Title

Uncertainties in Modeling of Natural Phenomena (MAS 4)

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Uncertainty in Modeling Natural Phenomena

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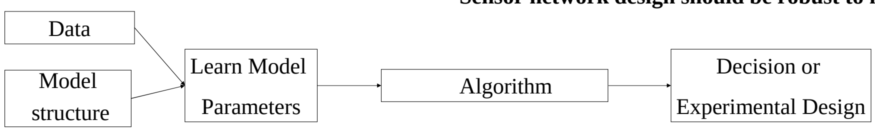
Introduction: Model Uncertainty affects our Experimental Design and Decision

Process of Modeling and Designing Experiments

- **A model is a representation of a system**
  Independence assumptions between the variables, the functional form of relationships between the variables, utility functions, constraints, etc.
- **Experimental Design and Decision making are based on the model assumed**



An Incorrect Model affects Decisions and Designs

- **Many sensor network applications deal with the detection of events.**
- **Each deployment is done in a new environment, and thus a precise model of the events is unknown.**
- **Sensor network design should be robust to model errors.**

Problem Description: Sensitivity Analysis and Robustness to Model errors

Uncertainty in Experimental Design

- **Experimental design problems can be written as optimization problems.**
- **Sensitivity analysis is an important area of optimization which allows us to know how perturbations in the model assumed will affect the outcome.**
- **If a model parameter is very sensitive, then the result of the optimization changes drastically when the parameter is slightly perturbed.**
- **This would require our learning process to be extremely accurate, which is not very often the case.**

Detection under incorrect models

- **The model-based detection problem is a hypothesis testing problem.**
- **Each event of interest can be represented by a hypothesis**
  Presence of an object in the field, location of a target (humans, cars), etc.
- **The nature of the problem makes the modeling process inaccurate**
  The hypotheses models assumed may be different from the true models of the events of interest.
- **Sensor networks require low-complexity algorithms**
- **Analysis of the probability of error in the system when the incorrect models are used will inform us on the robustness of our detection.**

Proposed Solution:

Sensitivity Analysis for Experimental Design

- **The standard form of an optimization problem:**

$$\begin{aligned} &\min && f_0(x) \\ &\text{subject to} && f_i(x) \le 0 \quad i = 1,\dots,m \\ & && h_i(x) = 0 \quad i = 1,\dots,p \end{aligned}$$

- **Perturbations can be introduced in the utility function or in the constraints to analyze the behavior of the solution**
  Example: For the perturbed constraints problem i.e.

$$f_i(x) \le u_i$$
$$h_i(x) = v_i$$

The optimal value can be lower bounded: $p^*(u,v) - p^*(0,0) \ge -\lambda^{*T} u - \upsilon^{*T} v$

- **A simple Experimental Design: Sensor Palcement**

$y_1,\dots,y_m$ are measurements, $x_1,\dots,x_m$ are locations (experiments), and $v_1,\dots,v_n$ are all possible locations (candidates) $(n > m)$

The measurements and the locations are related by the following linear model:

$$y_i = w^T x_i + \varepsilon_i \text{ where } \varepsilon_i \sim N(0,\sigma^2) \qquad i = 1,\dots,m.$$

w can be estimated by solving a LS problem: $\hat{w} = \arg\min_w \{J(w) = \sum_{i=1}^{m} (w^T x_i - y_i)^2\}$

The error $e = w - \hat{w}$, $e \sim N(0, \sigma^2 C_w)$ where $C_w = (\sum_{i=1}^{m} x_i x_i^T)^{-1}$

We can see that C_w (and the error covariance) depends on the choice of x_i

An optimization problem can now be formulated to minimize the error covariance (in some sense)

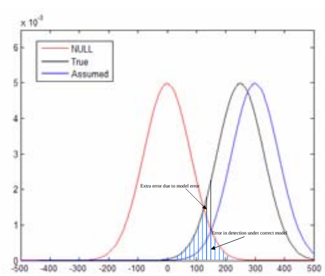
Example: A - optimal design (relaxed)

$$\begin{aligned} &\min_{l_1,\dots,l_n} && Tr\left[\left(\sum_{j=1}^{n} l_j v_j v_j^T\right)^{-1}\right] \\ &\text{subject to} && l \ge 0,\ 1^T l = 1 \end{aligned}$$

- **A sensitivity analysis of the experiment design helps in evaluating the confidence in the design of the networks.**

Robustness of Detection

- **Hypothesis testing is a classical problem in Detection theory**
- **Performance is measured by probability of error given the assumed model.**
- **Model errors affect the detection performance**
  When the hypotheses assumed do not include the correct model of the source, the classical error probability does not reflect the true performance.
- **Results about the reliability of detections exist for simple hypothesis testing problems under incorrect models.**
- **A simple illustration:**



- **Results about the robustness of detection algorithms in sensor networks help in the design of the networks.**
- **Robustness of the algorithm used determines the fidelity required in modeling the events of interest**
  When the algorithm is robust to model errors, a lower fidelity is acceptable
- **The fidelity required in the modeling process affects the number of sensors and the measurement frequency needed**