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Publication Date
1995-03-22

Asymmetric Square-wave Modulated Dicke–Switched Receivers

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Submitted to *IEEE Microwave Theory & ...*

22–MAR–1995

ABSTRACT — We find that a Dicke-switched radiometer is optimized for minimum output noise, if the square–wave modulation has a slightly asymmetric cycle. For a balanced receiver the optimum switching time ratio is ~ 1.5, and is larger for unbalanced receivers. The improvement in sensitivity over a symmetric square wave modulation is 10% for a balanced receiver, and increases for increasing imbalance.

1 Introduction

Differential radiometers are widely used for high-sensitivity applications, particularly in fields such as radioastronomy and remote sensing. The great advantage of differential receivers is the suppression of the effects of gain fluctuations. The primary concept is to compare target signal of interest, S, with a reference signal R, at a very rapid rate so that the receiver gain changes very little in one cycle. Residual effects due to gain fluctuations are proportional to the difference in input powers. Expressed in terms of antenna temperature,

$$\delta T_G = \frac{\delta G}{G}(T_S - T_R) \tag{1}$$

The simplest and best known differential radiometer concept is the Dicke–switched radiometer (Dicke 1946). Most applications of the Dicke receiver use square wave modulation, particularly when the two signals are comparable in power. Often, however, the two target antenna temperatures are not equal, thus reducing the suppression of gain fluctuations. When $T_S \neq T_R$ balance can be obtained either through modulating the gain or by adjusting the integration times on each target so the difference of the two semi-cycles of the switching is nulled (see e.g. Kraus 1966), or with a combination of the two methods.

In general, a Dicke receiver with system noise temperature T_N viewing two sources with antenna temperatures T_1 and T_2 will be balanced when

$$(T_N + T_1)f_1 = (T_N + T_2)f_2 g \tag{2}$$

where f_1 and f_2 are the fractions of integration time τ spent each cycle on T_1 and T_2, respectively, and g is the gain modulation factor – the ratio of the gains in each part of the switch cycle. The ratio of integration times f_1/f_2 is controlled by the duration of each cycle of the square wave modulation, and they sum to unity

$$f_1 + f_2 = 1 \tag{3}$$

For a traditional, symmetric Dicke radiometer $f_1 = f_2 = 0.5$.

2 Optimum Asymmetric Modulation

We now find the integration time ratio f_1/f_2 that minimizes the root mean square statistical noise, $\delta T_{rms} \equiv \langle (T - \langle T \rangle)^2 \rangle^{1/2}$. An "ideal" system – meaning most well-designed systems and neglecting gain fluctuations – with noise equivalent temperature, T_N, continuously looking at a target with antenna temperature T_t, has rms noise

$$\delta T_{rms-ideal} = \frac{T_N + T_t}{\sqrt{B\tau}} \tag{4}$$

where B is the system effective bandwidth and τ is the observation (integration) time.

First we consider a perfectly balanced system, so that equation (2) holds. The RMS statistical noise δT_{rms} of the output difference signal is given by the quadrature sum of the noise in each semi-cycle, i.e.:

$$\delta T^2_{rms} = \frac{(T_N + T_1)^2}{B f_1 \tau} + \frac{(T_N + T_2)^2 g^2}{B f_2 \tau}. \tag{5}$$

Substituting $(T_N + T_2)g$ from equation (2) and taking a square root, we find:

$$\delta T_{rms} = \frac{(T_N + T_1)}{\sqrt{B\tau}} F(f_i) = F(f_i)\ \delta T_{rms-ideal} \tag{6}$$

where $F(f_i)$ is

$$F(f_1) = \left[\frac{(1-f_1)^3 + f_1^3}{f_1(1-f_1)^3}\right]^{1/2}, \tag{7}$$

or equivalently, in terms of f_2:

$$F(f_2) = \left[\frac{(1-f_2)^3 + f_2^3}{f_2^3(1-f_2)}\right]^{1/2}. \tag{8}$$

The function $F(f)$ thus represents the increase in noise of the switched system over an "ideal" receiver spending full time looking at the target. The minima of the functions $F(f_1)$ and $F(f_2)$ represent the values of f_1 and f_2 which optimize the signal to noise ratio for given noise and target temperatures.

Figure 1 is a plot of the functions $F(f_1)$ and $F(f_2)$. It is found that their minima correspond to the values $f_1 = 0.39$ and $f_2 = 0.61$, rather than $f_1 = f_2 = 0.5$. The improvement achieved by using such integration time ratio is of order 10% over a symmetric modulation scheme.

It should be noted that this improvement does not depend on the system or target temperature. It does not matter which side of the input load has the higher integration time as long as equation (2) is satisfied. Thus, one can always take advantage of such asymmetric modulation in conjunction with the best configuration for balancing the system, i.e., by setting the higher integration time on the colder target load. The same rms noise provided by the symmetric case is reached for $f_1 \simeq 0.25$ and $f_2 \simeq 0.75$. Thus a highly asymmetric cycle (about 1/4 to 3/4) provides the same efficiency in terms of signal to noise of a standard symmetric square-wave Dicke radiometer. Therefore systems with target temperature ratios

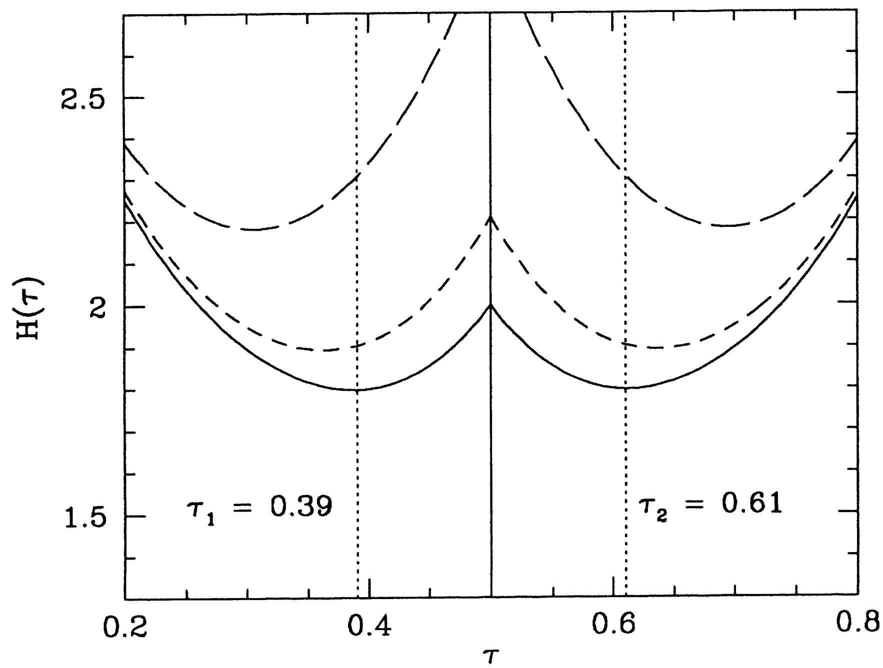

Figure 1: Plot of increase in noise over an ideal radiometer for an asymmetrical square-wave switching versus the fraction of the cycle time in one state. Note symmetrical switching has $f_1 = f_2 = 0.5$, for which, as expected, $F(f_i) = 2$. Curves are shown for three levels of balance: 1) solid line —- exact balance 2) short dash - - imbalance of 0.1 of the total receiver output power 3) long dash – – imbalance of 0.4 of the total receiver output power.

of a factor $T_1/T_2 \sim 4$ can be balanced with a noise comparable to a situation with equal temperature loads. If the noise figure T_N and reference temperature T_2 are accurately known, then one can deterime T_1 to a level set by δT_{rms} by measuring the gain g ncessary to null the output.

3 The Measurement of Small Signal Variation Case

Often the situation is such that one is actually trying to measure the small variations T_s of T_1 about its mean value $\overline{T_1}$. Examples are the remote sensing of the sky and surface temperatures. In this situation the target T_2 is a stable reference load used to switch against to remove gain variations. If the radiometer is balanced on average, then the radiometer output is the signal T_s plus receiver noise which is gaussian with standard deviation δT_{rms} derived in equation (6). The balancing gain ratio is

$$g = \frac{T_N + \overline{T_1}}{T_N + T_2} \times \frac{f_1}{f_2} = \frac{T_N + \overline{T_1}}{T_N + T_2} \times \frac{f_1}{1 - f_1} \tag{9}$$

The presence of a small signal imposed upon one of the targets power T_1 breaks the symmetry of the situation. In order to recover the signal and its error, one must put the gain ratio g on the T_2 side of equation (2) so that the error estimate on T_s does not have an explicit g dependence. Then the derivation of the optimum integration time is correct for the balanced case $f_1 = 0.39$. If the target temperature T_1 is less than the reference temperature T_2, this leads to a lower gain on the T_2 side both for the optimized and symmetric case.

4 The Unbalanced Case

So far we have considered a perfectly balanced system. In practice, even in the best situations, some residual unbalance (or offset ΔT) will be present, i.e.:

$$\Delta T = (T_N + T_1)f_1 - (T_N + T_2)f_2 g. \tag{10}$$

The mean square noise will be the quadrature sum of the noise in each switch cycle plus a term which is the square of the rms gain fluctuations $(\delta G/G)_{rms}$ times the offset ΔT. Neglecting the gain fluctuations but accounting for the offset term, equation (6) becomes:

$$\delta T_{rms} = \frac{(T_N + T_1)}{\sqrt{B\tau}} H(f_i) = \delta T_{rms-ideal} H(f_i) \tag{11}$$

where

$$H(f_1) = \left[F(f_1)^2 - 2\frac{\Delta T}{T_N + T_1}\frac{f_1}{f_2^3} + \frac{1}{f_2^3}\frac{\Delta T^2}{(T_N + T_1)^2} \right]^{1/2} \quad \text{where} \quad f_2 = (1 - f_1) \tag{12}$$

with f_1 and f_2 related by equation (3). The function $H(f_i)$ is the increase in rms noise fluctuations over an ideal radiometer including the effect of the offset and depends on the ratio $r \equiv \Delta T/(T_N + T_1)$. Note that one would get a similar result in terms of $r' \equiv -\Delta T/(T_N + T_2)$ by reversing the substitution after equation (4).

$$H(f_2) = \left[F(f_2)^2 + 2\frac{\Delta T}{T_N + T_2}\frac{f_2}{f_1^3} + \frac{1}{f_1^3}\frac{\Delta T^2}{(T_N + T_2)^2} \right]^{1/2} \quad \text{where} \quad f_1 = (1 - f_2) \qquad (13)$$

In fact, for given system noise temperature and integration time ratio, the noise depends on both target temperatures T_1 and T_2 through the offset term ΔT.

Table 1 gives the optimum integration time ratios and the decrease in noise factor, $\delta T_{rms-opt}/\delta T_{rms-1/2}$, corresponding to values of r in the range 0 to 1. We also show in figure 1 the shape of $H(f_i)$ for two values of r. For an unbalanced Dicke receiver the optimization of the modulation can have a relatively large effect. As expected (Figure 1) the overall efficiency decreases, since $H(f_i)$, and thus δT_{rms}, gets higher values. The optimum integration time ratio for increasing values of r tends to depart more and more from the conventional symmetric condition. For example, for $r = 0.1$ the optimized modulation requires $f_1 = 0.36$ and $f_2 = 0.64$; for $r = 0.4$, we find $f_1 = 0.30$ and $f_2 = 0.70$. The improvement in the signal sensitivity over a symmetric modulation increases roughly proportionally to r. However, switching, which was introduced to reduce the effect of gain fluctuations, is not likely to be appropriate, when the imbalance (offset) is very large.

5 Conclusion

We have shown that the ideal modulation in a balanced Dicke–type radiometer is obtained with a somewhat asymmetric cycle in the square wave pattern. If the system is significantly offset, even more asymmetric modulations is required to optimize the signal to noise ratio. The improvement in sensitivity by using optimum asymmetric modulation is a 10% factor for balanced systems, and about 30% for an offset $\Delta T/(T_N + T_2) \sim 0.5$. Asymmetric wave optimization can be used conveniently to balance a Dicke radiometer with $T_1 \neq T_2$. For example, in many radio or microwave astronomical observations one wishes to measure the sky temperature ($T_S \approx 5$ K), by difference with a stable reference load termination at moderately cooled temperatures. If $T_2 \approx 70$ K and $T_N \approx 120$ K, the system is well balanced for $f_S/f_2 \simeq 1.5$ without gain modulation (i.e. $g = 1$). This ratio (see Table 1) also optimizes the signal to noise ratio as due to the square wave cycle.

Acknowledgements We would like to thank Giovanni De Amici for reviewing this paper and John Gibson for working on a realization of this concept. This work supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

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Table 1

Optimized asymmetric modulation

r	f_2/f_1	$\delta T_{rms-opt}/\delta T_{rms-1/2}$
0	1.56	0.899
0.1	1.78	0.857
0.2	1.94	0.817
0.3	2.13	0.781
0.4	2.23	0.749
0.5	2.45	0.721
0.6	2.57	0.695
0.7	2.85	0.673
0.8	3.00	0.653
0.9	3.17	0.635
1.0	3.35	0.619