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K$^-$ –DEUTERON INTERACTIONS IN FLIGHT

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K^--DEUTERON INTERACTIONS IN FLIGHT

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ABSTRACT

Forty-two in-flight K^--deuteron reactions leading to the final-state Λ^0, π^-, and proton are investigated. It is shown that fewer of these reactions involved Σ hyperons in an intermediate state than was the case for kaons interacting at rest. Angular distributions in the K-d and π^--Λ^0 cm systems indicate P-wave as well as S-wave contribution to the reaction $K^- + n \rightarrow \Lambda^0 + \pi^-$. Cross sections are calculated for Λ^0's produced directly (30 ± 6.5 mb) and those produced via the intermediate Σ state (14.5 ± 4.5 mb).

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I. INTRODUCTION

In the spring of 1958, the Alvarez group at Berkeley exposed a 15-inch bubble chamber filled with liquid deuterium to a beam of slow (about 250-Mev/c) negative K mesons in order to study the elementary K-nucleon reactions. Of the various types of interactions observed in the chamber, one lent itself most cleanly to kinematic analysis, namely that in which a kaon and a deuteron interacted to give a negative pion, a proton, and a Λ^0 hyperon which subsequently decayed in the chamber into a negative pion and a proton. This is the only K-deuteron reaction (except for elastic K-d scatterings) in which all the final-state particles leave visible tracks, which makes the solution for the kinematic variables overdetermined by the four constraints of energy and momentum balance. This type of event was given the numerical designation 769. Upon analysis, a small fraction of these events (about 15%) were found to have occurred while the kaon was in flight. It is this subclass of events which is the subject of this paper.

II. KINEMATIC ANALYSIS

A. Hand Analysis

Included in the general category of Type 769 events are four subclasses, all of which have the same general appearance in the chamber (see Fig. 1): Λ^0 produced at rest, Λ^0 produced in flight, Σ^0 produced at rest, and Σ^0 produced in flight. A Σ^0 production appears similar to a Λ^0 because the Σ^0 decays in $\sim 10^{-19}$ second into a Λ^0 and a gamma of about 75 Mev energy. At this decay rate, even if the Σ^0 had the velocity of light, it could move only $\sim 10^{-8}$ centimeter before it decayed into a Λ^0 and a gamma. The only outward difference between the two types of events, then, is that the Σ^0 production has an energy and momentum unbalance of about 75 Mev, due to the missing gamma. All Type 769 events were analyzed by a hand-analysis technique which took as inputs the azimuthal and dip angles of each track, the momenta as determined by track curvature, or--for stopping tracks--the momentum as determined by track length. These data were obtained as output from an IBM 650 program which took digitized data from two stereo photographs of the bubble chamber event and printed out the azimuthal angle, dip angle, track length, momentum (from track curvature), and included angle between each pair of tracks in the event. The parameters were then adjusted by trial and error until a consistent solution was found which balanced momentum and energy at the Λ^0-production and decay vertices. Usually, this technique would distinguish unambiguously between the four possible subclasses.

B. Computer Analysis

Although it was found possible to separate out the Λ^0's produced in flight by this method, there are so many degrees of freedom in this type of event that it is difficult, if not impossible, to find by hand analysis a "best" solution which balances both momentum and energy. Fortunately, an IBM 704-computer program designed to perform this type of analysis has been developed by Dr. Horace Taft and Peter Berge.[1] The events involving in-flight Λ^0 production were analyzed with this program in order to determine the kinematic parameters more accurately than was possible by hand analysis.

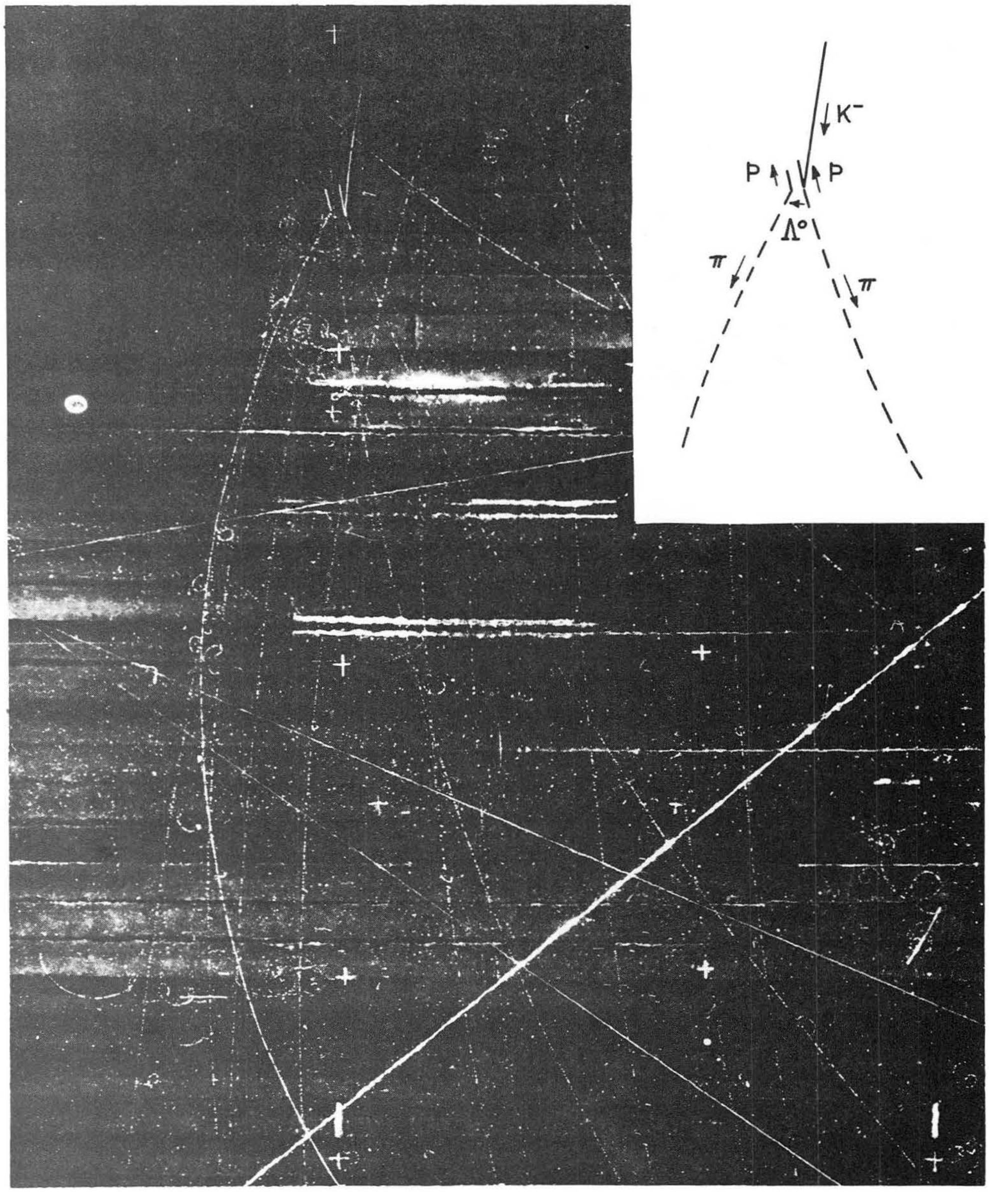

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Fig. 1. Representative Type 769 event in flight.

Input variables to the program for each of the four tracks at the production vertex were (a) the azimuthal angle; (b) tangent of the dip angle; and (c) $1/p \cos \lambda$, where λ is the dip angle and p is the track momentum as determined by curvature, by range (for stopping tracks), or--in the case of the neutral Λ^0--by fitting the parameters at the Λ^0 decay vertex on a kinematics chart. These variables were chosen because their errors have a nearly Gaussian distribution. The variance of each of the twelve variables was also an input to the program.

The problem confronting the computer is to calculate a fit to the input parameters which minimizes the χ^2--subject to the side conditions that the x, y, and z components of momentum must be balanced, and that energy must be conserved. A geometrical interpretation of the problem would be that in the 12-dimensional space representing all the possible values of the 12 input parameters, there is an 8-dimensional "surface" which includes only the values of the parameters allowed under conservation of momentum and energy. The input variables locate a point in the 12-dimensional space which will not in general be located on the surface of allowed situations, and this point, together with the input variance for each variable, determines the value of χ^2 for each point in the 12-dimensional space. The problem then is to locate the point on the 8-dimensional surface at which χ^2 is a minimum. The problem is solved by the introduction of Lagrange multipliers for the four constraint equations, giving 16 equations in 16 unknowns. Because of the nonlinearity of the constraint equations, an iterative procedure is used which assumes moderate linearity in the region containing the points corresponding to the initial and final values of the 12 parameters. The machine will continue to take "steps" to minimize χ^2 until the gradient of χ^2 is less than a given maximum acceptable value and the total momentum and energy unbalance is less than 1 Mev. At this point, the computer prints out the 12 fitted variables, a new variance for each, and the value of χ^2 for the event. Since in most events four constraints were applied to the problem, an average χ^2 of 4 was expected. In some cases, however, the proton from the production vertex had such low momentum that its track length was too short to be visible. The computer was then forced to use up three of the constraint equations in calculating the missing momentum vector, leaving the problem overdetermined by only one degree. For these cases, an average χ^2 of 1 was expected. The χ^2 for four-constraint cases averaged 4.44; for the one-constraint cases the average was 1.43.

Out of 47 events that were analyzed by the program as being probable Λ^0 events in flight, the analysis indicated that three actually occurred at rest. Two others with anomalously large χ^2's, it was concluded, were actually Σ^0 events.

Standard deviations in calculated momenta were typically $\pm$ 5 Mev/c for four-constraint cases, and $\pm$ 20 Mev/c for one-constraint cases. Angles were determined to within about 1/2 degree in all cases except for the calculated azimuth and dip of invisible protons, which typically had uncertainties of about 20°.

III. RESULTS

A. Direct vs. Indirect Λ^0 Production

1. At-Rest Case

Analysis of the at-rest Type 769 events indicates that a large fraction (about 60%) of the Λ^0 hyperons were produced by a two-step process in which the kaon reacts with one of the nucleons to give a Σ^0 or Σ^+ and a π.[2] The Σ hyperon then interacts with the other nucleon to give a Λ^0 and a proton. The evidence for this process is that the momentum distribution of the pion from the Λ^0 production vertex shows two pronounced peaks, one at about 250 Mev/c, and a larger peak at about 185 Mev/c (Fig. 2). The peak at 250 Mev/c is interpreted as being due to those events in which the Λ^0 was produced directly from a K^--neutron interaction, the proton merely being a "spectator" in the process. It is similar in form to the distribution calculated theoretically by Fujii and Marshak.[3] The second peak is considered to have arisen from the two-step process involving a Σ^0 or Σ^+ in the intermediate state,[4] since it corresponds

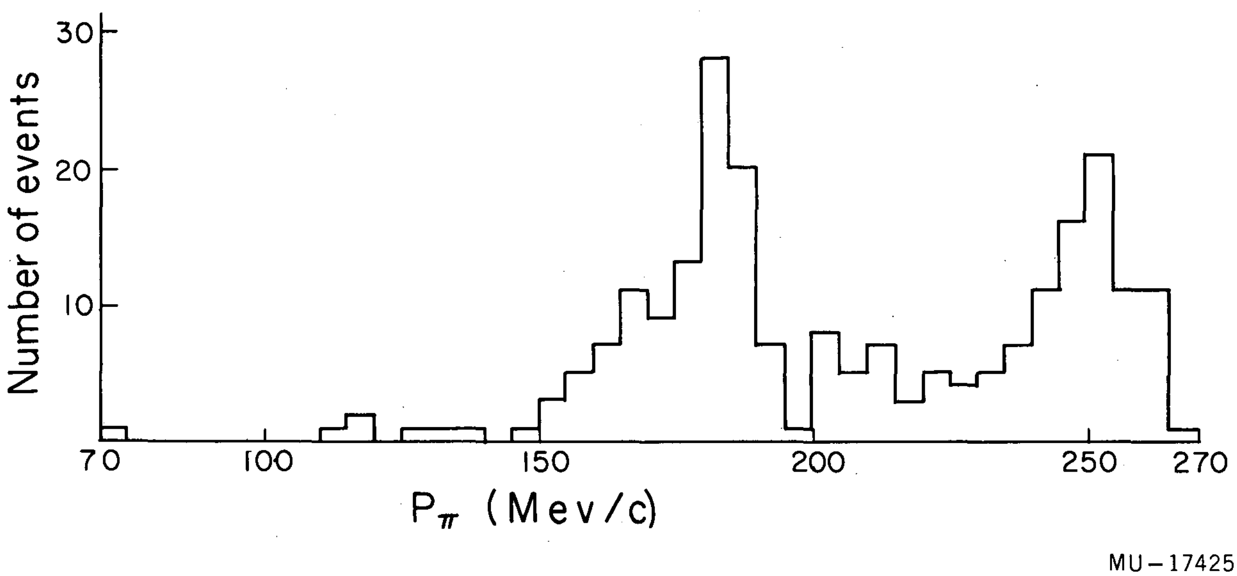

Fig. 2. Momentum distribution for pions for 228 at-rest events.

to the calculated pion-momentum distribution for a Σ and a π produced by a kaon in deuterium, yet the over-all kinematics for each event fits Λ^0 production. The at-rest Type 769 events were thus divided into two categories on the basis of the momentum of the production pion. Those with pion momenta greater than 215 Mev/c are considered to involve direct Λ^0 production and those with pion momenta less than this are considered to involve the two-step or indirect production. The indirect at-rest events taken on the basis of this separation outnumber the direct by a ratio of about 8/5.

2. <u>In-Flight Problem</u>

In order to determine this ratio for the in-flight events, clearly a new criterion must be found for separating direct from indirect events. The in-flight events differ from the at-rest case in that the center of mass is moving in the laboratory system of reference and the kinetic energy of the kaon brings a variable amount of additional energy into the center-of-mass system, some of which will go into increasing the pion momentum above what it would have been had the event occurred at rest.

3. <u>Method of Separation</u>

If one assumes that the impulse approximation is correct (i. e., in the direct events the proton is merely a "spectator" in the interaction), the distribution of proton momentum in the laboratory system can be calculated (see Appendix). This predicted momentum distribution is compared with the experimental distribution in Fig. 3. The comparison shows that there are too many protons with high laboratory-system momenta. One obvious interpretation of this fact is that these high-momentum protons were the ones involved in the Σ-to-Λ conversion process. Since this process is an exothermic reaction between Σ's and nucleons, the protons resulting are expected to have fairly high momenta. One might establish the criterion that all events with production proton momenta less than 200 Mev/c were direct events while all others were indirect. Since about 20% of the direct events that occurred at rest had proton momenta greater than 200 Mev/c (see Fig. 4), the classification of events on this basis establishes the lower limit to the number of direct events.

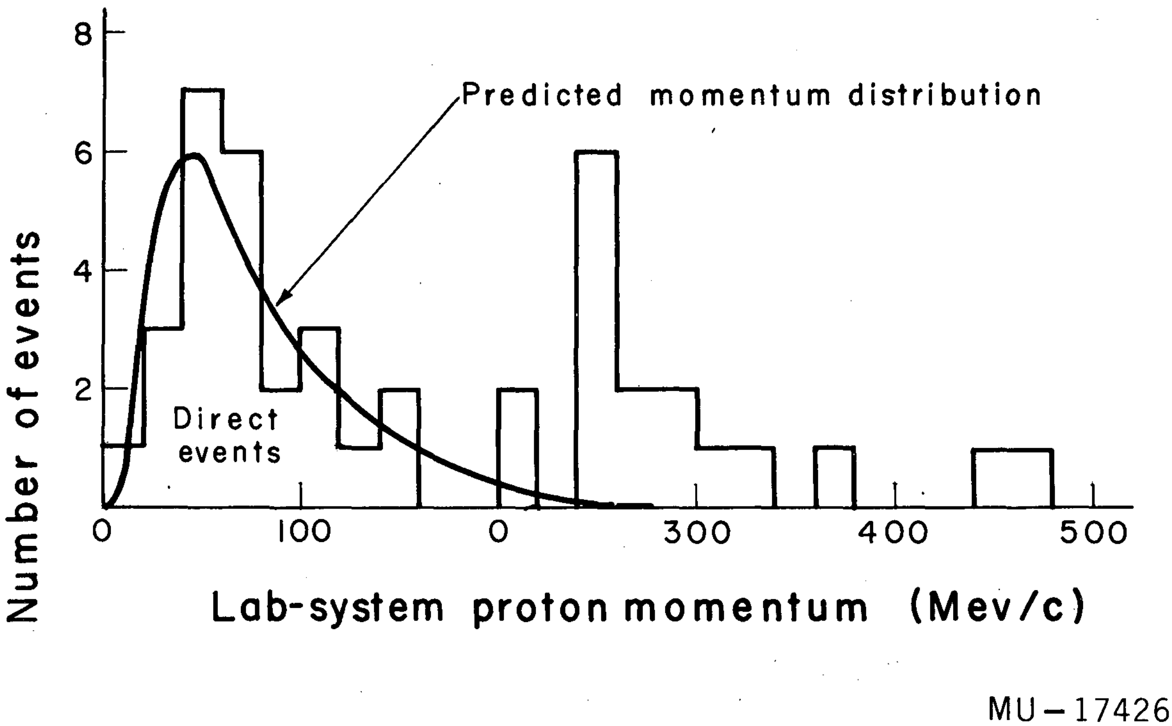

Fig. 3. Comparison of predicted and experimental momentum distributions.

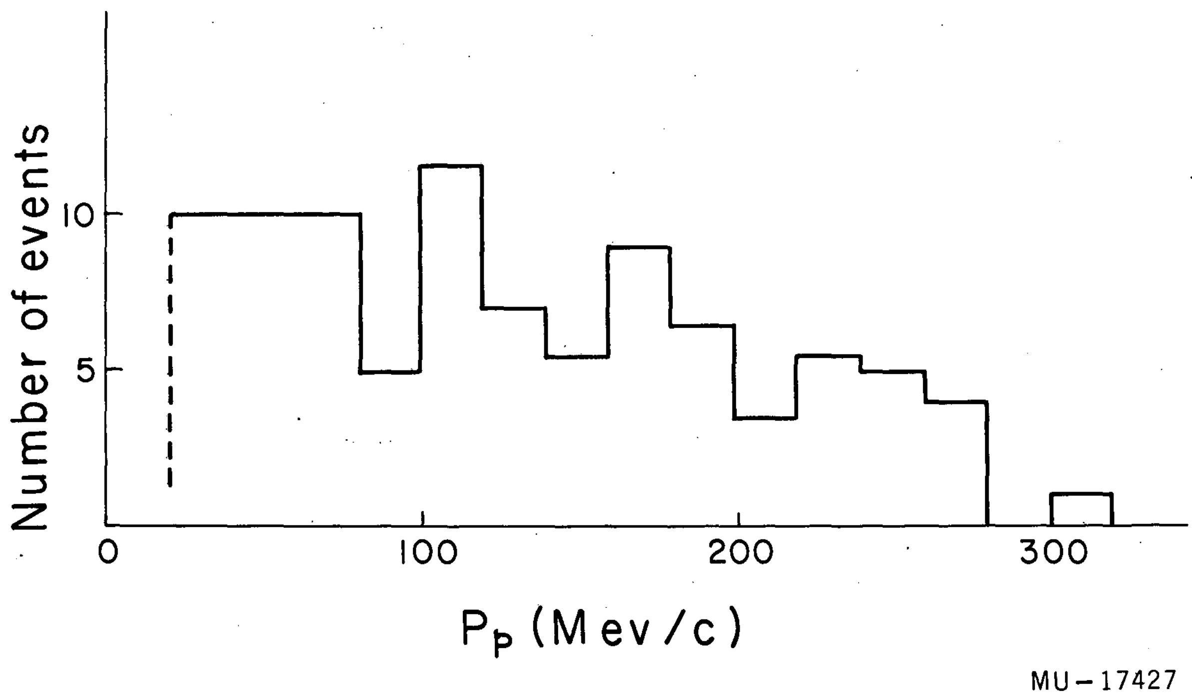

Fig. 4. Momentum distribution for protons for 97 direct events (at rest).

4. Results

The separation of events by proton momenta indicates 25 direct events and 17 indirect events. It is clear that the ratio of indirect Λ^0's to direct Λ^0's when the K interacts in flight is considerably less than the ratio when the K interacts at rest.

B. Angular Distributions

1. K-Deuteron Center-of-Mass System

If one regards the process $K + d \rightarrow \pi^- + \Lambda^0 + p$ as one involving a two-body initial state and a three-body final state, one can ask what partial waves in the K-d system are involved in the reaction. Since the momenta of the incoming kaons is low (< 250 Mev/c), there is little possibility that angular momenta higher than P-wave are involved. Under this hypothesis the angular distribution of any of the reaction products must have the form

$$\frac{dN}{d\Omega} = 1 + A\cos\theta + B\cos^2\theta,$$

where θ is the angle between the incoming kaon and the outgoing particle in the K-d center-of-mass system. If only S-wave angular momentum is involved, then A = B = 0 and the angular distribution of all reaction products must be isotropic with respect to the kaon. The histogram of $\cos\theta_{\pi K}$(K-d, c. m.) for 25 direct events chosen on the basis of a laboratory-system proton momentum less than 200 Mev/c is shown in Fig. 5. A least-squares fit of these data to $1 + A\cos\theta_{\pi K}\ B\cos^2\theta_{\pi K}$ gives $A = 1.04 \pm .52$, $B = 0.68 \pm .85$. Values of A and B for all events, direct and indirect, in the K-d system are

$$A = 0.58 \pm .31,$$
$$B = 0.33 \pm .51.$$

Nonzero values of A and(or) B for the distribution in the K-d center-of-mass system does not necessarily imply the existence of P-wave angular momentum in the K-neutron interaction, however. It has been calculated that 200-Mev/c kaons interacting only by S channel with neutrons would give a value A = 0.17.[5] This may be seen qualitatively by considering that since the proton must have a low

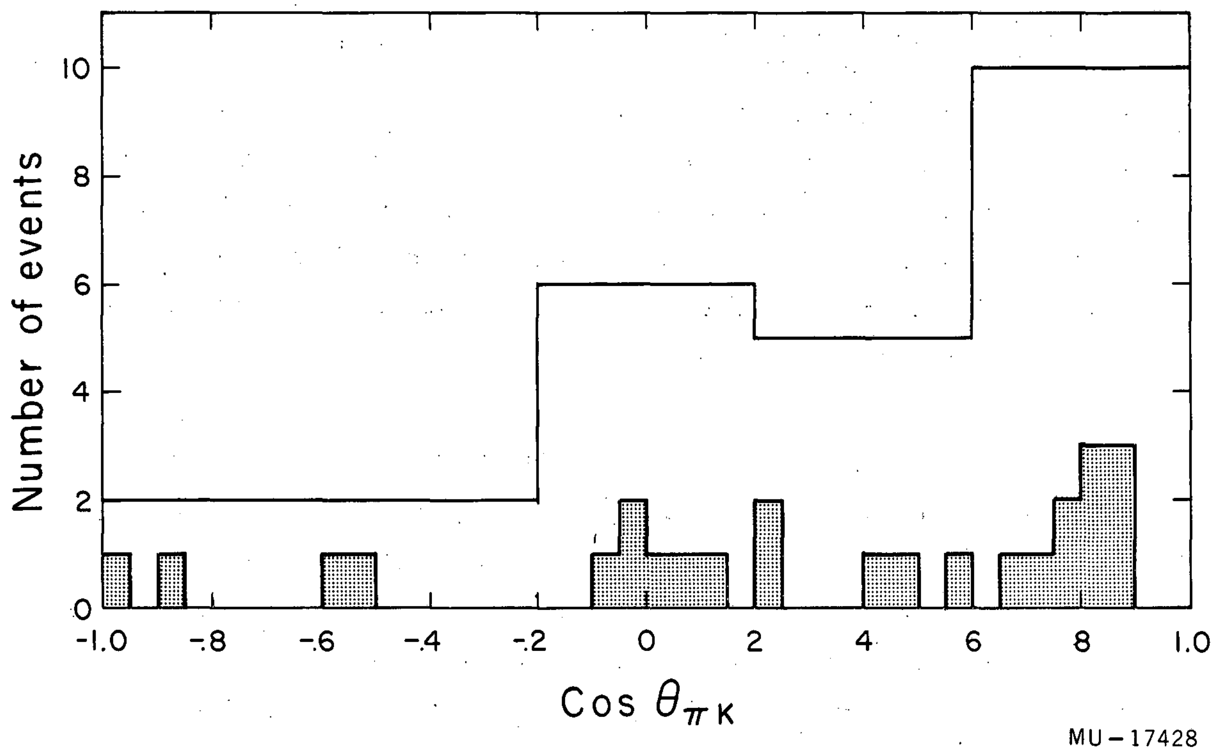

Fig. 5. Distribution of cos $\theta_{\pi K}$ in K-d center-of-mass system for 25 direct events (P_p < 200 Mev/c).

laboratory-system momentum, it tends to be directed backward in the K-d center-of-mass system; this backward proton momentum must be balanced by having the pion and Λ^0 tend slightly forward with respect to the kaon.

2. π-Λ^0 Center-of-Mass System

In order to determine whether there is a P-wave component in the K-neutron interaction, one would like to examine the distribution of the π-K angle in the K-neutron center-of-mass system instead of the K-deuteron system. Since the neutron is bound, it is in an unphysical region and the K-neutron system cannot be measured directly. To the extent that the neutron rest-mass energy is large compared with the deuteron binding energy, the π-Λ^0 center-of-mass system should have nearly the same velocity and total center-of-mass energy as the K-neutron system. In the impulse approximation, then, the distribution of the π-K angle in the π-Λ^0 system should be the same as that distribution in the K-neutron system. The effect of the inaccuracy of this approximation should be to smear the angular distribution out somewhat from what it would have been in the K-neutron system, but it should not introduce any "artificial" asymmetries. The distribution for the 25 direct events is shown in Fig. 6. The parameters calculated from this distribution are

$$A = 0.75 \pm .44,$$
$$B = 0.43 \pm .77.$$

From this, it appears that the direct Λ^0 production proceeds partially by P-wave even at these low kaon momenta.

C. Cross Sections

In estimating cross sections for the direct and indirect production in flight, it is necessary to exclude events that occur near the chamber boundaries, since the detection efficiency for those events is probably low and its value is unknown. Assuming 100% detection efficiency for those events occurring near the center (in the "defined" chamber region), it is then necessary to calculate the total kaon path for all kaons having tracks that enter the defined chamber. Since the tracks of kaons that passed completely through the chamber have not

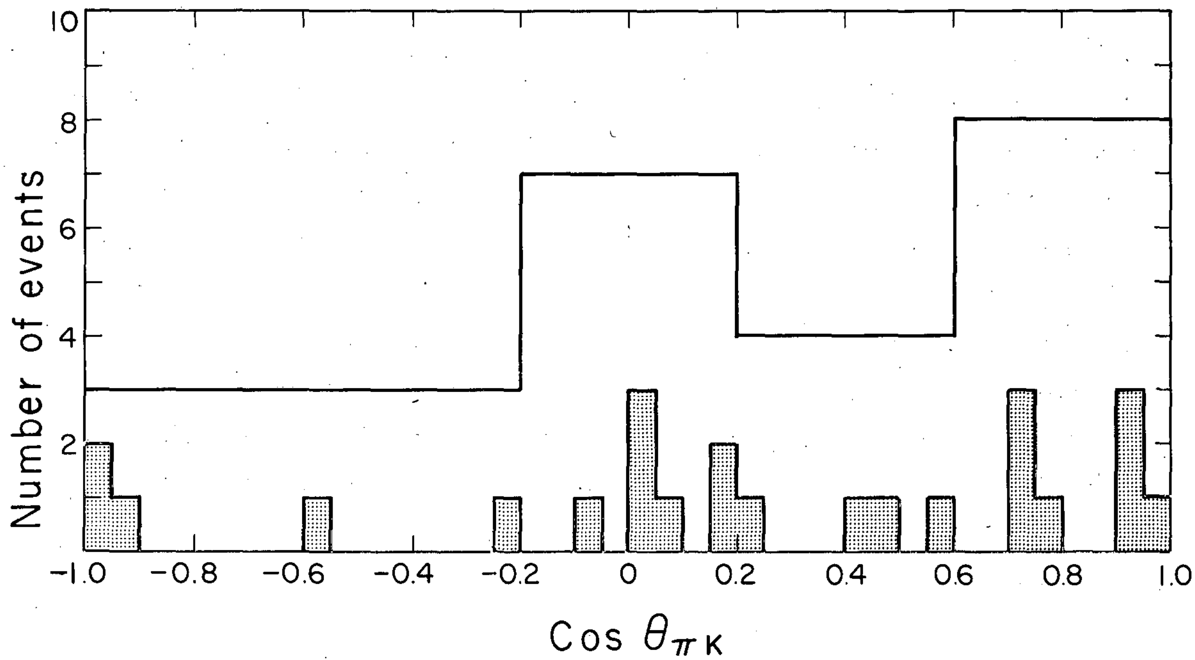

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Fig. 6. Distribution of cos $\theta_{\pi K}$ in $\pi^- - \Lambda^0$ center-of-mass system for 25 direct events (P_p < 200 Mev/c).

been analyzed, it is necessary to exclude a second class of events from consideration in determining cross sections: those which occurred in such a chamber location and with such kaon momenta that, had the kaon not interacted in flight, the kaon would not have stopped within the chamber.

The total kaon path length in various momentum intervals for the class of kaons which either stopped within the chamber or would have stopped had they not interacted in flight has been calculated in connection with another experiment.[6] The average cross section (in cm^2) for a given reaction, in a given momentum interval $p + \Delta p$ and p, is obtained by

$$\sigma_p = N_p/\rho L p,$$

where N_p is the number of events that occurred when the kaon momentum was in the range Δp, ρ is the number of deuterons per cubic centimeter, and L_p is the total path length in centimeters traveled by kaons with momenta between $p + \Delta p$ and p. Cross sections for direct and indirect Type 769 events are listed in Table I.

Table I

Cross sections for direct and indirect Type 769 events[a]

	Direct		Indirect	
K momentum (Mev/c)	No. of events	Cross section (mb)	No. of events	Cross section (mb)
50 - 100	3	49 ± 28	3	49 ± 28
100 - 150	5	21 ± 9	3	13 ± 8
150 - 200	13	25 ± 8	4	10 ± 5
50 - 200	21	Av. 30 ± 6.5	10	Av. 14.5 ± 4.5

[a]Errors are statistical only.

IV. CONCLUSIONS

In interpreting the results of this experiment, it is necessary to know whether the change in the ratio of direct to indirect Λ^0 production is due to a decrease in the percentage of Σ hyperons that convert to Λ^0's or to an increase in the ratio of Λ^0's produced directly relative to the number of Σ hyperons produced.

On the basis of the models of the conversion reaction considered by Karplus and Rodberg,[4] one would not expect the conversion process to be materially changed by allowing the kaons to interact in flight.[7] There is evidence that in K^--p reactions in hydrogen, direct Λ^0 production is substantially increased relative to Σ production when the kaon interacts in flight (300 to 400 Mev/c) instead of at rest.[8] The most likely explanation for the change in the direct-indirect ratio then appears to be that the K^--n reaction also is more likely to produce Λ^0's directly when the kaon interacts in flight.

V. ACKNOWLEDGMENTS

I am deeply indebted to Drs. Joseph Murray and Nahmin Horwitz for the many enlightening discussions, helpful comments, and suggestions offered in connection with this project. Appreciation is also due Peter Berge for his assistance in arranging the computer analysis of these events, and to Dr. Leonard Rodberg for his interest and encouragement.

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APPENDIX

In the calculation of the matrix element for direct Λ^0 production, the following assumptions are made:

(a) The kaon and neutron interact at a point.

(b) The pion and Λ^0 are created at the same point.

(c) No secondary interactions involving the proton occur (i.e., no π-p or Λ-p scattering).

The variables are expressed in units such that $\hbar = c = 1$. The origin of all spatial coordinates is the deuteron center of mass, and all momentum vectors are with respect to the laboratory system of reference, so that the conditions $r_p = r_n$ and $P_k = P_p + P_\pi + P_\Lambda$ apply. The initial state may then be described by

$$\psi_i(r_p, r_n, r_k) = \frac{1}{(2\pi)^3} \iint \delta(P-P_k) \exp(i\vec{P} \cdot \vec{r}_k) \phi_d (P') e^{iP' \cdot (r_p - r_n)} d^3P \, d^3P', \tag{1}$$

where ϕ_d is the momentum representation of the deuteron wave function. The final state is given by

$$\psi_f (r_\Lambda, r_\pi, r_p) = \frac{1}{(2\pi)^{9/2}} \iiint \delta(Q_1 - P_\Lambda) \delta(Q_2 - P_\pi) \delta(Q_3 - P_p) \cdot \exp\left[i (Q_1 \cdot r_\Lambda + Q_2 \cdot r_\pi + Q_3 \cdot r_p) \right] d^3Q_1 d^3Q_2 d^3Q_3 . \tag{2}$$

The matrix element for direct Λ^0 production is given by

$$M = \int_V \psi_f^* T \psi_i \, d\tau, \tag{3}$$

where T is the interaction operator specified in this approximation by

$$T = \delta(r_k - r_n) \delta(r_\pi - r_\Lambda) \delta(r_\Lambda - r_n). \tag{4}$$

Then, substituting Eqs. (1) and (2) into (3), one has

$$M = \frac{1}{(2\pi)^{15/2}} \int_V \delta(Q_1 - P_\Lambda)\, \delta(Q_2 - P_\pi)\, \delta(Q_3 - P_p) \exp[Q_1 \cdot r_\Lambda + Q_2 \cdot r_\pi + Q_3 \cdot r_p)]\, T$$

$$\cdot\, \delta(P - P_k) \phi_d(P') \exp[i(P \cdot r_k + P' \cdot r_p - P' \cdot r_n)]\, d^3P d^3P' d^3Q_1$$

$$d^3Q_2 d^3Q_3 d^3r_k d^3r_n d^3r_p d^3r_\pi d^3r_\Lambda. \qquad (5)$$

Integrating over P, Q_1, Q_2, Q_3 gives, from the delta functions,

$$M = \frac{1}{(2\pi)^{3/2}} \int_V \exp[-i(P_\Lambda \cdot r_\Lambda + P_\pi \cdot r_\pi + P_p \cdot r_p)]\, T\, \phi_d(P')$$

$$\exp[i(P_k \cdot r_k + P'(r_p - r_n)]\, \cdot\, d^3P^1 d^3r_k d^3r_n d^3r_p d^3r_\pi d^3r_\Lambda. \qquad (6)$$

Substituting for T and integrating over r_k, r_π, and r_Λ gives

$$M = \frac{1}{(2\pi)^{3/2}} \int_V \exp[i(P_k \cdot r_n + P' \cdot r_p - P' \cdot r_n - P_\Lambda \cdot r_n - P_\pi \cdot r_n - P_p \cdot r_p)]$$

$$\cdot\, \phi_d(P')\, d^3P' d^3r_n d^3r_p, \qquad (7)$$

$$M = \frac{1}{(2\pi)^{3/2}} \int_V \exp[i\{(P_k - P_\Lambda - P_\pi - P') \cdot r_n + (P' - P_p) \cdot r_p\}]\, \phi_d(P')$$

$$\cdot\, d^3P' d^3r_n d^3r_p. \qquad (8)$$

From the side conditions $r_n = r_p$ and $P_k = P_\Lambda + P_\pi + P_p$, one obtains

$$M = \frac{1}{(2\pi)^{3/2}} \int_V \exp\left[2i\,(P' - P_p)\cdot r_p\right] \phi_d\,(P')\,d^3r_p\,d^3P'. \tag{9}$$

Integrating over r_p gives

$$M = \frac{1}{(2\pi)^{3/2}} \int_V \delta\,(P' - P_p)\,\phi_d\,(P')\,d^3P' = \phi_d(P_p). \tag{10}$$

The momentum distribution for protons in the laboratory system for direct events is given by

$$\frac{dN_p}{dP_p} = |M|^2\,\frac{d\rho}{dP_p} = |\phi_d(P_p)|^2\,\frac{d\rho}{dP_p}, \tag{11}$$

where $\frac{d\rho}{dP_p}$ is the phase space factor describing the density of states. This distribution is plotted in Fig. 3.

FOOTNOTES AND REFERENCES

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