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Publication Date

2004-06-01

Trapping and Dark Current in Plasma-Based Accelerators

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Abstract. The trapping of thermal electrons in a nonlinear plasma wave of arbitrary phase velocity is investigated. The threshold plasma wave amplitude for trapping plasma electrons is calculated, thereby determining the fraction trapped and the expected dark current in a plasma-based accelerator. It is shown that the presence of a laser field (e.g., trapping in the self-modulated regime of the laser wakefield accelerator) increases the trapping threshold. Implications for experimental and numerical laser-plasma studies are discussed.

INTRODUCTION

Plasmas are capable of supporting large amplitude, space charge oscillations with phase velocities near the speed of light. Such plasma waves can support large electric fields, up to hundreds of GV/m, and can be used to accelerate charged particles. High-intensity lasers and charged particle beams have been proposed for the excitation of plasma waves, or wakefields, for plasma-based accelerators (for a review, see Ref. [1]).

Laser-driven plasma-based accelerator experiments [2–6] have typically operated in the self-modulated regime of the laser wakefield accelerator (LWFA). In this regime, a long (compared to the plasma wavelength), high power laser pulse propagating in a dense ($\sim 10^{19} \text{ cm}^{-3}$) plasma drives a plasma wave through Raman scattering. The plasma wave grows exponentially inside the laser pulse, via the Raman scattering instability, until the growth saturates nonlinearly or electrons become trapped in the plasma wave (subsequently damping the plasma wave). Such uncontrolled trapping results in the production of poor quality electron beams (e.g., with near 100% energy spread).

Several methods of controlled triggering of the trapping of background plasma electrons have been proposed for injecting electrons into the plasma wave for the production of high quality electron beams [7–11]. All optical injection methods [7, 8, 10], which rely on laser-triggered local phase space mixing are currently being explored experimentally and are promising candidates for the production of ultrashort ($\sim \text{fs}$) electron bunches.

Controlling the injection of background plasma electrons into the plasma wave is critical to the design of plasma-based accelerators. Unintended trapping of background plasma electrons can be considered a source of dark current in the plasma accelerator structure and is therefore undesirable for the production of high quality electrons beams [12]. In this paper we examine the trapping of thermal plasma electrons in a nonlinear plasma wave (i.e., the accelerating field) and calculate the threshold field for trapping.

PLASMA WAVE EXCITATION

Wakefield generation in the nonlinear 1D regime can be examined by assuming that the drive beam is nonevolving, i.e., the drive beam is a function of only the co-moving coordinate $\xi = z - v_\phi t$, where $v_\phi \leq c$ is the phase velocity of the plasma wave. Using the fluid momentum and continuity equations, the Poisson equation $\partial^2 \phi / \partial \xi^2 = k_p^2 (n/n_0 - 1 + n_b/n_0)$ can be written as [13–15]

$$\frac{\partial^2 \phi}{\partial \xi^2} = k_p^2 \gamma_\phi^2 \left\{ \beta_\phi \left[1 - \frac{\gamma_\perp^2}{\gamma_\phi^2 (1 + \phi)^2} \right]^{-1/2} - 1 \right\} + k_p^2 \frac{n_b}{n_0}, \quad (1)$$

where $k_p = \omega_p/c$ is the plasma wavenumber, ϕ is the electrostatic potential normalized to mc^2/e , $\beta_\phi = v_\phi/c$, and n_b is the beam density (nonzero if the plasma wave is excited by a particle beam driver). Here $\gamma_\perp^2 = 1 + a^2/2$ is the relativistic Lorentz factor associated with the electron quiver velocity in the laser field, where $a^2 \simeq 7.32 \times 10^{-19} \lambda_0^2 [\mu\text{m}] I_0 [\text{W}/\text{cm}^2]$ is the normalized laser intensity for a linear polarized laser pulse, with $\lambda_0 = 2\pi c/\omega_0$ the laser wavelength and I_0 the laser intensity. The 1D limit applies to broad drivers, $k_p r_\perp \gg 1$, where r_\perp is the characteristic radial dimension of the drive beam (laser or particle). Equation (1) is valid in the limit $(T/mc^2) \ll 1$ (i.e., nonrelativistic plasma temperature). In this analysis we also assume that beam loading effects are small. Beam loading effects will occur when a large fraction of the electron distribution becomes trapped in the plasma wave.

The axial electric field of the wake is given by $E_z = -E_0 \partial \phi / \partial \xi$, where $E_0 = cm_e \omega_p / e$. In the linear regime $E_{\text{max}} < E_0$, where E_{max} is the maximum amplitude of the axial electric field of the plasma wave, the plasma wave is a simple sinusoidal oscillation with frequency ω_p and a phase velocity v_ϕ determined by the driver, e.g., $\phi = \phi_0 \cos(k_p \xi)$ with $\phi_0 < 1$. When $E_{\text{max}} \gtrsim E_0$, the plasma wave becomes nonlinear and Eq. (1) indicates that the electric field departs from a simple sinusoidal form and the period of the nonlinear plasma wave increases as the amplitude increases [16–18].

Analysis of Eq. (1) indicates that the electrostatic potential oscillates between $\phi_{\text{min}} \leq \phi \leq \phi_{\text{max}}$ and the axial electric field oscillates between $-E_{\text{max}} \leq E \leq E_{\text{max}}$. The values ϕ_{min} and ϕ_{max} are given by

$$\phi_{\text{max/min}} = \gamma_\perp - 1 + \hat{E}_{\text{max}}^2 / 2 \pm \beta_\phi [(\gamma_\perp + \hat{E}_{\text{max}}^2 / 2)^2 - \gamma_\perp^2]^{1/2}, \quad (2)$$

where $\hat{E}_{\text{max}} = E_{\text{max}}/E_0$ and the \pm give ϕ_{max} and ϕ_{min} , respectively. Equation (2) is valid provided γ_\perp is slowly varying compared to ω_p (e.g., a laser driver in the long pulse, self-modulated regime) and for the plasma wave behind the driver. Behind the driver, $\gamma_\perp = 1$ and $\phi_{\text{max/min}} = \hat{E}_{\text{max}}^2 / 2 \pm \beta_\phi \hat{E}_{\text{max}} [1 + \hat{E}_{\text{max}}^2 / 4]^{1/2}$. In the linear regime $\phi_{\text{max/min}} \simeq \pm \beta_\phi \hat{E}_{\text{max}}$.

The cold plasma fluid equations break down when the plasma density becomes singular, i.e., from the Poisson equation, $\partial E_z / \partial \xi \rightarrow \infty$. From Eq. (1) this occurs at $\gamma_\perp = \gamma_\phi (1 + \phi)$. Hence the minimum potential satisfies $\phi_{\text{min}} = \gamma_\perp / \gamma_\phi - 1$, and Eq. (2) implies

$$E_{\text{WB}} = [2\gamma_\perp (\gamma_\phi - 1)]^{1/2} E_0, \quad (3)$$

where E_{WB} is the cold relativistic wavebreaking field including effects of a laser field [19, 20]. Equation (3) is a generalization to the Akhiezer and Polovin cold wavebreaking result [21] that includes the presence of a laser pulse. In the self-modulated LWFA, the plasma wave is driven by an instability (e.g., Raman forward scattering) inside the laser pulse. For relativistic laser pulses ($a \gtrsim 1$), Eq. (3) indicates that the maximum field achievable is significantly larger inside the laser field compared to the region behind the drive laser (where $a^2 = 0$ and $\gamma_{\perp} = 1$). For highly-relativistic laser intensities $a \gg 1$, the cold wavebreaking field scales as $E_{\text{WB}}/E_0 \simeq (2\gamma_{\perp}\gamma_{\phi})^{1/2} \simeq 2^{-1/4}(\omega_0/\omega_p)^{1/2}a$. Here we have assumed the phase velocity of the plasma wave is approximately equal to the nonlinear group velocity of the laser $\gamma_{\phi} \simeq [\gamma_{\perp}(\gamma_{\perp} + 1)/2]^{1/2}\omega_0/\omega_p$ [22].

TRAPPING OF THERMAL ELECTRONS

The dynamics of an electron in the presence of an electrostatic plasma wave (wakefield) and a laser pulse is described in the 1D limit by the Hamiltonian in the co-moving frame

$$H(u, \xi) = (\gamma_{\perp}^2 + u^2)^{1/2} - \beta_{\phi}u - \phi(\xi), \quad (4)$$

where ϕ is the plasma wave space charge potential given by Eq. (1), and u is the electron momentum normalized to mc . The Hamiltonian is time independent and therefore a constant of motion $H(u, \xi) = \text{constant}$. Equation (4) describes trapped and untrapped orbits, and the separatrix orbit separating the trapped and untrapped orbits is given by

$$H_s = H(u = \gamma_{\perp}\gamma_{\phi}\beta_{\phi}, \xi = \xi_{\min}) = \gamma_{\perp}/\gamma_{\phi} - \phi_{\min}, \quad (5)$$

where $\phi_{\min} = \phi(\xi_{\min})$ is the minimum wake potential and is related to the peak accelerating electric field of the wave by Eq. (2).

Consider a plasma electron with initial velocity v_t and normalized momentum u_t in the absence of any fields (i.e., before the passage of the driver and excitation of the plasma wave, $\gamma_{\perp} = 1$ and $\phi = 0$). The orbit of the electron will be defined by the Hamiltonian given by

$$H_t = (1 + u_t^2)^{1/2} - \beta_{\phi}u_t. \quad (6)$$

Note that, for an initially cold plasma, $u_t = 0$ and $H_t = H_{\text{cold}} = 1$ for all plasma electrons. Trapping of the electron will occur when the orbit defined by the Hamiltonian Eq. (6) coincides with a trapped orbit, defined by the separatrix orbit Eq. (5), namely, when $H_t \leq H_s$. Solving $H_t = H_s$ yields in the minimum initial electron momentum for trapping in the plasma wave,

$$u_t = \gamma_{\phi}\beta_{\phi}(\gamma_{\perp} - \gamma_{\phi}\phi_{\min}) - \gamma_{\phi} \left[(\gamma_{\perp} - \gamma_{\phi}\phi_{\min})^2 - 1 \right]^{1/2}, \quad (7)$$

where ϕ_{\min} is given by Eq. (2). Equations (7) and (2) can be solved for the peak field required for the onset of particle trapping as a function of the initial electron momentum:

$$\hat{E}_{\text{max}}^2 = 2\gamma_{\perp}(\gamma_{\phi} - 1) + 2\gamma_{\phi}^2 \left\{ (1 - H_t) - \beta_{\phi} \left[(1 - H_t)^2 + 2(1 - H_t)\gamma_{\perp}/\gamma_{\phi} \right]^{1/2} \right\}, \quad (8)$$

where H_t is given by Eq. (6). In the limit of an initially cold plasma $u_t = 0$, $H_t = H_{\text{cold}} = 1$ and Eq. (8) reduces to Eq. (3), i.e., trapping occurs only at the wavebreaking field $\hat{E}_{\text{max}} = E_{\text{WB}}/E_0$.

For a plasma consisting of electrons with nonrelativistic initial momenta ($u_t \ll 1$), Eq.(8) reduces to

$$\hat{E}_{\text{max}}^2 = 2\gamma_{\perp} (\gamma_{\phi} - 1) + 2\gamma_{\phi}^2 \beta_{\phi} \left\{ u_t - \left[(\beta_{\phi} u_t)^2 + 2\beta_{\phi} u_t \gamma_{\perp} / \gamma_{\phi} \right]^{1/2} \right\}. \quad (9)$$

Equation (9) determines the maximum plasma wave field before the onset of trapping for a plasma at a given initial temperature. For example, if the plasma momentum distribution is nonrelativistic and initially a waterbag distribution, then the maximum initial momentum is given by the normalized temperature (width of the velocity distribution) of the plasma $u_t \leq (2T/mc^2)^{1/2}$. Typically the temperature of plasmas used in short-pulse laser-plasma interaction experiments is of the order of a few 10s of eV [23, 24], or $u_t \sim 10^{-2}$.

The fraction of electrons trapped in the plasma wave can be computed for a given initial momentum distribution of the plasma electrons. For example, assuming an initial Gaussian velocity distribution of the plasma electrons with temperature T defined by the root-mean square (rms) velocity spread $(2T/m)^{1/2}$, with $(2T/m)^{1/2} \ll c$, the fraction of trapped electrons is

$$f_{\text{trap}} = \frac{1}{2} \left[1 - \text{Erf} \left(\frac{u_t}{2\sqrt{T/mc^2}} \right) \right], \quad (10)$$

where u_t is given by Eq. (7). Note that only electrons with momenta in the direction of the phase velocity of the plasma wave are trapped. Figure 1 shows the fraction of trapped electrons versus the initial temperature of a Gaussian plasma electron velocity distribution for three different nonlinear plasma wave amplitudes $\hat{E}_{\text{max}} = 1.5$, $\hat{E}_{\text{max}} = 2$, and $\hat{E}_{\text{max}} = 2.5$, with $\gamma_{\phi} = 10$ and $\gamma_{\perp} = 1$. The total number of trapped electrons (i.e., dark current in the plasma accelerator) can be estimated from Eq. (10). For example, for a plasma density of $n_0 = 10^{19} \text{ cm}^{-3}$, driver transverse size of $r_{\perp} = 10 \text{ } \mu\text{m}$, and accelerator length of 1 mm, a trapping fraction of $f_{\text{trap}} = 10^{-2}$ indicates $\sim 1 \text{ nC}$ of trapped charge. This trapping calculation neglects beam loading, which will be valid provided the wakefield of the trapped electrons is much smaller than the plasma wave, or $n_{\text{trap}}/n_0 \ll |\phi|$, where n_{trap} is the density of the trapped electron bunch.

Laser-driven plasma wave

A plasma wave driven by a laser-plasma interaction will have a phase velocity approximately equal to the group velocity of the laser pulse, typically $\gamma_{\phi} \simeq \omega_0/\omega_p \sim 10\text{--}100$ for laser propagation in an underdense plasma. Without some additional heating mechanism, laboratory plasmas used for LWFA experiments have temperatures of the order of $T \sim 10 \text{ eV}$ [23–25]. Therefore laser-driven plasma-based accelerators satisfy

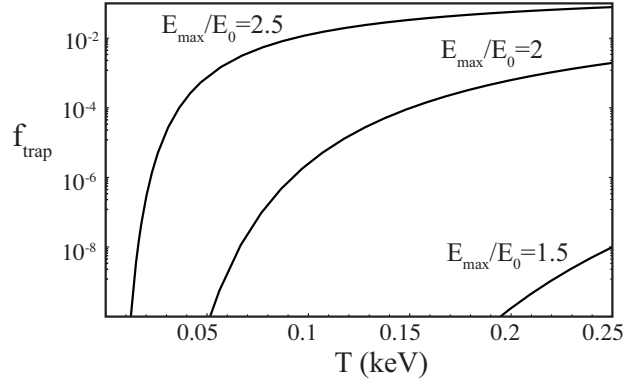


FIGURE 1. Fraction of trapped electrons f_{trap} [Eq. (10)] versus the initial temperature of a Gaussian plasma electron velocity distribution T (keV) for three different nonlinear plasma wave amplitudes $\hat{E}_{\text{max}} = 1.5$, $\hat{E}_{\text{max}} = 2$, and $\hat{E}_{\text{max}} = 2.5$, with $\gamma_{\phi} = 10$ and $\gamma_{\perp} = 1$.

$u_t \ll 1/\gamma_{\phi} \ll 1$. In this limit, Eq. (9) reduces to

$$\hat{E}_{\text{max}} \simeq [2\gamma_{\perp} (\gamma_{\phi} - 1)]^{1/2} - \frac{(\beta_{\phi}\gamma_{\phi})^{3/2}}{(\gamma_{\phi} - 1)^{1/2}} u_t^{1/2} + \frac{\beta_{\phi}\gamma_{\phi} (\gamma_{\phi} - 1)^{1/2}}{\sqrt{8}\gamma_{\perp}^{1/2}} u_t. \quad (11)$$

Equation (11) contains the cold relativistic wavebreaking field (generalized to include the influence of the laser field) with the lowest order corrections owing to the plasma temperature (initial electron momentum). Temperature reduces the trapping threshold from the cold wavebreaking limit.

For a flat-top laser pulse, the excited plasma wave amplitude can be evaluated analytically in terms of elliptic integrals [18]. For the optimal laser pulse length, the field behind the laser pulse driver is given by $\hat{E}_{\text{max}} = (\gamma_{\perp}^2 - 1)/\gamma_{\perp}$. Therefore the threshold laser intensity for trapping behind the laser pulse is given by $a^2 = \hat{E}_{\text{max}}^2 + (\hat{E}_{\text{max}}^4 + 4\hat{E}_{\text{max}}^2)^{1/2}$, where $\hat{E}_{\text{max}}(\gamma_{\phi}, u_t)$ is given by Eq. (9) with $\gamma_{\perp} = 1$.

High phase velocity plasma wave

For high phase velocity waves ($\beta_{\phi} \simeq 1$) such that $\gamma_{\phi}(1 - H_t) \gg 1$ (e.g., a plasma wave in a warm plasma driven by an ultra-relativistic particle beam), Eq. (8) reduces to $\hat{E}_{\text{max}}^2 \simeq \gamma_{\perp}^2 / (1 - H_t) - 2\gamma_{\perp} + (1 - H_t)$. For a nonrelativistic plasma temperature $u_t \ll 1$ in the high phase velocity limit ($\gamma_{\phi} u_t \gg 1$), this result reduces to

$$\hat{E}_{\text{max}} \simeq \gamma_{\perp} / u_t^{1/2}. \quad (12)$$

As indicated by Eq. (12), the field amplitude required for trapping in this limit ($\gamma_{\phi} u_t \gg 1$) scales as $\hat{E}_m \propto T^{-1/4}$ assuming a waterbag velocity distribution (which is the same scaling as the wavebreaking field derived by Katsouleas and Mori [26] and Rosenzweig [27] in this limit).

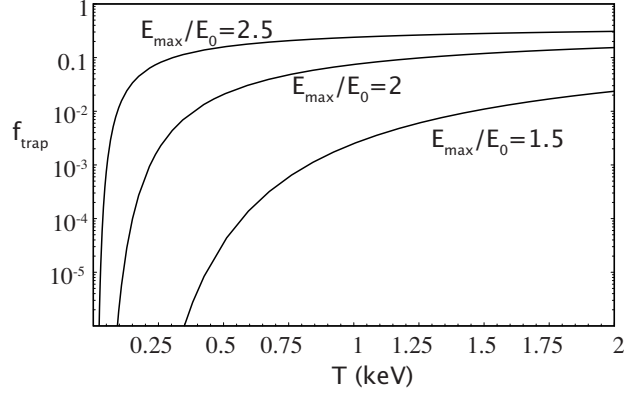


FIGURE 2. Fraction of trapped electrons f_{trap} versus the initial temperature of a Gaussian plasma electron velocity distribution T (keV) for three different nonlinear plasma wave amplitudes $\hat{E}_{\text{max}} = 1.5$, $\hat{E}_{\text{max}} = 2$, and $\hat{E}_{\text{max}} = 2.5$, with $\gamma_{\phi} = 10$ and $\gamma_{\perp} = 1$.

SUMMARY AND DISCUSSION

The threshold electric field for trapping background plasma electrons in a nonlinear plasma wave was derived. This result determines the fraction of background plasma electrons trapped in the wave and, therefore, the expected dark current in a plasma-based accelerator. Reduction of dark current is critical for producing high-quality electron beams in an accelerator. The trapping calculation included the presence of a laser field, allowing application of this analysis to the self-modulated regime of the LWFA. It was found that the presence of the laser field increases the trapping threshold and reduces the fraction of trapped electrons.

In addition to plasma-based accelerator design, the calculated trapping thresholds can have implications for the interpretation of numerical studies of trapping (or the detailed electron phase space structure). Kinetic effects (e.g., particle trapping) in short-pulse laser-plasma interactions is often simulated numerically using particle-in-cell (PIC) models. A well-known linear numerical instability is generated by the spatial grid in the PIC algorithm [28]. Owing to this numerical instability, the temperature of the plasma will grow until the Debye length λ_D is of the order of the size of the grid spacing Δz , where the Debye length is $\lambda_D = (T/4\pi n_0 e^2)^{1/2}$ or, in practical units, $\lambda_D[\text{cm}] \simeq 740 \sqrt{T[\text{eV}]/n_0[\text{cm}^{-3}]}$. This unphysical self-heating of the plasma in a PIC simulation results in a temperature given by $\lambda_D \sim \Delta z$. For typical short-pulse laser-plasma interaction parameters, plasma density $n_0 = 10^{19} \text{ cm}^{-3}$ and laser wavelength $\lambda_0 = 1 \mu\text{m}$, a spatial grid of $\Delta z = \lambda_0/10$ results in numerical self-heating to $T_{\text{num}} \sim 2 \text{ keV}$. Although this temperature is two orders of magnitude larger than plasma temperatures measured in the laboratory [23, 24], it is nonrelativistic $T_{\text{num}}/mc^2 \ll 1$, and therefore does not greatly influence the fluid response of the plasma [25], i.e., the collective hydrodynamic fluid response will be well-approximated by the PIC algorithm for these parameters. However, the detailed phase space structure and particle trapping effects (i.e., kinetic effects) will be poorly approximated at these unphysical plasma temperatures. For example, Fig. (2) shows the fraction of electrons trapped versus plasma temperature

for several nonlinear plasma wave amplitudes $\hat{E}_{\max} = 1.5$, $\hat{E}_{\max} = 2$, and $\hat{E}_{\max} = 2.5$, with $\gamma_{\phi} = 10$ and $\gamma_{\perp} = 1$. For laboratory plasma temperatures (e.g., 10s of eV) less than 1 part in 10^6 plasma electrons are trapped, while for 2 keV (i.e., the unphysical self-heating temperature in PIC using a grid of $\Delta z = \lambda_0/10$) over 10% of the electrons are trapped for a plasma wave of amplitude $\hat{E}_{\max} = 2$. One approach to reduce numerical self-heating in PIC simulations is refining the spatial grid. For the laser-plasma parameters $n_0 = 10^{19} \text{ cm}^{-3}$ and $\lambda_0 = 1 \text{ }\mu\text{m}$, a spatial grid of $\Delta z \lesssim \lambda_0/10^2$ is required to reduce the self-heating to $T_{\text{num}} \lesssim 10 \text{ eV}$.

ACKNOWLEDGMENTS

This work was performed under the auspices the U.S. Department of Energy, Office of High Energy Physics, under contract number DE-AC-03-76SF0098.

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