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Title

An Alternative Geometric Representation of the Correlation Coefficient

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expected to make the top league, first time. As Halberstam [3] says 'A series of ordinary papers is sometimes an unconscious preparation for something out of the ordinary'; and he goes on to state that some people can only produce their best work, when the *mechanical* side of writing has become a habit. It takes milk to make cream.

(iii) As reported in *Ecology* [6], there is however an evident cliquishness amongst researchers which allows conventional papers on orthodox topics, from members of the 'group', to readily find their way into print, but tends to prevent an 'outsider' ever getting a toe in edgeways. This 'inbreeding' is, I think, fairly evident even in some of the most prestigious statistical journals.

(iv) It is clear though, that both journal editors and referees are very hard pressed to cope with the rising tide of paper submissions. To a considerable extent this is due to the successive offering of individual papers to several journals, before they finally obtain an acceptance. Halberstam [3] suggests that, perhaps, up to 60% of typescripts are rejected by the first editor approached, but that eventually 80% of these get published.

There are various reasons for this, the most obvious being a mediocre paper getting accepted by a less fastidious journal only after having been first rejected by a top one, or a good contribution being initially directed to an inappropriate journal. However, there are also inexplicable vagaries in the whole process, and an author often finds that, say two very similar journals—call them A and B—are such that, frequently, if A rejects one of his papers B will still accept it, and vice versa. A very random element appears to operate. The result of all this is that there is much duplication of editorial and refereeing effort.

Would it not then be a good idea to institute some sort of clearing house, whereby authors sent their offerings to some central organisation, which arranged for a *once only* reviewing process? The resulting report could then be circulated successively to, say, two journals suggested by the author and, at the referees' discretion, say to a third, which they might think more suitable. Journals with space to fill could perhaps also request to see papers otherwise rejected.

This approach would mean that no really good submissions were wasted, due to more sensitive authors taking a first rejection as final; and that the many mediocre manuscripts were not independently fully refereed several times, before finally achieving acceptance. Also, it would help eliminate the 'itemising' of published research, mentioned by Ferber [2] and Searle [5], as each author would have a central record; and it would prevent the irritating, but currently understandable, malpractice of multiple submission. Of course, all this remains valid whether or not the idea of extended abstracts catches on.

(v) Finally, how are publishers to cope with the ever-increasing costs of presenting the accepted papers; and how are libraries to house the ever-proliferating quantity of material that cascades forth? As Searle [5] suggests, microforms may hold the answer. It is a solution that I am looking at, on behalf of *The Statistician*. The sort of savings to be expected are well demonstrated by AI[1], who makes his point by noting that the report [4] on disseminating technical information markets at \$10.25 in book form, but only costs \$3.75 on microfiche.

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AN ALTERNATIVE GEOMETRIC REPRESENTATION OF THE CORRELATION COEFFICIENT

An alternative geometric representation (see Leung and Lam (1)) of $\rho(X_1, X_2)$ can be given without reference to angle θ and its cosine.

Let the sum of the N values of random variable X equal $\sum n_i$. These same N values form the N elements of vector V_1 . There are exactly $N - 1$ additional distinguishable vectors, V_2, V_3, \dots, V_N that can be formed by permuting the N elements of V_1 such that $V_1 + V_2 + V_3 + \dots + V_N = (\sum n_i, \sum n_i, \sum n_i, \dots, \sum n_i) = V_s$. Multiplying V_s by the scalar $(1/N)$ gives V_m , the point in N -Space that is $(1/N)^{th}$ the distance from the origin to V_s . Now let D equal the pythagorean distance between V_1 and V_m . By multiplying D by the scalar $(1/N)^{1/2}$, the standard deviation of random variable X can be presented as $\sigma = (1/N)^{1/2}(D)$, while the variance of X is $\sigma^2 = (1/N)(D^2)$. This representation of the correlation coefficient between X_1 and X_2 can be used, for example, to show the following important relationships:

- (1) if $\rho(X_1, X_2) = +1$
then $D_{12} = ((D_1^2 + D_2^2) + (2D_1D_2))^{1/2}$
- (2) if $\rho(X_1, X_2) = 0$
then $D_{12} = ((D_1^2 + D_2^2))^{1/2}$
- (3) if $\rho(X_1, X_2) = -1$
then $D_{12} = ((D_1^2 + D_2^2) - (2D_1D_2))^{1/2}$

where $D_i = (1/N)^{1/2}\sigma_i$ and $D_{12}^2 = (1/N)\sigma_{1+2}^2$

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ON LEAST SQUARES ALGORITHMS

Kopitzke, Boardman and Graybill, in their article entitled "Least Squares Programs—A Look at the Square Root Procedure" [8], have once again called the attention of statisticians to a simple, stable method of solving the normal equations. Cholesky's square root procedure, first published in 1924 [1], has sometimes been neglected in the statistical community. Computer algorithms for this method, however, have been published, although Kopitzke *et al.* failed to mention that fact. Martin, Peters and Wilkinson [11; reprinted in 14] give several Algol procedures, and Healy [7] gives Fortran subroutines for the Cholesky method.

Kopitzke *et al.* made the following questionable statement: "In most cases (except perhaps in higher order polynomial regression) the normal equations can be compiled exactly if the original data are known exactly; thus, the least squares solution can be obtained using the square root decomposition of the normal equations with very little error." Simple examples (not high-order polynomials) have been published [9, 12, 10] which show that, when one is computing with a specified precision, problems exist which can be solved by applying an orthogonalization procedure to the original data matrix X even though the Cholesky decomposition of $X'X$ fails. Lawson and Hanson [10] give a detailed comparison of the pertinent computations for their example in both the Cholesky algorithm and the Householder transformation algorithm. Similar considerations show that the modified Gram-Schmidt algorithm [2, 13] and Givens transformation algorithm [4, 6] are as successful as Householder transformations in solving these ill-conditioned problems. All three of these methods operate directly on X (rather than $X'X$) to produce the Cholesky factor L such that $LL' = X'X$. The