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Wide channeling beam x-ray laser

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A channeling beam x-ray laser is proposed which is weakly dependent on the channeling length. The scheme is based on applying Bragg reflections to direct the amplified radiation toward a longitudinal cavity direction which is transverse to the beam propagation. This scheme implements a crystal distributed feedback cavity and is restricted to short radiation wavelength of the order of the crystal unit cell dimension.

Channeling x-ray lasers are based on a relativistic electron beam which propagates through axial or planar crystal channels and populates transverse bound states.[1] Transition between these discrete states yield narrow width, strongly forward peaked and tunable x-ray radiation.[2] The emitted radiation is limited by the range of the channeling length l_0, which is the length the beam particles stay in the bound channeling states. This length is affected by the particle interactions with the crystal and at room temperature its value can be of the order of 100 μm. In a single pass x-ray laser[1] the amplification length is limited by the channeling length and the requirement on the gain or beam current density is increased. In a distributed feedback concept based on multiple Bragg reflections of the radiation in the beam direction,[3] the channeling length limits the cavity longitudinal dimension.

In this letter we propose a scheme which is weakly dependent on the channeling length. The scheme is based on applying Bragg reflections to direct the amplified radiation toward a longitudinal cavity direction transverse to the beam propagation (see Fig. 1).

Two sets of crystal planes for Bragg reflections are considered. In Fig. 2 the first set of planes parallel to the z-y plane are presented. The channeling planes are parallel to the z-x plane. Primary radiation with wave vector $\mathbf{k}_1$ is emitted in the beam direction. The primary wave is tuned so it can be Bragg reflected from the z-y set of planes and form with the reflected wave $\mathbf{k}_2$ a two-beam Borrmann mode. This mode generate a standing wave with nodes on the atomic sites in the x direction and an energy flow in the $+z$ direction (see Fig. 2). The second set of planes parallel to the x-y plane are presented in Figs. 1 and 3. This set Bragg reflects the radiation waves $\mathbf{k}_1$ and $\mathbf{k}_2$ to form the waves $\mathbf{k}_4$ and $\mathbf{k}_3$, respectively. The sum of the waves $\mathbf{k}_3$ and $\mathbf{k}_4$ generate a two-beam Borrmann mode with standing wave in the x direction and a reflected energy flow in the $-z$ direction. Thus, the z-y planes acts as Borrmann planes and direct the energy parallel to these planes. The x-y planes couple the radiation flowing in the $\pm z$ directions and acts as an effective mirror structure with distributed feedback and generate a standing wave in the z direction.[4] That is, standing waves are generated in the x and z directions with nodes on atomic sites and the absorptions located close to the sites are strongly reduced.[5,6] The transverse dimension of the cavity, a, is selected to be of the order of the channeling length, $a < l_0 \sin(\theta)$. The channeling length limits only the transverse dimension of the cavity. The longitudinal cavity dimension, L, can be scaled up by many orders of magnitude by increasing the transverse dimension of the channeling beam independent of the channeling length. In the following we analyze the conditions to obtain amplified radiation modes with very low threshold gain.

We consider a two level system of channeling bound states $|1\rangle$ and $|2\rangle$, where W_0 and $\hbar\omega_0 = \epsilon_2 - \epsilon_1$ are the population and energy differences, respectively.[1,3] The Doppler up shifted radiation, $\omega = \omega_0/(1 - v/c) \approx 2\gamma^2\omega_0$ with v as the beam velocity, is emitted in the $\mathbf{k}_1$ direction and is tuned to closely match Bragg reflection conditions with the two sets of planes parallel to the z-y and x-y planes (see Fig. 3). The reflected radiation fulfills the Bragg relations: $\mathbf{k}_2 - \mathbf{k}_1 = \boldsymbol{\tau}_1$, $\mathbf{k}_3 - \mathbf{k}_2 = \boldsymbol{\tau}_2$, $\mathbf{k}_1 = -\mathbf{k}_3$, and $\mathbf{k}_4 = -\mathbf{k}_2$, where τ_1 and τ_2 are the reciprocal lattice vectors representing the set of planes z-y and x-y, respectively. For crystals like diamond or silicon the emitted radiation wave length is of the order 5 Å and for $\omega_0 = 3$ eV the beam energy is 10 MeV.

We consider the electric field of the radiation as a linear combination of the four partial traveling waves $\epsilon_i \exp[-i\omega(t - \mathbf{k}_i \cdot \mathbf{r})]$, where $i = 1, 2, 3, 4$, ϵ_i is the slowly varying part of the fields, $\hat{\mathbf{k}}_i = \mathbf{k}_i/k$ and $|\mathbf{k}_i| = k = \omega/c$. By applying the Maxwell-Bloch scheme for the radiation field and the polarization of the particle states a set of coupled equations for the partial fields can be obtained.[4] The coupling between the fields is due to the periodic reflection and absorption functions, and the set of four field equations can be written for ϵ_i, where $i = 1, 2, 3, 4$, as:[4,7]

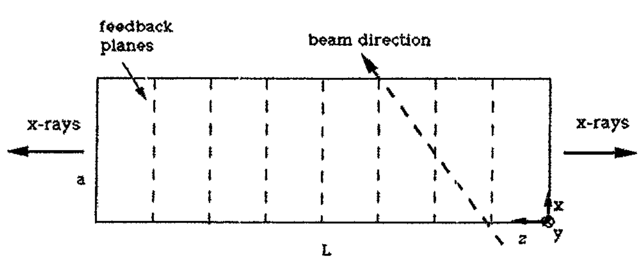

FIG. 1. Wide beam x-ray laser scheme.

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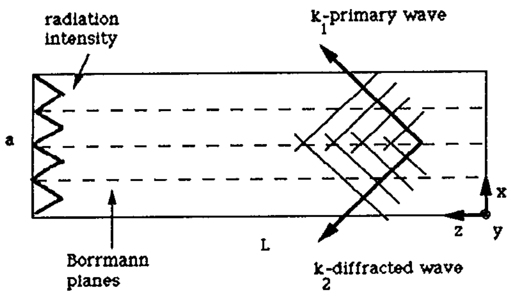

FIG. 2. Two beam Borrmann mode.

$$\frac{1}{c}\frac{\partial}{\partial t}\epsilon_i+\hat{\mathbf{k}}_i\cdot\nabla\epsilon_i+(v_0-g_i)\epsilon_i+v_i\epsilon_{i+1}$$
$$+\hat{v}_i\epsilon_{i-1}+v_i'\epsilon_{i+2}=0 \qquad (1)$$

and $i+4=i$. Here $v_0=\mu_0+i\kappa_0+i\delta$, where μ_0 and v_0 are the average absorption and reflection coefficients, respectively, and δ is the detuning of the waves from matching the exact Bragg condition for the reflection planes. For $i=1,3$ $v_i=v_z$ and $v_i'=v_x$, and for $i=2,4$ $v_i=v_x$ and $v_i'=v_z$, where $v_z=\mu_z+i\kappa_z$ and $v_x=\mu_x+i\kappa_x$. The factors μ_z and κ_z are the resonance absorption and reflection coefficients, respectively, due to the periodic set of planes *z-y*.[4] In a similar way we define μ_x and κ_x as absorption and reflection coefficients, respectively, relative to the set of planes *x-y*. The factor $v_i'=v$, $v=\mu+i\kappa$, where μ and κ are the back reflection and absorption coefficients, respectively. For simplicity v_i' is taken independent of i. The partial wave frequencies are detuned relative to the particle transition frequency as $\Delta_i=\omega(1-\mathbf{v}\cdot\mathbf{k}_i/c)-\omega_0$. The resonance condition is applied for the $\hat{\mathbf{k}}_1$ wave, where $|\mathbf{v}\cdot\mathbf{k}_i/c|=v/c$ and $\Delta_1\approx0$ or $\omega=\omega_0/(1-v/c)$. The other waves are out of resonance and $\Delta_i\approx\omega$. Thus the only interaction of the radiation with the beam is with the $\mathbf{k}_1$ wave and the gain factors can be written as $g_i=g\cdot\delta_{i1}$.[3]

The system at threshold is represented by the set of equations, Eq. (1), at steady state.[3] We use the transformations $x/\phi_x\to x$, $z/\phi_z\to z$, where $\phi_x=\sin\theta$ and $\phi_z=\cos\theta$. We further transform ϵ_i to $\epsilon_i\exp[g(x+z)/4]$ and define the four field variables as $\epsilon^{\uparrow}_{\pm}=\epsilon_1\pm\epsilon_4$ and $\epsilon^{\downarrow}_{\pm}=\epsilon_3\pm\epsilon_2$. The main spatial variations are in the narrow transverse x direction. The longitudinal variations in the z direction are small, and for $L\gg a$ the

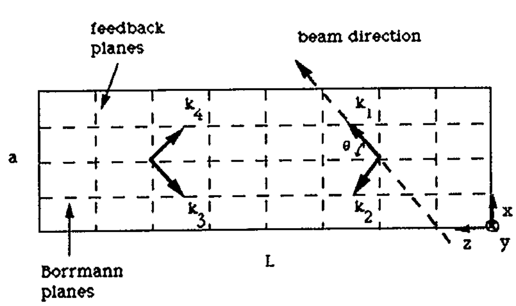

FIG. 3. Orientation of the four radiation waves.

derivatives with respect to z can be ignored in Eq. (1). The following set of four coupled equations $\epsilon^{\uparrow}_{+}$, $\epsilon^{\downarrow}_{-}$, $\epsilon^{\downarrow}_{+}$, and $\epsilon^{\downarrow}_{-}$ are obtained from Eq. (1):

$$\left(\frac{\partial}{\partial x}+v_0-\frac{g}{4}\pm v_x\right)\epsilon^{\uparrow}_{\pm}-\frac{g}{4}\epsilon^{\uparrow}_{\mp}+(v\pm v_z)\epsilon^{\downarrow}_{\pm}=0, \qquad (2a)$$

$$\left(-\frac{\partial}{\partial x}+v_0-\frac{g}{4}\pm v_x\right)\epsilon^{\downarrow}_{\pm}-\frac{g}{4}\epsilon^{\downarrow}_{\mp}+(v\pm v_z)\epsilon^{\uparrow}_{\pm}=0. \qquad (2b)$$

To obtain the modes with the lowest threshold gain we transform the set of four wave equations, Eqs. (2a) and (2b), to an approximate set of two effective waves. The set of coupled two waves are[3,4]

$$\left(\frac{\partial}{\partial x}+\mathbf{v}_0-\frac{\mathbf{g}}{2}\right)\epsilon^{\uparrow}+\mathbf{v}\epsilon^{\downarrow}=0, \qquad (3a)$$

$$\left(-\frac{\partial}{\partial x}+\mathbf{v}_0-\frac{\mathbf{g}}{2}\right)\epsilon^{\downarrow}+\mathbf{v}\epsilon^{\uparrow}=0, \qquad (3b)$$

where $\mathbf{v}_0=\boldsymbol{\mu}_0+i\boldsymbol{\kappa}_0+i\boldsymbol{\delta}$, $\mathbf{v}=\boldsymbol{\mu}+i\boldsymbol{\kappa}$. Here $\boldsymbol{\mu}_0$ and $\boldsymbol{\kappa}_0$ are the average absorption and reflection coefficients, respectively, and $\boldsymbol{\mu}$ and $\boldsymbol{\kappa}$ are the Bragg coupling resonance terms due to the periodicity in the absorption and reflection functions in the x direction. The gain factor $\mathbf{g}$ represents the amplification only in the forward, $\uparrow$, direction. The boundary conditions are $\epsilon^{\uparrow}(x=-a/2)=0$ and $\epsilon^{\downarrow}(x=a/2)=0$ and no external radiation sources are assumed. Following the treatment in Ref. 4 in the limit of strong reflection we obtain for the selectivity $\boldsymbol{\delta}$ and lowest threshold gain $\mathbf{g}$ that

$$\boldsymbol{\delta}=-\boldsymbol{\kappa}_0\pm\left[\boldsymbol{\kappa}^2+\left(\frac{\mathbf{g}}{2}-\boldsymbol{\mu}_0\right)^2-\boldsymbol{\mu}^2+\frac{6}{a^2}\right]^{1/2}. \qquad (4)$$

$$\frac{\mathbf{g}}{2}=\boldsymbol{\mu}_0-\frac{\boldsymbol{\kappa}}{(\boldsymbol{\delta}+\boldsymbol{\kappa}_0)}\left(\boldsymbol{\mu}\mp\frac{3}{[(\boldsymbol{\mu}^2+\boldsymbol{\kappa}^2)a^3]}\right). \qquad (5)$$

In the following we use the two wave solutions, Eqs. (4) and (5) to obtain the selectivity condition and the threshold gain for the four wave case. The lowest threshold gain is obtain for $|\epsilon_1|\approx|\epsilon_2|\approx|\epsilon_3|\approx|\epsilon_4|$, with the boundary conditions $\epsilon^{\uparrow}_{\pm}(x=-a/2)=0$ and $\epsilon^{\downarrow}_{\pm}(x=a/2)=0$ and no external radiation sources. Two possible cases are considered in the following with low absorptions: the case $\epsilon_1\approx\epsilon_4$ and the case $\epsilon_1\approx-\epsilon_4$.

Case A: $\epsilon_1\approx\epsilon_4$. For this case $\epsilon^{\uparrow}_{+}\gg\epsilon^{\uparrow}_{-}$ and $\epsilon^{\downarrow}_{+}\gg\epsilon^{\downarrow}_{-}$, thus $(\partial/\partial x)\epsilon^{\uparrow}_{-}$ and $(\partial/\partial x)\epsilon^{\downarrow}_{-}$ can be ignored in Eqs. (2a) and (2b). From the set of four equations we can solve for $\epsilon^{\uparrow}_{-}$ and $\epsilon^{\downarrow}_{-}$ to obtain an effective two wave coupled equations for $\epsilon^{\uparrow}_{+}$ and $\epsilon^{\downarrow}_{+}$ of the form of Eqs. (3a) and (3b). In the strong reflection case we obtain from Eqs. (4) and (5) the threshold gain g and the selectivity δ for this mode

$$g_1=2(\mu_0+\mu_x-\mu_z-\mu)+12(\phi_x/a)^3/(\kappa+\kappa_z)^2, \qquad (6)$$

and $\delta_1=-\kappa_0-\kappa_x+\kappa_z+\kappa$, where we used $a\to a/\phi_x$ and $\phi_x=\sin\theta_0$. For this mode $\epsilon^{\uparrow}_{+}\approx-\epsilon^{\downarrow}_{+}$ or $\epsilon_1\approx-\epsilon_2\approx-\epsilon_3\approx\epsilon_4$. The first term in g_1 represents the reduced absorption for $\mu_0\approx\mu_x\approx\mu_z\approx\mu$, and is generated when standing waves in the x and z direction are generated

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with nodes on the atomic sites.[7] The second term in g_1 is due to reflections[4] and can be written in terms of the channeling length by replacing ϕ_x/a by $1/l_0$.

Case B: $\epsilon_1 \approx -\epsilon_4$. In this case $\epsilon^{\uparrow}_- \gg \epsilon^{\uparrow}_+$ and $\epsilon^{\downarrow}_- \gg \epsilon^{\downarrow}_+$, thus $(\partial/\partial x)\epsilon^{\uparrow}_+$ and $(\partial/\partial x)\epsilon^{\downarrow}_+$ can be ignored in Eqs. (2a) and (2b). We can solve for $\epsilon^{\uparrow}_+$ and $\epsilon^{\downarrow}_+$ to obtain an effective two wave coupled set of equations for $\epsilon^{\uparrow}_-$ and $\epsilon^{\downarrow}_-$ of the form of Eqs. (3a) and (3b). For this case two possible modes are obtained with relatively low absorptions. Using the two wave solutions, Eqs. (4) and (5), the first mode is

$$g_2=4(\mu_0-\mu_x-\mu_z+\mu)+12(\phi_x/a)^3/(\kappa-\kappa_z)^2, \qquad (7)$$

and $\delta_2=-\kappa_0+\kappa_x+\kappa_z-\kappa$, where for this mode $\epsilon^{\uparrow}_- \approx \epsilon^{\downarrow}_-$ or $\epsilon_1 \approx -\epsilon_2 \approx \epsilon_3 \approx -\epsilon_4$.

The second mode is

$$g_3=2(\mu_0-\mu_x+\mu_z-\mu)+12(\phi_x/a)^3/(\kappa-\kappa_z)^2, \qquad (8)$$

and $\delta_3=-\kappa_0+\kappa_x-\kappa_z+\kappa$, where $\epsilon^{\uparrow}_- \approx -\epsilon^{\downarrow}_-$ or $\epsilon_1 \approx \epsilon_2 \approx -\epsilon_3 \approx -\epsilon_4$.

The three modes, Eqs. (6), (7), and (8), possess low threshold gain due to absorptions for $\mu_0 \approx \mu_x \approx \mu_z \approx \mu$, because of the near cancellation of the absorption coefficients.[7] This result is characteristic of the Borrmann effect, where standing waves are generated with nodes on the atomic sites which drastically reduce the radiation absorption.[5] In crystals with low atomic numbers, e.g., LiH, $\mu_0 \approx 10$ cm^{-1} and the Borrmann effect cancellation of the absorption can be $10^{-3}\mu_0$ and a threshold gain due to absorption of 2×10^{-2} cm^{-1}. The term in the threshold gain due to reflections depends on the transverse dimension of the cavity a, the strength of the back reflections κ, and the reflections κ_z from the *z-y* planes. For the channeling length $l_0=a/\phi_x$ the threshold gain scales as $(1/l_0)^3$. The lowest threshold due to reflection is obtained for the first mode, Eq. (6), where κ and κ_z are added. In this case for $\kappa=\kappa_z=10^4$ cm^{-1} and $l_0=90$ μm, we get $g=5\times10^{-2}$ cm^{-1} and low threshold values can be obtained. For a coherence length of 10 μ, beam energy of 10 MeV a high current density of 10^5 A/cm^2 leads to a threshold gain of 5×10^{-2} cm^{-1}. To reduce the high current density requirement for a practical system a further effort should be done to reduce the threshold gain.

It is possible to further reduce the threshold gain by increasing the number of diffraction sets of planes which satisfy the Bragg conditions simultaneously. In this case standing waves are generated in several directions relative to an atomic site, generating a larger nodal region around the atomic sites and reducing the radiation absorption. For some of the modes the effective reflection coefficient is increased by adding up the reflections from several sets of planes as in Eq. (6), and the threshold gain is reduced. For these modes it is possible to further reduce the required value of the channeling length.

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