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Analysis of High-Occupancy-Toll Lane Operation

THESIS

submitted in partial satisfaction of the requirements
for the degree of

MASTER OF SCIENCE

in Civil Engineering

by

Xuting Wang

Thesis Committee:
Professor Wenlong Jin, Chair
Professor Will Recker
Professor David Brownstone

2016

DEDICATION

This work is dedicated to my parents,
Mr Wang and Ms Dong,
who always support me
and
encourage me
in my life.

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ABSTRACT OF THE THESIS

Analysis of High-Occupancy-Toll Lane Operation

By

Xuting Wang

Master of Science in Civil Engineering

University of California, Irvine, 2016

Professor Wenlong Jin, Chair

In this thesis, we propose one approach to determine the real-time tolling strategy for high-occupancy toll (HOT) lane, and calibrate driver's value of time(VOT) as well. There are two goals of operating HOT lane, one is to maximize the freeway's, and another one is to maintain the free flow speed. We use queue length to track the traffic dynamics, and the point queue model is used. And with the application of a proportional-integral-derivative(PID) controller, we can calculate the pricing rate for HOT lane and calibrate driver's (VOT). Simulation results and comparison with previous studies are provided.

Chapter 1

Introduction

1.1 Background

In recent years, traffic congestion is becoming more severe in large cities and on highways and it affects economic competitiveness, driving safety, and air quality. According to the European UNITE project, the total social cost of congestion, on average, is equal to approximately 1% of GDP in Western Europe [33]. Chang(2003) [9]stated that the accident frequency on both freeways and arterial roads tends to increase with the congestion levels. TTI [42]reported that during the peak hour, the average annual delay for auto commuters in Los Angeles is about 80 hours, and the average congestion cost was 1711 dollars 2014. And CO_2 emission for a stop-and-go driving pattern would be higher than those with constant speed [5].

In the U.S., one type of managed lane called high-occupancy vehicle (HOV) lane(also known as carpool lane) is widely used on freeway to reduce congestion. This lane is available for cars with minimum two or three occupants. Many studies have concerned the low utilization of HOV) lanes. Kwon (2008) [21]collected peak hour speed and flow measurements from

700+ stations for 128 weekdays in 2005, and found 81% of HOV lane flow rates are below 1400 vehicles per hour per lane.

1.2 Motivation

Agencies have tried different strategies to ease congestion, such as expanding road capacity, optimizing signal timing and control, promoting voluntary reduction in driving at businesses and other large organizations. Recently, congestion pricing has received more and more attention both in the field of economics and transportation. Rand Corporation(2008) [43]indicated many strategies (such as raising fuel price) would provide short-term relief, but only pricing strategies could manage congestion in the long run.

Another type of managed lane is called high-occupancy toll (HOT) lanes. In HOT lane operation, we can charge for those single-occupancy vehicles(SOVs) if they want to use HOT lane to to reduce travel time. HOT lanes have been regarded as an improvement of HOV lanes in terms of better lane utilization.

Some studies have already discussed the strategies of HOT lane operations, but there are some deficiencies. First, many studies applied discrete choice model to determine driver's lane choice, and few used UE principle. In traffic assignment, Wardrop's principle is used more often, and can better capture driver's behavior. Instead, we will provide both discrete choice model and UE principle in this paper, and make a comparison between them. Second, Zhang(2008) [52]used the speed difference to update the pricing strategy, but it cannot capture the traffic dynamics, such as the queue evolution. In the step-wise function, multiple parameters are needed, but these parameters have no economic meaning. At the same time, Yin and Lou(2009) [50]applied a feedback-control approach to update pricing rate. The error term is expressed by the difference between measured and desired occupancy of the toll

lane. Third, Yin and Lou(2009) [50]used kalman filtering technique to calibrate the WTP, which involves much computation(because in each iteration, we need to update parameters, such as Kalman gain and error covariance matrix). At the same time, a well-designed PID controller can calibrate VOT more efficiently, and the basic idea can be easily understood.

So, in this paper, we will first describe the traffic dynamics using point queue, and calculate the travel time on two types of lanes. Both UE and discrete choice model will be used to model driver's choice. And a PID controller will be used to calibrate driver's VOT and the corresponding pricing rate on HOT lane.

Chapter 2

Literature Review

2.1 Congestion Pricing

Pigou(1920) [40]first proposed the idea of road pricing. In the book, he suggested that an optimal charge should be implemented against the congested road, so individual would cost the same when using either road. Knight(1924) [19]expressed the same idea by stating that the government should levy a small tax on each truck using the narrow road, so their ordinary cost, plus the tax, equals to the cost on the broad road. De Palma(2011) [13]pointed that individual traveler imposes delays on others, they do not pay the full marginal social cost of their trip and therefore creates a negative externality. In this case, congestion pricing should be imposed to internalize the costs of a negative externality.

2.1.1 Static Pricing

In static pricing, the space dimension is static. First-best pricing and second-best pricing schemes are widely used.

The goal of first-best pricing is to reach a system optimal (SO) flow, so the toll rate equals to the difference between marginal social cost and marginal private cost. Walters(1961) [46]considered a static model of tolling on a single road link. In this paper, he pointed out marginal social cost should be equal to the marginal private cost multiplied by the elasticity. So, if $C(X)$ represents the unit private cost at which the number of vehicles per hour is X , the toll should be set as $X * \frac{dC(X)}{dX}$. Dafermos(1972) [10]proposed an algorithm to find the SO flow for a general congested transportation network with multiple vehicle types. Yang(1998) [48] presented how marginal-cost pricing could be applied in a road network with elastic demand and queuing.

Unfortunately, it is not practical to toll on each link in a network. So, the second-best pricing has received more attention recently. Two key problems related to second-best pricing strategy are: 1) toll rate and location; 2) Different impacts brought by toll to different users[49].

Marchand(1968) [30]extended Levy-Lambert's(1968) work [25], and derived a second-best pricing strategy for two-route scenario to maximize a linear combination of the individual's utility. Verhoef(1996) [44]investigated the optimal one-route toll, and mentioned we should take account of the "spill-over" effects on the untolled route, comparing with first-best pricing. May(2000) [32]concluded four practical tolling alternatives: travel-time based charging, travel-distance based charging, link-based charging and cordon-based charging.

2.1.2 Dynamic Pricing

In static pricing, it's hard to capture the characteristics of traffic dynamics. In this case, dynamic pricing would be the solution. Vickrey(1969) [45]assumed a scenario that a bottleneck with finite capacity exists on the way for commuters to work, and a piece wise linear pricing structure is proposed to eliminate the queue.

Vickrey's bottleneck model has been extended to different topics. Arnott(1990) [1]proposed a toll time interval and a step rate for a simple bottleneck over the rush hour, and provided a comparison of no-toll equilibrium, SO and optimal coarse toll in average travel cost, scheduled delay costs and total travel time costs. Lindsey(2000) [26] considered heterogeneity in the trip-timing preferences and time costs of travelers. They applied the congestion delay indifference curve to capture the difference of no-toll equilibrium and system optimal in a morning peak period, and obtained the first-best pricing rate.

2.1.3 High-occupancy-toll Lane

In literature, Zhang(2008) [52]adopted a piece-wise feedback control model, based on the speed on HOT and GP lane, to calculate the optimal flow ratio for HOT lane. Then the toll rate is estimated backward based on discrete choice model. Yin and Lou(2009) [50]proposed two methods including feedback and reactive self-learning approach to determine the dynamic pricing rate regarding the arrival flow rate on GP and managed lanes. Based on the self-learning approach mentioned in Yin and Lou(2009) [50], Lou(2011) [28]applied multi-lane hybrid traffic flow model proposed by Laval's(2006) [24]to consider the impacts of the lane-changing behaviors. Laval(2015) [23]proposed a System Optimal(SO) toll for real-time arrivals under inelastic demand, but the solution is not well defined when flow rates reached capacity.

2.2 Value of Time

In economics, the value of time represents the opportunity cost of the time that a traveler spends on trips. It is the amount that a traveler would be willing to pay in order to save time, or accept as compensation for lost time. Vickrey(1969) [45]mentioned variation of value of

time(VOT) is important when deciding the pricing strategy. So, it is worth estimating the VOT.

In the last fifteen years, some economists have done some studies in VOT based on real world data. Lam(2001) [22] estimated VOT based on the survey data and loop detector data on SR-91. The VOT is obtained when maximum log-likelihood achieved. Brownstone(2003) [7] collected revealed preference data from drivers, loop detector and toll data on I-15, and unweighted maximum likelihood estimation is carried out for the mode choice. Then, the VOT for each respondent is calculated using the estimated parameters.

Yin and Lou(2009) [50] used logit model to formulate route choice in a simulation for a segment of freeway with HOT and GP lanes. But different from papers list above, they applied Kalman filtering technique, a method in control theory, to calibrate driver's willingness to pay(WTP).

2.3 Traffic Flow Theory

2.3.1 Point Queue Model

In road networks, if the demand is higher than the supply, there will be queues. In order to control these queues, we need to understand the characteristics of queues. In literature, some traffic flow models have been applied to describe queue evolution. Among these models are cell transmission model [11], link transmission model [51] and link queue model [16]. In this paper, we use Point Queue Model to study the dynamics.

Point Queue Model is developed based on the First-in-first-out (FIFO) rule. Because the queue has no physical length, the link travel time is the sum of constant travel time and delay time at the exit of the link. Vickrey(1969) [45] introduced point queue model to trans-

portation system by analyzing a bottleneck with dynamic arrival patterns. When Bell(1995) [6]studied stochastic user equilibrium assignment in network with queues, he applied point queue model to calculate link costs. Kuwahara(1997) [20]applied deterministic point queue model to evaluate the link travel time, and developed a many-to-many dynamic traffic assignment in a network with queues. Nie(2005) [35]compared point queue model with other macroscopic link models, including M-N model and CTM in network loading. Jin(2015) [17]derived point queue models as the limits of LTM and LQM, and provided analytical and numerical results.

2.3.2 Capacity Drop

The two-capacity or capacity drop phenomenon of bottlenecks has been observed and studied for decades, and it means”maximum flow rates decrease when queues form”. The existence and verification of capacity drop were first discussed in (Banks, 1990 [4]; Hall, 1991 [15]). Capacity drop can occur at a merge point of freeway, tunnels, lane drops. A road section with a lane-drop bottleneck is shown in figure 2.1. The upstream link 1 has m lanes, downstream link 2 has n lanes, and $m > n$. At the same time, capacity drop can happen where work zones and accidents appears. Later, Cassidy(1999) [8]reported 10% drop in capacity on two freeway bottlenecks in and near Toronto. Zhang(2004) [53]observed that capacity drops range from 3% to 12% in the 27 active bottlenecks in the Twin Cities metro area of Minnesota.

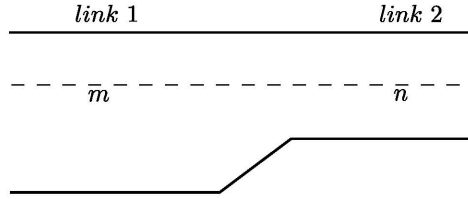


Figure 2.1: A lane-drop bottleneck

Newell(1993) [34] stated that a drop in the downstream bottleneck's discharge rate can further reduce the total discharge flow-rates of the whole corridor. In literature, many studies have investigated the mechanism of capacity drop. Hall(1991) [15] pointed out that the flow is reduced because the drivers accelerate from the queue. At the same time, heterogeneous drivers can also cause capacity drop [12]. However, the mechanism of capacity drop at active bottlenecks. Jin(2015) [18] proposed a phenomenological model with continuous fundamental diagram and discontinuous boundary flux function, and replicated some characteristics of capacity drop: first, when both upstream and downstream are not congested, the maximum discharge rate can be reached; second, the activation of bottleneck triggers capacity drop.

In order to reduce the delay caused by capacity drop, some control strategies have been developed, such as variable speed limit [27], and ramp metering[38][39].

2.4 Control Theory

Although control systems have been known and applied for over two thousand years, a detailed analysis of the system was not done before Maxwell(1867) [31]. In this paper, he analyzed the phenomenon of self-oscillation, which may cause the system unstable. By using differential equations, Routh(1877) [41] analyzed the stability of a system. Later, Lya-

punov(1892) [29], and Nyquist(1932) [36] made great contribution to the stability theory.

Most classical control problems deal with single-input-single-output, linear, time-invariant, finite-dimensional systems. While in modern control, the problems could be extended to multivariable, nonlinear, time-varying and high-dimension. Modern control theory uses the time-domain state space representation, and a set of input, output and state variables related by first-order differential equations. Modern control theory made the design of control systems simpler. However, it can also make the system sensitive to errors [37].

2.4.1 Basic Control Concepts

A control system is an arrangement of physical components connected or related in such a manner as to command, direct, or regulate itself or another system. And the basic elements of a control system is shown below.

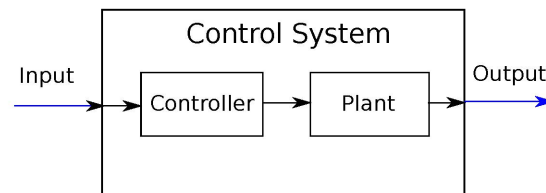
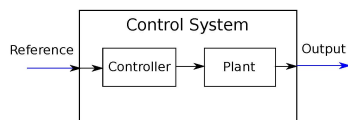


Figure 2.2: Basic elements of a control system

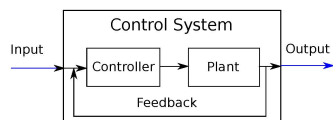
A block diagram of a system is a pictorial representation of the functions performed by each component and of the flow of signals [37]. In a block diagram, boxes show different process elements, and each box has both inputs outputs. The inputs denote the variables that influence a process, and the outputs are the signals that we are interested in [3]. For example, for a control system with acceleration rate as input, if the block diagram serves as

an integrator, the output should be speed.

There are two common classes of control systems. If the system is interconnected in a cycle, we call it closed-loop control, and open-loop otherwise. If the interconnection is broken, we refer it as open-loop control system. In this paper, we use closed-loop control, because comparing with open-loop control, it is more flexible, robust, and efficient.



(a) A open-loop control system



(b) A closed-loop control system

Figure 2.3: Two classes of control systems

2.4.2 PID Controller

In closed loop control system, we need feedback to make comparison between the actual and desired value. There are several feedback mechanisms, such as on-off control (also known as binary or bang-bang controller) and proportional-integral-derivative (PID) controller. The on-off controller is easy to implement, and no parameter is chosen. But the on-off controller often causes oscillations. Then, PID controller can be used to avoid this effects.

The first PID controllers were mechanical devices used to control windmills and steam engines. And PID controller is now commonly used in industry. According to Desborough (2002) [14], 97% regulatory controllers in refining, chemicals and pulp and paper industry use PID mechanism. In PID controller, the proportional action is proportional to the error, the integral term is proportional to both the duration and magnitude of the error, and the derivative term improves the closed-loop stability [2]. The block diagram of a PID controller

is shown in figure 2.4. For discrete time systems, we use controller is called proportional-summation-difference(PSD).

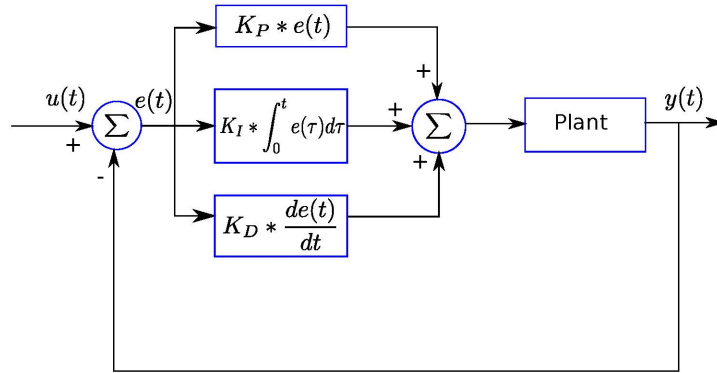


Figure 2.4: A block diagram of PID controller

So, the input/output relation for a PID controller is:

$$u(t) = K_P * e(t) + K_I * \int_0^t e(\tau) d\tau + K_D * \frac{de(t)}{dt} \quad (2.1)$$

However, in practical, the derivative action is seldom used. So, we are more interested in PI controller here.

For the choice of tuning method, it will depend on whether or not the loop can be taken "offline" for tuning, and on the response time of the system. Some common choices include manual tuning, Ziegler-Nichols method, Cohen-Coon method, software tools, and so on.

2.4.3 Stability of Control System

In practice, the basic requirement for a system is to be stable. If the system is unstable, even given a small input, the output may become infinite, and leading some problems.

In a dynamical system, equilibrium point is one of the most important feature. And if x_e is an equilibrium point, then $\frac{dx_e}{dt} = 0$, which means for all $t \geq 0$, $x(t) = x_e$.

There are different types of stability. If a solution is Lyapunov stable, then, for every $\epsilon > 0$, there exists a $\delta > 0$ such that, $\|x(0) - x_e\| < \delta$, for any $t > 0$, we have $\|x(t) - x_e\| < \epsilon$. If a solution is asymptotically stable, it is Lyapunov stable, and there exists a $\delta > 0$ such that, $\|x(0) - x_e\| < \delta$, we have $\lim_{t \rightarrow \infty} \|x(t) - x_e\| = 0$. These two types of stability are analyzed more often. At the same time, we can also observe unstable and marginally stable systems.

For a linear time-invariant system, written as $\frac{dx}{dt} = Ax, x(0) = x_0$, we can use its eigenvalue to help analyze the stability. If all eigenvalues of A have a strictly negative real part, the system is asymptotically stable [3]. Another way to study the stability is to check the poles of transfer function.

2.5 Route Choice Behavior

In this section, we will provide reviews on some principles or models on driver's route choice(or lane choice) behavior.

2.5.1 Wardrop's Principle

In deterministic path choice problem, we usually follow two principles, and both are proposed by Wardrop(1952) [47]. In this paper, Wardrop's first principle was proposed by "the

journey times on all the routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route”, and it results in user equilibrium(UE). Wardrop’s second principle indicated ”the average journey time is a minimum”, and it led to system optimal(SO).

Wardrop’s principle has been widely used in transportation system. And the first-best congestion pricing, which is already shown in chapter 2.1.1 is one of its applications.

2.5.2 Discrete Choice Model

Discrete choice models are first discussed in economics. These models can describe, explain, and predict choices between two or more discrete alternatives. Due to different types of random component, discrete choice models can be classified into logit model and probit model. If the random component is independent-identical-distributed(IID) Gumbel distributed, we can get a logit model; if normal distributed, the corresponding model is probit model.

In logit model, we have the following assumptions: individuals are utility maximizers; utility can be specified by the sum of a deterministic component and a random component; usually, the deterministic component is linear in parameters.

And in the literature of HOT lane analysis, both Zhang(2008) [52] and Yin(2009) [50] applied logit model to capture the lane choice behavior of SOVs. The multinomial logit model is enough to capture the choice between two type of lanes, because their random component is not correlated.

Chapter 3

Problem Statement

When there is congestion on freeway, people will experience longer travel time. In most cases, the managed lane is not used efficiently, so some SOV drivers have the incentive to pay to reduce the delay on regular lanes. But if too many drivers change lanes, vehicles on HOT lane cannot reach free flow speed. So, it is necessary to analyze the traffic dynamics on both lanes, and provide a effective control strategy.

This study starts from a simple scenario with two classes of lanes on freeway. The layout is shown in figure 5. In this segment, an instantaneous lane drop occurs on GP lane. In the upstream of the drop, there are two GP lanes and one HOT lane in this freeway segment with the same free flow speed, the capacity of HOT lane is C_1 per hour per lane, and capacity for GP lane is C_2 per hour per lane.

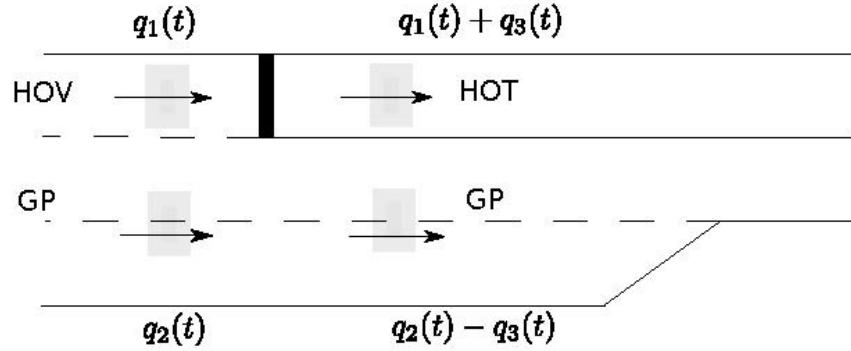


Figure 3.1: Layout of a freeway with HOT and GP lanes

There are two sets of loop detectors on the freeway. The first is set before toll reader to record the approaching flow rate on HOV and GP lanes, and the second is set behind the reader to detect the flow rates on HOT and GP lanes.

In this study, we have three types of vehicles. The first type is HOV, which has two or more passengers. The second type is "regular" SOV, and it uses GP lane all the time. The last type is "special" SOV, which has an incentive to pay to use HOT lane in order to reduce delay.

In Figure 5, $q_1(t)$ represents the flow rate of HOVs; $q_2(t)$ means the flow rate on GP lane before the toll reader, which is the sum of two types of SOVs; $q_3(t)$ represents the flow rate for the special SOVs. So, the flow rates after toll reader on GP lane and HOT lane should be $q_2(t) - q_3(t)$ and $q_1(t) + q_3(t)$ respectively.

There are several assumptions in this study. First, we initially do not know the true VOT, but we can use our framework to calibrate it. Second, HOVs do not need to pay when using HOT lane, while SOVs need to pay to get access. Third, we assume all the lane changes to HOT lane happen before the toll reader. Forth, homogeneous drivers follow the same lane

group choice behavior, such as logit model and UE.

When HOT lane is operated efficiently, vehicles on HOT lane move at free flow speed, and the throughput on HOT lane is maximized. A HOT lane pricing strategy is applied to achieve the goal. At the same time, in order to get the pricing strategy, we need to estimate VOT. In this study, we try to solve these problems.

In the next chapter, we will provide the framework of this study, and explain how to track traffic dynamics, calibrate driver's VOT and set the tolling rate in detail.

Chapter 4

Methodology

When implementing a congestion pricing strategy, we need to take driver's VOT into account. In literature, VOT has been collected through surveys, or estimation from data of freeways with toll lanes (such as CA-91, I-15 in California). In this study, we show that driver's VOT can be gradually learned through an operation simulation of a freeway segment with HOT lane.

In the simulation, the information available is the arriving flow rate on both HOV and General Purpose (GP) lane, and the capacity. We need to calculate turning proportion of SOVs from GP lane to HOT lane, queuing time on both GP and HOT lane, and the pricing rate at a certain time period.

In this part, we will show how to apply point queue model and PID controller to our model.

4.1 Block Diagram Description

In this section, we will show how models mentioned above are coordinated. A block diagram is used to illustrate the relation between these components.

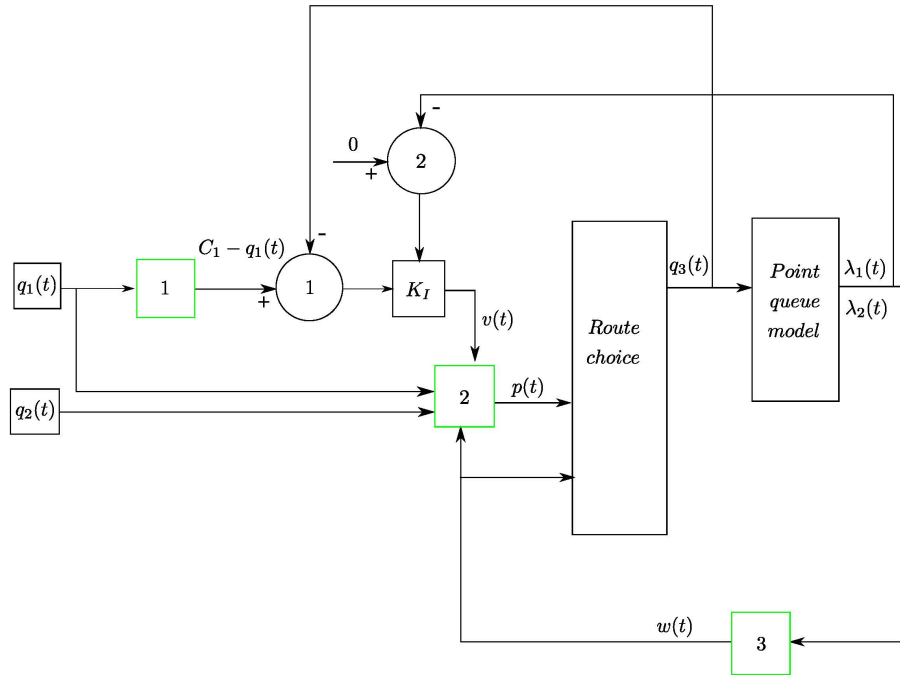


Figure 4.1: Block diagram of our approach

In the block diagram, the two goals of operating HOT lane serve as the reference level. And there are three control blocks (marked as green) in the diagram, and each represents a function. The first block converts the influx on the HOV lane to the optimal turning influx for SOVs, $C_1 - q_1(t)$. The second green block is used to get the pricing rate. For the third green block, the inputs are the queue length on two types of lanes. By applying queuing model, the output is the travel time difference on HOT and GP lanes ($w(t)$).

For the controller, one Integral (I) controller is used to reveal the true value of VOT. The first error signal is the difference between the actual influx turning from GP lanes and optimal

turning influx. And the queue length on the HOT lanes serves as another error signal.

We have two plants in the diagram. The first one reveals drivers' behavior, which can be based on discrete choice model, user equilibrium or some other principle. And the second plant shows the queue evolution on both lanes, and in this study, the point queue model is applied.

Below is a more traditional block diagram of this study.

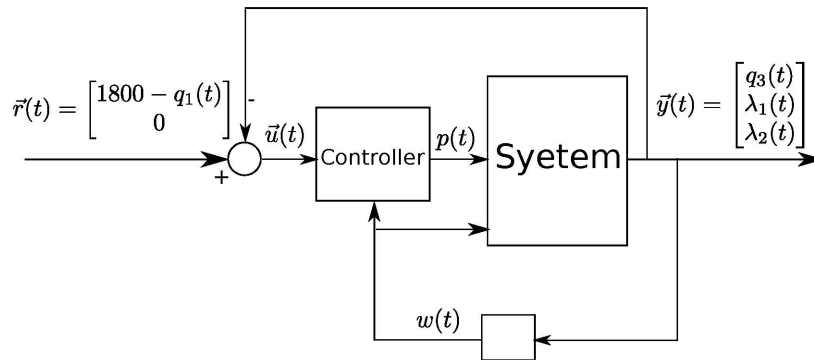


Figure 4.2: Another Block diagram of our approach

The inputs are optimal turning influx for SOVs, and optimal queue length for HOT lane, and the outputs are actual turning influx, queue length on both HOT and GP lane. By comparison, we can see the system has two subsystems, route choice model and point queue model.

In order to simulate the diagram, we need some initial and boundary conditions. For the initial condition, we need to give the initial assumption of value of time (i.e., $v(0)$), because when we are dealing with the numeric solution of ordinary differential equations (ODE), the initial value of v is needed. At the same time, in this case, we assume there is no queue

downstream at the beginning of the simulation, so the initial queue length on HOT and GP lanes is zero (i.e., $\lambda_1(0)=\lambda_2(0)=0$). And the boundary condition is the influx of HOV lane and GP lanes before the toll reader (i.e., $q_1(t)$ and $q_2(t)$).

And in each iteration, the solution should be $v(t)$, $\lambda_1(t)$, $\lambda_2(t)$ and an intermediate solution $q_3(t)$.

4.1.1 Control block 2

In the block diagram above, block 2 looks like a black box, and we will explain it in detail in this section.

Block 2 is used to calculate pricing strategy for HOT lane. Based on different driving behavior, we can have different formulations. If a logit model is applied, the inputs of this block are the influx on HOV lane and GP lanes, estimated VOT, and queuing time difference at time t , and the output, pricing rate, is $p(t) = w(t) * v(t) + \ln\left(\frac{q_2(t)-(C_1-q_1(t))}{(C_1-q_1(t))}\right)$. For the details, please refer to section 4.5.

4.2 Point Queue Model

In point queue model, the travel time is composed of the free flow travel time and the queuing time. For GP and HOT lane in a homogeneous road segment, the free flow travel time is the same, the only difference is the queuing time. And in this section, we will introduce the point queue model proposed by Jin(2015) [17], in both continuous and discrete form to capture the queue evolution.

4.2.1 Continuous Form

We consider a point facility, which can be regarded as a limit of a link with a storage capacity of Λ . $\delta(t)$ is the origin demand, $\sigma(t)$ is the destination supply, $d(t)$ and $s(t)$ are a link's demand and supply respectively. $\lambda(t)$ is the size of point queue, and it is used to model the dynamics of a queue.

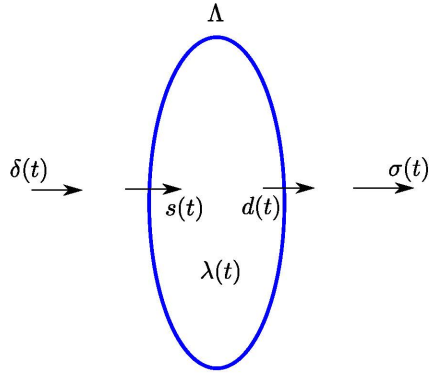


Figure 4.3: An illustration of point queue model

Given demand and supply function, we can have a point queue model:

$$\frac{d}{dt}\lambda(t) = \min\{\delta(t), s(t)\} - \min\{\sigma(t), d(t)\} \quad (4.1)$$

which means, the rate of change in queue length is the different between the upstream influxes and downstream out-fluxes.

When the storage is infinite, we have $s(t) = \infty$, and the rate of change is expressed by:

$$\frac{d}{dt}\lambda(t) = \delta(t) - \min\{\sigma(t), d(t)\} \quad (4.2)$$

For a very small time step Δt , we can have an approximate form of a point queue demand

as $d(t) = \delta(t) + \frac{\lambda(t)}{\Delta t}$, so the queue dynamics is:

$$\frac{d}{dt}\lambda(t) = \max\{\delta(t) - \sigma(t), -\frac{\lambda(t)}{\Delta t}\} \quad (4.3)$$

So, for HOT and GP lane in this study, the point queue model becomes:

$$\frac{d}{dt}\lambda_1(t) = \max\{q_1(t) + q_3(t) - C_1, -\frac{\lambda_1(t)}{\Delta t}\} \quad (4.4a)$$

$$\frac{d}{dt}\lambda_2(t) = \max\{q_2(t) - q_3(t) - C_2, -\frac{\lambda_2(t)}{\Delta t}\} \quad (4.4b)$$

4.2.2 Discrete Form

For the continuous point queue models, we use $\frac{\lambda_1(t+\Delta t) - \lambda_1(t)}{\Delta t}$ to replace $\frac{d}{dt}\lambda(t)$, and make it a discrete form, the queue evolution on both lanes is:

$$\lambda_1(t + \Delta t) = \max\{0, (q_1(t) + q_3(t) - C_1) * \Delta t + \lambda_1(t)\} \quad (4.5a)$$

$$\lambda_2(t + \Delta t) = \max\{0, (q_2(t) - q_3(t) - C_2) * \Delta t + \lambda_2(t)\} \quad (4.5b)$$

where $\lambda_1(t)$ and $\lambda_1(t + \Delta t)$ are queue length at time t and $t + \Delta t$ for HOT lane, and $\lambda_2(t)$ and $\lambda_2(t + \Delta t)$ are queue length at time t and $t + \Delta t$ for GP lane, Δt is the time interval.

And the figure below can graphically show the queuing time for a vehicle entering the queue at time t . It is clear that the geometric relation in the small triangle shows $C * \pi(t) = \lambda(t)$.

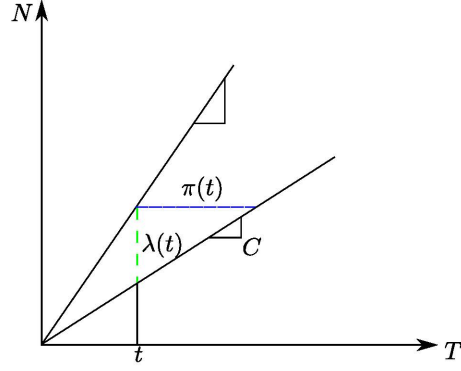


Figure 4.4: Queuing time

So, when a queue forms, the queuing time for a vehicle on HOT and GP lane is:

$$\pi_1(t) = \frac{\lambda_1(t)}{C_1} \quad (4.6a)$$

$$\pi_2(t) = \frac{\lambda_2(t)}{C_2} \quad (4.6b)$$

Then, after knowing the queue length on two type of lanes, the travel time difference, denoted as $w(t)$ can be calculated:

$$w(t) = \pi_2(t) - \pi_1(t) = \frac{\lambda_2(t)}{C_2} - \frac{\lambda_1(t)}{C_1} \quad (4.7)$$

4.3 PID Controller

In this paper, we will only use Integral(I) controller to generate the input to the plant, because we think the rate in change of price is influenced by the error signal. And here is the formulation of this controller.

4.3.1 Continuous Controller

For the Integral controller, we have :

$$\frac{d}{dt}v(t) = -K_I^1 * (C_1 - q_1(t) - q_3(t)) - K_I^2 * (0 - \lambda_1(t)) \quad (4.8)$$

where K_I^1 and K_I^2 are both non-negative, and denote the coefficients for the integral term.

In formula 12, we have two error terms, one is the difference between the current flow rate and the capacity of HOT lane, another is the queue length on HOT lane. When $q_1(t) + q_3(t) < C_1$, it shows HOT lane is under-utilized, the pricing rate is set too high. So, the rate should decrease when this difference increase, a negative sign is needed before the K_I^1 . For the second term, if there is a queue on HOT lane, it is obvious that the pricing rate is too low, and we should increase rate to eliminate the queue. When these two terms become zero, we can reach the steady state (or equilibrium state).

4.3.2 Discrete Controller

Then, we replace $\frac{d}{dt}v(t)$ by $\frac{v(t+\Delta t)-v(t)}{\Delta t}$, the discrete form is given by:

$$v(t + \Delta t) = K_I^1 * (C_1 - q_1(t) - q_3(t)) * \Delta t + K_I^2 * \lambda_1(t) * \Delta t + v(t) \quad (4.9)$$

4.4 Lane Group Choice

For the route choice, if we apply discrete choice model to capture the turning proportion of SOV to HOT lane, the flow rate on HOT lane is:

$$q_1(t) + q_3(t) = q_1(t) + q_2(t) * \frac{\exp(V_{HOT}(t))}{\exp(V_{HOT}(t)) + \exp(V_{GP}(t))} \quad (4.10)$$

Then the turning proportion is:

$$\frac{q_3(t)}{q_2(t)} = \frac{\exp(V_{HOT}(t))}{\exp(V_{HOT}(t)) + \exp(V_{GP}(t))} \quad (4.11)$$

where $V_{HOT}(t)$ and $V_{GP}(t)$ represents the deterministic utility of using HOT and GP lanes at time t .

4.5 Pricing Strategy

In our analysis, at time t , the utility for SOVs turning to HOT lane is $(t_f + \pi_1(t)) * v^* + p(t)$, and for those stay on GP lane is $(t_f + \pi_2(t)) * v^*$, where t_f is the free flow travel time, v^* is the true VOT. Based on logit model, the lane choice should be formulated as follows:

$$\frac{q_3(t)}{q_2(t) - q_3(t)} = \frac{\exp((t_f + \pi_1(t)) * v(t) + p(t))}{\exp((t_f + \pi_2(t)) * v(t))} \quad (4.12)$$

If we take the log on both side of equation (17), and simplify it, we can get:

$$\ln\left(\frac{q_3(t)}{q_2(t) - q_3(t)}\right) = p(t) - w(t) * v(t) \quad (4.13)$$

When HOT lane is operated at optimal, $q_1(t) + q_3(t) = C_1$, at time t , the pricing rate should be set as:

$$p(t) = w(t) * v(t) + \ln\left(\frac{q_2(t) - (C_1 - q_1(t))}{(C_1 - q_1(t))}\right) \quad (4.14)$$

4.6 Comparison With Yin & Lou's Two Approaches

The starting point of our paper comes from Yin and Lou's paper(2009) [50]. They proposed two approaches for determining dynamic tolls in response to real-time traffic condition, one is named feedback-control approach, and another is called reactive self-learning approach.

In the feedback-control approach, the tolling rate on HOT lane can be stated as:

$$p(t + 1) = p(t) + K(O_{HOT}(t) - O_{HOT}^*) \quad (4.15)$$

where $O_{HOT}(t)$ is the measured occupancy on HOT lane, O_{HOT}^* is the desired occupancy, and is usually equals or slightly less than critical occupancy. And K is a regulator parameter. And this method can be regarded as an application of ALINEA strategy [38]. And the block diagram is shown below.

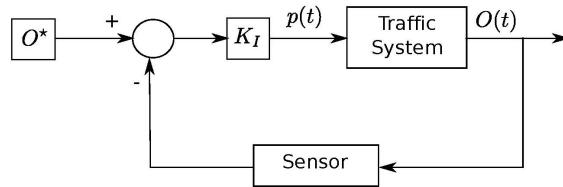


Figure 4.5: Block diagram of feedback-control strategy

In the self-learning approach, a logit model is applied to show the lane group choice. With the flow rates collected on GP, HOV and HOT lanes, the parameters in the model can be estimated recursively by Kalman filtering technique, and the WTP is obtained. And they applied point queue concept [20] to capture traffic dynamics. Then, the optimal tolling rate is determined. Here is the block diagram.

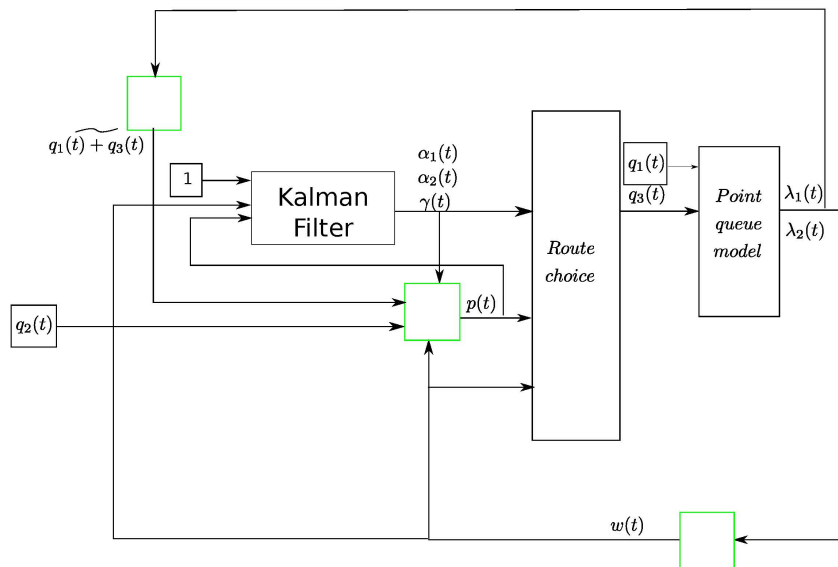


Figure 4.6: Block diagram of self-learning strategy

Our method can be regarded as a combination of these two ideas. Comparing with the feedback-control approach, our approach use both turning influx for SOVs and queue length on HOT lane as the error signals. And the parameter has an economic meaning, which is the advantage of the self-learning approach. At the same time, our method is computationally efficient., because we preset the value of two parameters in integral controller, and with the adequate analysis of the system, we can ensure convergence. But in Kalman filtering technique, for each iteration, the Kalman gain and error covariance matrix need to be updated, and there are interactions between them.

Chapter 5

Simulation Results

5.1 Simulation Result

For the simulation, the site is a freeway segment with capacity drop downstream of the GP lane (see figure 3.1), and the capacity for a lane is 1800 veh/h. An one-hour period is simulated, and the time interval is one second in order to match the requirement for point queue model. We assume the arrival rate of GP lane is 3600 veh/h, and 300 veh/h for HOV lane, and the simulation duration is one hour. We assume the true VOT is \$60 per hour, and the initial VOT is \$30 per hour. The simulation with $K_I^1 = 1$ and $K_I^2 = 10$ is shown below.

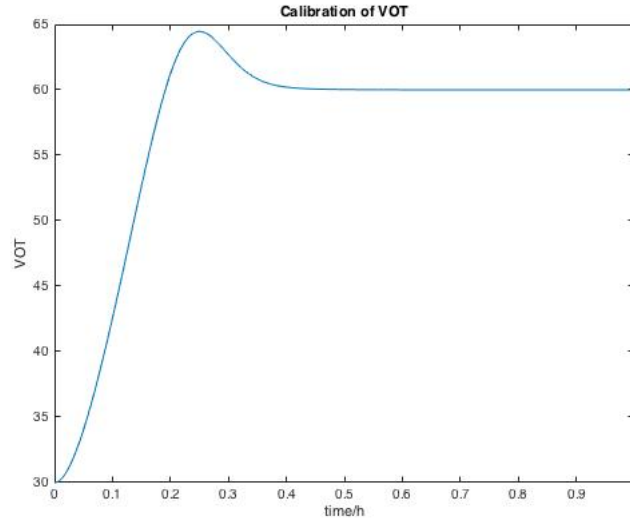


Figure 5.1: Calibration of VOT under uniform arrival

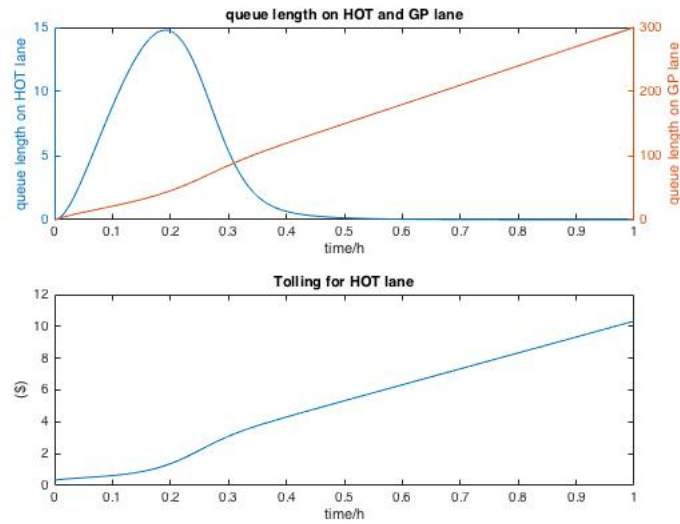


Figure 5.2: Queue length and pricing rate under uniform arrival

We can see VOT will converge at \$60 per hour. Then we test different combination of K_I^1 and K_I^2 . The result shows when we increase K_I^1 , the HOT lane will have a smaller maximum queue length and higher throughput. And the larger K_I^2/K_I^1 value will lead to a shorter queue, and faster queue elimination.

K_I^1	K_I^2	K_I^2/K_I^1	Maximum Queue Length(veh)	Converge Time(h)
1	10	10	14.79	0.97
1	20	20	11.42	0.46
1	30	30	9.64	0.44
5	50	10	4.06	0.92
5	100	20	3.36	0.49
5	150	30	2.94	0.21

Table 5.1: Performance of different controller

Here is the simulation result when the arrival rate is poisson with an average 3600 veh/h for GP lane, and 300 veh/h for HOV lane.

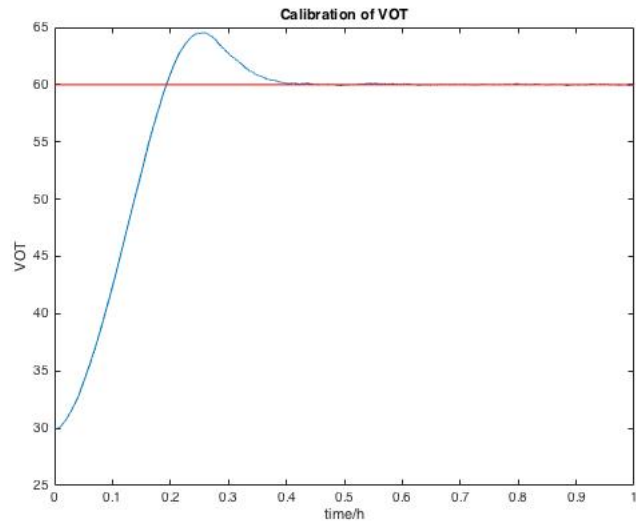


Figure 5.3: Calibration of VOT under poisson arrival

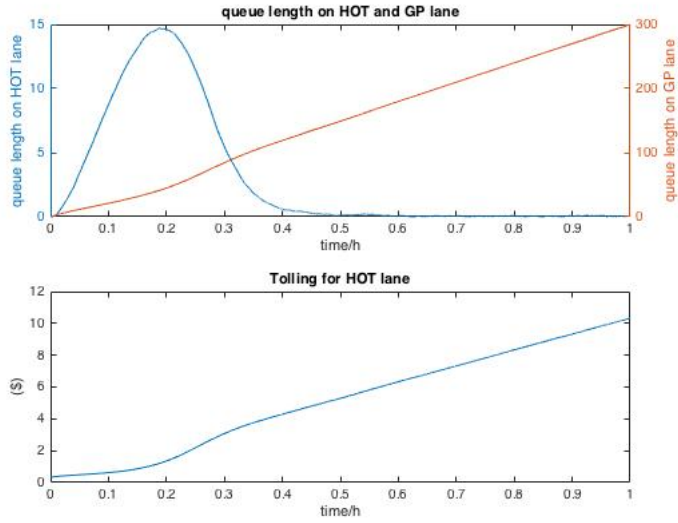


Figure 5.4: Queue length and pricing rate under poisson arrival

We also make simulation when the arrival rate is random with an average 3600 veh/h for GP lane, and 300 veh/h for HOV lane. As shown below, our proposed method still applies.

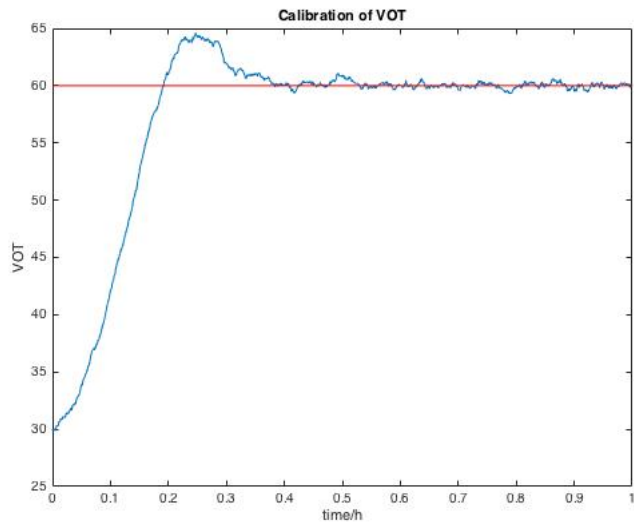


Figure 5.5: Calibration of VOT under random arrival

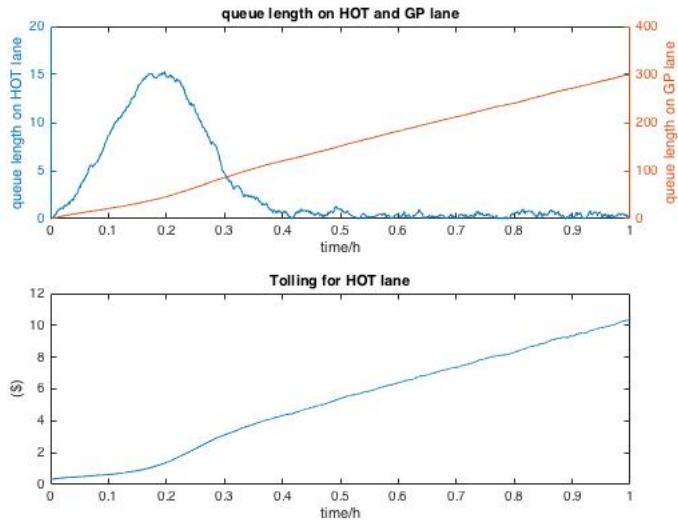


Figure 5.6: Queue length and pricing rate under random arrival

Finally, we set true VOT to be around a constant value to reflect heterogeneous drivers. And an average arrival rate is the same as the simulations above. Here is the result.

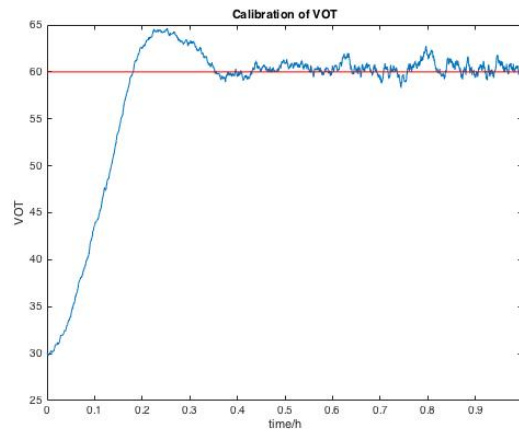


Figure 5.7: Heterogeneous VOT under random arrival

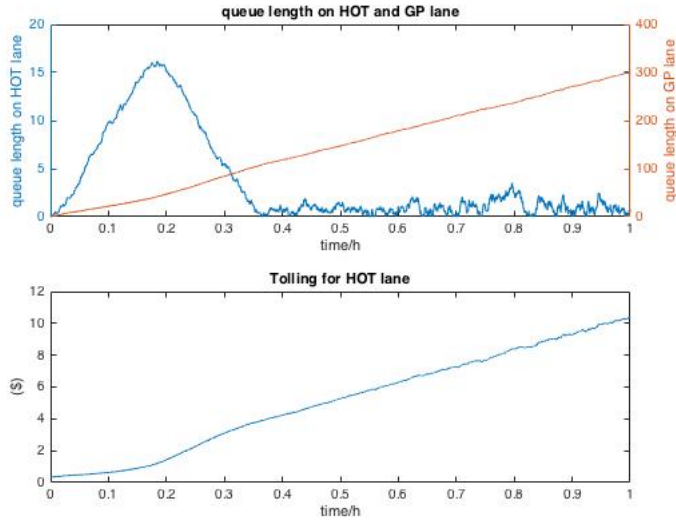


Figure 5.8: Queue length and pricing rate

At the same time, we setup the simulation in Simulink. Figure 5.9 is the setup under constant arrival rates. Function 10 represents the controller; function 11 shows the process of calculating tolling rate; function 12 is the lane group choice model; function 13 is the application of point queue model; function 14 calculate the travel time difference. And the VOT can finally converge to \$60 per hour as well.

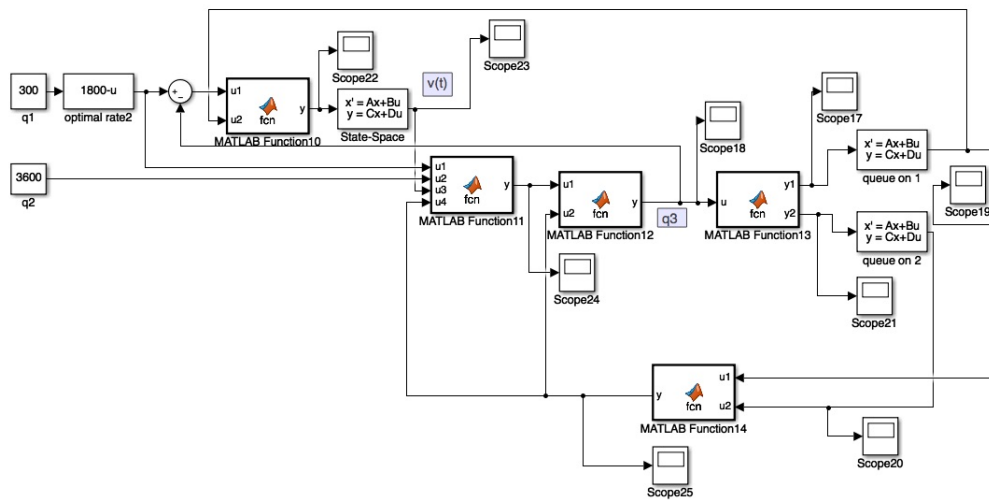


Figure 5.9: Simulink setup for constant arrival pattern

5.2 Comparison with Yin & Lou’s Result

In this section, we will compare our results with the simulation results provided by Yin and Lou (2009)[50]. In this paper, they stated that the self-learning approach works better than the feedback-control approach, so we just make comparison with the self-learning approach. We use the same setup as their first scenario.

When we have $K_I^1 = 5, K_I^2 = 50$ for a 20-minute simulation, the average queue length on HOT lane is 1.71 veh, and the average throughput is 1796 veh/h. While in the self-learning approach, the average queue length is 2.8 veh, and the average throughput is 1773 veh/h.

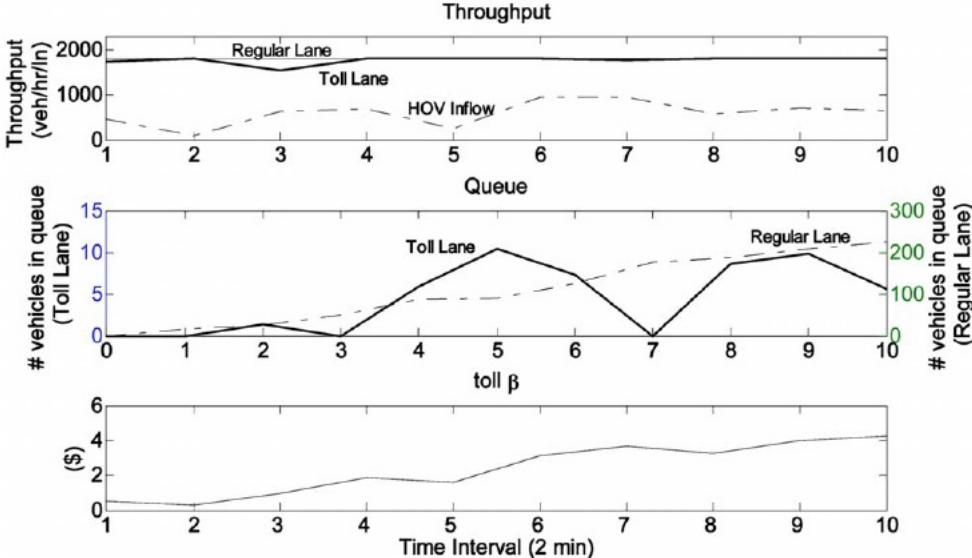


Figure 5.10: Performance of self-learning controller in Yin and Lou(2009)

And in our simulation, we can see the VOT gets pretty close to the true value at the end of simulation. While in their approach, there is still a gap between the estimated and true value (shown in figure 5.11), which means it may take a longer time to get convergence. So, our method is more efficient.

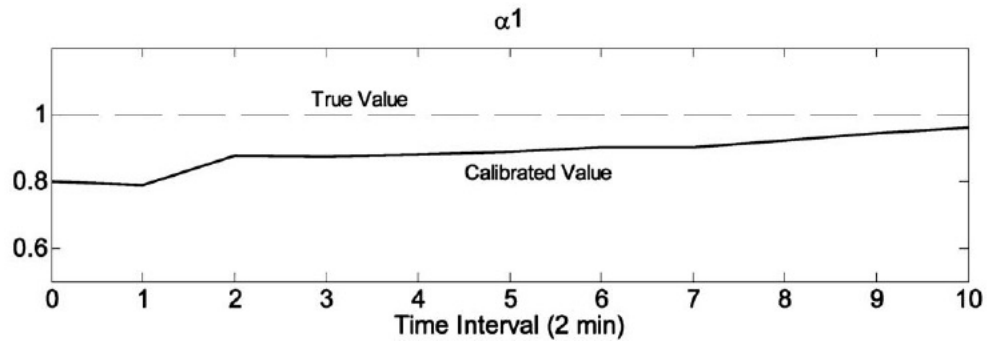


Figure 5.11: Calibrated VOT in Yin and Lou(2009)

Chapter 6

Conclusion

6.1 Summary

In this paper, we provide a new approach to calibrate driver's VOT. A unified approach of point queue model proposed by Jin (2015) is applied to capture traffic dynamics[17]. In this approach, the the rate of change in queue length is calculated by the difference between upstream influxes and downstream out-fluxes, and the flux through a boundary is determined by macroscopic junction models. For the system framework, it can be regarded as an integration of control and estimation, which is not common in literature. We use one block diagram to show the closed-loop control system, and we apply integral controller to control the system. The goal of operating HOT lane is to maximize flow rate while proving free flow speed to drivers. With this principle, we can determine the sign for each controller in order to reach system stability. The tolling rate can be calculated by multiplying travel time difference on two lanes by VOT.

And the simulation result on a simple freeway segment with capacity drop is provided. The results show that we can eventually get the true value of driver's VOT and corresponding

tolling rate when the HOT lane operates at optimal.

6.2 Future Study

The current study is set on a simple freeway segment with a lane drop on GP lane, and a logit model is used to capture driver's behavior. In the future study, we will first modify our formulation, and try to show how UE principle will influence the pricing strategy. Second, we will further discuss the meaning and role of VOT in this study, if it is already known, what will the system framework be?

Then, we will consider more complex traffic conditions. First, we need to consider the effect of on-ramp and off-ramps exist in the downstream. If SOV drivers are aware of the traffic changes ahead, will they have different lane group choices? Second, we need to think about lane changing, because it can create voids in traffic, and reduce the throughput. And we are also interested in how ramps can influence lane changing with the existence of HOT lane, and how to make a real-time pricing strategy.

Real-time ramp metering and variable speed limit (VSL) are widely used for the freeway control. We will see if there is any opportunity to coordinate HOT lane with one of these two strategies to provide a superior traffic condition.

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