working in a backward direction, so we begin by assuming a known entry vector from stage-1, and we let n be the number of members in that vector.

The subgame-perfect equilibrium requires that in the terminal stage-2, given an entry vector, the member firms in that vector reach a standard Nash equilibrium in production quantities. Also, the only difference between firms in this game is their forecasts, but these are of no importance during stage two because the forecasts have been replaced by accurate market information. Thus, all firms are essentially identical when they select a production quantity. It follows that the equilibrium production decisions will be the same as for a Cournot (i.e. quantity-setting) oligopoly of size n in which all oligopolists have accurate market information.

Specifically, suppose that each member firm i selects a production quantity q_i. Then the second-stage equilibrium condition is that i's production quantity is its revenue maximizing quantity given that its competitors will collectively produce quantity $\sum_{j \neq i} q_j$. It is straightforward to show (see appendix for derivations) using first order optimality conditions that the unique solution to this condition is for every member firm to produce quantity

$$\frac{a}{b\,(1+n)}. \tag{2}$$

Each member firm will then earn profits of

$$\frac{a^2}{b\,(1+n)^2} - C \tag{3}$$

Now moving to stage-1, firms here decide whether to enter the market, so we seek a subgame-perfect equilibrium entry vector. This is an entry vector that no firms are willing to enter or leave given that they anticipate that post-entry they will receive profits as in (3). But, firms' entry decisions are based not on actual but rather on forecasted reservation prices, so a firm with forecast f (i.e. a firm that anticipates a reservation price of f) believes that membership in an entry vector that has n total members will yield profits

$$\frac{f^2}{b\,(1+n)^2} - C$$

which will be positive when

$$n < \frac{f}{\sqrt{bC}} - 1$$

Or, since the number of entrants must be an integer, the firm anticipates positive (negative) profits from membership in an entry vector with

$$\left\lfloor \frac{f}{\sqrt{bC}} - 1 \right\rfloor \tag{4}$$

or fewer (more) firms[1]. Put another way, (4) is the maximum number of entrants that the firm believes that the market can profitably support. The firm is thus willing to enter any vector with strictly fewer than this number of members but is unwilling to enter any vector with this number or more members.

Let K denote the equilibrium number of firms that, unbeknownst to the stage-1 competitors, the market can actually support. This is calculated by replacing the forecast f in (4) by the actual reservation price a to get

$$K = \left\lfloor \frac{a}{\sqrt{bC}} - 1 \right\rfloor \tag{5}$$

Next define another value $\underline{f^K}$ as the infinum forecast such that a firm with this forecast would anticipate that the market can support K firms. It can be derived (see appendix) that

$$\underline{f^K} = \sqrt{bC} \left\lfloor \frac{a}{\sqrt{bC}} \right\rfloor \tag{6}$$

We can now characterize the equilibrium market structure for this first set of forecasting assumptions.

Proposition 1 *Necessary and sufficient conditions for equilibrium entry vectors are:*

(i) If $f_{hi} > \underline{f^K} + \sqrt{bC}$, that there are $\left\lfloor \frac{f_{hi}}{\sqrt{bC}} - 1 \right\rfloor$ members all of which are high-forecasters. The members earn negative profits.

(ii) If $f_{hi} \leq \underline{f^K} + \sqrt{bC}$ and $f_{lo} \leq \underline{f_K}$, that there are K members all of which either high- or accurate-forecasters. The members earn positive profits.

(iii) If $f_{hi} \leq \underline{f^K} + \sqrt{bC}$ and $f_{lo} > \underline{f_K}$, that there are K members. The members earn positive profits.

[1] $\lfloor \cdot \rfloor$ is the largest integer less than $(\cdot)$.

As stated in the proposition, there are three possible scenarios. In the first scenario (*(i)* in the proposition) the high forecasters are very overoptimistic about the size of the market. In fact, they are so overoptimistic that they oversaturate the market and lose money. In other words, too many fools rush into the market. Additionally, the accurate- and low-forecasters anticipate this happening and wisely stay out. In scenario *(ii)*, the accurate- and high-forecasters have such similar expectations about the market that they are indistinguishable with regard to their entry decisions. The low-forecasters, however, are much less optimistic than the other groups, so they will unwisely decline entry. Equilibrium vectors thus consist of any K firms from the two higher forecasting classes. In *(iii),* the groups all have such similar forecasts that they make their entry decisions as if they have exactly the same forecast, the accurate one.

Thus we see that two factors together determine whether a firm is a member of an equilibrium vector. The first and most obvious is the firm's own expectation of market size; firms with high expectations are more likely to enter the market. The second factor is the disparity between firms' forecasts. For example, we see in the proposition that whether an accurate-forecasting firm is a market entrant is determined by the degree to which its more rosy-eyed competitors overforecast. The proposition also implies several other results about the equilibrium structure:

1. low- and accurate-forecasters will never earn negative profits. They may, however, exclude themselves from the market if the high-forecasters are sufficiently overoptimistic.

2. Member firms, including the high-forecasters, make positive profits if and only if there is an equilibrium vector with an accurate-forecaster member.

3. Profits earned by each member firm weakly decrease with f_{hi}. This is because the number of equilibrium members increases with f_{hi}, and the members' profits decrease with the number of members. Also, the aggregate profits of all member firms weakly decreases with f_{hi}; this follows from the additional fact that aggregate oligopoly profits are maximized when there is only a single firm (i.e. the oligopoly is actually a monopoly) and are thereafter decreasing in the number of firms.

4. There are always at least K firms in any equilibrium. This occurs because we have assumed a large number of accurate- and high-forecasters. This result does not hold if the aggregate number of firms in these classes is less than K.

4 Entry and Structure Without Forecasting Classes

We now move to a more general case in which there are N potential entrants each whom has its own, possibly unique, forecast of the reservation price. In reality, different firms would be expected to use different forecasting systems and these systems would be populated by different data. Thus, it would be surprising to find multiple firms forecasting identical market conditions. To model this "dispersion" of forecasts across the firms, we take the forecasts to be independently chosen from a continuous "forecast sampling distribution" with cdf Γ and mean μ. This is not to imply that there is randomness built into forecasting systems; rather, it is a means of capturing the notion that multiple firms' individual forecasts will likely be scattered.

As before, a firm's forecast represents its belief about the post-entry market conditions as parameterized by the reservation price. The "bias" β of the sampling distribution is the difference between the mean of this distribution and the actual reservation price a; that is $\beta := \mu - a$. The distribution is "unbiased" ("negatively biased", "positively biased") if $\beta = (<, >)\ 0$. Of course, firms do not know the bias since they would then infer the actual market size.

The second-stage equilibrium conditions in the previous section have not changed because the differences between this model and the previous model are related to the dispersion of the forecasts, but these forecasts are again moot once the true market conditions are revealed at the beginning of the second stage. Thus, given an entry vector with n members, those members' equilibrium production quantities and profits are again calculated by (2) and (3).

Moving to the first stage, the forecasts are of great importance however. The following classification scheme, in which the forecasts are central, is the first step in identifying the equilibrium number of entrants and their profitability.

Classification:

1. As before, define K and $\underline{f^K}$ by (5) and (6).
2. Define $\underline{f^0} := 0$, and

$$\underline{f^j} := \underline{f^K} + (j - K)\sqrt{bC} \qquad \text{for} \quad j = \{1, \quad 2, \quad \ldots, \quad \infty\} \tag{7}$$

   $\underline{f^j}$ is the infinum forecast that would lead a firm to anticipate the market can profitably

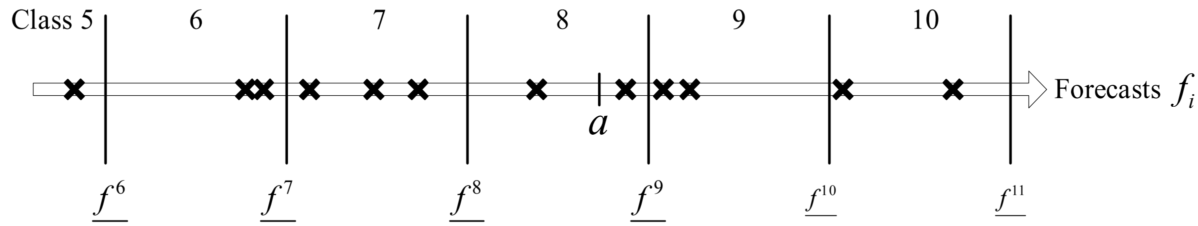

Figure 2: Classification of Firms

support exactly j members. Of course, since higher forecasts lead firms to anticipate profitable oligopolies of larger sizes, the $\underline{f}^j$ values are increasing in j.

3. The $\underline{f}^j$ values are then used to classify the firms; firm i is of class-j when $\underline{f}^j < f_i \leq \underline{f}^{j+1}$. Class-$j$ firms believe that the market will profitably support at most j firms; they will thus be willing to enter any entry vector with $j-1$ or fewer members only; they will be willing to leave any entry vector with $j+1$ or more members.

4. Let N^j denote the number of firms in class-j.

Figure 2 illustrates this classification scheme for an example with 12 potential entrants whose forecasts have already been drawn from the sampling distribution; the right-facing arrow represents a portion of the positive real line, and the crosses denote the forecasts.

The equilibrium industry structure is characterized in the next proposition. First define a new value $\underline{j}$ by

$$\underline{j} := \min \left\{ j \ ; \quad j \geq \sum_{m>j} N_m \right\} \tag{8}$$

$\underline{j}$ is the lowest class for which the class designation is greater than the aggregate number of firms in that class and all higher classes. The next proposition shows the market structure based on this classification of firms.

Proposition 2 *entry vectors are equilibrium vectors if and only if the following hold:*

(i) There are $\underline{j}$ entrants

(ii) No firms enter from any class $j < \underline{j}$

(iii) Every firm in all classes $j > \underline{j}$ enters

(iv) Exactly $\underline{j} - \sum_{m \geq \underline{j}+1} N^m$ firms from class $\underline{j}$ enter

(v) The entering firms in all entry equilibria will earn positive profits iff $\underline{j} \leq K$, and this occurs if there are more than K firms in classes higher than $\underline{j}$.

(iv) There are

$$\binom{N^{\underline{j}}}{\underline{j} - \sum_{m \geq \underline{j}+1} N^m}$$

equilibrium entry vectors.

We think of firms in class-$\underline{j}$ as good forecasters because they have forecasts that may not be perfect but are sufficiently accurate to induce the firm to behave in exactly the same manner as a firm that does have a perfect forecast. We think of firms in classes lower than $\underline{j}$ as conservative forecasters. From the proposition, and analogous to results from the previous section, good and conservative forecasters can never lose money because they will exclude themselves from entry if the market is not sufficiently concentrated.

We think of firms in classes higher than $\underline{j}$ as overoptimistic. From *(i)* and *(v)*, the profitability of these firms depends on the number of other overoptimistic firms and on the manner in which their forecasts are scattered. If $\underline{j}$ is greater than K, then an overoptimistic firm will be unprofitable if it enters, but it will not enter if there are sufficiently many firms that are even more optimistic (and therefore in higher classes).

It is also true that equilibrium profits are weakly decreasing in any firm's forecast. This is because: (1) an increase in one firm's forecast may push the firm into a higher class, (2) this may increase the equilibrium number of firms, and (3) this would reduce the profitability of every entrant.

Before moving on, it is interesting to note that a will always fall between $\underline{f^K}$ and $\underline{f^{K+1}}$. Thus, a firm that accurately forecasts demand to be exactly a will be in class-K; it follows from proposition 2 that if a perfect-forecaster such as this is a member of an equilibrium vector, then all firms in **any** equilibrium vector will earn positive profits.

Another interesting question is: what is the *a priori* probability that firms in an equilibrium oligopoly will be profitable? An entry vector will be profitable if and only if it has K or fewer members. This will be the case in equilibrium entry vectors iff K or fewer firms are in classes $(K+1)$ through ∞; or, equivalently, that K or fewer firms receive forecasts that are $> \underline{f}^K + \sqrt{bC}$. This probability, derived from the binomial distribution, is given in the following proposition the proof of which is in the appendix.

Proposition 3 *Prior to assigning forecasts to firms, the probability that any firms will be profitable at equilibrium is:*

$$1 \quad if \quad N \leq K$$

$$\textstyle\sum_{j=0}^{K} \binom{N}{j} \left[1 - \Gamma\left(\underline{f}^K + \sqrt{bC}\right)\right]^j \left[\Gamma\left(\underline{f}^K + \sqrt{bC}\right)\right]^{N-j} \qquad if \quad N > K$$

When $N > K$, this probability is decreasing in the bias of the sampling distribution β and the number of firms N; it is increasing in the market reservation price a.

Intuitively, positive equilibrium profits become less likely as β or N increases because this leads to larger equilibrium oligopolies and more intense post-entry competition. For rising β, equilibrium oligopolies get larger because firms become more likely to overforecast the market size. For an increased N, it is because the expected number of firms with optimistic forecasts increases. In contrast, a rising reservation price a does not change the size of equilibrium oligopolies; this is because entry decisions are based not on a but instead on the firms' forecasts of a. What does change is that profits increase with a for any entry vector which leads to a higher probability of profitable equilibria.

Now consider the special case in which the forecast sampling distribution is a normal distribution with mean μ and standard deviation σ. The probability that firms will be profitable at equilibrium is now

$$\sum_{j=0}^{K} \binom{N}{j} \Phi^j \left(-\frac{\underline{\underline{f}}^K + \sqrt{bC} - \mu}{\sigma}\right) \Phi^{N-j} \left(\frac{\underline{\underline{f}}^K + \sqrt{bC} - \mu}{\sigma}\right) \tag{9}$$

where Φ is the standard Normal cdf. In this case, we can make the additional claim that the probability that firms will be profitable at equilibrium decreases with the sampling standard deviation σ if and only if the sampling distribution bias is $< \underline{f}^K + \sqrt{bC} - a$. The proof of this claim follows from the arguments in the proof of proposition 3 (see appendix), the definition of β, and the fact that the probability of a success, as defined in the appendix, increases with σ if and only if the numerators in the above expression are positive.

Example 1 illustrates the model and results of this section.

Example 1: A new market with $N = 13$ potential entrants has $a = 250$ and $b = 1$. The potential entrants' forecasts of the reservation price a are sampled from a normal distribution with mean $\mu = 275$ and standard deviation $\sigma = 30$, and the investment required to enter this market is $C = 650$. By (5), this market is able to profitably support $K = 8$ firms. Applying (9), prior to the sampling of the forecasts there is a $\sum_{j=0}^{8} \binom{13}{j} \Phi^{j}(-.667)\ \Phi^{13-j}(.667) = 21.2$ percent probability that the members of any equilibrium will be profitable. To a large extent, this low probability is due to a positive bias of the sampling distribution of $\beta = \mu - a = 25$.

$\underline{f^K} = 229.5$ from (6), so *the bias is greater than $\underline{f^K} + \sqrt{bC} - a = 4.95$ which implies that this probability will increase with any increase in the sampling distribution's standard deviation σ. Indeed, this probability is quite sensitive to changes in σ; if σ is reduced to 20, the probability is only 4.2%; if σ is increased to 40, the probability increases to 37.1%.*

Table 1 gives the critical values of the classification scheme. Table 2 gives forecasts drawn from the sampling distribution for each of the 13 firms; the table also indicates each firm's classification.

Table 1

class j	critical value $\underline{f}^j$
0	*0*
1	*51*
2	*76*
3	*102*
4	*127*
5	*153*
6	*178*
7	*204*
8	*229*
9	*255*
10	*280*
11	*305*
$\vdots$	$\vdots$

Table 2

firm # i	forecast f_i	class
1	*234*	*8*
2	*247*	*8*
3	*226*	*7*
4	*304*	*10*
5	*232*	*8*
6	*212*	*7*
7	*272*	*9*
8	*275*	*9*
9	*248*	*8*
10	*287*	*10*
11	*282*	*10*
12	*276*	*9*
13	*300*	*10*

Table 3

class j	# of members N^j	$\sum_{m>j} N^m$
≤ 6	0	13
7	*2*	*11*
8	*4*	*7*
9	*3*	*4*
10	*4*	*0*
≥ 11	0	0

Table 3 shows that 8 *is the smallest number* j *which is greater than* $\sum_{m\geq j+1} N^m$*, so* $\underline{j} = 8$ *which implies, as per proposition 2, that* 8 *firms will enter at equilibrium. There will be* 4 *equilibrium vectors; each will contain all of classes* 9 *and* 10 *(i.e. firms* 4, 7, 8, 10-13*) and one firm from class-*8 *(i.e. one of firms 1, 2, 5, or 9). Finally, these firms will be profitable (since* $\underline{j} = 8 \leq K$*) despite the mere* 21.2 *percent probability of this occurring.*

5 Entry and Structure with Random Forecasts

To allow even more generality, we now consider the case in which: (1) each firm i's forecast of the reservation price a is not a single value but rather a distribution with mean λ_i and variance σ_i, and (2) for generality, we allow these distributions to be generated in any arbitrary fashion.

The second stage equilibrium conditions remain unchanged from the previous sections, again because the firms are only differentiated by their forecasts which are rendered moot by knowledge of the actual reservation price. That is, given an entry vector with n members, the equilibrium second-stage quantities and revenues are still given by (2) and (3). The first-stage entry equilibrium conditions conceptually remain unchanged in that equilibrium vectors are those entry vectors that no firms are willing to enter or leave. These decisions are now made on the basis of whether *expected* second-stage revenues will exceed the fixed cost C.

Given a first-stage entry vector with n members, a member firm i's expected second-stage revenues are $E\left[\frac{a^2}{b(1+n)^2}\right]$ which equals $\frac{E\left[a^2\right]}{b(1+n)^2}$ with the expected value taken over i's forecast distribution. As is true of all probability distributions, $\sigma_i^2 = E\left[a^2\right] - \lambda_i^2$, so we substitute $\lambda_i^2 + \sigma_i^2$ for the numerator to get expected revenues of

$$\frac{\lambda_i^2 + \sigma_i^2}{b\left(1+n\right)^2} \tag{10}$$

The fact that this increases with forecast mean λ_i is entirely intuitive; a larger market, parameterized by a larger a, equals stronger demand and higher profits, so a higher expectation of a translates into a higher expectation of profits. The fact that (10) increases with forecast std σ_i is less intuitive; in follows from the convexity of profits in a (see (3)) and Jensen's inequality.

This means that firm i believes, based on its forecast distribution, that member firms in an entry vector will be profitable (in expected value) if

$$\frac{\lambda_i^2 + \sigma_i^2}{b\left(1+n\right)^2} - C > 0$$

or, equivalently, if the number of members n is less than or equal to

$$\left\lfloor \sqrt{\frac{\lambda_i^2 + \sigma_i^2}{bC}} - 1 \right\rfloor$$

This now becomes the basis for a classification scheme as in the previous section. In fact we can use the previous scheme with a straightforward generalization to account for the forecast randomness. We now place a firm in class-j when $\underline{f^j} \leq \sqrt{\lambda_i^2 + \sigma_i^2} < \underline{f^{j+1}}$. Under this classification scheme:

1. The classes can be interpreted as before; class-j firms believe that the market can profitably support j or fewer firms.

2. Proposition 2 holds unchanged.

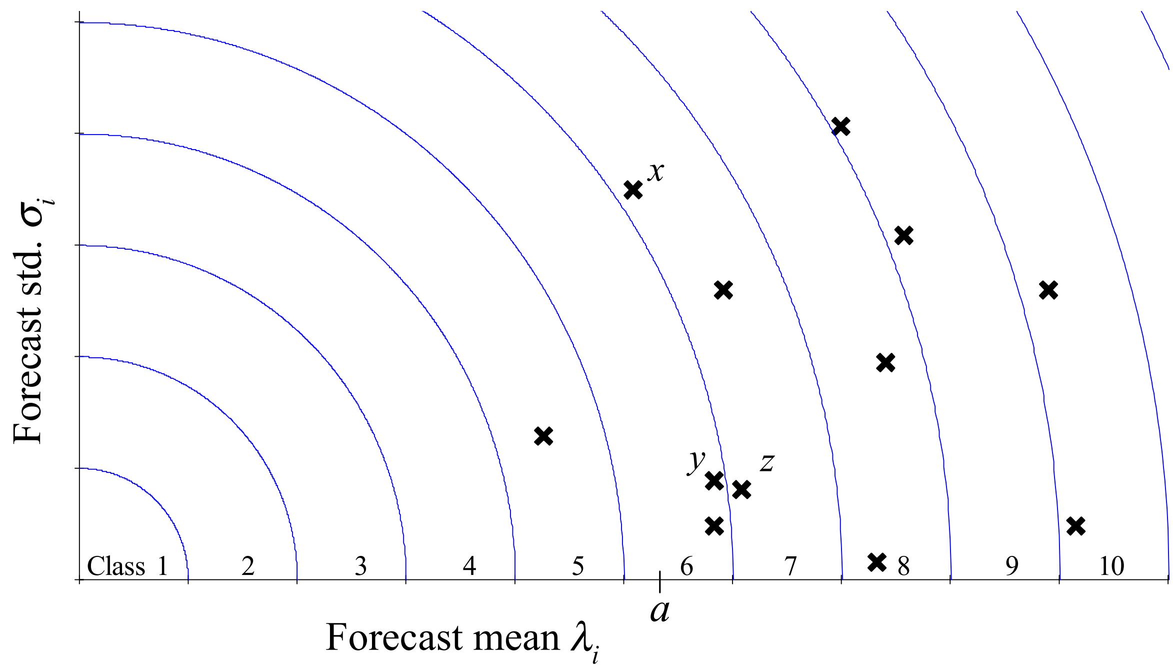

Figure 3: Classification with Disbursed Random Forecasts

3. Members' expected profits increase in both the mean and variance of their forecast distributions in any entry vector. This can be easily seen from (10) and includes all equilibrium entry vectors.

4. But, members' actual equilibrium profits weakly decrease in both the mean and variance of their forecast distributions. This happens because the increased expected profits (see previous point) may result in additional members at equilibrium and because the actual profits of the equilibrium members is decreasing in the number of members but unchanging in any firm's forecast (with the number of entrants fixed).

Since only the first two moments of the forecast distribution are relevant in classifying the firms, we can represent the classification scheme in a two dimensional plot of forecast mean versus forecast standard deviation. Figure 3 illustrates this for an example with 12 firms; the crosses in the figure again represent the firms' forecasts. The curved lines represent loci of mean/std pairs along which $\sqrt{\lambda_i^2 + \sigma_i^2}$ equals one of the critical $\underline{f}^j$ values of the classification scheme.

Each firm's class – labelled just above the horizontal axis – is determined by the region into which its forecast falls. By visually inspecting figure 3, it is easy to construct table 4 which shows the number of firms in each class. By (8) and proposition 2, $\underline{j} = 7$ is the equilibrium number of members. As is also shown in the figure, the actual reservation price a falls into the region of class-6 (i.e. $K = 6$). Thus, since $\underline{j} > K$, the equilibrium will not be profitable.

Table 4

class j	*# of members* N^j	$\sum_{m>j} N^m$
1 − 4	*0*	*12*
5	*1*	*11*
6	*2*	*9*
7	*3*	*6*
8	*2*	*4*
9	*2*	*2*
10	*2*	*0*

We also see in this example that while equilibria tend to be populated by firms who anticipate a large market size, there is not a monotonic relationship between a firm's forecast mean and its inclusion in an equilibrium. This is, we cannot say that if firms x and y have forecast means λ_x and λ_y with $\lambda_x < \lambda_y$ then any equilibrium that includes x also includes y. There are two reasons that x but not y might be a member of an equilibrium. First, it may be that x's forecast standard deviation is sufficiently high that x is in a higher class than is y. This case is illustrated by the labelled points in the figure. Firm x is in class-7 and there are 7 member firms at equilibrium, so there exists an equilibrium with x as a member. Not so for y, that firm is in class-6 and is thus excluded from any equilibrium. It is easy to see that x's large forecast standard deviation dominates the fact that its forecast mean is smaller than y's. The other possible scenario is illustrated by the points x and z. Both are in class-$\underline{j}$, and each equilibrium vector has one firm from that class. There is therefore an equilibrium vector with x but not z as a member even though $\lambda_x < \lambda_z$. Of course this also means that there is an equilibrium vector that includes z but not x.

6 A Sequential Equilibrium Selection Process

We have thus far simply presumed that all firms declare their entry decision simultaneously, but this might be a substantial departure from the manner in which real firms approach market entry decisions. It is quite possible that the entry decisions of real firms will be: (1) sequential – firms' decision-making processes might naturally be staggered in time; (2) phased – firms may announce an intention to enter prior to making an irreversible entry commitment; and/or (3) based on simpler criteria and/or less information than is presumed by the subgame perfect Nash equilibrium. We were thus interested in whether there is a process that includes these features and that terminates in an equilibrium entry vector as characterized by proposition 2. Indeed, the process described below is such a process. Additionally, this process typically terminates in a small number of steps (actually, "rounds" as described below) at a unique equilibrium vector, and it terminates in a quite natural fashion.

The process is initialized by specifying an arbitrary entry vector (e.g. a vector with no members) and an arbitrary sequence of the potential entrants. The process then consists of a number of rounds. In each round the firms sequentially decide whether membership in the vector is desirable. We presume that the firms make this decision based on a very simple criterion: a class-j firm will select membership in a vector if the number of other competitors who are already members is less than j and otherwise select non-membership. That is, the firm selects membership if and only if its membership will result in a total number of members that is $\leq$ the number that firm believes can be profitably supported.

The first round begins with the first firm making a membership decision based on the initial entry vector, and the vector is updated with that firm's membership decision. The next firm then chooses membership/non-membership based on this updated vector, and the vector is again updated. The process continues in this fashion with updating of the vector after each firm. Each round ends after the vector is updated with the N-th firm's decision, and the next round begins with the first firm reacting to that vector. The process terminates when the vector remains constant throughout an entire round.

It can be shown that this simple process terminates in a finite number of rounds in a unique equilibrium entry vector. The proof of this claim is omitted for brevity; the proof proceeds by showing that, after the first round and prior to the terminal round, the entry vector seen by the first firm is strictly monotonic under a form of lexicographic ordering. The set of possible entry vectors is finite, so termination in a finite number of rounds is guaranteed. The fact that the terminal vector is an equilibrium vector follows directly from the termination condition that the entry vector remains constant through an entire round; this would not occur if any firm has an incentive to unilaterally deviate from their membership decision in the terminal vector.

7 A General Formulation and Price Competition

The basic results about market structure have thus far relied heavily on specific assumptions about the nature of the investment (scalar and independent of any other decision), demand (price is linear in aggregate quantity), and mode of competition (quantity-setting). This section gives a more general formulation to illustrate that the results are actually quite robust to changes in the model. Under this much weaker set of assumptions the technique of mapping each firm's forecast into a belief about the largest number of firms that can be profitably supported, classifying the firms based on this belief, and using this classification to characterize the equilibrium market structure remains valid. As an example, we show how the model may be adapted to a setting in which firms compete by setting prices rather than quantities.

Let A be the set of possible market conditions. Contained within A are both the actual market condition (e.g. the market reservation price in the previous models) and also all possible forecasts of this market condition. The set A, and of course the actual market condition and all forecasts, may now be multi-dimensional Real vectors or could be taken from some more arbitrary set as in the price-competition example that follows.

Decision stage 1: Initially with N potential entrants into the new market, each firm i has a forecast $f_i \in A$. In the first decision stage, each firm then decides whether to enter the market by selecting and entry decision e_i from the set $\{enter,\ do\ not\ enter\}$. This gives an entry vector $(e_1,\ e_2,\ \dots\ ,\ e_N)$ each element of which is one firm's entry decision, and we let $\mathbf{E}$ be the set of all possible entry vectors.

Decision stage 2: Next, the actual market condition $a \in A$ is revealed and each firm selects a second stage strategy. S denotes the set of strategies that are available to each firm, and it includes the element 0. Thus, each firm i selects a strategy $s_i \in S$ to give a strategy vector $(s_1,\ s_2,\ \ldots\ ,\ s_N)$; note however that any firm that selected *no entry* in stage 1 is restricted to selecting 0 in the second stage. Define Σ as the N-dimensional $S \times S\ \times \ldots \times S$; this is the set of all possible strategy vectors.

Profits: Finally, profits are calculated by a profit function $\pi : \Sigma \times A \times \mathbf{E} \longrightarrow \Re^N$. This is a vector function, and its i-th element π_i represents firm i's profits. The following restrictions/assumptions are placed on π:

1. non-entrants receive zero profits – If $e_i = \{\textit{do not enter}\}$, then $\pi_i = 0$.

2. symmetry in profits – if both the first- and second-stage strategies of any two firms, say firms i and j, are exchanged, then π_i and π_j are also exchanged; the profits of all other firms remain unchanged.

3. unique second-stage equilibrium – For any entry vector, there exists a unique second-stage Nash equilibrium of firms selecting pure strategies from S. This condition may be satisfied in several ways. Most commonly, the equilibrium is derived in a unique closed form. this is the path taken in previous sections. Alternatively, one may show that the best response function (a vector function that maps the strategies of each firm's competitors into the firm's own optimal strategy) is a contraction; see Friedman [1990] for details or Carr et. al. [2000] for an example.

   This condition, together with condition 2 implies that the unique equilibrium will be symmetric with regard to entrants' profits; that is, all firms who enter will earn the same profits in the second-stage equilibrium. This allows us to write the equilibrium profits as a real function $\pi^e : n \times A \longrightarrow \Re$ where n is the number of firms who select entry in stage 1. π^e is the profits earned by each of the n entering firms under the second-stage equilibrium.

4. Given an entry vector, second-stage equilibrium profits π^e are decreasing in the number of entrants.

Moving backward from the second stage to the first stage, firms will make their first-stage entry decisions presuming that all firms will adopt equilibrium strategies in the second stage. Note that we have not, and do not, explicitly model a sunk cost of entry as would be analogous to the value C in earlier sections. An investment such as this is easily incorporated into the function π, but we do not do so explicitly because the model as specified allows more general investment models. For example, the sunk cost need not be a fixed quantity but could vary with the second-stage strategy chosen by a firm or the number of other entrants.

Forecasts: As before, each firm i makes it's stage-1 entry decision based on a forecast $f_i \in A$ and on the anticipated number of entrants. Firm i expects that if it does enter, and if the total number of entrants including i itself is n, then its profits will be

$$\pi^e(n, f_i)$$

Thus, i's equilibrium entry decision is to enter as long as this is > 0.

This brings us to the classification scheme. Firm i is placed in class-j when j is the highest integer for which $\pi^e(j, f_i) > 0$. It then follows from condition 4 above that a firm in class-j believes that equilibrium profits will be positive as long as there are j or fewer entrants. This is of course the same interpretation of the classes as in earlier section. Furthermore, proposition 2 still holds under the same logic as given in the appendix.

Example – price competition: To illustrate how this formulation generalizes the model, consider a case in which the entrants compete by each selecting a price, and these prices determine the demand experienced by each firm. This is of course a form of "Bertrand competition."

For simplicity, we assume a linear relationship between demand (i.e. quantity q_i) and price; when there are n entrants in the second-stage, the demand quantity observed by firm i upon selecting a price p_i is[2]

$$q_i = \alpha - \beta p_i + \gamma \sum_{j \neq i} p_j \qquad \text{with} \quad \alpha > 0 \quad \text{and} \quad \beta > (n-1)\gamma > 0 \tag{11}$$

where the p_j values are the prices selected by i's competitors who are also entrants. Firms i's profits are calculated by $\pi_i = p_i q_i - C$ where C is an irreversible investment required of every market entrant as in previous sections, and the parameters α, β, and γ are common to all firms which provides symmetry. The other restrictions on α, β, and γ ensure that firms will always select non-negative prices and that the reaction function is a contraction; this guarantees a unique second-stage equilibrium. As before, (11) presumes zero variable production costs, a condition that is easily relaxed. From first-order optimality conditions, it is easy to derive that the unique equilibrium has all entrants selecting a price of $\frac{\alpha}{2\beta-(n-1)\gamma}$ and earning profits of $\pi^e = \left(\frac{\alpha^2\beta}{(2\beta-(n-1)\gamma)^2} - C\right)$.

[2]Please note that β no longer refers to the sampling distribution bias.

In the quantity competition of previous sections, the demand parameters of equation (1) remain constant as the number of entrants changes. In contrast, the parameters α, β, and γ in (11) must change with the number of entrants. To see why, consider a case in which there is some set of entrants who have already reached an equilibrium in price. Now suppose that these firms' prices remain unchanged, that one more entrant is added (this will add an additional term inside the summation of (11)), and that the parameters α, β, and γ also remain unchanged. In this case (11) tells us that the original entrants will now each sell higher quantities at the original equilibrium price, so they will enjoy higher profits than at the original equilibrium. It can also be shown that if the firms adjust prices to reach a new equilibrium, this new equilibrium will have prices, quantities, and profits that are greater than in the original equilibrium. In other words, we have a situation in which a market becomes more profitable as it becomes less concentrated.

This is entirely unnatural in most settings. To preclude this scenario, α, β, and/or γ must change with the number of entrants, and this requires all elements of the set A, including the market condition and the forecasts, to take on a more complicated structure. More specifically, each element of A is now a function that maps every possible number of entrants (i.e. each member of the set $\{1, 2, ..., N\}$) into a triplet (α, β, γ). For any such function $a \in A$, and writing $\alpha_a(n)$, $\beta_a(n)$, and $\gamma_a(n)$ to highlight the dependence of these parameters on the number of entrants n for function a, we require that the equilibrium profit function, which is

$$\pi^e(n, a) = \frac{\alpha_a(n)^2 \beta_a(n)}{(2\beta_a(n) - (n-1)\gamma_a(n))^2} - C,$$

to be decreasing in n. This gives the natural situation in which firms' equilibrium profits fall as competition increases.

This model now fulfills all of the conditions listed above. Thus, once firms are endowed with forecasts (each of which is a function from the set A), whether randomly assigned or arising out of a variety of forecasting techniques, the firms can be classified as described above, and the market structure calculated as per proposition 2.

8 Conclusion

We have considered the manner in which firms' forecasts of market conditions are determinants of industry structure and profitability for four cases of increasing generality, and we have included cases of both deterministic and probabilistic forecasts. We show with analysis and example that industry structure and profitability can be computed by a straightforward scheme of classifying firms based on their beliefs about the size of the oligopoly that an emerging market can support. Furthermore, this scheme is adaptable to quite general assumptions including randomly assigned forecasts, forecasts that are themselves random, and different modes of competition including price-setting or Bertrand competition. It is also robust to changes in the underlying game-theoretic assumptions; for example, we may relax the assumption of simultaneous entry decisions without abandoning the insights drawn from the various models of this work.

We also consider the manner in which industry structure and profitability change with the underlying problem parameters, the number of firms, the nature of the firms' forecasts, and the manner in which the forecasts are dispersed or scattered.

9 Appendix – Derivations and Proofs

Derivation of (2) and (3): Prices are given by

$$a - bQ = a - b\left(\sum_{j\neq i} q_j + q_i\right),$$

and every member firm i will earn second-stage revenues $p_i q_i$ which equals $\left[a - b\left(\sum_{j\neq i} q_j + q_i\right)\right] q_i$. This is concave in q_i for every i which is sufficient to guarantee an equilibrium in pure strategies; this equilibrium can also be shown to be unique which together with the fact that all firms are symmetric guarantees a symmetric equilibrium. Firms i's first order optimality condition is that its best production decision follows

$$q_i = \frac{a - b\sum_{j\neq i} q_j}{2b} \tag{12}$$

By symmetry, at equilibrium $q_j = q_i$ for every $j \neq i$ and $\sum_{j\neq i} q_j = (n-1)\, q_i$. Substituting this into (12),

$$q_i = \frac{a - b\,(n-1)\, q_i}{2b}$$

Solving this for q_i then give the result of expression (2). (3) is then derived by substituting (2) into the profit expression $q_i p - C$.

Derivation of (6): The (possibly) fractional number of firms that a forecast $\underline{f_K}$ can profitable support is $\frac{\underline{f_K}}{\sqrt{bC}} - 1$. We wish to know the $\underline{f_K}$ value for which the number of firms that can be supported is exactly the integer K. That is, we wish $\underline{f_K}$ such that $\frac{\underline{f_K}}{\sqrt{bC}} - 1 = K$ or equivalently

$$\frac{\underline{f_K}}{\sqrt{bC}} - 1 = \left\lfloor \frac{a}{\sqrt{bC}} - 1 \right\rfloor$$

solving for $\underline{f_K}$:

$$\frac{\underline{f_K}}{\sqrt{bC}} = \left\lfloor \frac{a}{\sqrt{bC}} \right\rfloor$$

$$\underline{f_K} = \sqrt{bC} \left\lfloor \frac{a}{\sqrt{bC}} \right\rfloor$$

Proof of proposition 1: *(i)* First note that, by the definition of $\underline{f^K}$: (1) $K = \frac{\underline{f^K}}{\sqrt{bC}} - 1$ and (2) $\frac{\underline{f^K}}{\sqrt{bC}}$ is an integer. Let K_{hi} and K_{lo} denote the integer number of firms that the high- and low-forecasters respectively anticipate that the market can support. That is

$$\begin{array}{llll} K_{hi} = \left\lfloor \frac{f_{hi}}{\sqrt{bC}} - 1 \right\rfloor & \text{which is} \quad > & \left\lfloor \frac{\underline{f^K} + \sqrt{bC}}{\sqrt{bC}} - 1 \right\rfloor & \text{for this case (since } f_{hi} > \underline{f^K} + \sqrt{bC}) \\ & \text{and this} \quad = & \left\lfloor \frac{\underline{f^K}}{\sqrt{bC}} \right\rfloor & \\ & \quad\quad = & \frac{\underline{f^K}}{\sqrt{bC}} & \text{by (2)} \end{array}$$

This together with (1) gives that $K_{hi} - K \geq 1$. This means that high-forecasters believe that the market can support at least one more firm than do the other firms.

A first-stage entry vector is an equilibrium vector if and only if no firms are willing to enter or leave the vector. In this case, an equilibrium vector cannot have fewer than K firms, otherwise any non-member high-forecaster would be willing to enter. It cannot have greater than K members, or any of the members would be willing to leave. If it has exactly K members, all members are unwilling to leave, and all non-members are unwilling to enter. Thus, there must be exactly K members. Also, these members must all be from high-forecasters because the other firms do not believe, as evidenced by $K_{hi} - K \geq 1$, that the market can support K_{hi} firms; so any of these other firms would be willing to leave an entry vector with K_{hi} members. The claim of profitability follows from the fact that K is the maximum number of firms that the market can actually support.

(ii) Similar arguments apply in this case except that now $K_{hi} = K \geq K_{lo} + 1$, so high- and accurate-forecasters have the same beliefs about the number of firms that the market can support.

(iii) Similar arguments again apply, but now $K_{hi} = K = K_{lo}$ and all firms share the same beliefs about the number of firms that can be supported. $_{QED}$

Proof of proposition 2: We first show by contradiction that the conditions *(i)* through *(iii)* are necessary in any equilibrium vectors.

(i) First, assume that there exists an equilibrium vector with more than $\underline{j}$ entrants. There must be at least one entrant from class-$\underline{j}$ or from a lower class because definition (8) implies that there are at most $\underline{j}$ firms in all classes higher than class-$\underline{j}$. But, no firm in class-$\underline{j}$ or below will participate in an oligopoly of more than $\underline{j}$ total participants. This contradicts the assumption of an equilibrium vector.

Next assume that there exists an equilibrium vector with fewer than $\underline{j}$ entrants. Note that the "min" in (8) implies that $\underline{j} - 1 < \sum_{m \geq j}^{\infty} N_m$ (the definition of $\underline{j}$ would otherwise be violated). This implies that there are at least $\underline{j}$ firms in class-$\underline{j}$ and above, and at least one of them must therefore be excluded from the equilibrium vector. This excluded firm would be willing to enter this vector since it includes fewer than $\underline{j}$ other firms, and this again contradicts the assumption of an equilibrium vector.

Thus, there must be exactly $\underline{j}$ entrants in any equilibrium vector.

(ii) Assume that there exists an equilibrium vector that includes a participant from a class $j < \underline{j}$. As in the proof of *(i)*, this would exclude a firm from a higher class which would then violate the assumption of an equilibrium.

(iii) Assume that there exists an equilibrium vector in which a firm from a class $j > \underline{j}$ does not enter. As for the *(i)* and *(ii)*, this firm will enter the vector which violate the equilibrium assumption.

We now show that *(i)* through *(iii)* are sufficient for an entry equilibrium. Assume that these three conditions apply to an entry vector. This vector is an equilibrium because: (1) the firms in classes higher than $\underline{j}$ will not exit because they each anticipate that the market can profitably support $\underline{j}$ firms; (2) class-$\underline{j}$ that have entered will not exit for the same reason; (3) class-$\underline{j}$ who have not entered will not because they anticipate that the market cannot profitably support $\left(\underline{j}+1\right)$ firms; (4) firms from classes below class-$\underline{j}$ will not enter for the same reason.

(iv) follows directly from *(i)* though *(iii)*.

The first claim in *(v)* follows from *(i)* and the fact that the market can profitably support K or fewer firms. The other claim follows from the additional fact that $\underline{j}$ will be greater than K (as per (8)) if there are more than K firms in classes higher than $\underline{j}$.

(vi) All entry vectors include all of the firms in classes higher than $\underline{j}$ and none of the firms in classes lower than $\underline{j}$. Any entry vector for which those conditions hold and in which there are exactly $\underline{j} - \sum_{m \geq \underline{j}+1} N^m$ out of the $N^{\underline{j}}$ in class $\underline{j}$ is an equilibrium vector. The stated quantity is the number of combinations such as this. $_{QED}$

Proof of proposition 3: The case of $N \leq K$ simply follows from the fact that the market will be profitable even if every firm enters. The case of $N > K$ follows from well know results for the binomial distribution as described next.

As described in the text, the entering firms will be profitable if and only if K or fewer firms receive forecasts that are $> \underline{f^K} + \sqrt{bC}$. The probability that this will occur is calculated as follows: (1) the assignment of a forecast to an individual firm is a "trial;" (2) each trial is a success if that firm's forecast is $> \underline{f^K} + \sqrt{bC}$, so a success occurs with probability $1 - \Gamma\left(\underline{f^K} + \sqrt{bC}\right)$; (3) the probability that firms will be profitable at equilibrium is thus the probability of K or fewer successes out of N trials, and this is the cdf of a binomial distribution evaluated at K as given in the proposition.

The claim that the probability is decreasing with β follows because: (1) Γ is decreasing with both its mean μ and β (because Γ is a cdf and because $\beta = \mu - a$); (2) the success probability $1 - \Gamma\left(\underline{f^K} + \sqrt{bC}\right)$ is thus increasing in β; and (3) the result then follows because the binomial distribution is stochastically decreasing in the success probability. The claim that the probability of a profitable equilibrium is increasing in the market reservation price a follows similar logic: (1) Γ is constant in a, but $\underline{f^K}$ is increasing in a; (2) so the success probability is now decreasing in the success probability (because Γ is an increasing function); (3) and the result again follows because the binomial distribution is stochastically decreasing in the success probability.

Lastly, we need that the probability of a profitable equilibrium is increasing in the number of potential entrants. Suppose there are $N+1$ potential entrants and let M_{N+1} denote the number of these firms in classes $(K+1)$ through ∞. Also, select any subset of N of the potential entrants and let M_N denote the number of firms in that subset from classes $(K+1)$ through ∞. Of course, $M_{N+1} \geq M_N$. We need to show that $P[M_{N+1} \leq K] < P[M_N \leq K]$. Using conditional probability,

$$\begin{aligned} P[M_{N+1} \leq K] &= P[M_{N+1} \leq K\,|M_N > K]\,P[M_N > K] + P[M_{N+1} \leq K\,|M_N = K]\,P[M_N = K] \\ &\quad + P[M_{N+1} \leq K\,|M_N < K]\,P[M_N < K] \end{aligned}$$

In this expression, $P[M_{N+1} \leq K\,|M_N > K] = 0$ because $M_{N+1} \geq M_N$. Also, both $P[M_{N+1} \leq K\,|M_N = K]$ and $P[M_{N+1} \leq K\,|M_N < K]$ are ≤ 1 because they are probabilities. Thus,

$$P[M_{N+1} \leq K] < P[M_N + K] + P[M_N < K] = P[M_N \leq K]\,. \quad _{QED}$$

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