Title

Elementary Students’ Dispositions, Goals and Situated Actions in Mathematics Problem Solving

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Publication Date
2016

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA

Los Angeles

Elementary Students' Dispositions, Goals and Situated Actions in Mathematics Problem Solving

A dissertation submitted in partial satisfaction of the

requirements for the degree Doctor of Philosophy

in Education

by

Melissa Jane Denby Kumar

2016

ABSTRACT OF THE DISSERTATION

Elementary Students' Dispositions, Goals and Situated Actions

in Mathematics Problem Solving

by

Melissa Jane Denby Kumar

Doctor of Philosophy in Education

University of California, Los Angeles, 2016

Professor Noel D. Enyedy, Chair

Building on the construct of mathematical dispositions – ideas and values about mathematics and patterns of engagement over time – this study investigated which practice-linked goals students appropriated, their explanations of these goals, and the expression of these goals in situated actions in problem solving as a way to develop a more nuanced understanding of students' positive dispositions to mathematics. The study was conducted in a 3rd grade classroom in a school that actively supports the development of positive dispositions to mathematics over the 2013-2014 school year. Data sources included student surveys, student interviews and classroom observation. Analysis was conducted at the classroom and individual level to determine to what extent students held practice-linked goals aligned with the values of the reform-oriented mathematics education community. In addition, close analysis of case study students allowed analysis of how different goal profiles related to situated actions in problem

solving. The results of the study contribute to our field's understanding of how primary students orient themselves to mathematics and how the development of positive dispositions to mathematics can be supported in the classroom.

The dissertation of Melissa Kumar is approved.

Megan Loef Franke

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2016

Dedication Page

This dissertation is dedicated to Anush, Janaki and Meenakshi

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ACKNOWLEDGEMENTS

I begin by acknowledging my husband, Anush Kumar, for his unfailing support, love, and belief in me. I am profoundly grateful on my own behalf as well as that of our young daughters. I would like to thank our daughter Janaki for her patience through this process, and our little Meenakshi for her companionship over the last year. You two are the greatest joy of our lives.

I would like to thank my parents for a lifetime of support, love and joy. My mother Wendy Simons, for pouring her heart into mothering and for nurturing creativity, thought and learning from the beginning. My father Gordon Denby for inspiring hard work, determination, curiosity, and a love of play. My second mother Camille Denby for helping me navigate mothering and professional life, and for her generosity and affection. My father-in-law Ram Ramasubramanian for his support, and my mother-in-law Radha for her greatly appreciated care and companionship over the last several years.

I am profoundly grateful to my advisor Noel Enyedy, who offered me his dedicated guidance, support and mentoring throughout graduate school. He is one of the smartest people and one of the kindest people I know. I have been forever shaped by working with him and will, throughout my life, continue to draw from what I learned under his mentorship.

I am grateful to my dissertation committee, which include Noel Enyedy, Megan Franke, Jim Stigler and Noreen Webb. I am grateful to Megan for her enthusiasm for these ideas early on in my graduate career, and for providing some of the key pieces in shaping the proposal. I appreciate Jim and Noreen’s thoughtful perspectives on shaping the study both in the proposal and reporting phase.

I would like to thank the teacher in the study, who I call Ms.A ,for her generosity and enthusiasm in welcoming me into her classroom over the 2013-2014 school year. I would also like to thank all of the students, in particular the case study students for sharing their invaluable thinking and perspective during the study.

During graduate school I was privileged to work with many professors who generously shared their time and knowledge. I am grateful to Aimee Dorr and Bill Sandoval for their wisdom and feedback during many hours of the research apprenticeship courses. I also would like to thank Alison Bailey, Carollee Howes, Fred Erickson, Greg Tanaka, and Mike Rose for their guidance both in and outside of class.

I was fortunate to enjoy the mentoring and support of several "older sisters" during graduate school including Debbie Fields, Melissa Cook, Tesha Sengupta-Irving, Nina Neulight and Yvonne de la Pena. I would like to thank the many wonderful members of the LTRG and CONNECT research groups, especially fellow students David DeLiema, Jarod Kawasaki, Liz Redman, and Sihan Xiao for our many years of conversation and community. I would like to thank Joshua Danish for his role in my joining UCLA and for his mentoring during my first first few months. And I would like to thank Shilpa Baweja, for being a friend from the first moment we met and for sharing challenges, triumphs and life events during our time at UCLA.

I offer deep gratitude to my daughter Janu's amazing teachers. Zewdi, Mari, Asia, Sharmila, Hildy and Dianne, Jen and Helen, and Becca, Brian and Mary. Thank you for giving her your love, companionship and care.

Finally, I would like to thank Janice Osaka, my math teacher in high school, who helped me love math and solving challenging problems. Your teaching was the beginning of and inspiration for this journey.

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CHAPTER 1

INTRODUCTION

A strong mathematics practice is required in today's society for full participation in our economy and democracy. However, both international and national assessments have found that many students are not reaching the levels of problem solving that are central to advanced mathematical work (U.S. Department of Education, 2011; Gonzales, 2008). Advanced problem solving requires a confident, flexible use of mathematics (Boaler, 2008). Students must be able to innovate, make decisions and engage in mathematical discussions with others to be considered proficient by current standards for math education (CCSS, 2010; Kilpatrick, Swafford, & Findell, 2001; NCTM, 2010), in contrast to former conceptions that passing tests and mastering a set of skills was enough to advance. Helping students develop their problem solving skills has proven difficult within our current education system, despite a variety of reform efforts. Of particular concern is that many students are developing a passive, disengaged relationship with mathematics (Boaler & Greeno, 2000). In this study I argue that understanding a student's relationship with mathematics by asking them about their goals and watching their problem solving is critical to be able to fully support them in becoming advanced mathematical problem solvers.

There is growing consensus in the reform-oriented mathematics education community that examining students' learning of skills and knowledge alone cannot predict or explain their subsequent success in mathematics (Franke, Kazemi, & Battey, 2007). Rather, researchers and educators are including social, affective and motivational factors as central to what students learn and how they develop as mathematical thinkers (Gresalfi, 2009). Examining a student's relationship with mathematics is consistent with this view. I am interested not only in the ideas that students have, but the way these ideas play out in actual moments of problem solving. The construct of a disposition is useful here in that it encapsulates both a student's relationship with mathematics, and the connection between this relationship and their patterns of activity in math class. Formally, dispositions are defined in this study as "ways of being in the world that involve ideas about, perspectives on, and engagement with information which can be seen both in moments of interaction and more enduring patterns over time" (Gresalfi, 2009; Gresalfi & Cobb, 2006). As defined above, dispositions focus both on students' broader, longer-term ideas and perspectives about mathematics, as well as moments in which these ideas and perspectives are enacted in practice. These ideas and perspectives can also be thought of as a narrative that the student has about what it means to practice mathematics, and about themselves as mathematical thinkers. A small body of recent work has focused on students' dispositions in mathematics from different perspectives, many including a narrative perspective on students' relationships to mathematics (Boaler, 2008; Gresalfi, 2009; Langer-Osuna, 2012). My study contributes to this emerging body of work by examining students' dispositions specifically to problem solving in the classroom context. I coordinate several pieces of evidence to understand both *how* students think about their mathematics practice, and how their ideas get *translated* into their actions in problem solving. In this way I explore both the narrative that students have about mathematical problem solving and themselves as mathematicians, and how this narrative survives, changes or vanishes in the moment of actual math activity. In doing so I aim to advance our understanding of students' dispositions, to then better support students in fully developing as mathematicians.

In this study, students' relationships to mathematics are examined through their practice-linked and performance-linked mathematical goals. Goals are a useful way to examine both what drives students in problem solving and what they aim to accomplish. A goal is defined here as a motive that shapes human behavior in a specific activity. Work in Educational Psychology has focused on students' task vs. performance-oriented goals (Middleton & Midgley, 1997), demonstrating how adoption of task-focused (also known as mastery) goals is associated with more adaptive academic behavior (Ames & Archer, 1988). In this line of research, however, goals are construed as static and are not studied as connected to students' actions in the classroom. From the sociocultural perspective, beginning with early Soviet psychology, goals have been used as a way to understand the link between cultural activity and individual cognition (Wertsch, 1981). More recent work has examined goals as a mediating link between society, culture and thinking (Saxe, 1999, as cited in Nasir, 2002) and to understand the concurrent development of students' identities, mathematical goals, and learning in informal mathematics practices (Nasir, 2002).

In this study goals are viewed as part of a students' narrative about their mathematical practice. This includes both what the student thinks is important to practicing mathematics in their community, and what is important to them personally. This narrative is developed over a student's mathematical career. Specific goals emerge at particular points either because of experiences in the classroom or a student's own development, and then become relatively stable over time. These goals are then expressed in the classroom context. The expression of a student's goals is inhibited or supported by factors including a student's interpretation of goals and their social positioning, and these must be considered as well in understanding a student's narrative. I contend that studying students' goals can shed light on aspects of their dispositions because the more "enduring" aspects of their dispositions—the "ideas about" and "perspectives on" mathematics—must be transformed into situated activity in the moment. Goals are a likely construct to mediate this transformation from ideas and perspectives to action, and thus may illuminate what students think is important about practicing mathematics, why they are motivated to practice mathematics, and how aspects of the context and dispositions combine to direct student effort in problem solving. Looking at students' goals is therefore a useful lens in which to examine students' dispositions as they are manifested in mathematics classrooms.

There are particular goals that are considered "positive" by the reform-oriented mathematics education community, including wanting to understand the mathematics at a conceptual level (CCSS, 2010; Boaler, 2008), wanting to solve challenging problems (CCSS, 2010; Lampert, 1990), exercising agency and creativity in solving mathematics problems (CCSS, 2010; Boaler, 2002), and being viewed as a valuable contributor in classroom discussions (CCSS, 2010; Boaler, 2002; Boaler & Greeno, 2000). Reform-oriented educators and practitioners have identified these goals as supportive of developing strong mathematical understanding, and they have also been expressed as important by professional mathematicians (Boaler, 2008).

Studies of students' goals have tended to focus on middle and high school students (Middleton & Midgley, 1997; Nasir, 2002; Pintrich, 2000; Summers, 2006) and have found that students have considerable challenges in maintaining positive attitudes to mathematics during this time (Eccles & Midgley, 1989). Young students generally have a more positive attitude to mathematics, but some are already forming negative relationships with mathematics (Ramirez, Gundeson, Levine, & Beilock, 2013), and by fourth grade students' perceptions of self-efficacy and the value of mathematics have started to decline (Eccles, Wigfield, Harold, & Blumenfeld, 1993). Because the middle of elementary school appears to be an important turning point in a student's relationship with mathematics, this study focuses on the goals of third grade students to improve our understanding of the origins of this change. Identifying and cultivating positive goals earlier also affords greater opportunity to continue students' growth in their academic careers.

The main focus of this study is to describe students' goals and their related situated activity in problem solving, an important first step before designing changes to instruction. The study was conducted in an elementary classroom over the 2013-2014 school year in a school that actively supports the development of positive dispositions. I coordinate several data sources collected over the school year to examine students' dispositions. I use a student survey to assess students' broader appropriation of the positive goals listed above. I use interviews with case study students in which I ask them to explain their goal ratings, to understand how they interpret and think about these goals. In addition, I use questions about the importance of goals at specific moments of problem solving to explore how these goals are tied to a particular context. Finally, I use focused observation of small-group work for case study students to examine how goals were translated into action in problem solving, and how this translation was mediated by students' positioning and interpretation of the goals.

The study was guided by the following research questions.

Research Questions

RQ1a: To what degree do third-grade students hold practice-linked goals aligned with the values of the reform-oriented mathematics education community?

RQ1b: What kinds of explanations do students give for their ratings for each goal?

RQ2: How do different profiles of goals relate to students' situated actions in problem solving?

Significance

This study contributes to the field of Learning Sciences and Math Education by examining students' appropriation of practice-linked goals important to the reform-oriented mathematics education community, and their explanations for the importance of these goals. It also contributes a perspective on students' positive dispositions by connecting students' goal appropriation to their situated activity in problem solving. This is significant to understand what motivates student engagement in a mathematics class in order to design instruction that promotes understanding and supports positive dispositions to mathematics. Analysis of the alignment of students' goals with those of mathematicians, reform educators and researchers has implications for future curriculum and policy development. Finally, looking at dispositions and goals broadens the perspective on what students should gain during their education. Clarifying what positive goals for mathematics look like, how they translate into problem solving actions, and how they relate to other aspects of the classroom ecosystem will be valuable both to educators, researchers and parents, as well as students in their later years.

CHAPTER 2

LITERATURE REVIEW

In this section I discuss literature on dispositions and goals, present specific practice-linked goals derived from the reform-oriented mathematics education community, and discuss an example of a curriculum that fosters positive dispositions to mathematics. First, the construct of dispositions allows me to focus both on students' ideas and perspectives about mathematics, and their situated actions in problem solving. Second, the construct of goals allows me to see how those dispositions are translated into something that is more fine grained and useful for making choices during problem solving activity. Third, I describe positive goals for mathematics that emerged from mathematics education literature, the Common Core State Standards for Mathematics Practice, and from research on the practices of mathematicians. Fourth, I conclude my review of the literature by briefly reviewing Cognitively Guided Instruction (CGI), an existing curricula that sets out to foster both positive dispositions and positive problem solving in elementary aged children.

Dispositions

The construct of dispositions allows us to focus both on the moment-to-moment "interactions and engagement with the social environment" as well as the longer-term patterns which suggest more permanence and are suggestive of the future. Below I describe current conceptualizations of dispositions and recent research on students' dispositions in mathematics education.

Theoretical perspectives on dispositions.

The field of Psychology's perspectives on dispositions. The psychological perspective on disposition focuses on "thinking" and intelligence. Perkins and Tishman (1998) define *thinking* dispositions as "how people behave more or less intelligently governed not only by abilities but by predilections and tendencies" (p. 4). In this perspective, dispositions are considered stable, sometimes referred to as the "trait" view (Sengupta-Irving & Enyedy, 2015). Dispositions are considered to help explain the gap between what students are *able* to do and what they *actually* do. Perkins, Tishman, Ritchhart, Donis, and Andrade (2000) propose an alternate theory in contrast to the dominance of ability-centric measurement of intelligence in the field. In their theory of intelligence, disposition is composed of ability, sensitivity (detection of occasion) and inclination (motive or leaning). In a series of experiments, the authors found what they call the "disposition effect" (p. 16) in which people's ability is substantially greater than their observed performance.

In their review of work from both psychology and education, Perkins et al. (2000) found relative convergence toward the dispositions of "open-mindedness, reasonableness, curiosity, and metacognitive reflection" (p. 272) as characterizing intelligence. Like others in the field (Facione & Facione, 1992), they measured dispositions through clinical tasks or interviews.

Perkins and Tishman (1998) make the important point that the dispositions they examine are normative, in that they steer students toward conforming with culturally prescribed ways of thinking. These dispositions are valued because they are believed to be fruitful in achieving valued ends, such as increasing abilities or higher performance. In the current study, I also identified normative dispositions by examining students' appropriation of goals valued by the mathematics education community. My assumption was also that these goals were instrumental in helping students develop strong mathematical practices. However, I also asked students about explanations of the importance of their goals which allowed me to examine dispositions from the student's perspective. Consistent with qualitative research traditions (Creswell, 2009), students' perspectives are of great importance to shed light on what matters to them in math class**.** One of the intended contributions of the study is to provide further insight into what *students* think is important in math class, to make our support of dispositions more developmentally and student-centered appropriate.

The field of Education's perspectives on dispositions. The educational perspective can be summarized by Katz (1993) who defines dispositions as "habits of mind, tendencies to respond to situations in a certain way" (p 30). This definition, unlike the definition of "thinking dispositions" above, highlights the importance of the context to cue or elicit a 'response' of an individual to a specific situation. In this conception, dispositions develop in specific contexts "tied very closely to particular kinds of tasks, contexts and material" and then over time "come to appear in an ever-increasing number of domains and situations" (Carr & Claxton, 2002). Carr & Claxton (2002) stress that students must have opportunities to "deploy" dispositions in order for them to develop, similar to the idea that the mathematical thinkers that students become is closely tied to the practices in which they engage.

After reviewing educational literature investigating learning dispositions, Carr and Claxton (2002) identify and argue for three positive learning dispositions. These are 1) *Resilience*, the ability to stick with a challenging task despite an uncertain outcome, frustration and setbacks. 2) *Playfulness:* being ready, willing and able to perceive or construct variations on learning situations and thus to be more creative in interpreting and reacting to problems (p. 10) (further distinguished into mindfulness (the ability to notice detail and read the situation in different ways), imagination, and inventiveness). 3) *Reciprocity*: courage, awareness and inclination to communicate and collaborate with others. The authors argue that these dispositions are "positive" in giving students the "capacity and the confidence to engage in lifelong learning" (p. 9).

It should be noted that identifying "positive" dispositions is a source of some contention. Katz (1993) critiqued the idea that specific dispositions were always positive, citing examples of how each disposition proposed by Carr and Claxton (2002) could also lead to negative behavior and consequences for a student. However, the fact that dispositions "could" be negative should not be a deterrent for exploring which ones may lead to positive outcomes for students. Dispositions, alone, do not determine behavior just as ability cannot determine behavior. Rather, the context in which the person is thinking and learning are critical to understanding their dispositions, and for this reason students' dispositions and goals are studied in the classroom context in this study.

While the field of Education described above generally thinks of dispositions as a stable trait, other scholars sees dispositions as a narrative that a person has about him or herself, or a group of people have about themselves, that is then translated into practice. Identity is a useful construct for exploring this connection between "narrative" and "practice". Identity is defined as how a person thinks about themself, usually in relation to a group. Identities can be *narrative*, defined as "a set of reifying, significant and endorsable stories about a person" (Sfard & Prusak, 2005, p. 14) or *positional*, relating to a person's position in relation to a normative identity made available in a group (Cobb et al., 2009). Identities then influence how people act – people try to act in ways consonant with the type of person they imagine themselves to be. In this study, I draw from research and theory on identity and engagement to examine how a student's relationship with (and value of) mathematics is connected to their situate actions during problem solving, similar to the idea in identity that the "type of person" you believe yourself to be influences how you behave. In addition, I draw on the idea of "family resemblances" in identities (Sfard & Prusak, 2005) to look for similarities in students' dispositions, specifically the goals they embrace and their situated actions in problem solving.

The field of the Learning Science's perspective on dispositions. Recent work in the Learning Sciences has focused on students' dispositions as a way to more fully understand learning. The definition used in the introduction, "ways of being in the world that involve ideas about, perspectives on, and engagement with information which can be seen both in moments of interaction and more enduring patterns over time" (Gresalfi, 2009; Gresalfi & Cobb, 2006), comes from current research in the Learning Sciences.

In addition, several recent studies have focused on methodological issues of how to document students' dispositions in a classroom environment. Two studies of particular relevance are described further below.

Gresalfi (2009) examined how aspects of the classroom environment could make certain dispositions more likely to emerge, while also demonstrating that dispositions are neither determined completely by the classroom or individual. She used case studies of students who contrasted in academic achievement and patterns of engagement to show how dispositions might develop over time for different students in different types of classrooms (project-based vs. traditional). She used fine-grained analysis of video episodes of small group work at three points during the year, to show case study students' patterns of engagement with both peers and the mathematics for each "utterance moment". By also coding utterances for the "forcefulness" of the opportunity to participate, and whether a student took up the opportunity, renegotiated or did not take up, her analysis allowed her to make connections between students' engagement with math and with others, and their patterns of participation. She used the interaction of opportunities to engage and whether students took them up as an important indicator of students' dispositions to both engage with the mathematics and with others. Gresalfi (ibid.) showed that while classrooms do not *determine* students' dispositions, they do afford opportunities for certain types of dispositions to emerge.

Langer-Osuna (2013) examined dispositions in a different way by describing the development of a student who exhibited a great deal of change in a project-based classroom which used multiple 'key principles for student engagement'. Although her work focused on identity and engagement, it focused on many of the same concepts as dispositions. Like Gresalfi, she selected video episodes of group work at the beginning, middle and end of the school year, which she analyzed at the utterance level. Instead of descriptions of patterns at each time point, she used chi-square analysis to show shifts in function of talk both in terms of identity and engagement over the year, and micro-analytic discourse analysis to show the way that aspects of the classroom were resources for students in their engagement and construction of identity. She argued that key aspects of the students' ability to construct a positive identity with mathematics were, repeated opportunities to engage, and autonomy, including the "authority to address problems and have a sense of ownership in collaborative work" (p. 4). These opportunities to engage were resources (discussed more fully in the paragraph on Saxe below) that students could make use of in different ways (Saxe, 1999). She argued that in working in mathematics, students must coordinate both social and academic functions to "construct trajectories of identity and engagement" (Langer-Osuna, 2013, p. 9).

Both of these articles provide concrete examples of how students' dispositions can be documented in mathematics classrooms in terms of their situated actions while problem solving and their relationship with the field of mathematics – focusing either on disposition (Gresalfi,

2009) to persist and shape the environment, or mathematical identity (Langer-Osuna, 2013). I draw on the analytic methods of these articles in my proposed analysis of students' work, in order to answer the second research question, *RQ2: How do different profiles of goals relate to students' situated actions in problem solving?*

Goals

Motivation is often considered to be a distinct "emotional" or "affective" factor of learning in educational research, although more recent research argues for its central and critical role (Gresalfi, 2009). In this study, goals are a way to determine students' motivation to engage in mathematics within the context of their classroom. The construct of goals cuts across many fields and research paradigms. Here, I draw relevant ideas from several research traditions. The section concludes with a discussion of the connection between goals and dispositions as conceptualized in this study. I contend that studying students' goals can shed light on aspects of their dispositions because the more "enduring" aspects of dispositions—the "ideas about" and "perspectives on" mathematics—must be transformed into situated activity in the moment. Goals are a likely construct to mediate this transformation from ideas and perspectives to action, and thus may illuminate what students think is important about practicing mathematics, why they are motivated to practice mathematics, and how aspects of the context and dispositions combine to direct student effort in problem solving. Looking at students' goals is therefore a useful lens in which to examine students' dispositions as they are manifested in mathematics classrooms.

Theoretical perspectives on goals

The Educational Psychology perspective toward goals. Studies of goals in educational psychology have focused on the relation between students' goals and academic achievement. In many studies, researchers distinguish between a "task" goal orientation, in which the goal is to develop and improve ability, and a “performance” goal orientation, in which the goal is to demonstrate and prove ability (Dweck & Leggett, 1988). The general finding was that task-oriented goals were associated with higher positive perceptions of academic efficacy, adaptive behaviors such as effective learning strategies, and help-seeking strategies in studies across a range of ages (Middleton & Midgeley, 1997). However, because of inconsistent findings in the association of performance–oriented goals with academic outcomes and adaptive behavior, the authors added performance-avoidant goals, characterized by motivation to avoid failure and embarrassment, which they found to be positively associated with negative academic efficacy, positively avoiding help seek and greater test anxiety. (Middleton & Midgley, 1997). In more recent research, Pintrich (2000) looked at the presence of both types of goals in students (students were previously categorized as orienting to one or the other) and found that performance goals could also be adaptive when coupled with task oriented goals.

While this research points to important ways that broad classes of goals influence students’ overall performance, results are at a general level and do not describe how the expression of goals is tied to real-time activity of the classroom. In addition, academic efficacy is assessed through grades and teacher report rather than direct observations or evaluations of students’ problem solving. One of the findings of national and international studies (Gonzales et al., 2008; U.S. Department of Education, 2011) is that while students are acquiring skills and knowledge, they are missing the ability to solve more challenging problems. Finally, studies tend to focus on the middle school years, possibly because research has shown that students experience a decline in motivation for school during this transition (Eccles & Midgley, 1989). My study contributes to the field by relating students’ self-reported goals to their direct engagement in problem solving, as a more complete lens on their disposition towards mathematics, for students earlier in their academic careers.

The Sociocultural perspectives toward goals. The Soviet psychologists, the first to focus on goals from the sociocultural perspective, viewed goals as fundamentally linked to social activity and learning (Wertsch, 1998). In activity theory, developed by Vygotsky (1978) and later drawn into a coherent theory by his student Leont'ev (1981), human activity is analyzed at three different levels: at the level of *activities*, which are distinguished by motives and object of orientation, *actions*, distinguished by goals, and *operations*, distinguished on the basis of conditions under which they are carried out. All activity is driven by motives, a desire to realize a physical or ideal object, and the way these motives are pursued can be examined at different, interconnected levels. The activity is motivated by "overall" goals, which are carried out by an aggregate of "actions subordinated to partial goals" (p. 61). These partial goals are in turn realized by individuals' operations on their environment under specific conditions (Leont'ev, 1978). An important point is that actions and their goals must be viewed within the context of a specific activity. For example, in a classroom, a student's motive may be to increase their knowledge and abilities. The student may pursue this motive through different types of actions, such as seeking assistance from the teacher or peers, solving extra problems, or exerting sustained effort in their class work. The manner in which the student pursues these goals will depend on the conditions of the classroom environment.

A classroom is a complex social system, and within this system students are engaging in multiple activities on an ongoing basis. In this study I limit this complexity by focusing my analysis of student activity to collaborative problem solving, which is a central practice of mathematics and which affords many opportunities to see students pursue practice-linked goals for mathematics in specific conditions.

Empirical studies of goals and engagement in mathematics. Several studies have connected students' goals and their engagement in mathematics. In an ethnographic study of a bilingual Spanish-English 3rd grade classroom, Lave (1997) focused on the goals of children in an upper ability mathematics group. She found that students were deeply engaged in math work individually, but learned to adapt their own work to appear to take up the strategy of the teacher. Lave used this as evidence of how the practice-linked goals of the classroom shaped individual goals, namely to score well, please the teacher and appear competent.

In a similar vein, Nasir (2002) looked at the concurrent formation of students' goals and identities in informal mathematics environments of playing basketball and dominoes. She demonstrated that goals emerge from participation in specific contexts. She found that when given problems similar to those they encountered in basketball practice, middle school students were more interested in the goal of producing a mathematical answer while high school students wanted to find a solution that made sense in the context. Nasir connected student engagement to goals by characterizing engagement as pursuit of particular goals. She defined engagement both as how students related to each other and how they engaged in the practice. Finally, she looked at students' alignment of identity with professional basketball players, and how their goals were structured in alignment with their identification with the practice. For example, as students imagined themselves pursuing professional careers, they engaged more seriously and more often in developing skills and experience. She described two types of goal changes: in overarching goals of play and concurrent changes in related mathematical subgoals. She found that overall, both types of goals shifted in "increasing sophistication and differentiation". As individuals learned within the practice, they constructed new problem-solving goals, which were “in line with an increasingly sophisticated knowledge base and shifts in broader practice-linked goals” (p. 240). Thus individuals both shifted in what they strove to learn in order to participate in certain ways, and in their relationship with the field.

Connecting goals, dispositions and engagement. As argued by Saxe (1999) and Nasir (2002), goals are critical to understanding people’s activity and as demonstrated by Nasir (2002), are central to understanding the “mechanisms” for why students engage as they do in mathematics. The goals that students have will shape how they engage in their math class, as demonstrated in the studies above. Returning to the example above, a student who wants to understand the math may ask questions of peers in the middle of a problem about points they do not fully understand. They may continue the conversation until they feel they have reached understanding, and then attempt to convey their new insight in their class work. A student who values the challenge of problem solving may make the problem more complex by adding details, or trying to use an innovative solution, which may lead to several false starts which they persist through because they are motivated to find a unique solution. Goals relate to how students engage, and this engagement can be seen in the problem solving work of students.

Goals that students accept and pursue in activity speak to their dispositions. Students’ goals are not a complete account of their dispositions, as other factors affect their disposition such as group membership, group narrative, etc. However, looking at students’ goals does tell a story about how students align with the goals of the classroom and the reform-oriented math community, and allows this broader aspect of their disposition to be connected to “disposition in practice”, their situated actions in the practice of mathematics.

In this study, examining students' goals and situated actions was used as a window into their dispositions to mathematics by 1) Examining which goals students appropriate and linking it to the goals of the broader reform-oriented mathematics education community, 2) Examining how longer-term aspects of disposition such as practice-linked goals relate to shorter-term aspects such as situated actions in problem solving. Coordinating across levels of analysis will allow me to connect disposition "in narrative" (i.e. the stable stories we tell about ourselves as "math people") with disposition "in practice" (i.e., the fluid, minute-by-minute choices we make during mathematical activity) (Fields & Enyedy, 2013). Dispositions are meant to capture "emergent continuities" (Gresalfi & Ingram-Groble, 2008) in students' participation, and by looking at the connection between goals and situated actions in problem solving over time, I aimed to document this aspect of dispositions in a classroom setting.

Practice-linked goals in the current study. To choose a set of normative, productive practice-linked goals to look for in student activity, I have drawn upon the Common Core State Standards for Mathematical Practice (CCSSPM), the educational research literature, and from descriptions of the goals of mathematicians. I have included the Common Core State Standards for Mathematics Practices(CCSS, 2010) in Table 1 below, as I refer to them frequently in the following section.

Table 1

Common Core State Standards for Mathematics Practices

Standard	Description
1	Make sense of problems and persevere in solving them
2	Reason abstractly and quantitatively
3	Construct viable arguments and critique the reasoning of others
4	Model with mathematics
5	Use appropriate tools strategically
6	Attend to precision
7	Look for and make use of structure

8	Look for and express regularity in repeated reasoning

Goal 1: "Learning with Understanding". Learning with understanding is a fundamental goal for mathematics education. It is directly addressed in the educational psychology literature as adopting "task-oriented" goals. In the CCSSMP listed in Table 1, each standard reflects a different aspect of learning with understanding.

Learning with understanding is also a goal that mathematicians and professionals who use mathematics speak about in discussing what is important to them in mathematics. Mathematicians use their discipline to make sense of, *to understand*, phenomena in the world around us (Boaler, 2008). Burton (1999) notes that many mathematicians enjoy collaboration because it allows them to learn from each other. Understanding is also fundamental to the discipline of mathematics.

This goal is core and central to a positive disposition to mathematics in terms of a positive disposition. So math is about learning and really understanding what you are doing and wanting to engage at a deep level with the concepts, not just knowing how to follow the steps.

Goal 2: "Solve challenging problems". This goal refers to being motivated by the satisfaction and enjoyment from solving a "problem worth solving", one that cannot be solved right away and takes some effort and thought. This goal is different from learning and understanding in that it focuses more on the affect of overcoming, or accomplishing, a mathematical challenge. However, like the first goal, different aspects of what is required to solve challenging problems can be seen across all of the CCSSPM standards. In addition, the standard specifically emphasizes *perseverance* in solving problems. This is a core goal that is required for advanced mathematical practice – embracing challenge and seeing value in working through challenging problems.

To mathematicians, solving problems is the fundamental activity of their profession. As Hersch (1997) explains, “solving problems and making up new ones is the essence of mathematical life” (p. 18). Solving problems has a rich meaning for mathematicians beyond earning a living—many mathematicians describe mathematics and the solving of problems as ‘beautiful’. Words such as “euphoria” and “excitement” are common in describing their experiences (Boaler, 2008). Thus, wanting to solve challenging problems is also a central goal held by mathematicians.

Goal 3: “Agency and creativity”. This goal concerns the desire to be able to make decisions and bring aspects of oneself into math class. Students’ feeling that math is fixed and uncreative is linked to passive engagement and negative regard for mathematics (Boaler & Greeno, 2000). On the other hand, the CCSSPM and mathematics education research emphasize the agency and creativity, central to mathematics practice. A fundamental aspect of solving “problems worth solving” (goal #2) is that problems are truly challenging and require the thinker to invent or adapt strategies in a new way. Therefore the extent to which the students self-report or show evidence of using creativity in problem solving is of particular focus in the current study.

Agency is referred to directly in the CCSSPM. Procedural agency, being able to use the tools of the discipline, is referenced in Standard 5. Conceptual agency is indirectly referenced in Standard 1, about making sense of problems, Standard 3 about constructing viable arguments, and Standard 5 about using tools. Each of these standards suggests that students make decisions about how to structure their work, and how to use the tools at their disposal, for problem solving and mathematical communication with others. Creativity is similarly referenced indirectly in that students must decide how to make use of tools, and create their own arguments as well as help to shape the arguments of others.

Agency and creativity are central to the practice of mathematicians. Devlin (2000), a well-known mathematician and writer, describes math as "full of creativity' and "never dull" (p. 25). In addition, some mathematicians compare mathematics to the making of music. Devlin (2000) makes the analogy that just as music notation is not the same as "music", math notation is not the same as "math", but it is "in its performance that music comes alive … the same is true for mathematics" (p. 9). Boaler (2008) further elaborates that "math is a performance, a living act, a way of interpreting the world" (p. 29).

Goal 4: "Valued contributor". A positive disposition includes the perspective that math is a social activity and values participation in mathematics with others, including collaboration and communication with others around ideas. Therefore, one important goal for a positive disposition to mathematics is to be recognized within and to actively contribute to discussions in a community of practice.

The CCSSPM refer to this goal in Practice 3, which points to students' ability to explain and defend their own ideas and critique the reasoning of others on mathematical grounds. This standard refers more indirectly to participating in an ongoing way in math discussions. Communicating effectively in the language of mathematics with others is also emphasized in the NCTM standards for learning (NCTM, 2010) and the NRC strands for mathematical proficiency (Kilpatrick et al., 2001).

Being acknowledged for actively contributing to mathematical conversations is fundamental to developing within the practice. Mathematicians often refer to their conversations with colleagues as a valued means to improve their own ideas and share the excitement of problem solving, and central to their practice.

Summary. The goals listed above have been derived from the math education and educational psychology goal-orientation literature, from the CCSSPM, and from descriptions of the practices and values of mathematicians. These goals are meant to represent the motivation that students would have as part of a positive disposition to mathematics in alignment with the reform-oriented mathematics education community. The position of this paper is that all of these goals are important to be successful in mathematics practice. One of the questions of this study is to what extent students' appropriation of these goals is related to positive engagement in actual math practice, examined in the context of problem solving in this study. It may be that all goals must be fully appropriated by students for consistent success, but more likely students may have different profiles which stress particular goals from the list above.

Cognitively Guided Instruction (CGI) Curriculum

Curricula play an important role in defining what is culturally available to students in this study to appropriate. Cognitively Guided Instruction (Carpenter, Fennema, & Franke, 1996; Carpenter, Fennema, Franke, Levi, & Empson, 1999) is an example of a curriculum that attends both to fostering problem solving and fostering the dispositions of students to become problem solvers. The teacher in this study has been trained in CGI, which is considered to be a defining feature of the classroom environment. The following features of CGI are considered especially supportive for a positive disposition.

In CGI, teachers are provided with models of students' developing understandings as reference points. The intent is to organize and extend teachers' intuitive and spontaneous understanding of student thinking. In CGI, children are considered to bring a great deal of informal and intuitive knowledge of mathematics, which is purposively built upon in their primary years. The emphasis on understanding student thinking underscores the practice in CGI of actively listening to and building on students' articulations of their thinking about mathematical ideas. By placing an emphasis on understanding students' ideas and building upon them, the CGI approach promotes the goal of learning with understanding within students (goal #1).

The second feature of CGI that relates to this study is that teachers design their own problems for children to solve. By inventing problems, teachers can challenge and extend the kind of problems that students have been solving, and create problems that directly address the interests and needs of their students. By crafting problems for their own students, teachers can foster an interest in solving challenging and varied types of problems. In addition, by making problem solving a central feature of the curriculum, teachers support and make available the goal to engage in challenging problems for students (goal #2).

The third feature of CGI that relates to this study is that teachers value and build on student ideas rather than just introduce the standard strategies and procedures. In particular, teachers provide active support for students' invention and modification of strategies for problem solving. By providing support to invent and modify strategies and making this a regular part of classroom practice, teachers support a positive disposition towards agency and creativity (goal #3). In addition, it gives students the opportunity for conceptual agency in choosing how they would like to approach a problem. Finally, it allows students to bring aspects of themselves into their mathematics by inventing and modifying strategies, and being recognized for sharing these with the class.

The fourth feature of CGI related to this study is that teachers facilitate classroom discussions in which students publically share their strategies. The frequent opportunity, support and expectation to participate in classroom discussions to make recognized contributions in the community about mathematical ideas is supportive of students having the goal to be an active participant in discussions (goal #4).

Summary of This Study's Contribution

This study draws on the theoretical and empirical literature concerning dispositions, goals and engagement as described above. The central aim of the study is to investigate the extent to which students have appropriated goals in alignment with the reform-oriented mathematics education community, and how these goals relate to their engagement in problem solving. I chose to use dispositions as a framing construct because it addresses both students' affect and ideas about a field and their situated actions within that field. I have described above how dispositions have been thought of as habits of mind, and how some recent work has examined student's dispositions through student's math-related conversational moves in relation to the forcefulness of the opportunity and by examining the connection between identity and patterns of engagement with mathematics (Langer-Osuna, 2013).

However, in this study I argue that studying students' goals gives a more specific insight into the *motives* that students have for participating in math in the first place, and makes a unique contribution to understanding their relationship with mathematics. In doing so I am building on the approach used by Nasir (2002) in looking at how goals and relationship to the field are related. However, my study shifts this analysis by positioning goals as an aspect of students' dispositions, and examining how appropriation of different goals is translated into their situated actions in problem solving, which I argue is the heart of mathematics practice. In addition, I examine a unique set of goals which I have synthesized from the CCSSMP, the math education literature, and literature about the thinking of mathematicians. In this way I am using both relevant methodological approaches and making a contribution by my particular emphasis and age group. My study also makes a contribution by proposing four positive goals that are central to developing a positive disposition to mathematics, and describing the extent to which students appropriated these goals in a 3rd grade classroom.

CHAPTER 3

METHODS

Study Design

This study used a mixed-methods design to study students' practice-linked goals at the class and individual level, and to relate their goals to their situated actions in problem solving. This allowed the study to connect students' ideas and perspectives about dispositions with their situated activity in problem solving. In doing so the study aims to contribute a more nuanced portrait of students' positive dispositions situated within a classroom context.

I conducted research as a participant observer during the 2013-2014 school year, which allowed me to gather data about students' goals at the beginning, middle and end of the year. I collected data that allows me to answer research questions at the class and individual level. Data sources include student surveys at three time points during the year, case-study student interviews at three time points during the year, classroom observation throughout the year, and observation of 8 case-study students during small group work.

I used surveys and interviews to describe students' mathematical goals (RQ1a), and case-study students' interviews to describe their explanations for their ratings of each goal (RQ1b). I used the student surveys, student interviews, and classroom observations of small group work to investigate the connection between students' goals and their situated actions in problem solving (RQ2). I performed these surveys and interviews at three points during the year in order to investigate the stability and variability of the students' goals.

Study Site

I selected a third grade mathematics class for this study at a school with a progressive, constructivist teaching philosophy. The selected teacher was trained in and embraces the

Cognitively Guided Instruction (Carpenter et al., 1996; Carpenter et al., 1999) philosophy. The school follows a progressive philosophy with multi-age classrooms where students remain for two years. Three teachers each specialize in academic subjects for two connected classrooms, with the support of several classroom aides. The classrooms in the study frequently worked together. Although students usually worked in mixed-age groups, the classes separated by grade for mathematics. The mathematics class that I studied was composed of 3rd grade students. The school's ethnic and socio-economic diversity roughly mirrored that of the state of California, specifically Caucasian (36%), Latino (20%), Latino-Caucasian (12%), and Asian (9%), and 25% of students received financial aid.

School philosophy. The school in this study is a progressive reform-oriented institution which strives to cultivate students' critical thinking and expression, and employed collaborative learning and project-based instruction, all of which have been identified as factors that promote positive dispositions to mathematics in students (Boaler & Greeno, 2000). The goal statements of the school are presented in Figure 1.

- We are dedicated to addressing the needs of children from diverse backgrounds.
- We value teaching and learning environments that honor each child's natural joy of learning.
- We encourage creativity and support a disciplined approach to intellectual inquiry.
- We are a caring community of learners: students, teachers, staff, and families.
- We are committed to educating the whole child.

Figure 1: Goal statements of the school

Description of daily routine in mathematics class. The students began class by taking out their folders and notebooks. Students were expected to write a response to the prompt shown on the overhead in their notebook. The teacher circulated and spoke with individuals while other students worked quietly. Sometimes the teacher asked the group to stop so she could discuss a point or question that had come up before letting them continue working. After 5 to10 minutes the teacher usually asked several students to share a "mistake" or strategy/solution. [1] The student who was sharing put their notebook under the document projector and explained their work from the front of the class while the other students sat at their desks. After the student finished, the teacher led a discussion, inviting students to raise their hand and ask questions, asking the presenting student to think further about a point, or using the presentation to make a point of her own about the prompt. The teacher usually closed this section of the lesson with an observation about their work as a class or the mathematical concept.

Students were then asked to put their notebooks away and come to the rug for a mini-lesson with the teacher. The teacher began by reviewing work from the previous day. She then introduced the work for the day. The lesson on the rug lasted 10 to 20 minutes. The students then went back to work in pairs at their desks. Pairs were part of assigned seating that changed every few months. Occasionally students were asked to work in groups of four. Students usually worked at their desk for 15 to 20 minutes. Most days, the teacher brought the students back to the rug for a final discussion lasting 5 to 10 minutes. This usually began with several students sharing their work under the document reader and explaining their solution. The teacher either asked particular students to share before the group met, or asked for volunteers to share while they were gathered. Students who were not presenting were expected to listen and ask questions about the mathematics. The last five minutes of class before lunch were used to assign homework which students were expected to record in their calendars.

Participant Selection

Selection of classroom teacher. The teacher in this study was identified by math education experts in the UCLA School of Education as teaching in a way that supports the development

[1] There was a particular emphasis on mistakes and discussion of mistakes during the beginning of the year

and expression of positive dispositions to mathematics.

Selection of case study students. In order to get a fine-grained, detailed understanding of the connection between students' goals and their situated actions in problem solving, I chose eight focal students to follow throughout the year. This gave me an opportunity to develop relationships with students as well as to observe their situated actions in problem solving over an extended period of time. I hoped to see variation in students' degree of alignment with the goals of the reform-oriented mathematics education community, as well as variation in the connection between different goal profiles and student's situated actions; therefore I purposively sampled students with contrasting goal profiles and level of activity in problem solving. Looking across the class, I selected students who were well aligned to the goals valued by the reform-oriented mathematics education community, and those that were less well aligned. I also balanced for gender, and chose students who contrasted in how active they were in collaborative problem solving, both in how much they spoke with their partner and how much they were attending to their work versus distraction. I determined students' level of activity based on the first few weeks of classroom observation and by consulting with the classroom teacher. I chose students who represented the diversity of the school (which also reflected the diversity of the region). As reported in Chapter 4, most students rated the performance-linked goals as low to medium importance. Therefore, this was a less important factor in selecting case study students. The characteristics of the case study students are presented in Table 2.

Table 2

Characteristics of Case Study Students

Pseudonym and gender	Ethnicity	Alignment with practice-linked goals	4 Goals*	Performance-linked goals	Level of activity in problem solving	Academics
Tosha (F)	Caucasian	High	HHHL	Low	High	Strong
Jason (M)	Caucasian, other.	High	HHHH	Split	High	Strong
Blake (M)	African-American	High	HHLH	Low	Low	Developing
Miley (F)	Asian-American, other.	High Medium	HLMH	Low	Low	Strong
Will (M)	Latino, Caucasian	Low	HMLL	Low	High	Strong
Amaya (F)	African-American.	Low	HMML	Medium	High	Developing
Aaron (M)	American Indian, Caucasian.	Low	MLLM	Low	Low	Developing
Lauren (F)	Asian-American, American Indian, Caucasian.	Low	HMML	Low	Low	Strong

*The order of the goals listed here is 1) learning for understanding, 2) solving challenging problems, 3) having agency and creativity, and 4) being a valued contributor to discussions. H = High, L = Low.

Data Sources

Observation. I observed approximately 50 hours of mathematics lessons during the year. I used two cameras to capture whole class lessons, as well as case study students while in small group work. I tried to coordinate with the teacher so I could be present for problem solving sessions. I also tried to observe an entire unit in order to observe meaningfully bounded periods of work for students.

Video data. I created video recordings of all lessons that I attended. During whole class activity, I used two conventional video setups to focus on the teacher and classroom, respectively. During small group work, I focused the cameras on a case study student, rotating among the students each day. Each case study student was videotaped 10-12 times in small group work.

Field notes. As a participant observer in the classroom, I took field notes during whole class and group activities. I rotated between the focal students during group work so that I focused on the problem solving of one case study student per class period. I took notes specifically about decision points during the solving problem process. I paid particular attention to students' initial situated activity with the problem, including what they oriented to within the problem, how they selected their strategy and how they interacted with their peers as they began working on the problem. I also looked for decision points which occurred as students got new information that could be used for solving the problem, such as comments from peers or teachers, conversations about their strategies or plans for working on the problem, and when they reached a point of difficulty that required them to change or modify their strategy.

Stills of case study students' work. I used a digital camera to take stills of case study students' work during small group activity. This allowed me to see changes in their written work and work space during the class period.

Student surveys. In order to get a broad understanding of students' appropriation of the practice-linked goals, I gave students a survey about their goals at three time points during the year (see Appendix A for the winter survey). The survey required approximately 15 minutes to complete. Some questions were selected from the Patterns of Adaptive Learning Survey (PALS) which was developed to measure students' goals for achievement in mathematics (Midgeley et al., 2000). The scales have proven to be valid and reliable in both upper elementary (4^{th} grade and older) and middle school classrooms (Middleton & Midgley, 1997). I modified questions as needed to be developmentally appropriate for third grade students. I also selected questions from the "Math and Me" survey, developed and validated for elementary students, by Adelson and McCoach (2011), which is the only validated survey that addresses the attitudes to mathematics for students in third through sixth grade.

In addition to the practice-linked goals, I asked questions about the performance-linked goals of "feeling smart" in math class, and "being fast and correct". These goals address the educational psychology construct of a performance goal. I combined performance approach and avoidance goals for this survey. I refer to these goals as "performance" because they refer to students assessing and maintaining the appearance of performance in relation to others. Although performance goals can often be negative for students, they have been associated with positive outcomes (Pintrich, 2000), and I expected that at least some students would have appropriated these goals because of their pervasiveness in the traditional discourse about mathematics in our culture. In addition, I wanted to see how appropriation of these goals related to appropriation of the practice-linked goals and to their situated activity in problem solving.

In creating the survey, I generated three questions for each goal that were on a Likert-type scale. In the beginning of the year, I piloted the survey with several fourth students from the same pair of classrooms where I conducted my research. Then, I checked the questions with the teacher and finalized the survey before administering it to the class. The questions were slightly modified over the year based on student feedback in the first interview, in which we discussed their survey results. I added two questions about the goal of "helping others" because this came up as something important to students in the first interview, and I wanted to distinguish between this goal and being a "valued contributor".

Student Interviews. I conducted semi-structured interviews with students at the beginning, middle and end of the year. Table 3 below presents information about each round of interviews. See Appendix B for student interview protocols.

Table 3

Information about student interviews

#	Date range	Length	Recording method	Subjects	Content of interview
1	December 3-5	15 min	Audio	All students	Asked students about lowest and highest goals, contradictions in ratings, observable actions associated with goals
2	March 17-19	30 min	Audio	Case study	Asked students about goals at two kinds of moments of problem solving using cued video-recall
3	May 17-19	45 min	Video	Case study	Asked students to rate and explain goal ratings during beginning, trouble and preparing to share
4	June 3-10	30 min	Audio	Case	Asked students about lowest

				study	and highest goals, change of goal ratings across the year

First interview. The first interview occurred at the beginning of the year immediately after the first survey, and was conducted with all members of the class. I asked students about the goals they had rated as most and least important on the survey, and asked questions to probe about contradictions between survey questions and the ranking of goals at the end of the survey, to further understand nuances of the importance and meaning of each goal to them. I then asked students what kinds of things they did in class to accomplish the goals they identified as most important.

Second interview. The purpose of the second interview was to ask students about their goals in situ and develop greater understanding of their goals in practice. An additional objective was to probe inconsistencies between reported goals and situated actions. I selected several video clips of 30-60 seconds of each student problem solving that showed a decision point or a moment of trouble. I then compared clips in a table to ensure that I was asking about similar moments for each student. During the interview, I showed the student the clip once, then showed them again and invited them to press the “stop” button at any time to comment on something that had stood out to them in the video (Clarke, 2003). I also sometimes stopped the tape myself to ask them to clarify something that was happening the video. I had the transcript available if sound was poor. I then asked the student to explain to me what was happening in the video. I asked them to rate the importance of the 7 goals on an enlarged Likert scale. We repeated this process for an additional video showing a different type of moment. I initially planned to ask students about 3 clips, but because of the depth of conversation I only asked about 2 with most students.

Third interview. The purpose of the third interview was to ask students about their goals at specific phases of problem solving. I created a board with the Likert-type scale up the side and columns for three types of phases in problem solving - beginning a problem, encountering trouble, and preparing to share. During the interview I described each phase and asked students to place the goals on the scale, and explain why they had given that rating. In the last column I put their ratings of the goals from the last survey, and kept this covered until the end. After we had finished rating the different phases, I uncovered this column and asked them if anything surprised them, and if so why. I also used the placement of their ratings of different phases side by side to ask questions about why a goal had gone up or down from phases to phase.

Final interview. The purpose of the final interview was to get an end-of-the-year explanation of each student's most and least important goals, and to ask them about changes in goal ratings over the year. I created a table for each student that showed the importance of each goal for each survey, to facilitate asking them about change over the year, and why they thought this had occurred. This was also my final opportunity to ask them about any other topics that had come up in our conversations. For example, Amaya had spoken to me about her "zone" during the year. She had mentioned in the third interview that her zone had changed during the year and she now wanted to talk to people who were not in her zone in order to grow. I was able to check in with her about this during our final interview.

Challenge problems. I designed several challenge questions throughout the year in cooperation with the classroom teacher, presented in Table 4 below, and attached in Appendix C. The problems were designed to be worked on for one class period in partnerships, in keeping with the format of the class. Problems built on concepts that the teacher was addressing in the curriculum at that time. Problems were designed to have multiple possible solutions and entry points and to be accessible to all students in the class. The problems required students to make decisions not only about strategy, but about the numbers they would use, providing me with the opportunity to see students work through clearly defined decision points in the problem solving process.

The challenge problems were administered throughout the year; however, due to technical issues with the first administration only the second through fifth problems were analyzed. Problems were presented as regular classroom activity, and during interviews all case study students were surprised to learn that I had designed the problems, instead of their classroom teacher.

Table 4

Challenge Problems

#	Date	Lesson name	Mathematical concept	Description
2	1/27	Postage stamps	Multiplication	Decide how many 50c letter and 25c postcard stamps to buy with $20
3	2/6	Pencil problem	Division	Calculate number of pencils needed for a class of 27 to have 1 fancy and 2 plain pencils each. Then choose a number of volunteers to sharpen pencils and calculate how many each will sharpen.
4	3/12	Lasagna Problem	Fractions	Decide the most and least number of party guests who could be fed with 10 lasagnas
5	5/14	Scale up Royce Hall	Scale	Scale up a picture of Royce Hall to make a model that would fit on their desk

Analysis

Students' ratings of goals. I used class-level survey data to answer RQ1a**:** *To what degree do students' goals align with the values of the reform-oriented mathematics education community?* To calculate each student's overall rating for each goal at each time point, I averaged their rating for the three survey questions pertaining to that goal. Average ratings were then divided into three groups. An average rating of 1 – 2.3 was categorized as low importance, an average of 2.4 – 3.6 was categorized as medium importance, and an average of 3.7 – 5 was categorized as high importance, corresponding with the Likert-type scale used in the survey.

Students' explanations of goal ratings. I used interview data from case study students to answer RQ1b: *What kinds of explanations did students give for their ratings for each goal?*

Identifying codeable quotations in the interview data. Student interviews were transcribed and entered into the coding program Atlasti. From the transcript, I identified student quotations that contained an explanation of their goal ratings. I was less interested in how often a student mentioned an explanation, than the specific context for which they used that explanation. Therefore, I created a new quotation using the following guidelines:

1. The student started to talk about a different goal
2. The student started to talk about the same goal in a different context
3. The student gave a different explanation for a ranking for a goal

Sometimes quotations that had different codes overlapped in the transcript.

Coding student quotations. To examine students' different explanations of their goal ratings, I began by developing codes based on explanations in the fall interviews. After coding the fall interviews, I condensed codes that were similar and developed definitions and examples for each code. I then completed three more cycles of coding and revision by adding final, spring and winter interviews. After coding the transcripts, I checked for consistency between quotations assigned to a particular code, and combined codes that had low counts and referred to a similar explanation.

For the two codes “fast and correct” and “having agency and being creative”, students talked about the two elements of the code separately. They distinguished between “being fast” and “being correct” and also “having agency” vs. “being creative”. Therefore I created codes that were assigned to one of the elements within the larger goal.

From preliminary analysis of my codes I noticed several themes that cut across the codes. The following themes arose across all goals:

Social/ positioning
Future
Norms
Values
Personal benefit
Other

In a final check, I read through all interviews. Generally the interviews proceeded by talking about one goal at a time. In the winter and spring, I asked about each goal in 2 to 3 contexts. I edited the coding so a code was assigned only once to a unique context. The contexts were as follows:

Fall – Explanation of rating, advice to a younger student
Winter – Explanation of rating for situation 1, explanation of rating for situation 2
Spring – Explanation of rating for beginning, encountering trouble, and preparing to share.
Final – Explanation of rating, explanation of change over the year

I then analyzed counts of codes, and compared frequency of themes across the different goals. The results are presented in chapter 4.

Coding video of problem solving. I used video of case-study students’ problem solving, as well as their interviews and surveys, to answer the second research question, RQ2: *How do different profiles of goals relate to students’ situated actions in problem solving?*

Selecting the data set. Although students worked on problem solving throughout the year, they engaged in open-ended problems infrequently. In order to have a coherent set of data with which I could compare the situated actions of the case study students for the same problem, I chose to analyze the four [2] challenge problems that I designed in cooperation with the classroom teacher throughout the year. In this way I ensured that I had data for all eight case study students working on the same problems at the same time points. In addition, during challenge problems the teacher and I assigned the students to work with a different partner each time, which allowed me to see both consistent and varying patterns of interaction based on the problem and with whom they were working.

Using decision points to characterize students' situated actions in problem solving. In order to characterize students' situated actions in problem solving, I chose to identify and analyze decision points in problem solving. Decision points are defined here as moments in which students make decisions that affect the course of their work. They can be thought of as forks or crossroads that determine the work trajectory for that problem in the short term, and their trajectory of situated activity for the longer term. While working on solving problems, students moved through the following phases: deciding upon a strategy for the problem, executing the strategy, encountering trouble with the strategy/chosen course and resolving that trouble, being distracted and returning to the problem, and finishing the problem, although they could enter each phase multiple times and move between the types in different orders. In the analysis I focused on segments of work in which students were deciding upon a strategy, encountering and resolving trouble, or finishing the problem. I did not analyze sections of the transcript in which they executed a strategy unless trouble arose, because students were usually

[2] I designed five challenge problems but only analyzed four due to equipment issues with the first problem.

either working quietly or reiterating what they had decided during the beginning or trouble. I also did not analyze sections of the transcript in which students were off task, because only a few students had prolonged periods of off-task behavior, which I describe as applicable in the case-study profiles in Chapter 6.

During the beginning, trouble and finishing segments, students encountered such decisions as "what strategy do I use?", which number do I use?", "who is going to do what work?", "what do I do when I get stuck?", "what do I do when my partner needs help?", "how long do I let myself get distracted?", "how do I determine that I am finished?", "what do I do with feedback from the teacher/other adult at any time during the problem?". These decisions were used to divide the transcript into "events". These events were then analyzed for different behaviors that both characterized students' situated actions and could be compared with students' goals in order to understand the students' disposition. I follow Chi's pattern of analysis of verbal interaction and choose this grain size as best suited to reveal patterns of situated activity (Chi, 1997).

Creating codes. In order to create codes for students' actions, I first watched several videos of students' problem solving to look for decision points – places that students had to make choices about their problem solving – as a way to see their goals highlighted and enacted through situated actions. These situated actions included responding to a partner's request, deciding which strategy and numbers to use, responding to teacher feedback, deciding they had finished with work, and dealing with "trouble" in the strategy they had chosen. After watching the video, I generated a list of decision points and corresponding actions and created a table (see Chapter 5) outlining how these actions indicated the importance (or non-importance) of particular goals in that moment. Finally, I coded the video data in an iterative process, using a small set of video, developing codes, refining these codes, coding an additional set, and refining, etc. until I had coded the entire data set and checked that all quotes for each code were consistent with the code definition and in relation to other quotes.

Codes for students' actions. The actions that students took fell into the following categories (note "making decisions and working together" was further divided into subcategories). I briefly lay out the codes (in bold) within each category below, followed by more elaborate descriptions of each code in the following paragraphs:

- *Making decisions and working together*
  - *Making decisions*
  - *Diverging in work*
  - *Positioning*
  - *Focus*
  - *Norms*
  - *Interacting with the table*
- *Going Deeper*
- *One person stuck*
- *Deciding on a strategy/number*
- *Interacting with the teacher*
- *Finishing the problem*

Dynamic – Making decisions and working together. This category concerns the way that students interacted with their partners and made intellectual decisions about their work. Because of the size and complexity of this category, I divided it further into sub categories as described below.

Making decisions. When making decisions about their work plan, students discussed a proposed strategy or different perspectives on a strategy or number choice before moving forward. Sometimes students **argued different ideas,** in which each student would say their idea and back it up with reasoning or evidence. Students could also **share their ideas,** simply stating different ideas before deciding on which to take forward. Finally, one student might present an idea and the partner would **agree to the idea** without discussion. There were also occasions when one person would make a suggestion, which was **ignored** (not acknowledged) by the partner.

Diverging in work. Sometimes students might be working and discussing ideas together, but **chose to use different strategies** and numbers and therefore different solutions. Other times students **worked alone** and did not interact with each other.

Positioning. There were two ways that I observed students' using positioning while working together. One was to make statements about the **difficulty** of the problem, such as "this is easy" or "you might not be able to do that". [3] The other type of positioning was **monitoring** a partners' written work, pointing out mistakes, telling them what to write or even writing for them. This positioned the person helping as "more knowledgeable" and therefore entitled to make such corrections.

Focus. There were occasions when students were **distracted** by their partners or table mates while in the middle of work – students either chose to ***engage*** or ***defuse*** the distraction.

Interacting with the table. Although the students were expected to work with their partner, they sometimes interacted with the other pair at the table. One code was for **discussing work**, including students' interpretation of direction and comparing solutions. Another code was for the situation when one person would get up to **help** a person in another partnership, sometimes requested and sometimes not requested.

Norms. Students' sometimes **invoked classroom norms** about how they were expected to work together. These norms included talking about and working together on problems (although having different solutions seemed to be acceptable in some situations), making sure your partner was in the same part of the problem as you, making sure your partner knew what

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[3] On a few occasions students used difficulty to justify their strategy choice, looking for a way to make the problem "easier". This was called out in the event description.

you were doing, and explaining your work on your paper. The other person would either ***agree*** and comply with the norm, or ***disagree*** and not comply.

*Going Deeper***.** In a few cases, students either talked about different **possibilities** of numbers and solutions, or considered the **meaning** of the problem and their solution, in their discussions.

*One person stuck***.** This concerned cases how a partner reacted when the other was stuck and/or asking for help. Partners reacted in multiple ways - sometimes they **ignored** the request, sometimes they **offered an answer**, and sometimes they **offered an explanation**. On a few occasions the student who was stuck clearly needed help but **did not ask** for it, or the student **asked to copy** their partner's work.

*Interactions with the teacher***.** These moments included being **asked to work together**, **explaining work** to the teacher at her request and interaction with the teacher that **supported their thinking**. When the interaction supported their thinking, the teacher's support allowed the student to take a step forward in their strategy or thinking. These moments are discussed in greater detail in individual student profiles in the next chapter.

Finishing. Some students "**celebrated**" every time they finished a section or an entire problem, either making an announcement to the table that they were "done" or "have the answer", or doing a dance or shout. Other times students were especially **excited to get to games**, both on occasions when they used less than the time given to finish their work. Another code was students **checking their work**, either comparing written work with a partner, recounting representations, or using a second strategy to check their final answer. Sometimes students **stayed after** others went to the rug, sometimes they **went when the teacher called** them, without having checked their work, and sometimes they **checked their work with the**

teacher and then responded before turning in their work. The connections between these situated actions and practice-linked and performance-based goals are described in Chapter 5.

CHAPTER 4

STUDENTS' GOAL RATINGS AND EXPLANATIONS

This chapter reports on students' ratings and explanations of the importance of the four practice-linked and two performance-linked goals. Using data from student surveys and student interviews, I answer both parts of the first research question, RQ1a: *To what degree do third-grade students hold practice-linked goals aligned with the values of the reform-oriented mathematics education community* and RQ1b: *What kinds of explanations do students give for their ratings for each goal?* The purpose of this analysis is to contribute a more powerful way of examining students' overall goal profile, by both exploring their ratings of goals and their explanations of the importance of these goals. This allows a closer look at how students interpret these goals, and how their interpretations relate to broader themes such as social positioning, norms and values.

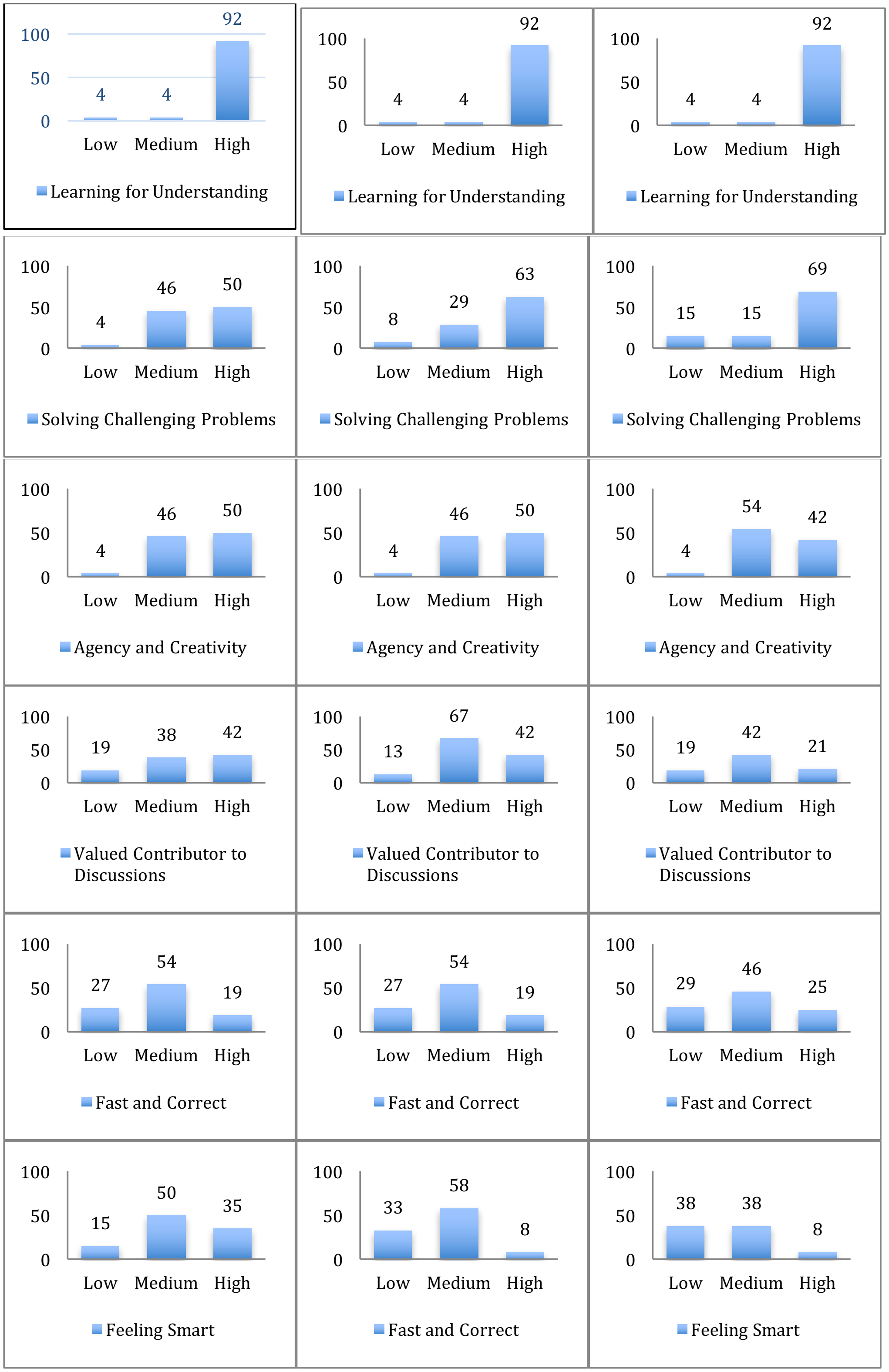
I first report on the class level data from the student surveys to show the ways that goals varied in importance. I then draw from the interview data to report on the eight case study students' explanations of goal ratings, showing how several themes showed up across explanations for rating different goals. These themes offer ways to see how students' make connections between different goals, and how these themes are part of a larger narrative that students have about doing mathematics, which is discussed in more detail in Chapter 6.

Findings at the Class Level

All students in the class completed a survey about the practice-linked and performance-based goals at three time points during the year. To calculate each student's overall rating for each goal at each time point, I averaged their rating for the three survey questions pertaining to that goal. I then divided students into three groups based on these ratings. An average rating of 1

– 2.3 was categorized as low importance, average 2.4 – 3.66 were categorized as medium importance, and average 3.7 – 5 was high importance, corresponding with the Likert-type scale on the survey. Figure 2 below shows the percentage of students in the class who rated each goal as low, medium and high importance on the fall, winter and spring survey.

Fall **Winter** **Spring**



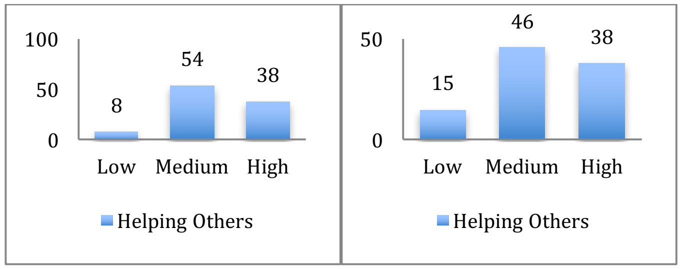


Figure 2: Percentage of students in class who rated a goal as low, medium or high importance

Class ratings of practice-linked goals. The goal of "learning for understanding" was of high importance to almost all students throughout the year. The goal of "solving challenging problems" was split between medium and high importance for most students in the fall. The percentage of students giving this goal a high rating increased over the year, ending at 69%. The goal of "having agency and creativity" was split between medium and high importance for the majority of students throughout the year. The goal of "being a valued contributor to discussions" split between medium and high importance for most students at the beginning of the year. By the end of the year, it was of medium importance to 42% of students and split between high and low importance for the rest of the students. These results indicate that the four practice-linked goals were at least somewhat important to the majority of students in the class throughout the year. They also show the variation in goal profiles, and that individual students did shift in their ratings of particular goals throughout the year.

Class ratings of performance-linked goals and "helping others". The goal of "being fast and correct" was of medium importance to half of the students, with the remainder of students split between low and high ratings throughout the year. At the beginning of the year, one third of students thought "feeling smart" was very important, half thought it was somewhat important and 15% though it was low importance. By the end of the year, it was split between low and medium importance for most students. The goal of "helping others" was added after the first survey because students placed emphasis on it during their debrief with me in the fall. Just over half of students rated this goal of medium importance in winter and spring. In spring, a greater percentage of students thought it was of low importance. Overall students rated the practice-linked goals as somewhat to very important. They rated the performance-based and helping goals as having lower importance.

Findings at the Case-Study Level

Explanations of Goal Ratings by Case Study Students. In order to better understand why students gave different ratings to the different goals, I asked case study students to explain their ratings during our one-on-one interviews. In this section I describe how students' different explanations for their goal ratings show the wide variation in how they interpreted each of the goals. These interpretations demonstrate that two students may have the same rating for a goal, but view the meaning of that goal quite differently, which in turn will likely be translated into different types of actions in problem solving. This study contributes a more nuanced and complete description of students' goals by including these explanations, which show that students think about these goals in different ways.

In addition, in developing codes for different explanations of goal ratings as described in Chapter 3, several themes arose that spanned explanations across the different goals. The themes were as follows:

Personal benefits – These explanations refer to a direct benefit for the student, including learning, feeling good, advancing or completing a task, or helping them with their work. Examples include having independence while working and getting done quickly to have time for games.

Norms – These explanations refer to a norm of the classroom both for the individual and the group. Examples include explaining work to their partner and taking turns to speak in group meetings.

Values – These explanations refer to a value that the student held individually. Examples include helping another student in need and sharing their learning with others.

Social – These explanations refer to a social aspect of classroom work. Examples include positioning and identity, obligations to others, and aspects of group work such as collaborating.

Secondary benefits – These explanations refer to secondary benefits that the students anticipated. Examples include success with future math problems and grades and admissions into college.

Other – These explanations refer to a range of topics, from unrelated comments to explanations of *how* the student would go about doing something, such as the steps they would take to solve a problem.

Not important – These explanations refer to why the goal was not important, such as other goals were more important or it was not relevant at that moment.

The fact that these themes arose across goals demonstrates that the students did not think of individual goals in isolation. Often several themes dominated their explanations of all of the goals, playing into a larger narrative that they had about what it meant to do mathematics, as described in the case study profiles in Chapter 6. Looking at these themes across goals allows us to see students' connections between goals as well as how they fit into students' larger narratives. Looking at students' explanations and how these relate to themes and the larger narrative offers a more complete picture of students' relationships to mathematics, and better explains how this relationship is translated into actions in problem solving, as explored in Chapter 6.

Explanations in all categories below except "not important" generally referred to medium and high goal ratings– when more than one student explanation in a category referred to a low rating, I indicated as such in the table notes. In the section that follows I discuss categories that contain at least 15% of student explanations for that goal, and are directly related to the meaning of the goal for students.

Table 5

Counts of case study students' explanations for goal ratings by category (percentages in italics)

	Personal benefit	Norms	Values	Social	Secondary Benefit	Other	Not important	Total
Learning for understanding	75 *(69)*	2 *(2)*		4 *(4)*	13 *(12)*	5 *(5)*	9 *(8)*	108
Solving challenging problems	40 *(33)*	2 *(2)*	11 *(9)*	11* *(9)*	12 *(10)*	25* *(21)*	19 *(16)*	120
Having agency	34 *(49)*	5 *(7)*		4 *(6)*		18 *(26)**	9 *(13)*	70
Having creativity	19 *(49)*					1 *(3)*	19 *(49)*	39
Being a valued contributor	25 *(22)*	16 *(14)*		33 *(29)*		21 *(19)*	18 *(16)*	113
Feeling smart	29 *(27)*	11* *(10)*		38* *(36)*		7* *(7)*	21 *(20)*	106
Being fast	15 *(35)*						28 *(65)*	43
Being correct	10 *(15)*			7 *(9)*	8 *(12)*	22* *(32)*	22 *(32)*	69
Helping others	18 *(23)*	19* *(24)*	16* *(20)*	9 *(11)*		7 *(9)*	11 *(14)*	80

*Some of these comments were associated with low ratings

General trends in categories across all goals. Overall, personal benefits dominated student explanations for why they valued these goals. The goals "valued contributor" and "looking smart" were socially motivated. Most explanations about the less productive goals "being fast and correct" were about why these goals were not important. The class was split about the value of creativity.

Explanations for ratings of the goal "Learning for understanding". Students' explanations for the importance of the goal "learning for understanding" were dominated by personal benefits, as shown in Table 5. The personal benefits that students described centered on different levels of learning.

Personal Benefits (75). Within this category, slightly more than one third of the explanations (28) were about *understanding the teacher's expectations for the problem*. Students wanted to understand the problem so they knew what the teacher expected of them.

> You have to pay attention to the teacher and know what they're doing because then you could fall off task or you don't know what you supposed to do, Ms. A will have to explain it all over again if you don't know what to do (Lauren, Fall, 09:59).

Here Lauren emphasized that you are supposed to know *what* to do vs. *how* to do it. This set of explanations focused on understanding what was being asked in the problem so that they could come up with a solution that met the teachers' expectations.

Another third of the comments (23) were about *understanding the math in the problem*. These explanations emphasized understanding the math for yourself vs. solving the problem without understanding what you were doing.

> I had to understand like the basics that went into … the word problems, and then that's, I had to solve multiple things, like how many, like how you'll cut this up into tenths, if you cut this up into tenths you'll get this answer (Jason, Winter, 24:34).

Here Jason discussed the mathematical concept of dividing by ten and his consideration of how "cutting by tenths" would result in a particular solution. He also mentioned understanding the "basics" indicating skills and concepts that were needed for that specific problem. This code is distinguished from *learning beyond the problem* in that students did not talk specifically about the learning extending beyond the current context.

The last group of students' explanations (24) was about *learning beyond the problem*. In these explanations, students said that "learning for understanding" was important because they wanted to learn new ideas and skills and be able to solve future challenges, "because if you don't increase your skills then you won't really get better, as a mathematician" (Will, Fall, 02:37). Some explanations connected the current problem with longer-term learning, "I wanted to learn more but I also wanted to understand things ... because I wanted to understand the problem, I wanted to learn new ways that can help me solve what I'm doing and stuff" (Lauren, Winter, 13:15). This group also included the idea of "moving up" in math, "So if I didn't do that I couldn't really move forward in math" (Will, Final, 04:16).

Summary. Students mostly explained their rating for this goal in terms of personal benefits related to learning, which ranged from understanding the teachers' expectations for the problem to learning beyond the problem.

Explanations for ratings of the goal "Solving challenging problems". The largest group of explanations for the importance of this goal was personal benefits, including learning, enjoyment and doing more challenges. The next largest groups, other explanations and why the goal was not important, were not directly related to the meaning of the goal to students.

Personal benefit (40). The largest group of explanations within this category (16) was about the idea that *doing challenging problems leads to learning*. This included explanations that solving challenging problems increased skills and strategies, "pushed" your brain, and "helps" math learning, as Jason explained in the fall, "To solve problems gets you better at math, that's, that's like one of the really important goals you need for math" (10:42).

Another group of comments (9) were more specific that *working through challenges allows you to work on progressively harder problems,* as Amaya explained in the final interview.

> Challenging means hard or difficult. You want to be able to pass the difficult line or else I should say put it in words "the zone". The easy zone and the hard zone... you want to be able to pass the challenging zone…like you know like a video game the next level, the next level, it gets harder and …You want be able to pass that level… The more challenging problems you'll do, it helps you a lot (06:52)

In this excerpt Amaya used the analogy of a video game to describe how doing challenging problems helped you reach progressively harder "levels" or challenges.

Another group of explanations in this category focused on the *enjoyment and good feelings* of working on and solving challenging problems (10). In her final interview, Lauren said she liked problems even if she could not find a solution, "some of them are really hard and you can't find a solution, but they are really fun" (02:20) and Tosha said it felt good when she solved a challenging problem, "It makes me feel really good when it happens" (Fall, 02:33). A small group of explanations (4) explained that the goal was important because they wanted to *solve the current problem*, "You're going to want to be able to solve challenging problems … so it can help you solve the problem, if you set a goal for yourself, that you can solve it" (Will, Spring, 05:15)

Summary. Most explanations of the importance of this goal were about personal benefits that included learning from doing challenges, being able to do harder problems and the enjoyment of working on (and solving) challenging problems.

Explanations for ratings of the goal "Having agency and creativity". In their explanations, students distinguished between having agency and having creativity, so I separated these in the codes, as shown in Table 5 above. Roughly two thirds of explanations were about "having agency" and one third were about "having creativity". Within the explanations about "having agency", half were about personal benefits and the rest were in the categories other or not important. Within the explanations of "having creativity", as many comments were about personal benefits as why creativity was not important while solving problems.

Personal benefit- agency (34). The largest group of explanations in this category was about the *efficiency* (18) of choosing your own strategy in solving problems. Efficiency included being able to choose approaches and strategies that were easier, simpler or personally best suited in solving the problem, as Will explained in his fall interview.

> I think if you get the same answer both ways, then (trying a new strategy is) not that important… when I'm learning a new strategy I want to make sure I got it, but if you get the same answer both ways, then maybe (another strategy is a) more quick and efficient way of doing it, or little faster, or just makes sense at the time that you're doing it (11:39).

Here Will explained how exercising agency was important to him as he weighed the benefits of trying a new strategy versus choosing a strategy that was "quick and efficient" and made more sense to him.

Another group of explanations were about the importance of *independence* (8) in choosing one's own strategy, as Amaya explained in the Fall, "because, you know like, not always, I want to do problems people's ways, sometimes I want to do it my own way, and get it, do it myself, and express my feelings with math" (02:27).

The last group of explanations were about how *trying new strategies led to learning (8)*, as Blake explained in Winter, "I think it'd be pretty important to me, cause, I like trying out new strategies, cause the more strategies I try, the more strategies I learn, and I want to have more than 2, 3, 4, 5 strategies" (26:47). As Blake pointed to in this passage, these explanations were specifically about trying *new* strategies vs. choosing which strategy to use.

Summary. The largest group of explanations were about the personal benefits of "having agency", which included efficiency of solving a problem, having independence in work, and learning by choosing something new.

Personal benefit – creativity (19). The largest group of explanations in this category was that "having creativity" *felt good* (9), "I think I was doing it for fun and it said be creative" (Miley, Winter, 28:13). Other explanations were about the direct result that *making up a strategy would help you solve that specific problem* (6), as Jason said in the spring, "Making up a new strategy would help you, make, answer, solve the problem" (20:52). The last group of explanations was that making up a strategy would add to their "*toolbox*" (4). In her final interview, Lauren imagined how her new strategy would be helpful to her in the future with other problems.

> Making up strategies, you can add that to what you already know and, like if you... there's this problem that doesn't match any of your strategies and you make up a new one, it could help you with other problems when you get older or as you go along (05:16).

Summary. Most explanations were about the personal benefits of "having creativity", primarily that it felt good to be creative, and making up a strategy could help solve a specific problem.

Explanations for ratings of the goal "Being a valued contributor to discussions". Unlike the goals above, the explanations for this goal were more evenly distributed among several categories. As expected, social explanations were the largest category. The next largest category of explanations was personal benefits, most of which were about getting help or ideas from others. Students also mentioned that this goal was important because of norms of working with others in their classroom. Explanations in the social category, including those about positive and negative positioning, were associated with medium and high ratings. This suggests that students' value of the goal was resilient even when their desire to share was tempered by concerns about the risks of sharing.

Personal benefit (25). While a few students said in their explanations that contributing to discussions was fun (3), most explanations were about the benefit that being an active contributor would lead to *getting help or ideas* (22). In this view, actively contributing was directly linked to getting feedback and ideas. In his final interview, Jason connected participation with learning through hearing the ideas of others, "If you're part of a discussion…you're learning about whatever they're talking about, you're learning for understanding and see "oh that's a mistake, oh I've got to fix it"" (Final, 27:19)

Social (33). The largest group of explanations in this category were about having one's *voice represented in discussions* (17), as Amaya expressed in her final interview, "it's my turn to show them I have strategies too... I have things too, I need to speak" (27:07). Other explanations were about the complexity of being *positioned in the classroom* while contributing (11), although these comments were still associated with medium and high ratings. Aaron said in spring, "I do want to share but I just get embarrassed a lot" (15:14). Miley said that she would only share when she was confident in her answer.

> Cause sometimes I don't have good ideas, and sometimes I get the answer wrong, and I don't like to get the answer wrong…no, it doesn't feel good… (but) sometimes do it, cause I'm not super super sure, all the time, but sometimes I am, I guess. But when I'm super super sure I'll raise my hand and, yeah (Final, 12:05).

Both Aaron and Miley's explanations highlight students' need to balance a desire to share with hesitation to be wrong or embarrassed in front of the group. A small group of explanations were about wanting to *help others learn* by sharing their ideas (5), as Amaya explained in fall, "because, you want to show your work, and it can make other, other people can learn from your idea" (14:19).

Summary. The most common explanations for the importance of this goal were the personal benefit of getting help and ideas, the social reasons of having a voice in discussions, and the desire to share being tempered by social positioning in the classroom.

Explanations for ratings of the goal "Feeling smart". Students expressed a complex relationship with this goal in their explanations of its importance. The largest category of explanations were social, which was not surprising given that the goal is often about being positioned in relation to others. The personal benefits cited were positive and focused on how internal confidence supported work. Explanations in relation to others included avoiding teasing, wanting to be perceived as smart, and rejecting positioning in relation to others. All but the category "personal benefits" were associated with a mix of low, medium and high ratings of the goal's importance, in contrast to the other goals discussed above.

Personal benefit (29). The main personal benefit students discussed was that *internal confidence could help them with their work* (20). This included helping stay focused and engaged, as Miley explained, "cause then you can keep up your confidence, and help you, think harder" (Spring, 09:35). While Miley linked confidence and thinking, Will connected confidence with perseverance, "I kind of want to feel smart cause like, just to, .. convince myself that I can think of it, cause, I think, … if you're stuck, you kind of want to feel like you can do it, there's always a solution, .. like you yourself can do it" (Spring, 20:21). A smaller group of explanations (6) cited the general *importance* of confidence (6), and a few explanations were that feeling smart just "*felt good"* (3).

Social (38). Explanations in this section were about students' comparison of themselves with others. One group of explanations expressed that being *perceived as smart was not important* (9), a second group expressed that *being perceived as smart* was important (10), and a third group expressed that *being in the middle* in relation to classmates was fine (11). A few explanations were that “feeling smart” was important to *avoid teasing* (3). About a third of explanations were associated with a low rating, perhaps reflecting students’ mixed feelings about this goal.

Summary. The most significant group of explanations of the importance of “feeling smart” were the personal benefit of internal confidence helping work, and the social explanations which were split between comparison with others being important or not important, or being fine to be “in the middle”.

Explanations for ratings of the goal “Being Fast and Correct”. Because students distinguished between “being fast” and “being correct” in their explanations, I developed separate codes for each as shown in Table 5. More of students’ explanations were about being correct than being fast. The main explanations given for why “being fast” was an important goal were the personal benefits of getting through the problem, and twice as many explanations were about why it was important *not* to be fast, including to avoid making a mistake. There were more varied reasons for the importance of “being correct”, ranging from the personal benefit of showing competence, to the importance of “being correct” as a core goal of mathematics, to avoiding embarrassment, but again one of the largest groups of explanations as about why this aspect of the goal was *not* important.

Personal Benefit – Fast (15). Explanations in this category centered around wanting to be fast to get the work over with. Students either wanted to *move on to another activity (7)* such as the math games, or *resolve the tension* (8) of the unsolved problem.

> I actually really like want to be getting done with this, because I don't want to be like,"Oh my gosh I’m in this challenging problem, I want to be done with this", I don't want to spend too much time on it …. since it's been troubling me a lot, so I want to be free from it, so I’m good and nice and clean (Amaya, Spring, 25:19)

In this excerpt Amaya expressed her interest in getting done to resolve the tension of the problem she said had been “troubling” her so she could be “free”. This was a contrast to other students’ experience of enjoyment during challenging problems, as discussed under the goal “solving challenging problems” above. It was interesting that no-one mentioned that getting done quickly felt good.

Not Important – Fast (28). Many of the explanations about why “being fast” was not important explained that rather it was important to *take your time* (15) while working, both to avoid making a mistake and because it was fun to think through the problem.

> I like to take my time and actually think about the problems…even if I do get the right answer when I rush, that's like coincidental. But I don't like rushing very much because you're most likely not going to get the right answer and, it's a lot of your thinking that you're putting on paper (Lauren, Final, 08:26).

In this excerpt Lauren both pointed out that she enjoyed thinking about the problem, and that rushing was not likely to lead to the right answer. Her explanation is a counterpoint to Amaya’s anxiety to “resolve the tension” above. The remainder of explanations for the goal’s lack of importance were that it was not important in general or at that specific moment of problem solving.

Personal Benefit – Correct (10). A small group of explanations put forth the idea that being correct was important because it *showed competence,* and *not* being correct meant you were *not* doing well (5), as Jason expressed in his spring interview, “cause if you get the right, if you get the right answer that means you're good” (19:15). A few explanations connected being correct with “moving up” (2) in math, and a few others connected getting the right answer with “a good feeling” (3).

Other – Correct (22). Half of the explanations in this category were simply that “being correct" was *important* in math. Will voiced a simple truth expressed by others, that it was important to get the right answer because this was the "main goal" of doing math, "I mean that's your goal, if you start a problem, your goal is to get the right answer, or and understand it, but like still get the right answer while you understand it" (Final, 11:36). Another group of explanations stated that the importance of being correct *depended on the context* (10) of the situation. For example, being correct on a test was more important than on a warm up or open-ended problem. A few students mentioned that there could be *multiple answers* (3) to problems, which was tied to specific situations we were discussing.

Summary. One third of explanations for the importance of "being fast" were the personal benefit of getting through the problem, and two thirds were why it was not important, including that taking one's time led to time to think and avoid unnecessary mistakes. Explanations were more varied for "being correct", with the biggest groups being other explanations including that it was simply a main goal of math, and that it was not important, and a smaller group of explanations about the personal benefits including showing competence, moving up and feeling good.

Explanations for ratings of the goal "Helping others". Explanations in this category indicated a mixed relationship with helping. Within the category "personal benefits" one half of comments were about students' own anticipated gain of getting ideas through helping others, and the other half of comments were about feeling good for helping. One quarter of explanations were about the norm of helping yourself before helping others, which explained the mixed ratings associated with this category. Finally, in the values category, explanations were about not wanting to abandon classmates, or to pass on the learning they had received.

Personal benefits (18). Roughly half of these explanations were about the benefit of *exchanging ideas* while helping someone (8) – you might learn a new idea or advance their own thinking while assisting someone else, “you can also learn from their strategies” (Miley, Spring, 07:46). The other half were about the *enjoyment of helping others* (10), “it just feels good (Lauren, Fall, 10:59).

Norms (19). Most explanations in this category were about the norm of *doing one’s own work first* (15). Students discussed the importance of understanding and completing the problem before helping others, “cause if it's a problem, and they're asking you to do it, you just don't go straight to go help somebody… you get it done with first, then you go help somebody else” (Jason, Final, 10:35). This group of explanations voiced a desire to help tempered by a firm belief that you could not do so until finished with your own work. Eight of these explanations were also associated with a low rating, indicating that helping was not a high priority in relation to taking care of one’s own work. A few explanations (4) cited collaborating as a norm of the classroom, including the idea that partners should work together, share their ideas and converge on the same answer.

Values (16). About half of these explanations were about the value of *being a good citizen* (9) by helping other students keep up and not letting them struggle alone. The other half of the explanations were about *sharing your learning with others* (7). There was focus on the learning and growth of others, the passing of knowledge, and the ways that they helped so that their fellow student could learn and think for themselves vs. just giving them the answer.

Summary. Explanations were divided between personal benefits, including feeling good and getting ideas through helping, norms of helping yourself first, and values of taking care of classmates and sharing your own learning. The fact that norms and values were associated with mixed ratings suggests that this category was tempered by a need to take care of their own learning, and while students wanted to help, this was not a primary goal during problem solving.

Chapter Summary

The results of the analysis reported in this chapter indicate that the four practice-linked goals were at least somewhat important to the majority of students in the class. Analysis of case-study students' explanations for goal ratings revealed that students interpreted the goals in varying ways. Across students' explanations of their goal ratings, several themes emerged that spanned the different goals. These themes show that students often think of the goals in similar ways, and that students' goals are best understood when looked at together and as part of students' larger narratives about what it means to do mathematics. Overall, personal benefits dominated student explanations for students' goal ratings. For the goals "valued contributor" and "looking smart", the largest category of explanations were social reasons, closely followed by personal benefits. These results together demonstrate the complexity and variability of students' goals, the interconnection between goals, and the connection of goals to students' larger narrative about what it means to do mathematics.

CHAPTER 5

CONNECTIONS BETWEEN GOALS AND SITUATED ACTIONS

The two main aims presented in this chapter are to explore connections between students' goals and situated actions while problem solving, and to explore the context-dependence and variability of these actions. In the first section I present a framework that connects students' goals and actions while problem solving. In the second section I illustrate how different partners affected a students' actions by comparing one student's experience with two different challenge partners.

The results presented in this chapter are meant to be exploratory. While identifying actions during problem solving that are suggestive of students' goals is relatively straightforward, determining which goals are actually important is much more difficult. One of the arguments of this study is that multiple goals are important to students at any one time, and likewise, while problem solving, their situated actions are suggestive of the importance of multiple goals, as shown in Table 6 below. As shown in Chapter 4, for many students (including the case study students) most of the 7 goals in this study were at least somewhat important. At any one moment during problem solving, some of these goals might be highlighted through their actions, and others may be shown dark, because of lack of evidence of the goal's importance. But to say that the goal is *not* important at that moment is not accurate. It is more likely that a particular goal is back-grounded because of another goal or because of the situation the student is in. Below I make inferences about how certain situated actions suggest the importance of certain goals in the moment by drawing on my knowledge of the students from spending time in the classroom for a year. One of the main results of this analysis is that there is often little direct evidence of students' goals in their actions. Therefore, inferences must be tentative.

Nevertheless, when comparing code counts between students and between partnerships, the analysis does *point* to interesting patterns that can be further explored through closer analysis of interactions during small group work, as presented in Chapter 6.

Analyzing students' actions during problem solving

As discussed in Chapter 3, in order to determine students' situated actions, I analyzed video of four class sessions in which students worked on a challenge problem that I designed with the classroom teacher. For each session I had 4 videos each focusing on a different pair of case study students. Focusing on these sessions allowed me to compare all eight case-study students working on the same challenging problem at the same time. In addition, the fact that they were paired with different partners each time allowed me to examine interactions between different partners, in contrast to regular work sessions in which they worked with the same assigned table partner each time.

I chose to focus on students' decisions during beginnings, encountering trouble and finishing the problem, as these were the times that their goals were most clearly revealed through the **decisions** they had to make about their work. I called each occurrence one of these phases an **event,** bounded by the emergence of an issue, decision or trouble and its conclusion. The first step in the analysis was to identify these events in the transcript. Then, within these events, I focused on students' **actions**. Actions are defined here as a student's statement or series of statements that form a response to an issue, decision or trouble. For example, one type of event was one student in the partnership being lost or stuck. The three actions that I coded for this event were the partner giving an explanation, the partner giving an answer, or the partner ignoring the request. This analytic process allowed me to look at events holistically, as a play by play of students' situated actions unfolding across the session, as well as to categorize and aggregate the occurrence of specific types of actions.

By looking at students' actions at decision points, when they have to decide which direction they are going to take or how they are going to respond to something in their environment, I hoped to see students' goals expressed in the classroom context. As discussed above, this approach did not give me information about *all* of a student's goals at any moment in time, or even comprehensive information about their goals during a problem solving session. However, it did give me an indication of how students' goals might show up while problem solving, which I could then compare with the goals that they said were important during their surveys and interviews (as discussed in Chapter 6).

Connecting students' actions to their goals

The next step in the analysis was to make connections between students' actions during problem solving, and the practice-linked and performance-based goals of the study. Because I did not ask students directly about their goals while they were working on the challenge problems, I use inference to connect their actions to which goal was important at that moment. In Table 6 below, I present my inferred connections between students' actions while problem solving and the practice-linked and performance-based goals in this study.

When connecting students' actions and goals, I first thought about what kinds of actions I would expect to see to indicate the possible importance of a goal while problem solving. I then took into consideration the way that students had spoken about the goals during interviews. For example, while "learning for understanding" was intended to probe the importance of understanding the mathematics, for some students it was important in terms of understanding the teacher's expectations for the assignment. Finally, I looked at the actions that were evident when analyzing video, and thought about how these related to the goals. In some cases the possible connection was relatively clear. For example, when a student discussed the different possibilities of numbers and resulting solutions they could use with their partner, the goal of “learning for understanding” seemed important because they were trying to understand how the mathematics in the problem worked. In other cases, the connection was less clear. For example, a student’s action of finishing early was a possible indicator of two goals – “being fast” and “feeling smart” in finishing quickly or before others. But it could also have been the case that because the student was efficient and focused, he or she found a solution in less time than the teacher had allocated for the problem. All connections presented here are tentative. I marked actions which suggested the importance of a goal with a “yes”. I also marked actions that seemed to contrast sharply with a reasonable action aligned with that goal with a “no”, in order to show that a goal was explicitly *not* important at that moment. For example, if a student chose to work alone during partner work time, then “being a valued contributor” seemed to be distinctly *unimportant* at that moment.

Table 6

Connections between Practice-linked and Performance-based Goals and Students' Actions while Problem Solving

Coded actions during problem solving		**Code Counts**	**Practice-Linked Goals**				**Performance-Linked Goals**			**Other**
Situation	**Students' situated actions**	**Incidents/ 8 hours total**	**Learning for Understanding ***	**Solving Challenging Problems**	**Having Agency and Creativity ****	**Valued Contributor**	**Being Fast**	**Being Correct**	**Feeling Smart**	**Helping Others**
Making decisions	***Discussion**: **argue ideas*** *– present an idea for a strategy or number and back it up with evidence or logic*	**19**		Yes	Yes	Yes				
	Discussion: share ideas – *share an idea for a strategy or number*	**33**			Yes	Yes				
	Suggestion ignored – *One partner ignores the suggestion of the other*	**18**				No				
Working together	***Work alone*** *–the partners work alone, without checking in with each other*	**23**			Yes	No				
	Pursue different solutions – *partners discuss work but decide to have different*	**5**			Yes					

	solutions									
	Monitor partner -*one student monitors and corrects the work of the other*	**11**							Yes	Yes***
	Easy Hard *–makes a comment about the work being easy or hard, or wanting to make things easy*	**11**							Yes	
	Table: discuss work - *discuss work with table mates including comparing solutions*	**11**						Yes		
	Table: Offer help *– offer help to tablemate*	**3**				Yes				Yes
Going Deeper	***Consider meaning -*** *consider meaning of problem and solution*	**8**	Yes (b, c)							
	Consider possibilities - *talked about different possibilities of numbers and solutions*	**4**	Yes (b, c)							
One person stuck	***Offer answer*** *– give part or an entire answer when asked for help*	**10**				Yes				Yes***
	Offer explanation – *offer an explanation of*	**16**				Yes				Yes

	the problem or own solution when asked for help									
	Ignore *– ignore request for help*	**8**				No				No
	Don't ask *– student who needs help doesn't ask*	**3**	No (a, b)							
	Ask to copy *– student who needs help asks to copy partner's work*	**3**	No (a, b)		No			Yes		
Finishing	***Celebrate*** *–announces or celebrates that they have part or all of a solution*	**26**		Yes				Yes	Yes	
	Check with T: respond *– responds to teacher feedback*	**7**	Yes (a)					Yes		
	Check work *–checks written work before turning in*	**13**	Yes (a)					Yes		
	When called to rug *– uses call to rug as cue to end work*	**4**								
	Before called to rug *– finish work before called to rug*	**8**					Yes			
	Work after called to rug *–continue to work on the problem after the group meets for discussion*	**5**		Yes				Yes		

*(a) Indicates the simplest level of this goal – wanting to understand what was being asked, (b) indicates wanting to understand the mathematics in the problem, and (c) indicates wanting to learn beyond the problem.
** The focus of this goal is "having agency" unless otherwise specified in this table.
***These actions may indicate that the student values helping, even if their help is not entirely appropriate.

Learning and understanding. Case study students talked about this goal in different ways during their interviews. The first way that students talked about this goal was about *understanding the teacher's expectations for the problem*. Another group of comments focused on *understanding the mathematics in the problem* and emphasized the importance of understanding what you were doing in order to learn. Finally, a smaller group of quotes were about *learning that would endure beyond the problem*. I begin by discussing actions associated with the simplest level of the goal.

One action that indicated the simplest level of *understanding the teacher's expectations for the problem* was choosing to check one's solution with the teacher before turning it in (8 occurrences). For example, as Lauren and Will were finalizing their answer during Challenge 4, Will asked Ms A, "What is the object of this exercise… to get the highest you can get?" (33:02). They engaged in discussion with her about how to decide on a realistic answer, possibly indicating that *understanding the teacher's expectations for the problem* was important at that moment. The pair then followed through and finalized their "realistic" solution before sharing with classmates.

Doing a final check of one's work (13 occurrences) was another action indicating that the student(s) wanted to *understand the teacher's expectations for the problem*. As Jason and Aaron were finalizing their work in Challenge 2, Jason checked his calculations before announcing, "I got the answer!" (26:30). Choosing to check his work before turning it in was a possible indicator that this simplest level of the goal was important to him at that moment.

The negative indicators of *understanding the teacher's expectations for the problem* were *not* asking for help when needed (3 occurrences), and asking to copy a partner's answer (3 occurrences), possibly indicating that this level of the goal was distinctly unimportant in the moment. When Will was working with Ravi on Challenge 5, he continually attempted to sync work with his partner, who was working alone. When Ms. A started calling students to the rug, Will was visibly frustrated and said “I don’t want to stay in” (during recess to finish work), and asked, “here let me see yours” (37:15). Will’s choice to diverge from his normal behavior possibly indicates that *understanding the teacher’s expectations for the problem* had become unimportant at that moment and “being correct” had possibly become a higher priority. These actions are most informative possible indicators that something in the moment has caused the goal to become unimportant, often due to frustration or social challenges.

The next level of the goal “learning for understanding” was *wanting to understand the math in the problem*. Two actions, considering possibilities (4 occurrences) and considering meaning (8 occurrences), were both possible indicators that the student wanted to “understand the math in the problem”. Considering possibilities was about exploring the range of numbers that could be used for the problem and their resulting solutions. Considering meaning was about considering what an emerging solution meant in real world terms. Both situated actions show a curiosity and willingness to explore a problem “from the inside”. When Will and Tosha worked on Challenge 3, they considered the different number of volunteers to sharpen pencils. Will said to Tosha, “We could have, this is like... there are two right answers... there are as many right answers for both”. Tosha then brought the conversation back to a practical level and said “yeah, so let’s pick a random number. That’s what I always do” (16:05 - 16:12). This interaction highlights the importance of both students’ participation in considering possibilities to move the conversation forward. While Will seemed interested in considering possibilities, Tosha was more focused on choosing a number and beginning work on the problem, possibly indicating that this level of the goal was important to Will but not Tosha at that time.

The action considering meaning (8 occurrences) was slightly more frequent. One example was when Will and Lauren discussed whether their answer is "realistic" with Ms. A at the end of Challenge 4. As they decided how many slices to put in each lasagna, Ms. A asked, "realistically, what do you think is the smallest number of people that you need to eat the 10 lasagnas?" Will responded, "probably like 20" and Lauren said, "ten if they're all very hungry men, 20 if it's just a moderate mix" (34:09). While Ms. A supported the conversation, it was Will and Lauren's *choice* to initiate and follow through with this line of thinking that indicates the importance of this level of the goal.

The actions "considering meaning" and "considering possibilities" are also possible indicators of the importance of the highest level of the goal "learning for understanding", which is *wanting to learn beyond the problem*. As students engaged in these actions, they were not only building understanding of that specific problem, but of the underlying mathematical patterns and relationships of the work they were doing. As Will and Lauren considered the meaning of the number of pieces in each lasagna, they were also exploring how the number of divisions to a unit influenced its size, a core concept of fractions. While neither of these actions are *always* indicators of wanting to learn beyond the problem, they are the strongest indicators that appeared in the data set.

As with the simpler level of *wanting to understand the teachers' expectations for the problem*, *not* asking for help when needed, or asking to copy an answer, are possible indicators that *understanding the math in the problem* was distinctly *unimportant* to a student at the moment, often due to frustration or social dynamics.

Solving challenging problems. The goal "solving challenging problems" was the most difficult to observe through students' situated actions in collaborative problem solving. There were only two actions that were possible indicators of this goal - staying at the desk after the group met to finish work (5) and celebrating when finished with part or all of a solution (26).

Staying at the desk to work after the group met on the rug (5 occurrences), was one action that possibly indicated the importance of this goal. For 3 out of the 4 challenge problems, Miley stayed longer at her desk to finish her work. When I asked her why she stayed, she said “because I didn't really have a good one yet, so I just, well I just wanted to do a bunch of strategies and a bunch of ways to do it, so I did that” (Winter 26:48). Here Miley’s choice to stay was a possible indicator that solving a problem to her satisfaction, and ideally with several solutions as she says, was an important goal for her at that moment.

Celebrating when finishing part or all of a solution (26 occurrences) was the second possible indicator of the importance of this goal. As Tosha and Amaya finalized their solution during Challenge 2, the stamp problem, Tosha bounced in her seat and said, “Ok. There we go. There we have our answer” (23:00). This action was a possible indicator that solving a challenging problem was an important goal at that moment.

Having agency and creativity. It was difficult to see evidence of students’ valuing the part of the goal “having creativity” while problem solving, without them explicitly saying so, with the exception of Miley who wrote “creativity” on her paper and presented several creative ways for solving the lasagna problem. When I asked her about her multiple strategies, represented with lively drawings, she said, “I think I was doing it for fun …and it said be creative … or explore different numbers of guests” (Winter, 28:13) Her work and her comments are possible indicators of the importance of “being creative” as she worked on this problem.

In contrast to the single action possibly indicating the importance of “being creative”, there were several actions that possibly indicated the importance of “having agency” while problem solving. “Having agency” was about choosing how to approach the problem – both the numbers used (which often determined difficulty) and the strategy. One set of actions that indicated the importance of this goal was the way students engaged in discussion with their partner. The strongest of these was arguing for ideas (19 occurrences), in which a student presented an idea and backed it up with evidence or logic. As Amaya and Jason began Challenge 3, Jason said, “I'm doing 28 times 2”. Amaya then said “it's 28 x 3 cause 2, 1 … so it would be 3” (16:25). Amaya not only put forward an idea but backed it up with logic (combining the 2 plain and 1 fancy pencil would result in multiplying the number of students by 3), possibly indicating that she wanted to exercise agency in deciding what strategy to use.

The next type of action for student discussion was sharing ideas (33 occurrences). Here a student simply put forward an idea about a strategy or number to use. In the following excerpt, Blake and Will were deciding how many letter stamps costing 50c each to use in the stamp problem:

1 Will: (to Blake) ok so how many 50's should we do?
2 Blake: can I write mine down
3 Will: let's just do, like, a lot
4 Blake: 2...
5 Will: but you can’t go over 20, so let’s get.
6 Blake: how about 19
7 Will: no, cause we already have -
8 Blake: 17

What the above excerpt shows is that both actively presented ideas about which number to use, although Will seemed to be considering the constraints of the problem more than Blake. However, each were actively putting forward ideas about what number to use in solving the problem, which was a possible indicator of the importance of “having agency” in their work.

One way that students could exercise agency in a partnership was to collaborate. Another was to withdraw from working with their partner. One possible indicator of this manner of exercising agency over one's work was working alone (23 occurrences) during what was expected to be a partner work time. This meant that at least one of the partners moved ahead without consulting or checking in with the other. This happened throughout Blake and Lauren's work together on Challenge 3. Lauren worked alone until she finished her challenge. Blake also worked alone, but seemed to need assistance. Therefore the action of working alone was a possible indicator that "having agency" was important to Lauren but not necessarily to Blake at that time.

Sometimes partners actively worked together but chose to have different solutions (5 occurrences). This action was also a possible indicator of the importance of "having agency". At a certain point while Miley and Lauren worked together on Challenge 2, Miley became frustrated and decided to use her own strategy and numbers. Both working alone and choosing different solutions were possible indicators that a student chose to exercise agency by withdrawing from work with their partner.

Like "learning for understanding", asking to copy one's partner work was evidence that the goal "having agency" was *not* important at that moment.

Being a valued contributor. This goal referred to contributing ideas and sharing strategies with partners or table mates in small group work, or with the class and teacher during whole group meetings. The main possible indicators of the importance of this goal were the different ways of engaging in discussion with a partner, discussed under "having agency" above. Just as with "having agency", arguing for an idea was the strongest possible indicator of the importance of the goal. The actions of sharing ideas, discussing ideas with the table and offering to help a table mate were also possible indicators of the importance of this goal. Finally, responding to a partner's request for help was a possible indicator of this goal. Although giving an explanation was more helpful to the other student, giving an answer also indicated that the student wanted their ideas to be valued and heard. In Challenge 2, Lauren and Miley were working on the stamp problem. After 18 minutes of work time, Miley said she still didn't know what to do. Lauren read the directions to her again, and suggested a possible solution,

> So like you could do, 15 fifty cents and then like I don't know, eight twenty five cents or, twenty five twenty five cents and three fifty cents. …after that you just add them altogether. Like the, how much it costs. Like this one was \$1, \$2, \$3, \$4, \$5, \$6, \$7, \$8, \$9, (pointing to rows?) and so on (27:31)

Here Lauren gave both an explanation and solution, possibly indicating that "being a valued contributor" was important to her in that moment.

Several actions suggested that this goal was *not* important at the time. Ignoring a partner's suggestion (18 occurrences), working alone (13 occurrences), and ignoring a request to help (8 occurrences) each showed passed-up opportunities to contribute or build on ideas and are therefore possible evidence that the goal of "being a valued contributor" was *not* important at that moment.

Feeling Smart. In their interviews, case-study students discussed two different kinds of "feeling smart" – internal confidence, and feeling smart in relation to others. Actions that showed internal confidence were difficult to observe through students' situated actions. The next chapter explores students' own explanations of how their actions reflected the importance of internal confidence to them. "Feeling smart" in relation to others was slightly easier to observe. Monitoring and correcting one's partner (11 occurrences), making statements about the work being easy (11 occurrences), and celebrating finding an answer (26 occurrences) were all possible indicators of the importance of feeling smart in relation to others (although celebrating could be about internal confidence as well). As they began the lasagna problem, Miley and Jason were discussing how many guests to feed with the 10 lasagnas. Jason proposed that they could

50 people, to which Miley said incredulously “people?”. When Jason kept writing Miley said impatiently, “how many people?”, to which he responded, “it's really easy, you just like dividing them among 1” (27:07). In this excerpt Jason seemed to use the statement “it’s really easy” to position himself as smart in relation to his partner, and perhaps prevail in the discussion of how many people to use. This action of saying the work was easy was also a possible indicator of the importance of “feeling smart” in that moment.

Being Fast and Correct. As discussed in Chapter 4, students differentiated between “being fast” and “being correct” in their explanations of the importance of this goal, so I continue this distinction in the analysis below.

Only one action, finishing before being called to the rug (5 occurrences), showed direct evidence of the importance of “being fast”. During Challenge 4, Amaya and Aaron were checking their work near the end of the work session. They confirmed their “biggest” and “smallest” number of guests and then Amaya said, “yes ok (turning her paper back to the front) let’s go play math games” (39:54) and left before the teacher called them to the rug. This possibly indicated that “being fast” for the purpose of finishing and getting to math games was an important goal to both of them at that time.

In contrast, there were several actions that possibly indicated the importance of “being correct”. One was when a student asked for an answer from their partner (3 occurrences). Here being correct seemed prioritized over other goals such as “learning for understanding”. In this data set, students only asked for an answer after making multiple attempts to understand the work of their partner. After Miley and Jason had worked together for 10 minutes on Challenge 4, Miley became frustrated and asked, “ok, how about we use your work and then I'll explain it, I don't understand mine” (35:53). Another action that was a possible indicator of the importance of “being correct” was “celebrating” after getting part or all of an answer on their solution sheet (26 occurrences). As mentioned above, it is important to examine the context to determine whether the student just wanted to be “correct” or was excited to have a viable solution. As Jason and Miley worked on Challenge 4, Jason announced, “I got my answer, it's 80” (29:26). This possibly indicated that finding a “correct” solution was important to him at this moment, although he could also have been excited to have solved the challenge and come up with his own solution.

The other three actions that were possible indicators of the importance of “being correct” occurred when students took steps to complete their solutions. Checking with the teacher and responding to feedback (7 occurrences), checking one’s work (13 occurrences), and staying to work at the desk after the group had gone to the rug (5 occurrences) were all possible indicators that the student placed importance on having a “correct” or viable answer, although all three actions showed evidence of the importance of practice-linked goals as well. On Challenge 3, Lauren and Blake checked their work with Ms. A. She asked them to do “more than one thinking of how many volunteers” (26:37) They went back to their table and worked on additional solutions. This action was a possible indicator that “being correct” was one of the students’ goals at that moment. As Tosha and Amaya finished their work on Challenge 4, they counted their pictures to confirm they had 16 stamps, checked that they had added the number of stamps correctly, and then co-constructed their explanation, confirming that they “put them in rows that would equal a dollar” (30:43). Similar to the action “checking with the teacher and responding to feedback”, this action possibly indicates that “being correct” is at least one of the goals important to the student at that moment.

Helping Others. There were several actions that possibly indicated that helping others was important to a student during an event. Two of these actions, offering an answer to a partner's request for help (11 occurrences), and monitoring and correcting the work of a partner (11 occurrences), showed an undeveloped understanding of how to support the learning of others. While these actions are "helping" in the sense that the partner received feedback and ideas, they included a power differential and positioned the person offering the help as "more knowledgeable". In this way, these actions seemed to possibly indicate the importance of helping in terms of being positioned in relation to others. During Challenge 3, Lauren checked in with Blake after she had finished her own solution. He said "I'm trying to figure it out". Lauren then said, "I did 2, 4 and 10 – you can do the same thing and I'll tell you the answers". (30:31). While Lauren seemed to be offering help to Blake so that he would have a solution by the end of the class, giving him the answers was not helping him advance his own learning or problem solving skills. Similarly, monitoring a partner's work (11 occurrences) could be considered a possible indicator that the person wanted to help their partner. As Tosha and Will checked their work on Challenge 3, Tosha offered to show Will how to do the algorithm. She then led him through the steps, (without explaining what was happening), corrected his paper and even wrote some steps out for him. Like the action "giving an answer", the action "monitoring a partner's work" can be considered a possible indicator of wanting to help the other student for the reason of being positioned as more knowledgeable.

The actions that possibly indicated the importance of helping in a manner more consistent with the classroom culture were offering an explanation in response to a partner's request (16 occurrences), or offering to help a table partner (3 occurrences). As Miley and Jason worked on Challenge 4, Miley asked, "wait, what? Is that 60 people or 60 lasagnas?" Jason responded, “We're trying to get to 60, we're trying to divide 10 upon 60 so - so we have, that's 20 lasagnas I have” (26:07). Here Jason offered an explanation of his own current understanding of the problem, in which he appeared to be trying to feed 60 people with 10 lasagnas. His action of pausing his own work to explain his thinking to his partner was a possible indicator that the goal “helping others” was important to him at that moment.

While giving an explanation occurred within partner work, helping others at the table required the student to leave their partnership to respond to a student across the table. During Challenge 3, Jason (who was working with Amaya) asked Lauren, “I'm trying to figure out 84 divided by two”. Lauren got up and went around to see his work. He then announced “42” and Lauren nodded in acknowledgement, before returning to her chair (23:27). Her willingness to get up and help a peer was a possible indicator of the importance of “helping others” at that moment. It is interesting that Lauren voluntarily helped Jason, when she had virtually ignored her own partner Blake during the work session. The complexity of who students help, how and when, is further explored through the case of Lauren in Chapter 6.

Ignoring requests for help (8 occurrences) showed that the goal “helping others” was *not* important to the student at that moment.

Summary. The decisions that students made while problem solving were categorized as different types of actions in response to situations such as encountering trouble, deciding which strategy to use, and responding to a partner’s request or help. It was difficult to determine students’ goals only by observing their situated actions in problem solving. In this section I have identified actions that were possible indicators of goals, and these actions were often possible indicators of multiple goals. In addition, when being rigorous about identifying solid evidence of students’ situated actions, there were surprisingly low counts across a dataset of 8 hours of small-group work. As mentioned in the beginning of this chapter, this analysis is meant to be exploratory and the connections I suggest between goals and situated actions are tentative. One of the findings of this part of the analysis is that, even in a progressive classroom, there is often little evidence of the four practice-linked goals in students' problem solving. In the next chapter, I more fully explore how students' actions in context can be used to understand the importance of different goals at different moments.

The impact of partnerships on students' goals

In this section I discuss how partners can affect each others' situated actions while problem solving. I begin by discussing the case study students' comments during the third interview in which I asked about how they thought different partners affected the ratings they gave each of the goals during trouble. Their responses were as follows.

Aaron – If he was working with his best friend, his goals wouldn't change but he would ask for more help. "*Like if it was a super duper challenge, I would just ask him for a little help and say, "hey can you help me on this one, it's sorta challenging for me" and he would just go like "sure" and then like 4 seconds later he would like give me a hint"*.

Amaya – If she was working with Tosha (someone focused vs. someone not focused) her goals wouldn't change but she would get done fast and have a really good discussion. "*Tosha would have these really good ideas and we would be done like really quickly … because like, we talk it over pretty quickly,… like she'll say something, and then I'll get it, and then I'll say "oh actually this is a really good idea, we could do this" and we'll be done, you know what I mean?"*

Blake – If working with someone who thought differently, his goals wouldn't change. He added that he might learn from them but he also might run into trouble if he didn't understand their work. "*Yes and no. Because I'm going to see how they do it, and maybe I can learn from them but if they don't do it in a way I get, I'm going to, I'm going to, run into trouble"*

Jason – If working with a friend, his goals wouldn't change but it would relieve the pressure of resolving trouble. *"That makes it fun for me when I'm doing it with somebody I like .. and it makes me not have a lot of pressure on me when I'm in like ..a challenging (problem)".* When working with someone who thought differently, the goal "learning for understanding would change from 2.5 to 4 .*"Cause you're making a new strategy, you're making a new strategy if you compromise it …. yeah you would (think) about how they think the strategy should be, so you're going to learn for understanding"*

Lauren – If working with a good friend, there would be no change in goals but they would "work really well together". If she was working with someone who thought differently and was not as sure about the math, then being a valued contributor would go from 2 to 4. "*There would be a lot more discussions and contributing, so a 4. I'd say it's a 4 because, I would have to explain… make them not confused… and I would have to make them understand what the problem is asking, or what it wants, and then we can get it"*

Miley - If she was working with her best friend Ailee, "helping others" would become more important, from 1 to 4, "*I think helping others would be more important when you're working with (your friend)".*

Tosha – no change for working with different people

Will –Initially he said that if he was working with someone who thought differently, "being a valued contributor" would move down a little, but then said it would stay the same because, "*if they, think a totally different way than I think, then… actually … we can work it out, so I think it's still the same".*

In summary, when working with someone who was a good friend or they got along with, students' comments suggest that working with a friend was more comfortable and led to more discussion, interaction and helping. When working with someone who thought differently, the comments suggest that students saw working with someone who thought differently as potentially challenging but also a learning opportunity. Working with someone who did not understand the math as much as themselves could be either a burden or an opportunity to help, depending on the student's own goals.

Changes in action codes for different partnerships

In this section I use the case of Miley to illustrate how working with different partners resulted in different types of situated actions during problem solving. Generally, Miley tended to take a more passive part in decision making with partners. She either participated *with* her partner in making decisions (vs. leading), or her partner made decisions and she asked for help. She often asked for help or expressed that she did not understand what the other person was doing, and only pursued her own solution (different from her partner) when she had exhausted efforts to work together. Miley's situated actions that varied by partnership are presented in Table 7.

Table 7

Miley's' Situated Actions That Varied by Partnership

Code	Challenge 2 Lauren	Challenge 3 Aaron	Challenge 4 Jason	Challenge 5 Tosha
Work alone	3	0	9	0
Ask for help – receive explanation	2	0	2	0
Ask for help – receive answer	2	1	0	0
Suggestion ignored	1	2	2	0
Request for help ignored	2	0	1	0
Different solutions	2	1	0	0
Argue ideas	0	1	1	2
Share ideas	0	0	0	2
Agree to idea	0	0	1	1
At desk until done	1	1	1	0

I chose to further explore Miley's work sessions with Lauren and Jason based on the differences in situated actions shown in the table above, in particular working alone, receiving an explanation in response to a request for help, having a request for help ignored, and arguing for ideas. While the differences in counts may seem small, as shown in the analysis below they often point to patterns that can be more fully seen through closer analysis of students' interactions.

Working with Lauren. Miley and Lauren worked together on Challenge 2, the postage stamp problem, in which students had to choose how many 25c and 50c stamps to buy with $20. As shown in Table 7 above, Miley's work with Lauren was marked by more requests and receipts of assistance than her other partnerships. In addition, the girls decided on different solutions, which happened with Aaron but not the other partnerships. Miley and Lauren started out working on the problem together. Over time, however, Lauren began to work on her own and Miley seemed stuck, partly because she was trying to coordinate her work with Lauren, and partly because she did not seem to know how to solve the problem. After Lauren finished her own work, she offered Miley possible solutions. Eventually Miley decided to do her own solution, and stayed after Lauren and the group went to the rug to complete her work.

One of the features of Miley’s work with Lauren was that attempts to work together, including suggestions and requests for help, were often ignored. In the beginning, Miley made a suggestion to do 3 postcards, but Lauren did not respond and proceeded with her own work. After this Miley asked for help with a calculation, and Lauren again did not respond.

Miley: What is 19 minus 75? $19 minus 75 cents? (no answer from Lauren)
Lauren: (to self) ok now they have 19, so (paper sound)

Miley was then harassed by the male partnership across the table (18:24). Notably, Lauren did not come to her defense. Miley then asked Lauren for help with “100 divided by 75” (21:14). Lauren asked her to “hold on”, then proceeded with her work and did not come back with a response. After a few more minutes (23:40) Miley asked for a new paper from the teacher so she could start over. Lauren noticed this and instead of discussing possible ideas with her, offered her an answer.

Miley: I'm starting over
Lauren: Why are you starting over? (looks at back of her sheet)
Miley: so what do I do? I don't get what you did
Ms. A: we are stopping in about 4 minutes
Lauren: Like, just put fifteen... Wait no... what's half of fifteen? (thinks)
Miley: What?
Lauren: (thinking)
Miley: Half of fifteen?
Lauren: 13 .... plus twelve and a half. Wait half of fifteen, oh...
Miley: I don't know .. (can't hear)
Lauren: Half of fifteen
Miley: I don't get what ...
Lauren: No, Wait why am I asking what's half of fifteen? (kids are going to rug)
Here just put fifteen (drawing on her sheet)

In Line 3, Miley makes an indirect request for Lauren to explain her work, but instead Lauren offered her an answer (Line 5), different from what she had on her own paper. Again Miley asked questions in Lines 8 and 12 which were also ignored. A few minutes later Miley again said she didn’t know what to do. This time Lauren offered an explanation in the form of rereading the directions, repeating her suggestion of doing 15 50c stamps.

Miley: I don't get what I'm supposed to do.
Lauren: Here so, (reading) "MSA wanted to buy stamps, she had $20 to spend". So like you could do, 15 fifty cents and then like I don't know, eight twenty five cents or, twenty five twenty five cents and three fifty cents.
Lauren: After that you just them add them altogether. Like the, how much it costs. Like this one was $1, $2, $3, $4, $5, $6, $7, $8, $9, (pointing to rows?) and 7 so on.
Miley: (writing - Lauren watches)
Lauren: Write ten fifties and three twenty fives. Add how many, how much in dollars and cents that all equals. Then write that at the bottom.

In this excerpt Lauren does offer an explanation of how to do the work, although the numbers she suggests are different than her own work. Miley still seemed to be confused and asked Ms. A about the point of the work.

Miley: (to Ms. A) we need help on this
Lauren: 5 dollar, 6 ... (bouncing pencil on page as she works) 13 dollar .
Miley: I don't get what we're supposed to do ....are we supposed to get as many stamps as possible?
Lauren: Within $20.
Miley: We have $20
Lauren: You don't have to use up all $20. You can just do $8 if you wanted.
Miley: I want to use up all $20 to get all the stamps.
Lauren: That's going to take a while.
Miley: Fine, I'm only going to use 10, $10. $10 ...

Part of the issue here seemed to be Miley’s mixed relationship with the goal “solving challenging problems” that made her averse to taking on more challenging solutions such as what Lauren suggested (Lauren had chosen 34 50c and 12 25c stamps to equal $20). Although Miley said she wanted to use all $20 in Line 8, when Lauren said “that will take a while” (Line 9), Miley quickly changed to using $10. She seemed to want to pursue a “simpler” solution and could not convince her partner to do so. After Miley’s announcement that she was going to use up $10, Lauren went to the rug and Miley stayed 10 more minutes to complete her work. Her final solution was using $5 to buy 5 50c and 10 25c stamps.

Summary. In her work with Lauren, Miley did not have many opportunities to have her ideas considered or to discuss the work itself, as Lauren worked relatively independently throughout. When Miley did ask for help, she first received possible answers that she did not understand, and only got an explanation as the group was going to the rug to meet. On the other hand, Miley chose to wait until the end of the work session to pursue her own solution which was more comfortable and she could understand.

Working with Aaron. Miley and Aaron worked together on Challenge 3, the pencil problem, in which they were asked to calculate the number of plain and fancy pencils needed for 28 students, choose a number of volunteers to sharpen pencils and calculate how many each would sharpen. Miley’s work with Aaron was in marked contrast to her work with Lauren. Their interaction was marked by friendly banter, and they worked together until they both had written solutions. Neither ignored each other’s suggestions or requests for help. Aaron went to the rug 5 minutes before the teacher called them to the rug, and Miley stayed 5 minutes after the group to finish her solution for Part 2 (volunteers).

As Miley and Aaron began work, they discussed the numbers they were going to use. Miley made a suggestion about Aaron’s calculations, and then explained her own thinking.

1 Aaron: this is what I made up (looking at own paper). So this is, it would be 60
2 Miley: but
3 Aaron: no I'm not ... I'm adding both of them together. So
4 Miley: But you're not supposed to do that. Did you read it?
5 Aaron: and now someone tells me
6 Miley: I didn't know what you were doing (watching him erase)

Miley: ok so basically, 28 kids, equals one fancy pencil
Aaron: (speaking while writing) 28 equals 28. So that's how many fancy pencils there are?
Miley: mm hmm (nods)
Aaron: ok, and so what's the second one? The second one's going to be harder
Miley: yeah. How many "new" pencils total. So then,

Miley's correction occurred in Line 4, which Aaron accepted in Line 5. Miley then explained her thinking in Line 7. The pair coordinated their work by checking frequently with each other about their strategies and emerging solution.

As they moved on to calculate the number of plain pencils needed, Aaron observed Miley using icons and suggested that she could use the grouping strategy instead to calculate 28 x 2, which she considered and then accepted.

Aaron: you know there's a faster way?
Miley: Yes I know but
Aaron: you can just do 2, and put 28
Miley: What? (both laugh)
Aaron: now you have to put 2 in all of them
Miley: (nods) yeah

Aaron presented his logic in Line 1 and his idea in Line 3, and further explained his idea in Line 5. This appeared to be understandable and useful to Miley, as she verbally accepted the idea in Line 6 and the strategy appeared on her final written solution.

As they were finishing calculating the total pencils needed, they checked and discussed their solutions.

Miley: 6 plus 6 is 12, plus 2 is 14. Great
Aaron: 14? (laughs) Miley, do the math please.
Aaron: it is, (excited?) I found out what the answer is . Be quiet (to Miley)
Miley: yeah it's 14 dude.
Aaron: the answer's not 14.
Miley: yeah it is. I got it
Aaron: 56 plus 28 is not
Miley: yes 84! (celebrates) yeah
Aaron: it's not 84
Miley: yeah it is

11 Aaron: (shows paper) look. 56, plus 28, fif- oh yeah, 84
12 Miley: you just got dissed (can't hear)
13 Aaron: I'm finished! (celebrates, loud) oy, oy oy

There is some initial confusion as Miley was adding "6" and "8" from 56 (2 x 28) and 28 to get the total pencils needed in line 4. Aaron then argued in Line 7 that this didn't' make sense. Miley continued her calculation and announced in Line 8 that she had "84". Aaron argued again in Line 9, but then they compared their papers and agreed on the answer, ending with teasing and celebration.

Summary. In her interaction with Aaron, Miley was both confident in asserting her own ideas and open to considering his suggestions. In contrast to Lauren, Aaron was an engaged partner who listened to her questions and discussed ideas with her. Miley was also in the position of helping Aaron on several occasions, instead of being the one asking for help throughout as she was with Lauren. Together, these interactions show how a partner can affect the situated actions that a student takes while problem solving. Partners affect the opportunities that students have to offer and receive help, discuss and share ideas, and work collaboratively. As demonstrated, Miley engaged in different types of actions in the different partnerships. Although the differences in counts in Table 7 seem small, these play out in larger patterns when examining the full interaction between students.

Chapter Summary

Students engaged in different types of situated actions while problem solving which were marked by their decisions in response to issues that arose in the course of activity, such as being asked for help, deciding how to proceed in a problem with a partner, and deciding whether they were finished. In this exploratory analysis, these actions are used as possible indicators of students' practice-linked and performance-oriented goals. When applying rigorous criteria to situated actions within small group work on problem solving, the counts for actions were relatively low across an 8 hour data set. While the connections between situated actions and goals are suggestive and tentative, they can point to more interesting patterns that can be explored through closer analysis of students' interactions, as presented in Chapter 6. Finally, a student's relationship with their partner affected the types of situations and therefore actions that a student engaged in, as shown by Miley's engagement with two of her challenge partners.

CHAPTER 6

THE MEDIATION OF EXPRESSION OF STUDENTS' GOALS TO SITUATED ACTIONS IN PROBLEM SOLVING

There is often a gap between students' practice-linked goals – the motives that shape their activity in mathematics– and their situated actions in problem solving. As shown in Chapter 4, students hold a complex array of goals that they think about in varying ways, which must be translated into actions during problem solving as explored in Chapter 5. The expression of their goals into actions while problem solving is further complicated with the many factors that mediate the "expression" or realization of these goals. In this chapter I focus on how two of these factors – students' interpretation of goals and their subsequent positioning of themselves and others– mediated the expression or inhibition of their goals in their situated actions in problem solving. I do so by examining the goal profiles and related actions of four case study students. I use students' comments from their interviews to build a portrait of how they thought about the goals in the study, including the themes discussed in Chapter 4. I also discuss how each student's goal explanations related to his or her overarching narrative about what it means to practice mathematics. I then explore how this led the student to position him or herself and other students in certain ways during problem solving and how this and students' interpretations of the goals mediated the expression of their goals into problem solving actions through analysis of excerpts from the challenge problems sessions.

Interpreting Goals as Performance and Position – The Case of Tosha

Tosha's case demonstrates the power of a narrative that is brought from outside the classroom that clearly influenced the interpretation of the goals in the study. Her narrative about herself and what it meant to practice mathematics was a clear strand throughout her talk about goals and problem solving. Tosha had medium to high ratings of the practice-linked goals and low ratings of performance-oriented goals in general, as shown in her goal profile below.

Table 8: Tosha's goal profile

Goal	Fall	Winter	Spring	Beg	Troub le	Finish
Learning for Understanding	4.3	4.7	4.7	4	3	5
Solving Challenging Problems	5	4.7	4.7	5	1	1
Agency and Creativity	4.5	2.7	2.3	1C 2A	4A 1C	1
Valued Contributor	2	1.85	1.7	5	5	5
Feeling Smart	3.7	1	1	1	1	1
Fast and Correct	1.3	2	2.2	1F 3C	1	1F 3C
Helping Others	NA	3	4	5	2	2

Key: pink is high, orange is medium, blue is low rating on scale of 1 to 5

Much of what Tosha seemed to believe was consistent with the mainstream view of what it meant to practice mathematics – that math is about performance, both in class and in the future. Tosha's approach was a reasonable and competent way to navigate the demands of the classroom, and she experienced success at what she seemed to consider the rules of the game – finding an answer, recording her work, and keeping the teacher aware of her progress. ,When I asked Tosha if and why she valued the goals "learning for understanding" and "solving challenging problems", she said she did, but when I looked deeper it seemed her understanding of these two goals were in the service of performance, rather than the goals in and of themselves. While it is practical and reasonable to interpret these goals this way, the fact that they were driven by performance meant that Tosha pursued them only until she felt she had satisfied her goal for performance. Once she had "an answer", she stopped pursuing, questioning and inquiring the mathematics of the problem in front of her.

In her interviews, Tosha talked about the importance of using mistakes to learn (not valuing "being correct"), trying to understand how the mathematics worked ("learning for understanding"), challenging herself ("solving challenging problems") and learning from her peers ("being a valued contributor"). On the surface her explanations of the practice-linked goals seemed to be consistent with the culture of the classroom. However, her narrative that mathematics was about performance manifested itself in the way she positioned herself and others in her peer interactions. Part of her orientation toward performance was showing that she was doing better than others. Thus as she engaged with fellow students, Tosha had to be the "more knowledgeable" peer. She did not position herself as a receiver of ideas, except from the teacher. This led her to shape conversations with her peers in particular ways, including declining invitations to go deeper with her thinking. In the sections that follow, I first explore how Tosha's narrative about performance appeared in her talk about goals during the interviews. I then show how she seemed to transform her explicit talk about the practice-linked goals towards insuring future performance through situated actions that revealed her tendency to try to position herself socially in the classroom in certain ways.

Emphasizing "moving up". In explaining her goal ratings, Tosha placed a lot of emphasis on the theme of secondary benefits, learning math now to aid in future performance. She spoke of moving forward, moving up and getting smarter. Her explanations of the importance of the goals "learning for understanding" and "solving challenging problems", were in the service of insuring future performance. That is, the way she thought she could achieve future competence, was to challenge herself now. In our first interview, in explaining why "learning for understanding" and "solving challenging problems" were her most important goals, she cited her parents' expectations, "my parents always tell me to push myself… and that way I can be prepared for the next grade… it will help me …when I grow up" (Fall, 01:27). Again at the end of the year, she said, “if I don't really learn for understanding then I won't, I don't think I'll really get that far” (Final, 00:43).

The personal benefit of moving up was also apparent as Tosha talked about “solving challenging problems”, “you'll never go anywhere if you just stay at the same level… you'll have the same kinds of math problems …and you're never good at them, and then, I’ll just stay there, you won't go farther” (Fall, 06:23). Tosha’s attitude toward actual problems in math class was consistent with her ideas about performance but inconsistent with her professed embrace of challenges. The challenge problems in the study were meant to be open to multiple entry points and to invite students to explore different solutions. However, *because* they could be answered quite simply, Tosha did not consider them challenges. As soon as she had “an answer”, her engagement with the problem was done. In her winter interview, she explained why she did not consider the lasagna problem a challenge.

Tosha: Well I was just thinking, I know multiples of 10 and I already knew the answer from the start but I don't think Blake did so I asked him and yeah...

MK: What do you mean the answer... because you were just...

Tosha: Because I knew, there would be, ...each person would get... it's 20 people came, each person would get 1/2 because 10 times 2 equals 20.

MK: But what if he had said he wanted 50 people to come what would have happened?

Tosha: Well I would have known, like 1/5 because 10 times 5 equals 50.

MK: So you already knew basically how... if you give any number what would happen? Ok.

Tosha: And I knew that there would be... that you wouldn't be able to do it if there were like 25 persons (MK: oh) because there's no multiple of 10 and they wouldn't split up equally. (Winter)

Tosha’s comments revealed her search for the “answer”. In Line 1 and 2, she said, “I already knew the answer from the start” and in Line 8 she said, “I would have known” the answer for the number of guests I asked her about. Sometimes students interpret school as a game of figuring out the strategy or concept to be practiced – in this excerpt Tosha seemed to think that the objective of the problem was to practice multiples of 10, as she says in Line 12 in explaining why 25 cannot be an answer, “there’s no multiple of 10”. What this perspective obscured was the interesting math question she had stumbled upon – can 10 lasagnas feed 25 people? She did not pursue it. Her assertion that “I already knew the answer from the start” (Line 1 & 2), revealed her stance that there was an “answer” to be found.

Tosha said that it felt good to solve challenges, “it makes me feel really good when it happens” (Fall, 2:33), and that she liked to challenge herself. However, this also seemed to be in service of her goal of “moving ahead”. The idea that learning more and solving challenging problems allowed you to “do more” and “go farther” is consistent with the way the goals were intended. However there was an edge of getting ahead that was very focused on performance vs. really understanding the math. Performance could have motivated Tosha to engage with the mathematics at a deep level. However, what actually seemed to happen was that her focus on performance and getting ahead trumped the goal of deep understanding and engagement with the ideas that would have served her best in reaching her goal of advancement.

Learning interpreted as “learning the steps”. Another strand in Tosha’s narrative was that math was about learning the steps or “how to do” a math problem. In explaining the importance of “learning for understanding”, she said “if I don't learn how to do it and understand it, when I get older … and I come into an everyday problem... I won't know what to do, like how to solve it” (Winter, 07:50). Her comments about “what to do” or “how to solve it” suggest a logic that if she solved a particular problem then she would encounter a similar type of problem in the future. While mathematical concepts definitely build on each other, there is a subtle difference between developing a flexible “toolbox” and learning a “right” way to do a problem. Truly challenging problems often require a fair amount of interpretation and organization on the part of the mathematician, and students must be prepared to do this work.

Tosha's statements about the goal "having agency" also suggest that she thought of math as 'steps to be learned'. Because her main emphasis seemed to be about performance, self-expression and developing her own tool kit were secondary objectives. Tosha seemed less interested in choosing strategies than with finding the "answer" to the problem. She initially rated "having agency" as very important for the social reasons of standing out and having something to offer, "you might know some stuff that other people don't… so you could help other people and have other people help you" (07:23). By Spring her rating had dropped to not important (2.3) because Ms. A had given them more strategies and she actually preferred to be told which to use, "I liked that better than … trying to figure out which strategy to use." Tosha's lack of interest in deciding how to do a problem for herself is in stark contrast to other case study students, who cited "independence", "efficiency", "interest" and "learning" as reasons to choose their own strategy. Her apparent willingness to surrender agency suggests a problematic stance for her long-term growth as a mathematical thinker. While this approach may work for her in the short term, over time a lack of interest in coming up with strategies and finding different ways to solve a problem will make it hard for her to approach and work with truly challenging problems.

Tosha did acknowledge the importance of agency to problem solving on two occasions. In spring when talking about beginning a problem, while she gave the goal a low rating, she said that deciding how to approach a problem was important, "you need to decide what you're going to do, and I think that's kind of important" (Spring, 28:59). When she encountered trouble, she gave the goal a high rating, and explained, "when you run into trouble, I think deciding how to solve the problem is … more important than starting a problem, because you need to find out how to solve a problem" (28:59). While these comments suggest some awareness that agency was important for problem solving, choosing different strategies or doing something in a different way still seemed to be beyond her narrative about what it meant to practice mathematics.

Positioning as "better understander". Tosha had a complex relationship with her positioning in relation to her classmates. Her emphasis on "getting far" when talking about the goals "learning for understanding" and "solving challenging problems" was one indication that she valued moving up and ahead. In her discussion of the goals "valued contributor" and "feeling smart", she revealed more about how she thought of herself in relation to her classmates. Overall Tosha rated "feeling smart" as not important. She saw small benefits of having the teacher "know what she could do", but asserted that she liked math because it was fun and helpful, not because feeling smart felt good, "well feeling smart just feels weird to me, the reason why I do (math) is because it's fun and it will help me" (Fall, 03:23). However, in Winter, Tosha said while the goal was not important, she wanted to be the "best understander" and to "feel a step ahead of everybody".

> I don't think about feeling smart, I think about understanding, not really being smart but being the best understander. That kind of is like feeling smart …to me… Because if you're understanding it, I feel like I'm a step ahead of everybody else (9:48).

Here Tosha revealed how performance played out as being seen as "a step ahead". While she rejected wanting to "feel smart", which may reflect the taboo of bragging or being "too smart" mentioned by several students in her classroom, she openly said she wanted to be the "best understander". She seemed to be searching for a way to perform without negatively impacting others, yet her performance was still linked to positioning in relation to her peers. Again in Spring she said that positioning was not important, "I like to learn, and make mistakes, but I don't really like always being the best, like always feeling smart doesn't really feel comfortable to me" (06:27). Her mixed comments reveal a complicated relationship with being positioned as knowing more than others.

Similarly, Tosha had a complex relationship with the goal "valued contributor". Sometimes she said the goal was not important because she thought she should give others a turn to speak and could learn from them, "because, well sometimes I raise my hand too much, and I think it's better for me to listen to other people for what they say, so I can learn from them" (Fall, 04:49). Other times, such as when talking about the specific moments of the lasagna problem, she said that it was her responsibility to help others if she knew more than them.

> Because I always don't want to leave my partner like, blank minded and don't know what to do and sometimes if I think that they're going down the wrong direction I want to have something so they kind of lean to the right direction (Winter, Encountering Trouble, 18:31).

In this excerpt she connected her own contributions to the learning of others, by helping "lean to the right direction", but the way she talked about this positioned herself as knowing more, "I don't want to leave them blank-minded" and "if I think that they're going down the wrong direction". Her positioning is complex because she seemed to believe that knowing more obligated her to help others and not leave them stranded. Indeed as she worked with others she frequently elicited their participation. In the Winter interview, she spoke about the **norm** of making sure everyone got to speak in small groups, "I guess I like other people to have a say in what we're doing" (14:19). She never worked alone or chose a different solution than her partner. However, another manifestation of performance and being positioned as "knowing more" transformed into a lack of obligation to seriously consider the ideas that she invited her partner to share.

Summary of Tosha's narrative. When considering Tosha's explanations of her goal ratings, we see a picture of a student with a distinct orientation towards performance with an assumption that in math there is often a right way of doing things. On the positive side, Tosha seemed to believe that the route toward competent performance ran though doing challenging problems and understanding the mathematics. On the less positive side, her orientation towards future performance limited and constrained how she engaged with challenging problems. It also led to her position herself and others in certain ways during problem solving, as explored in the next section.

Tosha's situated actions while problem solving. In this section, I explore how Tosha's goals were expressed in complicated ways when collaborating with others to solve mathematical problems. In particular, her narrative about performance led her to position herself as "more knowledgeable other" in interactions with peers, limiting the extent that she could receive ideas from them. In the following excerpt, Tosha guided their collaborative work toward using the algorithm to check their solution. Her value of helping seemed to motivate her attempt to create a joint solution with Will. However, her focus on "an answer" and "learning the steps" was translated into leading her partner through the steps without trying to help him understand what they were doing.

The excerpt below is from their work on Challenge 3, the pencil problem. Students were asked to calculate how many pencils were needed so that each student in a class of 28 had 2 plain and 1 fancy pencil, choose a number of volunteers to sharpen the pencils, and calculate how many pencils each would sharpen. Tosha and Will already had a solution of 4 volunteers sharpening 21 pencils each. Tosha then proposed they use the algorithm to check their work. She did not, however, help Will understand the decomposition, instead she focused on how he could record the use of the strategy on his paper.

Challenge 3, Tosha and Will, Finished 2

Tosha: Do you know how to do the algorithm?
Will: Uhh yeah, only for 2 digits
Tosha: Ok you can do this with two digits just to check your work. If you have (writing) times...
Will: (inaudible)
Tosha: because 4 times 20.
Will: oh yeah yeah yeah
Tosha: I don't want to confuse you. ok, take 1, 1 x 4 (pointing to his paper) right?
Will: Yeah
Tosha: and then, but then you have to put a circle, because part of this is going there, like that, so put a circle there
Will: (writes on paper)
Tosha: so that's 84
Will: (inaudible)
Tosha: put the 8 right next to the 6, I mean the x,
Will: (looks at her paper, then writes)
Tosha: now draw a line - x equals zero. So if the x equals zero, (drawing on his paper) No you have to add... If the x equals 0, 0 plus 4...
Will: ohh ohh... you want that there?
Tosha: (writing on his paper) No you add the bottom one. Yeah 84. But since that's kind of hard for you ... Ok so 21...

In this excerpt, Tosha led Will through the steps without explaining what each one meant. She told him what to write and where on his paper (Line 8, 11, 16, 18 and 21) and wrote *for* him at the very end (Line 21). She did not explain why the algorithm worked. The goal was simply to help him "do" it. This, incidentally, was at odds with her own professed desire to "be able to do it herself" at a later point. The "leading through the steps" occurred in interactions with other partners as well. With each partner, she actively shaped the flow of activity by focusing on the decision and execution of a strategy and entry point, phrasing her ideas as suggestions to be "confirmed" by her partner.

Tosha's stance toward performance and finding an answer also seemed related to her declining invitations to go deeper in thinking about a problem with a partner. Earlier in the same interaction, Will and Tosha had just figured out how many total pencils were needed for 28 students to each have 1 fancy and 2 plain pencils. Will started to talk about more than one right answer, initiating the consideration of possibilities, but Tosha firmly brought the discussion back to the practical work of deciding on a number.

Tosha: Ok so we got that we got that answer, now we have the back (reads)
Will: We could have, this is like... there are two right answers... there are as many right answers for both.
Tosha: yeah, so let's just pick a random number. That's what I always do.
Will: But if we really wanted to make it easier we could just pick 1
Tosha: Yeah, yeah.
Tosha: So pick a random number.

Will initiated considering possibilities in Line 2, but Tosha politely declined in Line 4 and instead advocated for her strategy of picking a “random” number in Line 4 and again in Line 7. Her focus on finding an “answer” shaped her interaction with her partner by inhibiting her from further exploring the mathematical concepts, or even acknowledging that there could be multiple solutions to the problem.

Tosha’s positioning seemed to entitle her to disregard her partner’s comments if she could not make sense of them. After watching a video clip in which she had asked Blake how they should split up the lasagnas, she said, “I had no idea what he was saying so I just went with my idea” (Winter, 18:11). In fact, when he did make a suggestion she immediately disregarded his comment and decided on the number and strategy that they would use.

Challenge 4, Tosha and Blake, Beginning 3

Tosha: let's split this, how do you want to split it?
Blake: do you want to split-
Tosha: wait wait, how many guests do we want?
Tosha: (looks briefly) how many guests do you want?
Blake: I honestly don't care, because I'm not sure ...
Tosha: ok, so let's split this in half, we know
Blake: if we split it into 8, we have 16, 24..
Tosha: what do we do if we split it in half?
Blake: we get 2 plus 2
Tosha: here, split every one in half, and I'll (inaudible) (splits in half on paper)
Blake: (does the same)
Tosha: now let's count, will that give us 20, count how many you have

Blake: counts by 2's - gets 20. Yeah

Tosha: 20, that will get us 20 people, right? So that would be ... so if 20 guests came, let's write our answers up here, how about that? (top of page) let's write a number sentence. (writing) if 20 people came (Blake writes too) each person would get a 1/2 of a lasagna

Tosha: (checks Blake’s paper, points out mistake and has Blake erase)

Tosha first asked Blake how they should split the lasagnas (Line 1) but then switched to asking about the total number of guests (Line 3). Blake said that he “doesn’t care” so she proposed splitting the lasagnas in half (Line 6). Blake then suggested they split the lasagnas in 8, and started counting by 8’s (Line 7) but Tosha immediately brought their focus back to her idea of splitting by 2 (Line 8) by asking him questions and directing their work (Line 10 and 13). So although she had invited Blake to suggest an idea, when he did suggest a number and started using a strategy of skip counting, she disregarded it and continued with her own chosen strategy and number.

While working with partners, Tosha sometimes made comments that positioned her as the “better understander” and legitimized her instruction. In the interaction with Will above, first she said, “I don’t want to confuse you” (line 8). Then, she said, “since that’s kind of hard for you” (Line 21 and 22) justifying why she had written on his paper for him. This interaction further points to her positioning as “better understander” and her self-assigned role to instruct other students. Her sense of obligation to work with her partner, as well as her perception of herself as “knowing more”, seemed to combine to lead to this pattern of interaction.

Summary of Tosha’s goal profile and situated actions. Tosha’s focus on math as a performance with steps to be learned and “an answer” to be found, combined with her sense of obligation to help others, manifested in her “instructing” others in the knowledge she herself seemed to pursue. However, this did not result in a productive situation for herself or her partner. She frequently missed opportunities to deepen her understanding, or to learn from working with the ideas of her peers. Tosha's focus on performance also limited how she expressed the goals "learning for understanding" and "solving challenging problems". She only pursued these until she found "an answer", her measure of performance. Tosha's profile also illustrates how a student's interpretation of the practice-linked goals is critical in understanding their disposition. When looking at her goal profile, she appears to be a student with a positive disposition to mathematics. When looking at her goal profile in combination with her explanations, we see a much more complicated array of goals and relationship with mathematics that do not always play out as expected in her situated actions in problem solving.

Focusing on Understanding and Reciprocity – The Case of Lauren

Lauren and Tosha were both students who had high scores on unit assessments and seemed comfortable in math class. Both participated in the "pull-out" group for students who needed extra challenge once a week. However, they contrasted in their relationship with mathematics in terms of what they thought it meant to do mathematics, and how they thought of themselves as mathematicians. While Tosha seemed to think about math as "learning the steps" and moving forward, Lauren seemed to think about it as really understanding how the math worked and focusing on "thinking and ideas". In addition, they contrasted in their sense of obligation and relationship to their peers. While Tosha placed high importance on helping others and seemed to enjoy doing so, she also positioned herself as a "better understander" than most peers. Lauren, on the other hand, focused on internal confidence but rejected feeling better than peers. She also placed less emphasis on helping others and seemed to have a firm belief about the reciprocity of contributions in partner work, that mediated her interactions with peers. Not surprisingly, the girls had quite different patterns of situated actions while problem solving.

Understanding the problem – think, discuss, understand. Like Tosha, Lauren rated the goal "learning for understanding" highly on her surveys, as shown in her goal profile below.

Table 9: Lauren's goal profile

Goal	Fall	Winter	Spring	Beg	Trouble	Finish
Learning for Understanding	4.3	4.3	4.7	5	2.5	1
Solving Challenging Problems	3	4.3	3.7	3	1	1
Agency and Creativity	4	3.7	3.7	3	1	1
Valued Contributor	2	2.7	3.3	2	2	4.5
Feeling Smart	1	1	1.7	3.5	3.5	1
Fast and Correct	1.5	2.3	3	1	1	1
Helping Others	NA	2.5	2.5	1	1	1

Key: pink is high, orange is medium, blue is low rating on scale of 1 to 5

In her explanation of her rating, Lauren emphasized that understanding both *what* the problem was asking, and the *underlying concepts*, was critical before even starting work.

> You have to understand the problem in order for you to actually solve it, and you have to know what you're doing, you have to think, you have to, like discuss… before you can even get started … before you discuss what it means you have to understand it, and you have to think about it (Spring, 8:17).

Lauren's focus on thinking about the problem, thinking about the meaning and discussing the problem with a partner is in contrast with Tosha who only spoke about "choosing a strategy". Lauren seemed to think about the goal "learning for understanding' in a different way. Her orientation was toward a deep engagement with the ideas and process of solving problems, in contrast to Tosha's search for the implicit math to be learned and answer to be found.

Lauren rated "solving challenging problems" as very important in winter but somewhat important in fall and spring. In explaining her rating, she spoke about enjoying challenging problems because they stretched her thinking.

> I like fun challenging problems, and ones with multiple answers, but it just tends to screw me a little, and I like when it does that because I get to think in different directions that I can't really do when it's a straightforward problem …yeah, it pushes me in a different direction in a way, like makes me go to another way of thinking that I would have never thought of when I was doing it straightforward (Lauren, Winter, 32:53)

Laurens' explanation suggests that she saw the value of challenging problems not as a means to improve her performance, but a way to stretch her thinking and expand her understanding of math in new ways.

Another aspect of Lauren's approach to problem solving, also in contrast to Tosha, was her value of having agency in deciding how to solve problems. Lauren said that "having agency" was most important at the beginning of the problem when she was deciding on a strategy.

> I have a lot of strategies, I kind of look at the problem, read it over and see which strategy I have that would fit best with the problem, that would probably make it easier and then I do the problem, and I use another strategy that would fit good with it also and I check my work (Winter, 10:06)

In this excerpt Lauren explained that "having agency" was important for her in order to select the strategy that best suited the problem, and also mentioned efficiency. Later in the interview she further elaborated that, "deciding how to solve problems, like basically starts the whole way you solve the problem" (13:54, Winter). In her comment, she pointed to how the careful selection of a strategy affected how problem solving would unfold. This is in contrast to Tosha who seemed to be looking for a way to solve the problem, without necessarily considering the match between the problem and the strategy she was using. Some of Lauren's language was consistent with Tosha's about trying to interpret what the problem was "about" from the teacher's perspective, "I like to think about what they're trying to tell me " (2:21, Final) and "I have to know what they're trying to tell me or else I'm not going to know how to do almost anything that has to do with math and solving problems" (3:02, Final). In these comments Lauren positioned knowledge as coming from the teacher, and linked understanding what she was supposed to be learning to her future success. However, it took on a different flavor absent Tosha's focus on future performance and her practice of social positioning. Otherwise, Lauren's stance toward understanding was an emphasis on learning new things vs. learning "how" to do things, "I wanted to learn more but I also wanted to understand things ... I wanted to learn new ways that can help me solve what I'm doing" (Winter 13:15).

Sharing ideas and reciprocity. Lauren's relationship with sharing in the group, like many of her peers, was complicated. Although she rated "being a valued contributor" as somewhat important during group discussions, she said she did not particularly like it, "I have stage fright, I'm not a big fan of sharing .. in front of the whole class, I'm quiet and stuff" (Winter, 22:09) or identify with it, "just not that type of person" (Spring, 36:01). At the same time she felt an obligation to share a strategy if it had not been mentioned in whole group discussion, "if no-one says my strategy I will share in discussions " (05:39, Fall) or if she thought she had something "significant' such as her work with Will on the lasagna problem, "I… wanted to share my strategy, I thought it was pretty significant" (Winter, 22:30). Lauren showed awareness that contributing could lead to the development of her own ideas, like Tosha, and she also saw sharing ideas as a way to assist the learning of others. "Participating your ideas can lead to a new strategy or a better way to solve a problem or improvement in your math work and in everybody's" (08:04).

Lauren's discussion of partner work also involved a level of obligation, but whereas Tosha spoke about helping a partner "lean in the right direction", Lauren spoke about sharing the workload, "I can't just be silent while he does all the work" (07:44, spring). Her work ethic for herself and caution in not taking advantage of others indicate that she saw herself as obligated to participate. But while Lauren had a sense of obligation to the community, she also did not feel compelled to help everyone, or at least she abided by the norm that you should get your own work done first. One of the benefits she mentioned to "helping others" was getting ideas and helping herself: “if they tell me, like a strategy that I didn't know, then, when I go over to see their work, and they have something else that I don't know about, I can have another strategy that I can use some other time” (04:31). This expectation that she would get ideas may explain why she participated more with some partners than others. When discussing her work with Will, she said “helping others” was somewhat important because they could “exchange” ideas, “We were just kind of discussing about that and I wanted to help him and he wanted to help me” (Winter, 28:23). These comments suggest that Lauren saw helping as a two way street that should benefit both partners. Although she did not say it explicitly, Lauren may have thought that some students were more likely to be a source of ideas, interesting conversation and assistance than others. This would explain why she did not help Blake, as explored in the situated actions section below.

Summary of Lauren’s narrative. In her explanation of the goals, “learning for understanding” and “solving challenging problems”, Lauren expressed the importance she placed on understanding how the math worked, focusing on “thinking and ideas”, and using challenges as opportunities to stretch her thinking and learn. While she did not identify with or enjoy sharing in a group, she felt obligated to make sure an idea was represented in a group discussion. In terms of working with partners, she stressed “getting your own work done first” before helping others, saw helping as a two way street and seemed to prefer helping and interacting with partners from whom she thought she could learn.

Exploring the problem from within: situated actions. In this section, I explore how Lauren’s goals were expressed in complicated ways in her interactions with peers around problem solving. Lauren’s emphasis on the importance of understanding, thinking and discussing seemed to be most fully expressed in her interaction with Will during the lasagna problem. This relationship seemed to facilitate and support the kind of learning and engagement that Lauren said she valued. Lauren’s willingness to engage and discuss ideas with Will indicates that she considered him someone who thought in a similar way and made equal contributions to discussions. The two were both in the pull-out math group and scored high on unit assessments.

In the excerpt below, Lauren and Will had just begun work on the lasagna problem, in which students were asked to determine the most and least guests they could feed with 10 lasagnas. The pair quickly agreed on 1 person as the least number of guests, although Lauren joked, “I can't imagine someone eating 10 lasagnas” (26:13). Then they moved into discussion of the most number of people.

Challenge 4, Lauren and Will, Beginning 2

Will: Most, now this is where it gets hard
Will: I think you could do all the way to 68, and get 5 billion
Lauren: yeah, I don't think we can do that
Will: (laughing) how about, let's go
Lauren: but we could split them up into pieces, as much as we want, we'll count up all the pieces for 10 lasagnas, that will tell us how many people
Will: oh yeah, I get it, that's a good idea (both smiling and laughing)
Both (drawing and counting)
Will: 6, 7, 8, . 10 trays, right? and then how many slices per tray?
Lauren: If the pieces are not that big ..
Will: how about, well since I didn't make my trays really big, we can't put that many slices, I think making all of them 15 would be nice
Lauren: you don't say we're going to put only 15 in each tray
Will: maybe ..that would kind of work - let's try 10, 10 in each tray
Lauren: (inaudible)
Will: so how many people
Will: oh no I can just write 10. (both draw)
Lauren: this makes it easy
Will: "10" "10"
Lauren: that's one hundred. That's one hundred people. it's too little
Will: why don't we change it to 15

While they were still talking about the least number of people, Lauren brought up the criteria of “realistic” when she said, “I don’t think anyone can eat 10 lasagnas”. As they considered the most number of guests, Will explored large numbers (Line 2). Lauren commented in Line 3 that it would be “too much”, but then built on his idea in Line 5 and 6 and applied it to their problem, grounding their work in a known context by talking about "cutting slices" while still exploring different sizes of pieces. Rather than focusing exclusively on searching for a solution, the two were playful and exploratory, and derived obvious enjoyment from their conversation. Their discussion also led them to explore the relationship between the divisor and the size of the pieces – an important consideration for a real world understanding of fractions.

As they began to talk about dividing up the pieces, Lauren made two comments about the meaning of their emerging work. In line 10, she talked about the size of the pieces. In Line 20, she talked about how many people they could feed. The pair had moved their work from exploration to crafting an answer, alternating with considering the meaning of each proposal. Lauren and Will's interaction during the lasagna problem contained the majority of the code counts for considering meaning and considering possibilities, indicators of the higher levels of "learning for understanding" - understanding the math in the problem and learning beyond the problem. These situated actions are consistent with Lauren's statement that she had to "think" about the math and what it meant, and strong evidence that she wanted to understand not only *how* to come up with the solution, but how her mathematical choices impacted the *meaning* of her solution.

Patterns of discussion with others. While Lauren had a mathematically rich conversation with Will, in other situations, she had quite different types of conversations, or sometimes no conversations at all. Her peer interactions appeared to be mediated by her values about reciprocity and her positioning of her partner as either an equal or unequal contributor to discussions.

In the following excerpt, Lauren and Blake were working on the stamp problem, in which students were asked to choose a combination of 25c and 50c stamps to buy with $20. In this interaction, Lauren hardly spoke to Blake until she had finished her work. However, she did get up to help Jason across the table with some of his multiplication.

(Challenge 3, Blake and Lauren, Trouble 1)
1 Jason: I got 83
2 Lauren: 84, it's 84
3 Amaya and Blake: it's 84
4 Amaya: you must have messed up or something (Lauren comes around to see his
5 paper)
6 Blake: working on his paper - front side
7 Jason: 3 pencils are 28, so that's how I got ... (Lauren reads)
8 Lauren: yeah, you got 84
9 Jason: (looks at her) oh yeah, it's, I screwed up on the count (Amaya is watching) 10
oh yeah I screwed up on this one (Lauren nods, returns to desk)

The fact that Lauren got up to help Jason with a calculation, while ignoring her own partner, is further evidence that Lauren's situated actions for the goals "helping others" and "being a valued contributor" were mediated by her positioning of herself and her partner related to her ideas about reciprocity.

At the end of the problem solving session, Lauren offered Blake a solution to "copy" onto his paper. While this form of assistance is not truly helpful to a partner, it is consistent with Lauren's narrative about her obligation to "share an idea" and her lower priority of helping others.

(Challenge 3, Blake and Lauren, Trouble 8)
1 Lauren: what else do you want to do now?
2 Blake: I don't know, I
3 Lauren: you have to do 3 strategies
4 Blake: I know I'm trying to figure out
5 Lauren: I did 2, 4 and 10
6 Blake: looks at her
7 Lauren: you can do the same thing, and I'll tell you the answers

Summary of Lauren's goal profile and situated actions. While her situated actions showed that she valued and was able to engage with the mathematics at a deep level, she seemed to reserve this for times when she considered her partner an equal or interesting participant. If she didn’t anticipate that her partner would be a source of ideas and conversation, she preferred to work alone.

Barriers to Realizing Goals for Learning and Solving Challenges – The Case of Blake

Blake had some of the highest ratings of the goals “learning for understanding” and “solving challenging problems” of the class, as shown in his profile below.

Table 10: Blake’s goal profile

Goal	Fall	Winter	Spring	Beg	Trouble	Finish
Learning for Understanding	5	5	5	5	5	5
Solving Challenging Problems	5	3	5	5	5	3
Agency and Creativity	3	4	4.3	5A 3C	3A 1C	3
Valued Contributor	4.3	4.7	4.3	5	4	5
Feeling Smart	3	4.7	3.7	3	4.5	5
Fast and Correct	4	4.3	4	3	1F 4.5C	5
Helping Others	NA	3.5	4	3	2	1

Key: pink is high, orange is medium, blue is low rating on scale of 1 to 5

Blake gave eloquent explanations for why these goals were important to him. He was also one of the case-study students who had the greatest mismatch between goals and situated actions during problem solving. Blake did not seem to make choices that would assist him in pursuing these goals in problem solving. While he valued the goals “learning for understanding” and “solving challenging problems”, he also spoke about math as “finding the answer”. Blake sometimes seemed to struggle with math, and was one of the students in the class who often scored 50% or below on unit assessments. Blake seemed to position himself as “in the middle” and a recipient of help, and pointed to the resultant benefit of frequently being offered help by others. In interaction, Blake often took a passive role and did not advocate for his own suggestions. Below I illustrate how these factors of positioning and a focus on finding the answer seemed to inhibit the expression of Blake's goals for "learning for understanding" and "solving challenging problems".

The importance of understanding it for myself. In his interviews, Blake said that "learning for understanding" was a highly important goal both in general and during moments of problem solving, "you really need to understand, if it's a really hard problem, you need to listen well, you need to listen carefully, you have to learn what the problem's about" (Final, 6:42). Blake's words, "what the problem's about" point to both *understanding the teacher's expectations* but also possibly *understanding the math in the problem*. Blake spoke frequently about the importance of understanding the work for himself, "I want to learn something to understand it, and when I understand it, I can do my work, and .. I'm, I'm going to do a really good job. But, .. if I don't understand it and I just copy my friend's, I don't know if they understand it.. it's just that I need to understand for myself" (Spring, 7:07). Here and elsewhere, Blake contrasted understanding it for himself with not copying from others. In explaining his rating for the goal "agency and creativity", he talked about the importance of "doing one's own work", and elaborated, "don't cheat, don't look at others cause if you learn from them, you're not going to learn" (Fall, 11:14). However, while he wanted to understand the work for himself, he also admitted that he liked to get done quickly so he could play math games, "you really want to play with your friends, but I think you should really take your time, try to get the answer right, and then you can go play" (Spring, 14:56). These conflicting goals played out further in the way he positioned himself and was positioned by others.

Positioning as "in the middle" and recipient of help. Blake's positioning of himself and by others was a strong mediating factor of the expression of his goals "learning for understanding" and "solving challenging problems". This was seen clearly in the way he talked about the goal “valued contributor”. Blake focused mainly on the benefit of “getting help and ideas” through participating, “I'm going to see how they do it, and maybe I can learn from them”. In his explanation, he positioned himself as a recipient of knowledge from others.

> Well, I like (contributing) cause Tosha is smart, and I'm, smart, so if I worked with her, then, if she, if we did this strategy, that made me a little smarter than before because, she's smarter than me, and, trying out new strategies is good for your education (Winter, 19:02)

In this excerpt, Blake first explicitly said that Tosha was smart, although he said, “I’m smart” too. Then he started to say, “if she” but changed to, “if we” did a strategy together, initially positioning her as the active party before including himself. Finally, he said that if they did a strategy together, that would make him smarter than before. This idea that he would benefit from working with others is different from “copying”, in that he emphasizes that doing the strategy *with* her will lead to learning. However, when they work together, it seems to be a more dependent relationship than he expresses in his narrative about what it means to do mathematics (see Challenge 4, Blake and Tosha, Beginning 4 below).

Positioning also came up in the way that Blake spoke about feeling smart. In explaining the importance of the goal, Blake cited the **social** reason that being positioned as “in the middle” as compared to peers, could result in receiving help.

> If people know like, I'm somewhat true in smart, so I'd ask them, "can you teach me a little bit of math", well I wouldn't ask them that, but they know I'm struggling with math I guess, so .. they'd help me out some more. So then I'd feel smart and I'd be a little smarter than I was before (Winter, 17:16).

In this excerpt, Blake asserted that receiving help could improve his own understanding. He also observed that if people saw him struggling, then “they’d help me out more”. This suggests that Blake may have realized the advantage of being perceived as struggling. While this may be a prudent position to take, especially when working with more confident (and assertive) peers, it may also inhibit his assertion of his own ideas and his accountability to himself to push for his own understanding.

Later in the interview, Blake illustrated how this positioning might play out, using a situation from his work on Challenge 4 with Tosha.

> I don't need to feel smart in that moment because I already have someone really smart (Tosha) next to me, I'm not saying that she's going to do all the work for me, but she kind of did because I didn't have anything to think about (29:02).

Here Blake made no objection to Tosha helping him, and even relinquished his own responsibility for solving the problem, “I don’t need to feel smart in that moment …she did all the work for me …I didn’t have anything to think about”. Here Blake pointed to a catch 22, that by being perceived as struggling he received help, but by receiving too much help he did not have to “think” and therefore lost an opportunity to learn.

Summary of Blake’s narrative: While Blake emphasized in his interviews that he wanted to understand the math for himself, in accepting this level of help, he was giving up both the agency and chance for his own understanding, that he said was important to him. His positioning by himself and others as a recipient of help seemed to be an inhibiting factor in the expression of these goals.

Blake’s situated actions in problem solving. In this section I explore how Blake’s goals and narrative where expressed during problem solving, mediated by his positioning in relation to others.

While Blake placed a high priority on understanding the work for himself in his ratings and explanations of the goal “learning for understanding”, this was not reflected in many of his situated actions in problem solving. In his narrative he emphasized not copying and understanding the math for himself, however, on several occasions he seemed thrilled to have overheard an “answer” from a peer. In the following excerpt, Blake and Will had just started work on Challenge 2, the stamp problem. In this problem, students were asked to decide how many 50c and 25c stamps they would buy with $20. Amaya, sitting across the table, had added the two prices and told her partner Lauren the result.

(Challenge 2, Blake and Will, Beginning 2)

Blake: ok, (reads to self, smiles as finishes) (Amaya is talking quickly with Lauren)
Will: (watches girls)
Amaya: Yeah it would be 75
Blake: Thank you for giving us the answer, ... which I already knew, but thank you

Here Blake seemed gleeful to have overheard “an answer”, which was actually just the sum of the two stamp values. He seemed to seize on “an answer” without investigating how the pair across the table had come to it, both of which are counter to his statements about the importance of understanding the math for himself. This incident reveals Blake’s mixed relationship with pursuing his own understanding. On the one hand, he wanted to do his own work, on the other, he seemed happy to “discover” “an answer” through one of his peers. This language of “an answer” is similar to Tosha’s focus on finding an answer and the expectation that there be “an answer” to be found. Interestingly, like Tosha he also tied getting the answer to his future, “I want to get into a good school, and a good college, so I want to understand the work so I can .. at least get close enough to the right answer” (Winter, 16:30). This interpretation of math as performance may have similarly limited his pursuit of understanding the math rather than finding “an answer”.

A gap between appreciating challenges and pursuing them. Blake’s actions were not consistent with his high rating and explanations about the importance of challenges. In his explanation of his high rating of the goal “solving challenging problems”, he said that he identified as someone who enjoyed challenges, and that "(challenges) fit me well". However, his counts for the possible indicators of the importance of this goal were low to none – he did not stay after the group went to the rug to finish his work, celebrate when he was finished, or engage in arguing for different ideas with a partner. Nor did he seem to leap into solving a problem or engage actively with his partners in discussion about the problem. Blake's agency during problem solving may shed some light on his lack of persistence in pursuing his own understanding of the math and solving challenges. Some of the main possible indicators for "having agency" were the conversational moves that students made during discussion with their partner. The most assertive of these was arguing for different ideas, which Blake only did on one occasion when he was working with one of his more supportive partners, Jason. When this occurred, Blake and Jason were working on Challenge 5, Scale up Royce Hall, in which they were asked to scale up a picture of Royce Hall to a model that would fit on a desk. In the beginning of the problem, Blake took an active role in measuring and proposed a strategy for measuring the bottom dimension of the building. However, when Jason started talking about scaling up their measurements, Blake stopped responding, indicating that either agency had become unimportant or its expression had become inhibited.

Challenge 5, Blake and Jason, Beginning 2

Blake: Jason, hang on Jason, we have to measure it this way too, Jason
Jason: I know, I already did. 2 3/4, so
Blake: half, half (pointing to bottom edge of building image and dividing it with his hand)
Jason: The bottom thing is 2 3/4, so all we need is this (pointing to bottom edge of building on paper), so, how many times should we times it?
Blake: (doesn't answer)
Jason: probably, like, we should make it go to 9 inches (looks at Blake) ok?
Jason: let's go to 9 inches (starts erasing)
Blake: (looks down at his paper)

Blake presented his idea for a strategy in Line 1, and argued for it in Line 3, claiming they could divide the building symmetrically as indicated with his comment “half half” and his hand movement dividing the picture on the page. However, when Jason countered with his measurement in Line 5, “all we need is this” and suggested they move to a discussion of scale in Line 6, Blake stopped contributing go the discussion, despite 3 attempts by Jason to elicit his participation (Line 6, 8, and 9). Blake could have asked a question or offered a different answer, but instead he focused on representation (and not mathematical ideas) during the remainder of the interaction, indicating that agency had become less important or inhibited for him.

This excerpt suggests that Blake became discouraged when his idea was not considered, rather than engaging in negotiation of whose idea would be used. Blake seemed to think of having agency as “all or nothing” – one person’s idea was accepted – rather than a conversation, and co-development of ideas with his partner. This exchange also highlights the importance of a supportive partnership for students like Blake, who may not be as confident in math. The fact that it was so quickly deflected may have a greater impact on his willingness to put forth and argue for his ideas the next time.

A more frequent conversational move for Blake was sharing an idea in which a student put forth an idea for a strategy or number to be used in solving the problem, without presenting evidence or logic for its use. Blake shared an idea 3 times, all at the beginning of a problem. However, as shown above in his work with Jason, when his partner offered a different idea, Blake did not advocate for his own idea or counter with another but instead seemed to retreat into a more passive role. In the passage below, Blake and Will were working on Challenge 2, the stamp problem, in which students were asked to decide how many 25c postcard and 50c letter stamps to buy with $20. Will began by asking how many postcard stamps they should buy (he mistakenly used the value for letter stamps - both students have moments of confusion about the numbers during this passage).

Challenge 2, Blake and Will, Beginning 2

Will: (writes a little on Blake's paper, back to his own) how many thingies, postcard stamps should we buy?? so each of them are 50 ... 2 is a dollar,
ok, so let's do ..
Blake: Let's buy all of them
Will: Let's buy 6. Want to buy 6?
Blake: Yeah (nods) 6 is an equal number
Will: So 6, how much is 6 dollars?
Blake: 6 dollars ...
Will: Oh no wait how much is, 50 is a dollar, oh yeah we have this blank space, so let's .. (writing) so we want to buy ...
Blake: 60 plus 50 would be 11, dollars .. (looking across table). 6 dollars is ..
Will: Wait wait wait (erasing and writing)
Blake: 60 cents plus 50 cents equals .. (looks at Will) ah .. (writes on paper) 60 plus 50 would be, 60 cents plus 50 cents would be 11 dollars

Blake's suggestion, "let's buy all of them" in line 4, could have meant that he only wanted to buy letter stamps (Will says postcard in line 1 but uses the value for letter stamps), or it could have been a "random" suggestion. In response, Will suggested "6" in Line 5 which Blake agreed to in Line 6. Blake commented, "6 is an equal number" perhaps meaning that an even number would work well for making groups of stamps that cost $1. At this point the boys started doing work separately. Will worked on calculating the total cost, making an initial error in line 7, and correcting himself in line 9. Blake added 60 plus 50, seeming to mix up number of stamps with the money units. Some of Blake's statements suggest that his difficulty in interpreting what the problem was asking made him somewhat dependent on his partner. However, the **choices** that Blake made about how to advocate for his own ideas and engage with his partner, show the gap between his professed importance of the goals "having agency" and "learning for understanding" and his pursuit of these goals during the work session.

In the excerpt above, Blake's partner considered his suggestion. In other cases, Blake's idea was ignored or diverted by his partner and his responses were additional evidence of the gap between his rated importance of the goal and his actions during problem solving. As mentioned above, Blake seemed to think of agency as “all or nothing” rather than a process of negotiation. In the excerpt below, from work with Will later in the problem, Blake proposed using the pull-down strategy and argued for its merits, “it shouldn’t take too long”. Will did not respond and proposed another solution, which the partners ended up using.

(Challenge 2, Blake and Will, Trouble 3)

1 Will: she wants us to get as many (stamps) as we can, but it's going to take a really 2
long time, so ...
3 Blake: well, we can do that pull down, so, shouldn't take too long
4 Will: I say, (looking at paper)
5 Blake: 3 dollars plus 4 dollars equals 7 dollars so far (Will doesn't respond)

This interaction is further evidence that Blake saw “having agency” as “all of nothing” instead of a process of negotiation. This may have also been affected by the way Blake positioned himself and was positioned by others, as discussed in the next section

While being perceived as struggling could sometimes result in Blake receiving a range of help, at other times it resulted in him being abandoned by a partner. As discussed in Lauren’s profile above, when she and Blake were partners, they worked alone for the majority of the work session. Blake could have been waiting for help, which did not come until the end. As discussed in Lauren’s profile above, she saw helping as a two way street, so when she observed that Blake was not an active participant who contributed his own ideas, she may have decided that working actively with him would not be as productive as working alone.

After Lauren got up to help someone across the table, Blake indicated that he needed help but did not ask for it directly.

(Challenge 3, Blake and Lauren, Trouble 6)

1 Blake: (to Jason, who just got helped by Lauren) Wait you're on the second one?
2 Jason: Yeah, there's only the last problem you have to do and that's it
3 Blake: Why are we even doing this?
4 Blake: (gets up, to look at Jason's paper)

5 Jason: 84 divided by 2 equals 42
6 Blake: (returns)
7 Lauren: (looks away, looks back at paper)
8 Blake: Oh oh I know how you can make one um

Blake's comment in Line 1 indicates that he is not where he wants to be in working on the problem. His comment in Line 3 seemed to express frustration and exasperation with the work. Blake may have sensed that Lauren was not interested in taking the leading role unless he reciprocated, and therefore avoided asking her directly for help. This was an opportunity for him to draw on other resources or pursue his own strategy, but he did neither. Notably, though, when she later offered him an answer, he declined, maintaining his value to not copy. Here Blake's positioning by himself inhibited the expression of his goal "learning for understanding", in that he seemed to be waiting for his partner to take the lead and offer help, rather than trying to engage with his partner or pursuing his own solution.

Blake's positioning was sometimes directly referred to by himself and his peers. In the following excerpt, Blake and Tosha were partners for Challenge 4 and Jason was working across the table. Jason chimed in an answer to one of the "challenges" that Tosha was giving Blake. Blake countered back "that's easy!" as if to reject that he needed Jason's help and could answer the question on his own. Jason then pushed back and teased Blake about getting "educated" by Tosha.

(Challenge 4, Blake and Tosha, Beginning 5)
1 Tosha: ok so let's try thirds. What's 3 times 10?
2 Jason: 9 10. 3 times 10 is thirty
3 Tosha and Blake look at Jason
4 Tosha: (to Jason) I was asking him
5 Blake: that's easy, it's 30
6 Tosha: So if we had 30
7 Jason: sorry, I didn't mean to spoil Blake's education
8 Blake: Dude, that's like the simplest question
9 Jason: (laughing) I didn't mean to spoil your education Blake, I really didn't.

Here Blake positioned himself as “smart” by claiming that Tosha’s question was easy (Line 5). Jason then countered in Line 7 that Blake was getting an “education” from Tosha, positioning him as knowing less and therefore being taught by Tosha. Blake rejected the evidence for this in Line 8 but did not explicitly reject that he was receiving help. It seemed like Blake was positioning himself as competent while not protesting to being a recipient of help from others, consistent with the way he talked about “feeling smart” in his interviews. As discussed above, Blake held the conflicting goals of “learning for understanding” which he explained as “understanding it for yourself”, but he also valued “feeling smart” for being positioned as “in the middle” and being a recipient of help. In this moment, he managed this conflict by refuting the idea that he was not understanding it for himself by saying “that’s easy, it’s 30”, while not denying that he was receiving help from someone else.

Summary of Blake’s goals, narrative and situated actions. While Blake said in his explanations that “learning for understanding” was important to him in order to understand the math himself and know what the teacher was asking, and also that he valued solving challenging problems, in practice the expression of these goals seemed inhibited by his conflicting goals of learning and “finding the answer”. In addition, his positioning by himself and others as “in the middle” and a receipt of help led to others helping him and sometimes his relinquishing of responsibility for his work. Blake’s situated actions during problem solving suggest that he may have become used to his position and found it beneficial enough to maintain even though it constrained his own growth.

Pursuit of Understanding "In my zone" – The Case of Amaya

Amaya was similar to Blake in that she had high ratings of all goals, including "learning for understanding" and "solving challenging problems", both in general and during moments of problem solving as shown in her profile below.

Table 11

Amaya's goal profile

Goal	Fall	Winter	Spring	Beg	Troub le	Finish
Learning for Understanding	5	4.7	5	5	5	5
Solving Challenging Problems	3.5	2.3	4.7	5	4	5
Agency and Creativity	4.25	3.3	5	5	3	5
Valued Contributor	3	2.7	4.3	4	4	5
Feeling Smart	3.3	3	4.3	3	5	5
Fast and Correct	3	4	4.7	5	5	5
Helping Others	NA	4.5	4.5	4	5	5

Key: pink is high, orange is medium, blue is low rating on scale of 1 to 5

Like Blake, Amaya sometimes struggled to understand the math concepts and tended toward simpler solutions in her work. She also had several unit assessment scores of 50% of less. However, in contrast to Blake, Amaya's interpretation of the practice-linked goals in a generally positive and consistent way led her to adopt practices consistent with her high ratings of "learning for understanding" and "solving challenging problems". Amaya participated actively in discussion with her partners, frequently arguing for and sharing ideas. While she also saw herself as "in the middle" and recognized the benefits of receiving help from more confident/capable peers, she also valued her own participation and positioned herself as just as capable of helping others as receiving help. She had a strong value of her own ideas and voice, and an internal drive to be an active participant and be heard by others. Like Tosha, Amaya valued performance, in terms of "looking like she knew what she was doing", but this seemed more a protective measure than to outperform others. Although the current study did not focus on students' change over the year, Amaya's change in confidence and relationship with mathematics was revealed in stark relief in her interviews. While she began the year talking about her "comfort zone" in math class that did not include challenging problems, by the end of the year she was speaking about her zone moving up so that she could interact with and learn from peers who thought differently (i.e. knew more) and identifying herself as someone who loved challenges.

Learning for understanding and solving challenging problems – in her words. In her interviews, Amaya explained that the goal "learning for understanding" was important both for *understanding what the teacher expected for the problem*, and *learning beyond the problem*. She explicitly connected these two levels: "I want to start off understanding what my problem is, but still learning it at the same time" (Spring, 2:36) and "when I understand the math, then I can learn new things and I might learn new skills" (Fall, 2:09). Like Blake, Amaya seemed to be partially motivated to *understand what the teacher expected for the problem* in order to avoid a negative outcome. While Blake wanted to avoid copying, Amaya wanted to avoid the social consequence of having to ask others: "if you don't know what the problem is, then you're not going to know what to do, you're just going to be like: "can someone help me"" (10:21), and extra homework: "if you're getting a special homework packet today …. You're going to be like: "what do I do?" … and you have to keep it for extra homework" (23:46).

One prevalent strand of Amaya's narrative was a desire to resolve the tension of an unsolved problem or not knowing what to do.

> Like if I'm stuck, I definitely want to go straight (tunnel gesture) to the understanding zone, I definitely don't want to be stuck in this crazy middle zone (churning gesture) where I can't like understanding anything (cross gesture),…so I know boom (hand comes down on table) in the middle that I am understanding this (Spring, 18:35)

Staying in a comfortable "zone" was another strand of Amaya's narrative .At the beginning and middle of the year, Amaya rated "solving challenging problems" as somewhat important, but in the final survey, and moments of trouble, it was very important to her. In the Fall, she explained her rating as an issue of identity, which she expressed in terms of her comfort zone for math: “I have zones and my zone for math is just like, it's not too challenging, it's like right in the middle” (07:38). When I asked her further about this she said, “because, (laughing, sheepish) I'm not really good at math… (smaller voice) … I need to learn more” (08:38). She also said she found challenging problems uncomfortable: “like a little bit difficult for me .. and so I just feel weird and different about them” (14:42). However, in the final interview, something had shifted, and she was eager to move to new “zones”.

> You want to be able to pass the difficult line or else I should say put it in words "the zone". The easy zone and the hard zone... you want to be able to pass the challenging zone…like you know like a video game the next level, the next level, it gets harder and …You want be able to pass that level… The more challenging problems you'll do, it helps you a lot (06:52)

In this excerpt Amaya mentioned both wanting to go to a higher zone and the benefits of doing so, including learning from doing challenging problems: “I'd really like want to be able to solve that problem, … cause I, like you know, what your difficulties are in math, right? … we want to be able to solve the challenging problem so we can learn from it” (Spring, 14:52). Her interpretation of being able to solve challenging problems was to be able to solve them quickly: “if the challenging problems are challenging for me .. I would want to solve them, easily, snap” (Spring, 16:26) which seems related to her desire to “resolve’ the tension of an unsolved problem.

Positioning herself as an important contributor to discussion and of help to her classmates. While Amaya spoke about being aware of other people being smarter than herself, she also positioned herself as an important contributor to discussion. This emerged through her discussion of the goals, “valued contributor”, “having agency”, and “feeling smart”. When talking about the importance of being a valued contributor, she mentioned that contributing was important so others could learn from you: “you want to show your work, and … other people can learn from your idea” (Fall, 14:19). A major explanation that she gave for the importance of this goal was having her voice represented in discussion.

> I would definitely want to be a part of …this small group and if …the four people of them aren't letting me speak, then it's kind of like "am I invisible or something?" … I want to share my ideas. … I want to be in the discussion … I want my voice to be in there. I mean I won't mind if they have like a really good strategy … I'd let them keep going but then I want to share my idea. Maybe it would add on or something (Final, 14:55)

In this excerpt Amaya spoke about wanting to have her voice “be in there”, and wanting to share her ideas. Amaya’s value of her own ideas and professed commitment to advocating them was in contrast to Blake. In the end Amaya said that maybe her idea would add on (to another person’s idea). This suggests that she saw discussion as a collaboration and mixing of ideas, rather than the competing of one idea with another, as Blake seemed to do. This is consistent with her additional explanation of “getting help and ideas” through contributing, “let's say I make a mistake when I share, then they can just tell me that mistake, and I'll be learning something new” (Fall, 06:30). She seemed to see discussion as a reciprocal process in which she was just as much a contributor as beneficiary.

While she wanted to be an active participant in discussion, at the beginning of the year and through the winter Amaya said she was much more comfortable being with students in her own zone.

> The reason I like smaller groups is because for some reason I feel like ..they're all like your same. You know how like people talk different... the teachers take different groups who are different levels? You're all in your same level so if you make a mistake … it's going to be alright … that's your level so they will understand what you did (Winter, 20:29)

In this excerpt she mentioned both that she would feel more comfortable about making mistakes, and that it would be easier to be understood by people at her level. However, by the spring survey, and final interview, she was talking about benefiting from people in a “higher zone”.

> I want to go up to a higher zone so like I could contribute with others that are like... they have more... they're in a different zone than me so I can learn more, learn more from them... actually I can contribute more if I learn more from people that are in a different zone than me (30:31)

Amaya seemed to have experienced a dramatic shift in her comfort zone, as well as her perception of the benefit of working with others who thought differently from herself. She also talked about asking more of herself: “then when you go up higher, when you get this new age, then now you're saying to yourself, ok now I'm older and I need to like step it up… I need to get the courage to like feel smarter." (Final, 28:57).

In contrast to Blake, Amaya mentioned many reasons for helping others besides receiving help and ideas. Some of the reasons for this were that it felt good, helping others made you a scholar, and helping facilitated the spread of knowledge.

> You would feel good if you helped others. That means that you are scholar-... you're helping others that need help and that means they're taking your ideas from your work (that) they can carry on to other people (Winter, 22:40)

Amaya also talked about help being reciprocally beneficial for both participants.

> If this was a really challenging problem, and that person, is like stuck behind it, the same way as me, we could help each other… (tunnel gesture)…we could check over each other's problems and see what we did wrong (Spring, 28:02)

In this except and the one above, Amaya positioned herself as both a giver and recipient of help, in contrast to Blake who consistently positioned himself as a recipient only.

Like Blake, Amaya positioned herself as “in the middle” for most of the year, until her shift in “zone” in the spring. However, her reasons for feeling “somewhat smart” were slightly different – she wanted to avoid bragging, and not get overconfident: “ You want to feel smart, but you just don't want to feel too smart, because if you feel too smart you're going to be all braggy, and stuff, and it's just not very nice “(Fall, 23:04). Amaya’s comment reflects the complexity of her relationship with “feeling smart”. While she cautioned against feeling “too smart” in relation to other students because this could result in bragging, she also referred to the importance of internal confidence and how that could help her work: “I want to be feeling smart, like, oh this is going to work", giving positive actions (Final, 8:29).

Amaya’s situated actions in problem solving. Like Blake, Amaya rated the goals “learning for understanding” and “solving challenging problems” as very important. However, she contrasted with him in that her actions were much more consistent with her ratings of these goals. For “learning for understanding”, Amaya participated in all of the actions that were possible indicators for this goal. She checked her work with the teacher, checked her work while finishing, and discussed work with her table. While Amaya did not always have the leading idea or the idea that was carried forward, she actively participated in discussions and was a much more active advocate for her own ideas.

In the following excerpt, Amaya and Tosha are working on the stamp problem, in which students are asked to decide how many 25c postcard stamps and 50c letter stamps to buy with $20. The partners are deciding how many postcard stamps to buy, after deciding on $12 of letter stamps (24 stamps).

Challenge 3, Amaya and Tosha, Trouble 1

Tosha: wait, so how many number of postcard stamps? the number of the postcard stamps?
Amaya: there's 7
Tosha: we got, we got we've got 12 dollars right
Amaya: mm hmm
Tosha: because there are 12 of these?
Amaya: But we have... oh that's for, (both turn papers over)
Tosha: (to self) that's 12 ...
Amaya: How many, how many - postcard stamps?

10 Tosha: yeah, right, because we have 12 dollars worth.
11 (Amaya writes on paper) And then we have 8 letter stamps, right?
12 Amaya: yeah
13 Tosha:. 8 dollars. (both write)
14 Amaya: And total number of stamps, 20.
15 Tosha: Well wait. Total number of stamps...
16 Amaya: Well if you want to know the total number of
17 stamps of all of these (pointing to her paper)
21 Tosha: Wait, I need to ask Ms. A about something. I don't know if she's asking
22 how many we put or how many dollars (hand raised)
23 Amaya (pointing to her paper) yeah I think, I'm wondering if it's this or it's this, 25
cause I know that this is 20, but this is 75... (hand up too)
26 Tosha: (pause) wait I'm not sure (both have hands up)
27 Amaya: me neither
28 Amaya: cause it wouldn't make sense, unless you put both .. (Tosha puts hand
29 down)
30 Amaya: well we should just ask her (Tosha hand back up) wait, it also says total
31 cost ... (Tosha hand down) so maybe total number of stamps means these two, and 32
total costs, means 20, or vice versa (both hands go up again) but still, we should 33
ask her

In this excerpt Amaya actively participated in discussion, alternating turns of talk with Tosha and making contributions to their emerging solution. She was open to ideas from her partner, and seemed motivated to follow her partner's thinking. In the first half of the excerpt, the girls talked about the "trouble" of how to decide how many letter stamps to buy. In the second half, Tosha proposed they ask the teacher, but Amaya noted that the directions had a possible answer (Line 30-33). In contrast to Blake, Amaya did not wait for assistance or leadership from her partner. Rather, she was an active participant of the discussion, consistent with the goals "learning for understanding" and "solving challenging problems".

Contributing and helping – situated actions. Consistent with her positioning of herself as helping others, Amaya helped and guided her partner on several occasions. In the following excerpt, she and Jason were working on the pencil problem, in which students have to calculate how many pencils are needed for each of 28 students to have 2 plain and 1 fancy pencil. The partners were just beginning their work and sharing strategies with each other.

Challenge 3, Amaya and Jason, Beginning 1

Jason: I'm doing 28 times 2
Amaya: no it's not 28, it's … (pointing to his paper) it's 28 x 3 cause 2 , 1….so it would be 3

Here Amaya suggested an idea to her partner and followed up with logic (multiplying 28 students by 3 as the total number of pencils for each student). Amaya argued for her ideas 7 times across the four challenge partnerships, in contrast with Blake who only did so once. However, while Amaya was an active participant, she did not always contribute her ideas. In the following excerpt, Amaya and Jason were finalizing their work on the pencil problem, deciding how many volunteers should sharpen 84 pencils and how many they would each need to sharpen. They had already recorded the solution for 2 volunteers.

(**Challenge 3, Amaya and Jason, Finished 5)**

Jason: I got the answer Amaya! (pounds table) it's 4!
Amaya: wait, for which one?
Jason: 21 divided by 4
Amaya: let's do 1 more
Jason: 21 divided by 4 equals 4
Amaya (gets up, looks at his sheet) ok, no you have to do one more (both reading)
Jason: well I did 1 2 3. 1, 2
Amaya: what? 4 divided by 4?
Jason: no 84 divided by 2 equals 44, equals 42
Amaya: (sitting down) ok
Jason: and then, 42 divided by 2 equals 21

Here Amaya played an active role in monitoring the progress of their work, however, she was less active in contributing her own ideas in this interaction. She had already recorded the solution for dividing by 2 volunteers, which was a simple entry point for her engagement with the problem.

Amaya's situated actions with partners could range from equal turns of talk, to letting her partner take the lead, to her taking the lead. In the following excerpt, she and Aaron were working on the lasagna problem.

(Challenge 4, Amaya and Aaron, Trouble 2)

Amaya: what are you doing (to Aaron who is drawing 5 lasagnas) each person gets one lasagna
Aaron: yeah, but there's only 5 lasagnas
Amaya: no, look. A family .. (pointing to paper) it's 10 lasagnas
Aaron: (looks)
Amaya: oh my gosh Aaron
Aaron: why did you tell me it was 5
Amaya: I didn't tell you it was 5. I said if there were 2 people, each person could get 5. But we changed it to 10 people (both write again)
Amaya: (looking) laughing
Aaron: (scribbling)
Amaya: what are you doing Aaron?

In this excerpt, Amaya took the leading role, monitoring and correcting her partner (Line 1 and 12) and explaining the problem to him (Line 4) and her thinking to him (Line 8 and 9). The fact that she could play a variety of roles, depending on her partner, seemed to be one of her strengths.

Summary of Amaya's goals, narrative and situated actions. Like Blake, Amaya sometimes struggled in math class and scored 50% or less on several unit-assessments. However, while Blake had a narrative that included finding "the answer" and positioning himself as receiver of help, Amaya's narrative included the idea that she was equally capable of giving as receiving help, and a firm belief in the importance of her own ideas and contributions to discussion. This narrative translated into her positioning herself as a valued contributor to discussions and both receiving and giving help to others. Her positive goals were more directly translated into situated actions in problem solving because of the narrative that she held.

Chapter Summary

In this chapter I have used four case-study students to illustrate the complexity of the process by which students' goals for mathematics are expressed in their situated actions in problem solving, and how this process seems to be mediated by their interpretation of the goals, their overarching narrative about what it means to practice mathematics, and their positioning by themselves and others. The first case of Tosha illustrates how some positive goals can be subverted by how they are interpreted or how they are made to relate to another goal. In her case, “learning for understanding” and “solving challenging problems” are placed in the service of performance. In addition, her drive for performance leads her to position herself in certain ways in relation to her classmates that actually inhibits productive, reciprocal engagement. Tosha’s case also demonstrates the importance of examining a student's explanations and interpretation rather than just their rating of the goal. While it looks like her actions are inconsistent with her goal ratings, they are in fact consistent with her interpretations of the goals.

In the second case of Lauren, we see a student who interprets the goals “learning for understanding” and “solving challenging problems” in a way more consistent with the perspective of the reform-oriented mathematics education community. She values thinking, deep engagement and discussion with her partner vs. performance or finding the answer. While Tosha positioned herself as “helper” and “more knowledgeable” to others, Lauren saw helping and contributing to discussion as reciprocal. This translated into her having active, thoughtful discussions with partners who she seemed to think could also engage and contribute ideas (Will), but withdrawing from interaction with partners who she thought would not do so (Blake).

The third illustrates how a student’s goals could be in conflict with each other. Unlike Tosha who had a consistent interpretation of the goals, Blake both claimed that he wanted to “understand things for himself” and “solve challenging problems”, and positioned himself as “in the middle” and a recipient of help from more capable peers. His comments during his interviews revealed his awareness of this conflict, but he did not seem to resolve it. Rather, his positioning and interpretation of "contributing" as receiving help seemed to inhibit his advocating for his own ideas or pursuit of his understanding of the math.

The fourth case of Amaya illustrates how a consistent and positive interpretation of the practice-linked goals led a somewhat struggling student to adopt the right sorts of practices, which helped her deal with or stay positive in spite of her struggles with math. In addition, her positive interpretations seem to have contributed to her dramatic shift in confidence and her embracing challenges from the beginning to the end of the year. Amaya's case puts in high relief the potential impact of practice-linked goals on a student's situated actions in problem solving.

CHAPTER 7

DISCUSSION

> "Solving problems and making up new ones is the essence of mathematical life", (Hersh, 1997, p.18)

> "I like fun challenging problems, and ones with multiple answers, but it just tends to screw me a little, and I like when it does that because I get to think in different directions that I can't really do when it's a straightforward problem …yeah, it pushes me in a different direction in a way, like makes me go to another way of thinking that I would have never thought of ",(Lauren, Winter, 32:53)

My goal in attending graduate school was to find ways to help students be successful in mathematics, particularly in problem solving which I and many others believe to be the heart of a vibrant mathematical practice. The way I have chosen to do this is to explore the role of students' goals in supporting their problem solving. This includes what students think is important about mathematics and how they relate to it. Perhaps students may form a relationship to mathematics as Lauren describes above, of enjoyment and expectation to be "go to another way of thinking". In exploring students' goals I hope to provide some insight into their relationships with mathematics, and how this might impact their problem solving, as my contribution to the ongoing conversation about how to best support students in developing as mathematical thinkers.

I have organized this final chapter into three sections. In the first section I review the major findings of the study and discuss how they relate to current research in the field. In the second, I discuss limitations of the study and in the third I consider how the findings can be used to inform instruction that supports a positive disposition to mathematics. I close with a discussion of future directions for research in Mathematics Education and the Learning Sciences.

Findings

Examining students' relationships to problem solving through their goals. In this study I have argued that developing a positive relationship to mathematics, and in particular problem solving, is critical to becoming an advanced mathematician, but remains a challenge for many students. This relationship encompasses why students think mathematics is important and what their aims are in pursuing problem solving. Students' relationship with mathematics in turn affects the ways that they develop as mathematical thinkers and how they engage in mathematical practice. In order to examine students' relationships with mathematics, I focused on four positive goals widely held to be important by mathematicians and reform-oriented mathematics educators, which address important aspects of problem solving (Boaler, 2002; Boaler, 2008; Boaler & Greeno, 2000; CCSS, 2010; Lampert, 1990; NCTM, 2010). I chose to examine these goals as a way to translate students' relationships into aims valued by the mathematics education community, and as a way to examine how these goals drive students' actions in the classroom. By understanding the extent to which students hold these positive goals, and how they think about them, I argue that we can better prepare ourselves as educators to support students' development of a positive disposition to mathematics.

In order to examine students' thinking about these four goals, I analyzed their goal ratings on a student survey and their explanations of their ratings in the interviews with the case-study students. The first main finding of the study was that the majority of students rated the positive goals as "somewhat to very important" across the three time points in the year, while they rated performance goals as "less important", as reported in Chapter 4. As expected, individual students varied in the importance they assigned to goals and the classroom contained a variety of "goal profiles", as described in Chapter 6.

Complexity revealed through explanations. The results of students' goal ratings confirmed that the positive goals identified in the study were at least "somewhat important" to most students. However, examining the case study students' explanations of their goal ratings revealed a critical aspect of understanding students' goals: the complexity and variation in the ways that they think about and interpret the goals. Some of these explanations were more consistent with the values of the reform-oriented math education community than others. For example, the goal "learning for understanding" could be important for a variety of reasons, including understanding the teacher's expectations for the problem, or understanding the mathematics in the problem, or for anticipated secondary benefits such as gaining admission to college. Thus two students could both rate a goal as highly important, but what this meant to them could differ greatly. These different interpretations then mediated the translation of that goal into action. A student who valued the goal of understanding the teacher's expectations for the problem, might pursue very different actions than a student who wanted to understand the mathematics in the problem, as reported in Chapter 6. These findings demonstrate that only looking at students' goal ratings paints an incomplete picture of students' thinking about these goals. By exploring how students actually think about and interpret these goals, we get a more complete picture of their relationship to mathematics.

The role of themes and the narrative. Another finding about students' explanations of goal ratings was that their explanations of both practice-linked and performance-oriented goals were organized around several broader themes, as reported in Chapter 4. The themes included personal benefits, social reasons, secondary benefits, norms, and values for why a goal was important. The themes suggest that students do not think about the goals in isolation. Rather, students' goals are interwoven into a narrative that spans several of the most important goals, and these goals overlap and influence each other. Students' goals were best understood by considering all of the goals together, and how students made connections between them. For example, as reported in Chapter 6, Blake placed emphasis on the theme of the personal benefits of learning in explaining the importance of the goals "learning for understanding" and "solving challenging problems", but also placed emphasis on the theme of the social benefit of receiving help for the goals "valued contributor" and "feeling smart". In his case these two themes often competed and resulted in actions that were inconsistent with his goal ratings. The underlying reasons for his inconsistent behavior are only seen when examining his goals in terms of the greater themes, and this is true for the other case study profiles in Chapter 6 as well.

The overarching narrative that students followed also seemed to affect how they interpreted the goals and the meaning they ascribed to them. For example, in the case study of Tosha as reported in Chapter 6, mathematics was about performance. For her, goals were interpreted and repurposed in service of the larger narrative about what was important in mathematics (performance) and what it meant to perform (complete the work). In contrast, for Lauren, mathematics was about increasing her understanding of how the mathematics worked. Therefore, she interpreted the goals in purpose of increasing her understanding of the mathematics.

Viewing students' goals as part of a narrative rather than in isolation further highlights the importance of exploring the meaning of the goals to students, and using the goals in service of understanding their *relationship* with mathematics. Similar to findings in recent work that examines both "mastery' and "performance" goals together, this study affirms that different types of goals must be examined together in order to fully understand students' motivation and thinking about mathematics. In addition, both types of goals can have more or less adaptive expression in problem solving depending on students' interpretation of the goal and how it is pursued in service of their larger narrative about mathematics.

Students talked about similar issues and themes across the goals "learning for understanding", "solving challenging problems" and having agency such as performance, learning, and enjoying the mathematics. They also talked about similar issues and themes for the goals of "feeling smart", "being a valued contributor", and "helping others", such as positioning, obligations to others, and exchanging ideas. While these groups of goals are consistent with the usual categories of "social" and "academic" dimensions of students' dispositions, (Gresalfi, 2009, Langer-Osuna, 2015), this study examines the goals in greater depth both through students' explanations of individual goals, and by examining the way the themes and greater narratives related to students actions. These multidimensional goal profiles and related actions contribute a more nuanced portrait of students' dispositions to mathematics.

The role of positioning in the expression of practice-linked goals in situated actions of problem solving. While students held certain goals and interpreted their importance in certain ways, these goals had to be translated into situated actions in shifting classroom contexts. The expression of students' practice-linked goals was mediated by their interpretation of the goals, as discussed above, and their positioning by themselves and others, discussed in the following section.

The second factor that emerged as important for the expression of students' goals was the way they positioned themselves and were positioned by others in the classroom, in particular during partnership work. This positioning was directly related to their narrative about themselves and what it meant to do mathematics. For example, Tosha's narrative of mathematics as about performance was connected to her pursuit of being a "better understander" and her positioning of herself as the more knowledgeable peer in a partnership. Her positioning in turn inhibited her from exploring the ideas of others or trying to merge her ideas with those of others. In the case of Amaya, her narrative about herself as a source of ideas and just as able to give help as receive it, led her to position herself as an active legitimate contributor to discussion in spite of her feeling that she was "in the middle" as compared to her peers. In the case of Lauren, her narrative that contributing should be reciprocal led her to only position others whom she considered equal in contribution and ideas as worthy of interaction. This resulted in her missing opportunities to explore the ideas of others who thought differently from her, or expand her own understanding by articulating her own ideas and negotiating decision making with others.

Limitations of the Study

One limitation was that while the surveys were developed in student-friendly language, students often did not always interpret the goals as they were intended, as evidenced by the variety of their explanations of the goal ratings. The student survey could be revised to be more effective at capturing the range of meanings that students ascribed to the goals.

Another limitation was the small number of case-study students (8). A larger sample of students would afford more information about the range of students' explanations of goal ratings and distribution of their ratings.

In order to characterize students' situated actions in problem solving, I chose to analyze data from four challenge problems I had designed with the classroom teacher. While these videos ensured that I had comparable problem solving sessions between case-study students at three time points throughout the year, they represented only a small sample of students' overall problem solving throughout the year. Analyzing a greater number of problem-solving sessions in a variety of contexts, including the long-term project at the end of the year, would have yielded greater insight into students' situated actions while problem solving.

In connecting students' goals and explanations of goal ratings with their situated actions in problem solving, I had to use a degree of subjectivity and make assumptions in inferring goals from their actions. This section of the study is exploratory and tentative, and could be further improved by either asking students directly about their goals, while they were problem solving, or member checking connections with students at a later time.

Implications for Classroom Practice

In this section I present several ideas for supporting the development of students' positive dispositions to mathematics based on the findings of this study.

Supporting the development of practice-linked goals. As discussed above, the goals that students hold are part of a larger narrative they have about themselves and their practice of mathematics. One of the major finding of this study was that students' goals are multidimensional and their explanations of the importance of these goals vary widely, with some explanations more consistent with the values of the reform-oriented community, but others not so. One way to support students in developing a positive disposition would be to consciously cultivate a classroom narrative that supports the practice-linked goals in this study. This was already occurring to some extent in the classroom for this study, based on my observations over the year. Students were expected to be able to articulate and advocate for their ideas, collaborate with others around the mathematics, and challenge themselves in their problem solving. In spite of this, students like Tosha were able to appear highly successful and yet maintain a narrative of mathematics that was not entirely consistent with that of the classroom. Two possible steps that teachers could take to support students' positive dispositions to mathematics are first, to determine what narratives students hold, and second, to help them re-examine these narratives and move toward a more positive orientation to mathematics.

One way to determine what narratives students hold, is to expand the student survey used in this study to include the range of explanations that students gave for each goal's importance. In particular, surveys could probe for the themes that emerged in examining students' explanations. Students would be asked to select which of the themes was most important in rating a particular goal. In addition, the survey could ask about some of the narratives that emerged in examining the case studies in Chapter 6. For example, there could be questions about how important it was for partners to contribute equally, students' feelings about helping and being helped, what kind of mathematics person they saw themselves as, and other questions that would address issues that arose from the narratives of the students in this study. Developing a survey in this way would provide the classroom teacher with a relatively easy way to know the goal ratings, goal explanations and narratives for all students in the class. For example, with a student like Tosha, the survey would reveal that although she thought of the goal "learning for understanding" as important, she was not interested in exploring possibilities or the meaning of mathematics, nor did she value "having agency". This would be a signal to the teacher that Tosha needed support in engaging more deeply with the mathematics and taking more responsibility for choosing the strategy she used in a problem.

Another way to explore students' goals and explanations would be to have a class discussion toward the beginning of the year about these goals, and have students share their interpretations of what the goals meant. This list could then be used for ongoing conversations about the importance of these goals, in which the teacher could "check in" about circulating narratives about practicing mathematics.

After identifying which of the practice-linked goals students had appropriated and their interpretations of the goals, the teacher could start to consciously cultivate a narrative about what it means for each of their students to practice mathematics. One approach to cultivating a narrative could be through class discussions, in which the teacher shared stories and profiles about professional mathematicians, as well as fictitious students with positive and less positive dispositions to mathematics. Students could then have an opportunity to write their own profile about their goals and problem solving style in mathematics, which they could revisit and revise over the year. The teacher could write a response to their profile, as a way to support and add to their thinking. This would be a self-reflective tool to show students their own development over time.

Students could also be put in groups and tasked to come up with a list of qualities of a mathematician. The teacher could match up students who had contrasting interpretations of the goals, such as Lauren and Tosha, to encourage them to grapple with different perspectives.

Supporting productive engagement with peers while problem solving. A final recommendation to support students' positive dispositions would be to find ways to develop their productive engagement around solving problems with their peers. One way to address this would be through the classroom narrative mentioned above. Part of the narrative could be about what it means to work with another mathematical thinker, both considering their ideas and being able to negotiate around strategies and entry points. Another part of the narrative could be emphasizing the importance of doing one's own thinking. This might be a gentle redirect for students like Blake who have become accustomed to getting help and therefore giving up their responsibility for coming up with their own strategies and solutions.

Another way to support students' productive engagement would to have students brainstorm possible issues that might arise while working together, and use role-play to explore how to address these issues. For example, students might mention that sometimes they felt that one person wanted to make all the decisions. The group could generate possible responses, and then a pair could role-play some of these in front of the group, allowing students to “try out” different ways of interacting in a supportive environment. Supporting students’ productive engagement in problem solving with peers is an active area of mathematics education research today (c.f. Webb et al., 2014). The findings of this study emphasize the importance of supporting students’ productive interactions to facilitate the expression of their positive dispositions in their situated activity in problem solving.

Future Directions for Research

The results of this study point to several possible lines of future research. For this study, the focus was to *document* students’ goals, goal explanations and situated actions as a first step in understanding students’ dispositions in the classroom. A next step would be to further develop the survey to ask students specifically about the themes and narratives that arose among their explanations for their goal ratings, as discussed in the Implications section above. Another step would be to connect students’ *change* in goals, goal explanations and situated actions over the school year to events in their classroom, including the goals established and promoted by the teacher. A longer-term study could follow students over several years in a school that supported positive dispositions, to document key milestones in the development of their relationships to mathematics and related actions in problem solving. The elementary years of 2nd grade through 5th grade seem an important period to focus on given research about the potential decline of students’ attitudes to math during this period (Eccles et al., 1993; Ramirez et al., 2013). These studies would support a design experiment for a curriculum or approach that specifically targeted supporting the development of students’ positive dispositions using the practice-linked goals.

APPENDIX A

Student Survey (Winter)

I am interested in understanding what is important to students your age as they do math. Please think about yourself in math class this year. Your answers will only be seen by the research team and do not affect your grade. Please let us know what you really think! Thanks for your help.

Part 1: For each sentence, circle the number that best describes you.

1**. One of my goals in class is to learn as much as I can**.				
1	2	3	4	5
NOT AT ALL TRUE		SOMEWHAT TRUE		VERY TRUE
2. It's important to me to feel smart in math class.				
1	2	3	4	5
NOT AT ALL TRUE		SOMEWHAT TRUE		VERY TRUE
3. It's important to me that other students think I have interesting ideas to share.				
1	2	3	4	5
NOT AT ALL TRUE		SOMEWHAT TRUE		VERY TRUE
4. I choose to work on problems that challenge me.				
1	2	3	4	5
NOT AT ALL TRUE		SOMEWHAT TRUE		VERY TRUE
5. It's important to me to get the right answer to a problem.				
1	2	3	4	5
NOT AT ALL TRUE		SOMEWHAT TRUE		VERY TRUE
6. It's important to me that other students ask me for help.				
1	2	3	4	5
NOT AT ALL TRUE		SOMEWHAT TRUE		VERY TRUE
7. I want to be able to work on more challenging problems.				
1	2	3	4	5
NOT AT ALL TRUE		SOMEWHAT TRUE		VERY TRUE
8. It's important to me that I really understand the work I do during class.				
1	2	3	4	5
NOT AT ALL TRUE		SOMEWHAT TRUE		VERY TRUE

9. It's important to me that other students think I am smart at math. 1 2 3 4 5 NOT AT ALL TRUE SOMEWHAT TRUE VERY TRUE
10. It's important to me to have ideas to share in discussions. 1 2 3 4 5 NOT AT ALL TRUE SOMEWHAT TRUE VERY TRUE
11. It's important to me to get good grades and scores . 1 2 3 4 5 NOT AT ALL TRUE SOMEWHAT TRUE VERY TRUE
12. It's important to me to make up new strategies for solving math problems. 1 2 3 4 5 NOT AT ALL TRUE SOMEWHAT TRUE VERY TRUE
13. It's important to me to help other students. 1 2 3 4 5 NOT AT ALL TRUE SOMEWHAT TRUE VERY TRUE
14. It's important to me to look like a smart mathematician in class. 1 2 3 4 5 NOT AT ALL TRUE SOMEWHAT TRUE VERY TRUE
15. I try out new strategies that the teacher gives us or that other students made up. 1 2 3 4 5 NOT AT ALL TRUE SOMEWHAT TRUE VERY TRUE
16. Working on challenging problems is fun even if I don't find a solution. 1 2 3 4 5 NOT AT ALL TRUE SOMEWHAT TRUE VERY TRUE
17. I don't like being told which strategy to use to solve a problem. 1 2 3 4 5 NOT AT ALL TRUE SOMEWHAT TRUE VERY TRUE
18. It's important to me that the teacher thinks I have interesting ideas to share. 1 2 3 4 5 NOT AT ALL TRUE SOMEWHAT TRUE VERY TRUE

19. It's important to me that I improve my skills .				
1	2	3	4	5
NOT AT ALL TRUE		SOMEWHAT TRUE		VERY TRUE
20. I like to get my work done as quickly as I can.				
1	2	3	4	5
NOT AT ALL TRUE		SOMEWHAT TRUE		VERY TRUE

****

Part 2: Please read the sentences about goals for learning below. Think about yourself and what is important to you as you learn in class. Put a 1 for the sentence that is most important to you, a 2 for your second most important, 3 for your third most important, and so on up to 8 for your least important goal.

Please circle what kind of discussion you are thinking about for the last goal.

Goal for learning	Number
Learning for understanding	
Making up strategies and deciding how to solve problems	
Being able to solve challenging problems	
Feeling smart in math class	
Getting done quickly and getting the right answer	
Being able to help others	
Being a valued contributor to discussions (whole group) (small group) (both)	

Please explain why the goal you gave a "1" is your most important goal:

____

____

____

APPENDIX B

Student interview protocols

1st Student Interview Protocol (Fall)

Introduction: My name is Ms. Kumar. I am really interested in learning more about your goals in math class. I have some questions for you about the survey that you did. I have your survey so we can look at some of your responses if we need to. Do you have any questions for me before we begin?

Part 1*:* Questions about survey

1. You said that your most important goals in math class are (responses). I'd like to know more about that.

2. You said that you that these goals are least important (responses). I'd like to know more about that.

3. I saw that you wrote that you would tell a new student (insert response). Can you tell me more about that?

4. I want to ask you about these two responses (show responses that contradict). Can you tell me more about this?

5. Is there anything else you would like to tell me about doing mathematics in your class?

Part 2: Questions about actions in class

6. You said that your most important goals are (responses). What kinds of things do you do in math class so that you can (insert response. Ex: learn more, understand more, etc.?)

2nd Student Interview Protocol (Winter)

Discussion of cued video episodes

Introduction: My name is Ms. Kumar. I am really interested in learning about your goals and your problem solving in math class. Today I would like to talk about your goals while we watch some video of you problem solving in class. Do you have any questions for me before we begin?

Procedure:

1. Remind student of challenge problem. Ask about their solution
2. Watch cartoon as practice
3. Watch clip. Rewatch and pause.
4. Do ranking
5. Watch clip again and member check
6. Ask student to rank goals for that episode

Probe 1: "Can you tell me more about what was happening here?" "what were you thinking when that happened?" "What did you mean when you said that?".

3rd Student Interview Protocol (Spring)
Discussion of cued video episodes for case study students (n=8)

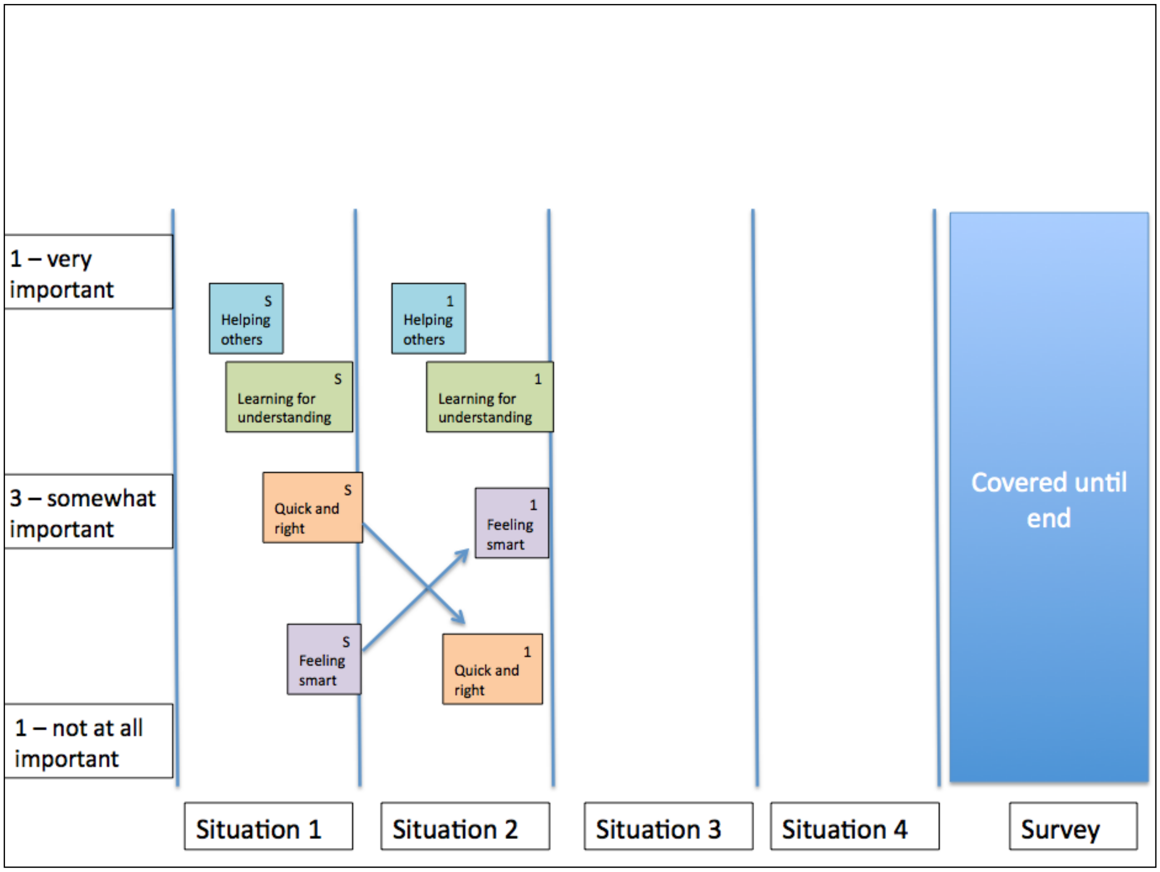
Preparation: Ahead of time I will look over the previous interview in which I asked students about their goals during specific situations in class using video based recall. I will note any goals that the student said were consistent across situations. In addition, I will assemble ranking of goals for each moment I asked them about and be prepared to present these to students if we talk about a similar situation.

Introduction: My name is Ms. Kumar. As we have discussed before, I am really interested in learning about your goals and your problem solving in math class. I'd like to continue our conversation from last time when we used video clips to talk about your goals at different moments while you solving a challenge problem. I have some notes about your goals from our conversation last time. This time I would like to ask you about your goals at different *kinds* of situations during problem solving. I'll ask you to rank your goals for each situation on this big board, and then I'd like to talk to you about your ranking. Do you have any questions for me before we begin?

I will explain that we'll be using the chart to talk about their goals during the different situations. We will go through the situations one by one. *Ahead of time I will place their goals from their survey in the right most window, but I will keep this covered up until the end.*

First, I will write the situation at the bottom of the board. I will be using the following situations: deciding on a strategy for a problem, the strategy you are using doesn't work (come to a bypass), you are working with someone who thinks differently from you, you are preparing to share a strategy in front of the class. Then, I will ask the student to place the goals on the scale in that window. *If there are goals that they have said are consistent across situations in a previous interview, I will place that goal ahead of time and check in with them during the interview. This will allow me build on previous conversations and focus on goals that change.* I will ask students to put the goal at the top of the scale if it is very important, at the bottom if it is not at all important, and in the middle if it is somewhat important (see image below). I will then ask them to talk about their placement of each goal.

Example Board:



We will then move on to the next situation. I expect students to naturally transfer over some of their goal positions from the previous window. *Each goal will be a different color to allow comparison across windows. Each set of goals will have a number to associate it with that window.* After the student ranks their goal for a situation, we will talk about why some goals shifted and others did not from other situations.

At the end, I will show the student their ranking of goals from the survey, and together we will compare this with their ranking of goals in different situations. We will discuss why some goals stayed the same and others changed.

4th Student Interview Protocol
Final Interview

Introduction: My name is Ms. Kumar. I am really interested in learning more about your goals in math class. I have some questions for you about your responses to the surveys that you did this year. I have your last survey so we can look at some of your responses if we need to. Do you have any questions for me before we begin?

1. You said that your most important goals in math class are (insert from student's file). I'd like to know more about that.

2. You said that you that these goals are least important (insert from student's file). I'd like to know more about that.

3. I noticed that the importance of this goal changed over the year (insert from student's file). Do you agree with that? Why did this goal change over the year?

4. I want to ask you about these two responses (insert from student's file). Can you tell me more about this?

5. Is there anything else you would like to tell me about doing mathematics in your class?

APPENDIX C
CHALLENGE PROBLEMS

Names ____________
____

Stamp Challenge Problem

Mrs. Acosta wanted to buy stamps. She had $20 to spend.

Postcard stamps cost 25¢.

Letter stamps cost 50¢.

How many of each kind of stamp should she buy?
____

____
Number of
Postcard stamps

____
Number of
letter stamps

____
Total number of
stamps

____
Total cost

Please explain how you solved the problem:

____
____
____

Names ____________________

____

Pencil Challenge Problem

Part 1

Our class needs some new pencils! We're going to get 2 plain pencils for each student, and 1 fancy pencil for each student. Ms. Acosta wants new pencils too so we have 28 people total.

How many plain pencils and fancy pencils do we need? How many new pencils total?

Part 2

We need volunteers to sharpen the pencils. How many volunteers should we ask for? How many pencils will each volunteer sharpen?

Names ____________________

____

Lasagna Challenge Problem

Your class is planning dinner for guests at an upcoming class celebration. You are not sure how many people are coming.

A family plans to bake and bring 10 lasagnas.

If everyone gets an equal piece, how many people can eat lasagna for dinner? Explore different numbers of guests. What is the least number you need to eat the lasagna? What is the most it could feed?

This is a thinking exercise. Feel free to be creative! Please explain your thinking in pictures, numbers and words below.

Names ___________________
____

Scale Challenge Problem

The picture below shows one dimension of a building downtown. Describe how you would scale up the building (Royce Hall) below to make a model that would fit on a desk (it doesn't have to take up the whole desk).

Please explain how you came up with your solution:

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