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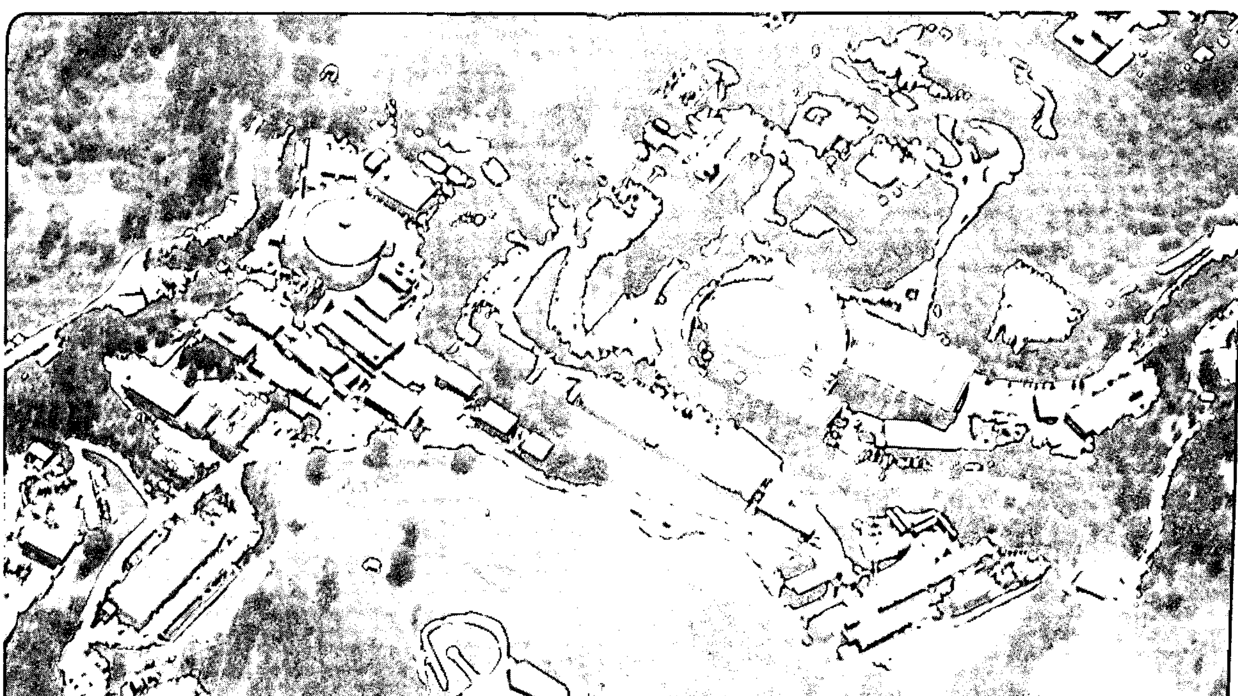

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Studying Non-calculable Models of Dynamical Supersymmetry Breaking *

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Abstract

There are supersymmetric gauge theories which do not possess any parameters nor flat directions, and hence cannot be studied anywhere in the field space using holomorphy ("non-calculable"). Some of them are believed to break supersymmetry dynamically. We propose a simple technique to analyze these models. Introducing a vector-like field into the model, one finds flat directions where one can study the dynamics. We unambiguously show that the supersymmetry is broken when the mass of the vector-like field is small but finite, and hence Witten index vanishes. If we increase the mass of the vector-like field, it eventually decouples from the dynamics and the models reduce to the original non-calculable models. Assuming the continuity of the Witten index in the parameter space, one can establish the dynamical supersymmetry breaking in the non-calculable models.

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Supersymmetry is an attractive possibility to stabilize the hierarchy between the weak- and unification- or Planck-scales. Especially interesting is the case where the electroweak symmetry cannot be broken in the supersymmetric limit, such as in the minimal supersymmetric standard model, since one can understand the smallness of the weak scale in terms of the smallness of the supersymmetry breaking effects. However, the origin of the hierarchy itself remains unexplained in lack of understanding why supersymmetry is (weakly) broken. Dynamical supersymmetry breaking is a natural idea to explain the smallness of the supersymmetry breaking scale [1]. Indeed, a class of chiral gauge theories were shown to break supersymmetry dynamically [2], and can be used to construct realistic models [3]. Since then, there was a substantial progress in the technique to analyze dynamics of supersymmetric gauge theories based on holomorphy [4, 5, 6, 7]. The technique was also applied to build new models which break supersymmetry dynamically [8].

The earliest models of the dynamical supersymmetry breaking [9, 10], however, cannot be analyzed using the holomorphy. The known examples are SU(5) theory with $\mathbf{5}^*$ and $\mathbf{10}$, and SO(10) with a single $\mathbf{16}$.[‡] These models do not have any adjustable parameters nor any flat directions, and hence "non-calculable". They were argued to break supersymmetry dynamically because of the following reason. These theories possess an $\mathrm{U}(1)_R$ symmetry. If the low energy theory preserves $\mathrm{U}(1)_R$, the low energy particle content should saturate the anomalies of the fundamental theories. There are possible candidates of such low energy particle contents. But it was argued such particle contents are "implausible" because of the complicated charge assignments. Then it is more "plausible" to have $\mathrm{U}(1)_R$ symmetry spontaneously broken, and one needs its non-linear realization. However, it tends to require a flat direction in the low-energy theory. This is also argued to be "implausible" since the fundamental theory did not possess any flat directions. Even though a strong case was made, it is still desired to have a method to analyze these models where one can explicitly see the breakdown of supersymmetry.

The purpose of this letter is to point out there is a simple method to study the "non-calculable" models by introducing additional vector-like field (field which transforms under a real representation of the gauge group) to the models. The original models are understood as the limit where the

[‡]There are also models with non-abelian flavor symmetries [2] even though we do not discuss them in this letter. The framework in [11] should be useful to analyze such models.

	H	ψ	$\psi\psi H$	H^2	λ	M
$U(1)_R$	1	−3	−5	2	7	0
$U(1)_M$	2	−1	0	4	0	−4

Table 1: Charges of the fields and parameters under non-anomalous global symmetries in the SO(10) model with $\psi(\mathbf{16})$ and $H(\mathbf{10})$.

vector-like fields decouple. When the vector-like field is massless, there are flat directions and the models can be analyzed using by-now well-known technique of holomorphy. Once we turn on the mass of the vector-like field, the models break supersymmetry spontaneously, and hence Witten index vanishes. Assuming the continuity of the phase as we increase the mass of the vector-like field, Witten index vanishes in the limit where the vector-like fields decouple. There is no sign of supersymmetry restoration when one gradually raises the mass of the vector-like field. Then one can conclude that the original models break supersymmetry dynamically.

We discuss an SO(10) model with a single $\psi(\mathbf{16})$. It was shown that this model does not have any flat directions [10]. We introduce a vector-like field H which transforms as a $\mathbf{10}$. The non-anomalous global symmetries of the model are listed in Table 1. There are two non-anomalous symmetries in this model, an R-symmetry $U(1)_R$ and a non-R symmetry $U(1)_M$. In the absence of the superpotential, the most general flat direction of this model (up to gauge transformations) is parametrized by three complex scalar fields $H^\pm$ and χ,

$$H^1 = \cdots = H^8 = 0, \tag{1}$$

$$H^9 = \frac{i}{\sqrt{2}}(H^+ - H^-), \tag{2}$$

$$H^{10} = \frac{1}{\sqrt{2}}(H^+ + H^-), \tag{3}$$

$$\psi = (\uparrow \otimes \uparrow \otimes \uparrow \otimes \uparrow)\chi, \tag{4}$$

with the D-flatness condition

$$|H^+|^2 - |H^-|^2 - \frac{1}{2}|\chi|^2 = 0. \tag{5}$$

See appendix for notation. The low-energy theory is a pure SO(7) supersymmetric Yang–Mills theory with two singlet chiral superfields,§ which can be identified with gauge-invariant composite fields $\psi\psi H$ and H^2 (the gauge indices are contracted in an obvious manner). There is a unique possible superpotential generated non-pertubatively by the condensate of SO(7) gauginos [12],

$$W_{\text{n.p.}} = c\frac{\Lambda^{21/5}}{(\psi\psi H)^{2/5}} \,. \tag{6}$$

The coefficient c is a constant of order unity. There is no ground state in the absence of a tree-level potential.

There are two possible terms in the superpotential at the tree-level,

$$W_{\text{tree}} = \lambda\psi\psi H + \frac{1}{2}MH^2. \tag{7}$$

Charges of the parameters λ and M under the global symmetries are also listed in Table 1. The total superpotential is the sum of W_{tree} and $W_{\text{n.p.}}$ and is exact in the sense of [6] as shown below. Because of the $U(1)_R$ and $U(1)_M$ symmetries, the superpotential has to take the form

$$W_{\text{total}} = c\frac{\Lambda^{21/5}}{(\psi\psi H)^{2/5}}\, F\left(\frac{\lambda\psi\psi H}{c\Lambda^{21/5}/(\psi\psi H)^{2/5}}, \frac{MH^2/2}{c\Lambda^{21/5}/(\psi\psi H)^{2/5}}\right), \tag{8}$$

where $F(x,y)$ is a holomorphic function with $F(0,0) = 1$. For small x and y, it has to behave as $F(x,y) \sim 1 + x + y$ plus terms with higher powers in x and y to be consistent with a perturbation in terms of λ and M. However, higher powers in x and y lead to a singular behavior when $\Lambda \to 0$ (weakly coupled limit), and are not allowed. Therefore, $F(x,y) = 1 + x + y$ *exactly*, and hence $W_{\text{total}} = W_{\text{n.p.}} + W_{\text{tree}}$.

We first discuss the case where H is massless, *i.e.*, $M = 0$. In this case,

§When SO(10) breaks down to SO(7), $45 - 21 = 24$ chiral superfields are eaten. It leaves $16 + 10 - 24 = 2$ chiral superfields massless.

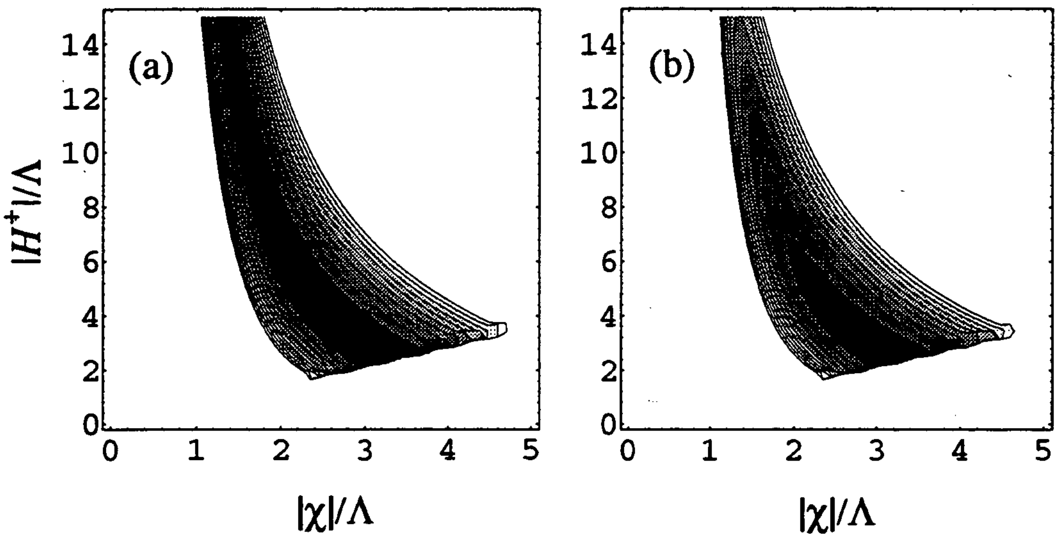

Figure 1: Contour plots of the potential in the SO(10) model. See the text for the definition of the fields χ and H^+. We eliminated H^- using the D-flatness condition Eq. (5). Darker region has lower energy. The white region has a potential energy larger than $0.1\Lambda^4$. The sharp cutoff from below is due to the D-term constraint $|H^+|^2 \geq |\chi|^2/2$. The choice of the parameters is (a) $(M, \lambda) = (0, 0.01)$, (b) $(M, \lambda) = (0.01\Lambda, 0.01)$. The vacuum energy is $V \simeq 6 \times 10^{-4}\Lambda^4$ in case (b).

there is a moduli space of the supersymmetric vacua defined by[¶]

$$\psi\psi H = \sqrt{2}\chi^2 H^+ = \left(\frac{2c}{5\lambda}\right)^{5/7} \Lambda^3. \tag{9}$$

When λ is small, the moduli space is far away from the origin, and one can study the potential explicitly with perturbative Kähler potential. See Fig. 1(a) for the moduli space Eq. (9) which extends to the infinity $H^+ \to \infty$. Note that the D-flatness requires $|H^+|^2 - |\chi|^2/2 = |H^-|^2 \geq 0$ and the moduli space does not extend to the the region $|H^+|^2 < |\chi|^2/2$.

Now we turn on the mass term $\frac{1}{2}MH^2 = MH^+H^-$. The conditions for the supersymmetric vacua are

$$\left(-\frac{2}{5}\frac{\Lambda^{21/5}}{(\sqrt{2}H^+\chi^2)^{7/5}} + \lambda\right)\chi H^+ = 0, \tag{10}$$

$$\left(-\frac{2}{5}\frac{\Lambda^{21/5}}{(\sqrt{2}H^+\chi^2)^{7/5}} + \lambda\right)\sqrt{2}\chi^2 + MH^- = 0, \tag{11}$$

$$MH^+ = 0, \tag{12}$$

along with the D-flatness condition Eq. (5). It is easy to see that the supersymmetry is spontaneously broken once there is non-vanishing M as follows. Eq. (12) requires $H^+ = 0$, and then the D-flatness requires $H^- = \chi = 0$. But this is inconsistent with Eqs. (10), (11). Therefore, any finite M breaks supersymmetry dynamically. For small M, the flat direction in Eq. (9) is still almost flat, with M raising the potential for larger H^+. The minimum lies along the direction Eq. (9) with smallest possible H^+, and hence $H^- = 0$, $|H^+| = |\chi|/\sqrt{2}$ (see Fig. 1(b)). There is a unique ground state with no massless scalars.[‖] For larger M, the vacuum is pulled towards smaller values of H^+ and χ. We studied numerically that the vacuum energy becomes larger for larger M.

[¶]The moduli space can be parametrized by the gauge-invariant superfield H^2. Note that $U(1)_M^3$ anomaly is saturated by H^2: $2^3 \times 10 + (-1)^3 \times 16 = 64 = 4^3$. $U(1)_R$ symmetry is explicitly broken by $\lambda \neq 0$ and hence does not give us useful constraints on the low-energy particle content. Even though one should be able to discuss the dynamics with this composite field H^2, we prefer to use the elementary fields $H^\pm$ and χ as in [2] because the Kähler potential has a much simpler form. Such a treatment is valid when both λ and M are small.

[‖]Since $U(1)_R$ is explicitly broken by $\lambda \neq 0$, there is no R-axion in this case, similar to the model with a massive singlet in [5].

	H	ψ	$(\phi, \bar{H})$	$\psi\phi\bar{H}$	$\psi\psi H$	$(\bar{H}H, \phi H)$	f	h	M
$U(1)_R$	8	0	−6	−12	8	2	14	−6	0
$U(1)_M$	−4	2	−1	0	0	−5	0	0	5
$U(1)_Y$	3	1	−3	−5	5	0	5	−5	0
SU(2)	1	1	2	1	1	2	1	1	2

Table 2: Charges of the fields and parameters under non-anomalous global symmetries in the SU(5) model with $H(\mathbf{5})$, $\psi(\mathbf{10})$, $(\phi, \bar{H})(\mathbf{5}^*)$.

We can never study the limit $M \to \infty$ exactly using the elementary fields because one enters into an intrinsically strongly interacting regime. In fact, all the field values approach $\sim \Lambda$ from above as we gradually raise M, and we lose our handle on the Kähler potential. However, this analysis does show that the Witten index of the SO(10) model with a **16** and **10** vanishes for any small but finite values of M.** Assuming the continuity of the phase as we gradually increase M, we obtain vanishing Witten index for $M \gg \Lambda$, which is equivalent to an SO(10) model with a single **16**. If the Witten index of a model vanishes, the model generically breaks supersymmetry, even though one cannot logically exclude the possibility of having equal number of supersymmetric zero-energy states for both bosonic and fermionic states. Therefore, we confirm the conclusion in Ref. [10] that the SO(10) model with a single **16** breaks supersymmetry dynamically.††

A similar analysis can be done for the SU(5) model with $\psi(\mathbf{10})$ and $\phi(\mathbf{5}^*)$. Again we introduce vector-like fields $H(\mathbf{5})$ and $\bar{H}(\mathbf{5}^*)$. There are flat directions which can be parametrized by four chiral gauge-invariant fields $\psi\phi\bar{H}$, $\psi\psi H$, $\bar{H}H$, ϕH. The low energy theory along the flat directions is a pure SU(2) Yang–Mills theory with four singlet chiral superfields. The exact su-

**Note also that the superpotential W_{total} breaks supersymmetry if we regard $\psi\psi H$ and H^2 as true degrees of freedom, since $\partial W/\partial(H^2) = M/2 \neq 0$. However one needs to discuss the singularities in the Kähler potential to justify this argument.

††Another interesting point is that this analyis confirms the spontaneous breakdown of $U(1)_R$ symmetry in this limit as conjectured in [9, 10]. If we take $\lambda = 0$, there is an exact $U(1)_R$ symmetry. Having $M \neq 0$ leads to a well-defined vacuum with dynamical supersymmetry breaking. One can easily see that $U(1)_R$ symmetry is broken spontaneously at this vacuum, and there is an R-axion.

perpotential including the non-perturbative effects is*

$$W = c\frac{\Lambda^6}{(\psi\psi H)^{1/2}(\psi\phi\bar{H})^{1/2}} + h\psi\psi H + f\psi\phi\bar{H} + MH\bar{H}. \tag{13}$$

In the limit of $M \to 0$, there is a moduli space of supersymmetric vacua defined by†

$$\psi\psi H = \sqrt{c}\Lambda^3\frac{(4fh)^{1/4}}{2h}, \tag{14}$$

$$\psi\phi\bar{H} = \sqrt{c}\Lambda^3\frac{(4fh)^{1/4}}{2f}. \tag{15}$$

Again, the combination of the D-flatness and small but non-vanishing M leads to the spontaneous breakdown of supersymmetry.

In summary, we proposed a simple method to analyze non-calculable supersymmetric gauge theories. In the SO(10) model with a single **16**, there is no flat direction and it cannot be analyzed with the holomorphy. When we introduce a **10**, the model has flat directions and can be analyzed unambiguously. By introducing a small mass of **10**, one can show the supersymmetry is spontaneously broken, and hence Witten index of this model vanishes. Assuming the continuity of the phase, Witten index remains vanishing for larger M. In the limit $M \to \infty$, the model reduces to the original SO(10) model with a single **16**, and is expected to break supersymmetry.

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*We can always make an SU(2) rotation between $\bar{H}$ and ϕ to allow mass term only for $\bar{H}$. All other terms in W are invariant under this SU(2) rotation.

†The moduli space is parametrized by SU(2)-doublet chiral superfield $\bar{H}H$ and ϕH. Both $U(1)_R$ and $U(1)_Y$ are broken explicitly by f and h, but SU(2) and $U(1)_M$ are not broken. The anomalies match with this particle content. $U(1)_M^3$: $(-4)^3 \times 5 + 2^3 \times 10 + (-1)^3 \times 5 \times 2 = -250 = (-5)^3 \times 2$, $U(1)\times SU(2)^2$: $(-1) \times 5 = -5$.

APPENDIX

This appendix summarizes the notation, and shows the flat direction explicitly. It is known that the gamma matrices of SO(8) can be chosen real, $\gamma^1, \ldots \gamma^8$, and γ^9 is also real in this basis (this is why there is Majorana–Weyl spinor in SO(8)). We define the SO(10) gamma matrices by

$$\begin{aligned}
\Gamma^1 &= \gamma^1 \otimes \sigma^1 \\
&\vdots \\
\Gamma^8 &= \gamma^8 \otimes \sigma^1 \\
\Gamma^9 &= \gamma^9 \otimes \sigma^1 \\
\Gamma^{10} &= 1 \otimes \sigma^2 \\
\Gamma^{11} &= 1 \otimes \sigma^3.
\end{aligned}$$

The Weyl spinor $\psi(\mathbf{16})$ is defined by $\Gamma^{11}\psi = +\psi$, and hence can be written as $\psi = \tilde{\psi} \otimes \uparrow$ with $\tilde{\psi}$ having 16 comonents while ψ has 32 components. The charge conjugation matrix C in SO(10) spinor is nothing but $C = \Gamma^{10}$ in this basis.

Without a loss of generality, we can always make an SO(10) rotation to bring $H(\mathbf{10})$ to the form

$$H = (0,0,0,0,0,0,0,0,H^9,H^{10}). \qquad (16)$$

Then all D-terms vanish except that for the rotation between 9th and 10th components. This leaves SO(8) gauge symmetry unbroken.

Since SO(8) spinors are real, the contribution of ψ to the D-terms for SO(8) generators vanishes when one takes $\tilde{\psi}^* = \tilde{\psi}$ up to a phase. Note that the generators of SO(8) rotations commute with γ^9 while those of $(i,9)$ and $(i,10)$ rotations anti-commute for $i = 1, \ldots, 8$. Therefore, all D-terms vanish for $(i,9)$ and $(i,10)$ rotations if all the non-vanishing components in $\tilde{\psi}$ have the same chirality under γ^9, and we take $\gamma^9\tilde{\psi} = +\tilde{\psi}$. For convenience, we fix our basis such that $\gamma^9 = \sigma^3 \otimes \sigma^3 \otimes \sigma^3 \otimes \sigma^3$. Recall SO(8) real spinor is equivalent to a vector representation up to an outer automorphism (triality), and we can take $\tilde{\psi} = (\uparrow \otimes \uparrow \otimes \uparrow \otimes \uparrow)\chi$ using an SO(8) rotation without a loss of generality. The unbroken symmetry is SO(7).

Now the only D-term which we have to discuss is that of the (9,10) rotation. It is convenient to define $H^\pm = (H^{10} \mp iH^9)/\sqrt{2}$ so that $H^\pm$ have eigenvalues ± 1 under (9,10) rotation. χ has an eigenvalue $-1/2$. Therefore the D-flatness requires

$$|H^+|^2 - |H^-|^2 - \frac{1}{2}|\chi|^2 = 0, \tag{17}$$

as in Eq. (5). The gauge invariant fields are

$$\begin{aligned}
\psi\psi H &\equiv {}^t\psi C\Gamma^\mu \psi H^\mu \\
&= {}^t\tilde{\psi}\tilde{\psi}H^{10} + {}^t\tilde{\psi}(-i\gamma^i)\tilde{\psi}H^i \\
&= \sqrt{2}\chi^2 H^+, \qquad (18)\\
H^2 &\equiv H^\mu H^\mu \\
&= 2H^+H^- \qquad (19)
\end{aligned}$$

for $\mu = 1, \ldots, 10$ and $i = 1, \ldots, 9$.

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