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Extended Left-Right Supersymmetry with Radiative Neutrino Mass and Multipartite Dark Matter

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ABSTRACT OF THE DISSERTATION

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The extended left-right supersymmetric model (eLRSUSY) has the correct particle content to have gauge coupling unification, also three coexisting dark matter candidates; a neutral Fermion n in a $SU(2)_R$ doublet, an exotic Higgsino $\tilde{eta}_R$, with no gaugino mixing, and the lightest neutralino χ_1^0 with MSSM-like behavior, the three candidates have different interactions. We study their possible phenomenological impact on present and future experiments. The model also has a neutrino ν that acquires a small radiative Majorana mass from dark matter via a loop in a Ma-model.

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Chapter 1

Introduction

This thesis was written as a practical application of a theoretical model, knowledge of quantum field theory, the Standard Model (SM) and basics of supersymmetry (SUSY) have been assumed.
A good introduction to SUSY can be found in Martin's "A Supersymmetry Primer"([1]) and another good review of dark matter in the context of SUSY can be found in G. Jungman, M. Kamionkowski, K. Griest's "Supersymmetric Dark Matter" [2].
Extended Left-Right Supersymmetry (eLRSUSY) is a supersymmetric extension to Left-Right model, with a gauge symmetry $SU(3)_c \times SU(2)_R \times SU(2)_L \times U(1)_X$, considered to be a subgroup of $SO(10)$, with X is no longer the hypercharge. Left-Right models have the benefits ([3–9]), of allowing neutrinos to acquire masses , give a Dark Matter candidate (If a discrete symmetry is conserved), and with the condition of no tree-level flavor changing neutral currents, will have a non SM Gauge Boson(Z') in the TeV scale.
In SUSY R-parity conservation allows for the possibility of a stable particle, including extra conserved symmetries will increase the DM candidates. There are three conserved quantities in our model([11]): S, a global $U(1)$ and the discrete Z_2 symmetries M and H. The usual R parity is then $R \equiv MH(-1)^{2j}$. Leading to three stable particles: the lightest neutralino $\tilde{\chi}_1^0$, the lightest scotino n, and the exotic $\tilde{\eta}_R^0$ fermion.
This thesis is constructed in the following way:
In part I, the construction of model is explained, it is divided in 5 chapters. Chapter 1 is the introduction written here, in chapter 2 the Lagrangian symmetries and particle content are introduced. Chapter 3 has the scalar potential, in chapter 4 we calculate the masses of the gauge particles and some Z' interactions that would be required to calculate the scattering of DM. In chapter 5 we will study the neutralino and chargino mass matrix, and find out that in this sector eLRSUSY is not an extention of the MSSM, but with numerical calculation we can find parameters space where the lightest neutralino behaves MSSM-like.

Part II contains two important implications of the model, in chapter 6 we will calculate the neutrino mass with 1 loop using the Ma-scotogenic model. And, in chapter 7 we will calculate the 1 loop RGEs for gauge couplings, it will be shown that the model has the precise content to have unification a mass scale $\sim 10^{16}$ GeV.

In part III we will address the question of dark matter in the model, chapter 8 has the explicit calculations of the cross sections and relic abundances for 2 of the DM candidates of the model, in chapter 9 the direct detection of this candidates vs the experimental results of XENON. Chapter 10 we use computer simulations to calculate the direct and indirect detection of the third DM candidate in the model, an MSSM-like neutralino.

Finally, part IV holds the conclusions, including an example of one point in the parameter space of the model, the bibliography and appendixes.

Chapter 2

eLRSUSY

2.1 Symmetries

Our model is a supersymmetric extension of dark left-right model (DLRM) [3], where the gauge symmetry is an Left-Right extension of the Standard Model given by $SU(3)_c \times SU(2)_R \times SU(2)_L \times U(1)_X$, which as it will be seen in chapter 7 is assumed to be a subgroup of $SO(10)$. Also, three extra symmetries are used, S, a global $U(1)$ symmetry two discrete Z_2 symmetries M and H. R parity (as defined in supersymmetric theories) will be now a combination of M and H given by $R \equiv MH(-1)^{2j}$. $S \times M \times H$ is used to differentiate some of the particles in our model, and have have the added benefit of giving one DM candidate for every unbroken symmetry.

A new generalized unbroken "lepton" number $S' = S + T_{3R}$, will arise form the spontaneous breaking of $SU(2)_R \times S$.

2.2 Particle content

In the Minimal Supersymmetric Standard Model (MSSM), two Higgs doublets are required [1], nevertheless in supersymmetric models with a bigger symmetry(*i.e* E_6 or $SO(10)$) this number can increase, and bidoublets and triplets might be added [12–16]. In our model we will follow ([45]),where it was found, that by adding extra particle content it's possible to obtain gauge coupling unification. Therefor our model will contain 8 Higgs doublets, 2 Higgs bidoublets, 2 charged singlets and and a neutral singlet (all of this superfields) will be added. Also following DLRM there is new exotic quark "h" and more important for our study, an extra neutral Fermion "n" which is NOT a Dirac partner of the SM neutrino. If "n" is a DM candidate, then it will be called a scotino [10], in addition in eLRSUSY we will also add a heavy neutral fermion N, necessary for radiative neutrino masses [17–32]. Being a supersymmetric theory all of these Fermions will have scalar superpartners.

Using the fields with the symmetry assignments $SU(3)_c \times SU(2)_R \times SU(2)_L \times U(1)_X \times S \times H \times M$ the superfields are:

$$
\begin{aligned}
\Delta_1 &= \begin{pmatrix} \delta^0_{11} & \delta^+_{12} \\ \delta^-_{11} & \delta^0_{12} \end{pmatrix} \sim (1,2,2,0;1/2,+,+),\ \Delta_2 = \begin{pmatrix} \delta^0_{21} & \delta^+_{22} \\ \delta^-_{21} & \delta^0_{22} \end{pmatrix} \sim (1,2,2,0;-1/2,+,+) \\
\Phi_{L1} &= \begin{pmatrix} \phi^0_{L1} \\ \phi^-_{L1} \end{pmatrix} \sim (1,2,1,-1/2;0,+,+), \quad \Phi_{L2} = \begin{pmatrix} \phi^+_{L2} \\ \phi^0_{L2} \end{pmatrix} \sim (1,2,1,1/2;0,+,+) \\
\Phi_{R1} &= \begin{pmatrix} \phi^0_{R1} \\ \phi^-_{R1} \end{pmatrix} \sim (1,1,2,-1/2;-1/2,+,+),\ \Phi_{R2} = \begin{pmatrix} \phi^+_{R2} \\ \phi^0_{R2} \end{pmatrix} \sim (1,1,2,1/2;1/2,+,+) \\
\eta_{L1} &= \begin{pmatrix} \eta^0_{L1} \\ \eta^-_{L1} \end{pmatrix} \sim (1,2,1,-1/2;0,+,-), \quad \eta_{L2} = \begin{pmatrix} \eta^+_{L2} \\ \eta^0_{L2} \end{pmatrix} \sim (1,2,1,1/2;0,+,-) \\
\eta_{R1} &= \begin{pmatrix} \eta^0_{R1} \\ \eta^-_{R1} \end{pmatrix} \sim (1,1,2,-1/2;1/2,+,-),\ \eta_{R2} = \begin{pmatrix} \eta^+_{R2} \\ \eta^0_{R2} \end{pmatrix} \sim (1,1,2,1/2;-1/2,+,-) \\
s_1 &= s_1^- \sim (1,1,1,-1;0,+,-), \quad s_2 = s_2^+ \sim (1,1,1,1;0,+,-), \quad s_3 = s_3^0 \sim (1,1,1,0;0,+,-) \\
\psi &= (\nu, e) \sim (1,2,1,-1/2;0,-,+), \qquad \psi^c = (e^c, n^c) \sim (1,1,2,1/2;-1/2,-,+) \\
N &\sim (1,1,1,0;0,-,-), \qquad n \sim (1,1,1,0;1,-,+) \\
Q &= (u,d) \sim (3,2,1,1/6;0,-,+), \qquad Q^c = (h^c, u^c) \sim (3^*,1,2,-1/6;1/2,-,+) \\
d^c &\sim (3^*,1,1,1/3;0,-,+), \qquad h \sim (3,1,1,-1/3;-1,-,+)
\end{aligned} \tag{2.1}
$$

2.3 eLRSUSY Lagrangian

Before writing the Lagrangian with all the possible interactions, first we should consider the fields that we require, fermions fields ψ_i will be written using left handed components (for right handed fields the charge conjugate is used), ϕ_i is used for complex scalar fields (sfermions and squarks are included in this category), the gauge Bosons are included in the field tensor given by, $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$ and in the covariant derivative $D_\mu = \partial_\mu - \imath g T^a G^a_\mu)$, λ_a are the gauginos.

The Lagrangian is:

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}_{gauge-scalar} + \mathcal{L}_{gauge-fermion} + \mathcal{L}_{scalar-fermion} + \mathcal{L}_{gaugino-fermion-scalar} \\
&\quad + \mathcal{L}_{gaugino-gauge} + \mathcal{L}_{gauge} + +V(\phi, \phi^*) + \mathcal{L}_{soft}
\end{aligned} \tag{2.2}
$$

The first term gives interactions between the scalar particles and the gauge Bosons, after symmetry breaking the gauge Bosons will get a mass.

$$\mathcal{L}_{gauge-scalar} = |D_\mu \phi|^2 \tag{2.3}$$

The second term has interactions between two fermions and a gauge particles,

$$\mathcal{L}_{gauge-fermion} = \imath \overline{\psi} \sigma^\mu D_\mu \psi \tag{2.4}$$

the third teem has interactions between gauginos, fermions and scalars,

$$\mathcal{L}_{gaugino-fermion-scalar} = \imath \sqrt{2} g \phi_i^* \lambda_a \psi_i \tag{2.5}$$

the forth term has interactions between two fermions and a scalar, is clear that after symmetry breaking, fermions will acquire mass.

$$\mathcal{L}_{scalar-fermion} = \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \Psi_i \Psi_j + h.c \tag{2.6}$$

W is the superpotential, a function that in general, will contain all the square and cubic terms of the superfields that can be written down as allowed by the symmetries imposed by the model (linear terms can also be written down, but they will not have an impact in the equations of motion, and can be ignored).

the fifth term is the same as in the SM and has cubic and forth order interactions between Bosons.

$$\mathcal{L}_{gauge} = F^a_{\mu\nu} F^{\mu\nu a} \tag{2.7}$$

the sixth interaction is between gauginos and gauge Bosons.

$$\mathcal{L}_{gaugino-gauge} = \imath \overline{\lambda}^a \sigma^\mu D_\mu \lambda_a \tag{2.8}$$

V is the scalar potential, explicitly will be written as

$$V(\phi,\phi^*) = W_i^* W^i + \frac{1}{2}\sum_a D^a D^a = \sum_i |\frac{\partial W}{\partial \Phi_i}|^2 + \frac{1}{2}\sum_a g_a^2 (\phi^* T^a \phi)^2 \tag{2.9}$$

Where $W^i = \frac{\delta W}{\delta \phi_i}$ and $D^a = -g(\phi^* T^a \phi)$. The first term in the scalar potential will be called the F-term, and the second the D-term.

The term $\mathcal{L}_{soft}$, is the soft breaking term, it will be explain in more detail in section 2.5.

2.4 Superpotential

The superpotential will be separated in two, it will make explicit the fact that the second term includes the SM fermions, in this terms the Yukawa couplings will be used to give mass the the leptons and quarks.

$$\begin{aligned} W_1 &= -\mu_L \Phi_{L1}\Phi_{L2} - \mu_R \Phi_{R1}\Phi_{R2} - \mu_\Delta Tr(\Delta_1\Delta_2) \\ &\quad -\mu_{L2}\eta_{L1}\eta_{L2} - \mu_{R2}\eta_{R1}\eta_{R2} - \mu_{s12}s_1 s_2 - \mu_{s3}s_3 s_3 \\ &\quad +f_1\Phi_{L1}\Delta_2\Phi_{R2} + f_2\Phi_{L2}\Delta_1\Phi_{R1} \\ &\quad +f_3\eta_{L1}\Delta_1\eta_{R2} + f_4\eta_{L2}\Delta_2\eta_{R1} + f_5\Phi_{L1}\eta_{L1}s_2 + f_6\Phi_{R1}\eta_{R1}s_2 \\ &\quad +f_7\Phi_{L2}\eta_{L2}s_1 + f_8\Phi_{R2}\eta_{R2}s_1 + f_9\Phi_{L1}\eta_{L2}\chi_3 + f_{10}\Phi_{L2}\eta_{L1}s_3 \end{aligned} \tag{2.10}$$

and

$$\begin{aligned} W_2 &= \mu_N NN + f_{11}\psi\Delta_1\psi^c + f_{12}Q\Delta_2 Q^c + f_{13}Q\Phi_{L1}d^c \\ &+ f_{14}n\psi^c\Phi_{R1} + f_{15}hQ^c\Phi_{R2} + f_{16}\psi N\eta_{L2} + f_{17}\psi^c N\eta_{R1} \end{aligned} \tag{2.11}$$

A tree level neutrino mass would come for a term like $\psi\Phi_{L2}\nu^c$, which is non-existent in the superpotential (ν^c does not exist in the model). Nevertheless, a 1 loop neutrino mass can be generated as it will be seen in Chapter 6.

The Superpotential can be rewritten explicitly in terms of the separate isospin components

$$
\begin{aligned}
W_1 \;=\; & -\mu_L(\phi^0_{L1}\phi^0_{L2}-\phi^-_{L1}\phi^+_{L2})-\mu_R(\phi^0_{R1}\phi^0_{R2}-\phi^-_{R1}\phi^+_{R2}) \\
& -\mu_\Delta(\delta^0_{11}\delta^0_{22}-\delta^-_{11}\delta^+_{22}-\delta^+_{12}\delta^-_{21}+\delta^0_{12}\delta^0_{21}) \\
& -\mu_{L2}(\eta^0_{L1}\eta^0_{L2}-\eta^-_{L1}\eta^+_{L2})-\mu_{R2}(\eta^0_{R1}\eta^0_{R2}-\eta^-_{R1}\eta^+_{R2})-\mu_{s12}s^-_1s^+_2-\mu_{s3}s^0_3s^0_3 \\
& +f_1(\phi^-_{L1}\delta^0_{21}\phi^+_{R2}-\phi^0_{L1}\delta^-_{21}\phi^+_{R2}-\phi^-_{L1}\delta^+_{22}\phi^0_{R2}+\phi^0_{L1}\delta^0_{22}\phi^0_{R2}) \\
& +f_2(\phi^0_{L2}\delta^0_{11}\phi^0_{R1}-\phi^+_{L2}\delta^-_{11}\phi^0_{R1}-\phi^0_{L2}\delta^+_{12}\phi^-_{R1}+\phi^+_{L2}\delta^0_{12}\phi^-_{R1}) \\
& +f_3(\eta^-_{L1}\delta^0_{11}\eta^+_{R2}-\eta^0_{L1}\delta^-_{11}\eta^+_{R2}-\eta^-_{L1}\delta^+_{12}\eta^0_{R2}+\eta^0_{L1}\delta^0_{12}\eta^0_{R2}) \\
& +f_4(\eta^0_{L2}\delta^0_{21}\eta^0_{R1}-\eta^+_{L2}\delta^-_{21}\eta^0_{R1}-\eta^0_{L2}\delta^+_{22}\eta^-_{R1}+\eta^+_{L2}\delta^0_{22}\eta^-_{R1}) \\
& +f_5(\phi^0_{L1}\eta^-_{L1}s^+_2-\phi^-_{L1}\eta^0_{L1}s^+_2)+f_6(\phi^0_{R1}\eta^-_{R1}s^+_2-\phi^-_{R1}\eta^0_{R1}s^+_2) \\
& +f_7(\phi^+_{L2}\eta^0_{L2}s^-_1-\phi^0_{L2}\eta^+_{L2}s^-_1)+f_8(\phi^+_{R2}\eta^0_{R2}s^-_1-\phi^0_{R2}\eta^+_{R2}s^-_1) \\
& +f_9(\phi^0_{L1}\eta^0_{L2}s^0_3-\phi^-_{L1}\eta^+_{L2}s^0_3)+f_{10}(\phi^0_{L2}\eta^0_{L1}s^0_3-\phi^+_{L2}\eta^-_{L1}s^0_3)
\end{aligned} \tag{2.12}
$$

$$
\begin{aligned}
W_2 \;=\; & Y_e(e\delta^0_{11}e^c-\nu\delta^-_{11}e^c-e\delta^+_{12}n^c+\nu\delta^0_{12}n^c)+Y_u(d\delta^0_{21}h^c-u\delta^-_{21}h^c-d\delta^+_{22}u^c+u\delta^0_{22}u^c) \\
+\; & Y_d(u\phi^-_{L1}d^c-d\phi^0_{L1}d^c)+Y_n(ne^c\phi^-_{R1}-nn^c\phi^0_{R1})+Y_h(hh^c\phi^0_{R2}-hu^c\phi^+_{R2}) \\
+\; & Y_{N1}(N\nu\eta^0_{L2}-N\eta^+_{L2}e)+Y_{N2}(N\eta^-_{R1}e^c-N\eta^0_{R1}n^c)
\end{aligned} \tag{2.13}
$$

We have renamed the parameters $f_{11}-f_{17}$ in the equation above, to show this terms are Yukawas responsible for the mass of the Fermions. From (2.6) the mass m_e comes from $\langle\delta^0_{11}\rangle = u_1$, m_u from $\langle\delta^0_{22}\rangle = u_4$, m_d from $\langle\phi^0_{L1}\rangle = v_{L1}$, m_n from $\langle\phi^0_{R1}\rangle = v_{R1}$, and m_h from$\langle\phi^0_{R2}\rangle = v_{R2}$). Note that $\langle\phi^0_{L2}\rangle = v_{L2}$ doesn't contribute to fermion masses, but is involved in the scalar and vector masses. A diagonal Yukawa structure guarantees the absence of tree-level flavor-changing neutral currents.

2.5 Soft Term

Supersymmetry must be broken, if not particles and their superpartners would have the same mass. This mean the Lagrangian should include new terms have to be added that break SUSY, with the restriction that the quadratic divergence needs to remain canceled, and these terms must also be renormalizable. There are many ways to break SUSY, being the most common mechanisms ([34–36]) : Gravity mediated (SUGRA, mSUGRA, cMSSM), Anomaly Mediated and Gauge Mediated (Although there are more), a good review can be found in ([33]).

It's also possible to write the soft SUSY breaking terms without a theoretical explanation, this explicit breaking will consist on the following terms:

1. Gauginos, in the eLRSUSY there will be 4 of these masses corresponding to the bino, left and right winos and the gluino, be aware given the extended gauge symmetry the bino soft mass M_B will not be the same as the one in the MSSM.

2. Scalars, this will be for all Higgs partcles and also the scalar superpartners of the fermions.

3. Bilinear, these are in the form $B_{ij}\phi_i\phi_j$, and are included for every square term in the superpotential.

4. Trilinear, these are in the form $A_{ijk}\phi_i\phi_j\phi_k$, and are included for every cubic term in the superpotential.

Chapter 3

Scalar Sector

The Higgs sector in the model is extremely rich, and therefor extremely non trivial. There are 2 charged singlets, a neutral singlet, 8 complex doublets, and 2 bidoublets. Naively, for every complex singlet there are 2 fields (If is neutral there will be a real and imaginary component, and if is charged there will be a positive and a negative particle). For doublet there will be 2 neutral scalars and 2 charged ones, and for the bidoublet twice as much than a doublet. This means that the model will have 26 neutral scalars and 28 charged ones. As mentioned this is a naive count since any charged scalar will have the same mass as its conjugate, also only 6 of the neutral scalars will acquire a vacuum expectation value (vev), that will make the real and imaginary components different. Over there will be then 21 neutral scalars and and 14 charged ones.

3.1 Scalar potential

As mention in the last chapter, the scalar potential will depend on three terms

$$V_{scalar} = V_{soft} + V_D + V_F \tag{3.1}$$

3.1.1 Soft term

We will write the soft potential in two different terms, the first one will include only the Higgs particles, while the second one will also include sfermions.

$$
\begin{aligned}
V_{1soft} = \quad & -m_{L1}^2(\overline{\phi_{L1}^0}\phi_{L1}^0 + \phi_{L1}^-\phi_{L1}^+) - m_{L2}^2(\overline{\phi_{L2}^0}\phi_{L2}^0 + \phi_{L2}^-\phi_{L2}^+) - m_{R1}^2(\overline{\phi_{R1}^0}\phi_{R1}^0 + \phi_{R1}^-\phi_{R1}^+) \\
& -m_{R2}^2(\overline{\phi_{R2}^0}\phi_{R2}^0 + \phi_{R2}^-\phi_{R2}^+) - m_{\eta_{L1}}^2(\overline{\eta_{L1}^0}\eta_{L1}^0 + \eta_{L1}^-\eta_{L1}^+) - m_{\eta_{L2}}^2(\overline{\eta_{L2}^0}\eta_{L2}^0 + \eta_{L2}^-\eta_{L2}^+) \\
& -m_{\eta_{R1}}^2(\overline{\eta_{R1}^0}\eta_{R1}^0 + \eta_{R1}^-\eta_{R1}^+) - m_{\eta_{R2}}^2(\overline{\eta_{R2}^0}\eta_{R2}^0 + \eta_{R2}^-\eta_{R2}^+) - m_{s1}^2 s_1^- s_1^+ - m_{s2}^2 s_2^- s_2^+ - m_{s3}^2\overline{s_3^0}s_3^0 \\
& -m_{\Delta 1}^2(\overline{\delta_{11}^0}\delta_{11}^0 + \delta_{11}^+\delta_{11}^- + \overline{\delta_{12}^0}\delta_{12}^0 + \delta_{12}^+\delta_{12}^-) - m_{\Delta 2}^2(\overline{\delta_{21}^0}\delta_{21}^0 + \delta_{21}^+\delta_{21}^- + \overline{\delta_{22}^0}\delta_{22}^0 + \delta_{22}^+\delta_{22}^-) \\
& -B_L(\phi_{L1}^0\phi_{L2}^0 - \phi_{L1}^-\phi_{L2}^+) - B_R(\phi_{R1}^0\phi_{R2}^0 - \phi_{R1}^-\phi_{R2}^+) - B_\Delta(\delta_{11}^0\delta_{22}^0 - \delta_{11}^-\delta_{22}^+ - \delta_{12}^+\delta_{21}^- + \delta_{12}^0\delta_{21}^0) \\
& -B_{\eta_L}(\eta_{L1}^0\eta_{L2}^0 - \eta_{L1}^-\eta_{L2}^+) - B_{\eta_R}(\eta_{R1}^0\eta_{R2}^0 - \eta_{R1}^-\eta_{R2}^+) - B_{s12}s_1^- s_2^+ - B_{s3}s_3^0 s_3^0 \\
& +A_1(\phi_{L1}^-\delta_{21}^0\phi_{R2}^+ - \phi_{L1}^0\delta_{21}^-\phi_{R2}^+ - \phi_{L1}^-\delta_{22}^+\phi_{R2}^0 + \phi_{L1}^0\delta_{22}^0\phi_{R2}^0) \\
& +A_2(\phi_{L2}^0\delta_{11}^0\phi_{R1}^0 - \phi_{L2}^+\delta_{11}^-\phi_{R1}^0 - \phi_{L2}^0\delta_{12}^+\phi_{R1}^- + \phi_{L2}^+\delta_{12}^0\phi_{R1}^-) \\
& +A_3(\eta_{L1}^-\delta_{11}^0\eta_{R2}^+ - \eta_{L1}^0\delta_{11}^-\eta_{R2}^+ - \eta_{L1}^-\delta_{12}^+\eta_{R2}^0 + \eta_{L1}^0\delta_{12}^0\eta_{R2}^0) \\
& +A_4(\eta_{L2}^0\delta_{21}^0\eta_{R1}^0 - \eta_{L2}^+\delta_{21}^-\eta_{R1}^0 - \eta_{L2}^0\delta_{22}^+\eta_{R1}^- + \eta_{L2}^+\delta_{22}^0\eta_{R1}^-) \\
& +A_5(\phi_{L1}^0\eta_{L1}^- s_2^+ - \phi_{L1}^-\eta_{L1}^0 s_2^+) + A_6(\phi_{R1}^0\eta_{R1}^- s_2^+ - \phi_{R1}^-\eta_{R1}^0 s_2^+) \\
& +A_7(\phi_{L2}^+\eta_{L2}^0 s_1^- - \phi_{L2}^0\eta_{L2}^+ s_1^-) + A_8(\phi_{R2}^+\eta_{R2}^0 s_1^- - \phi_{R2}^0\eta_{R2}^+ s_1^-) \\
& +A_9(\phi_{L1}^0\eta_{L2}^0 s_3^0 - \phi_{L1}^-\eta_{L2}^+ s_3^0) + A_{10}(\phi_{L2}^+\eta_{L1}^- s_3^0 - \phi_{L2}^+\eta_{L1}^- s_3^0)
\end{aligned}
$$

$$
\begin{aligned}
V_{2soft} = \quad & m_\psi^2(\tilde{e}\tilde{e} + \tilde{\nu}\tilde{\nu}) + m_{\psi^c}^2(\tilde{e^c}\tilde{e^c} + \tilde{n^c}\tilde{n^c}) + m_N^2\tilde{N}\tilde{N} + m_n^2\tilde{n}\tilde{n} && (3.2) \\
& +m_Q^2(\tilde{u}\tilde{u} + \tilde{d}\tilde{d}) + m_{\psi^c}^2(\tilde{h^c}\tilde{h^c} + \tilde{u^c}\tilde{u^c}) + m_{d^c}^2\tilde{d^c}\tilde{d^c} + m_h^2\tilde{h}\tilde{h} \\
& +A_e(\tilde{e}\delta_{11}^0\tilde{e^c} - \tilde{\nu}\delta_{11}^-\tilde{e^c} - \tilde{e}\delta_{12}^+\tilde{n^c} + \tilde{\nu}\delta_{12}^0\tilde{n^c}) + A_u(\tilde{d}\delta_{21}^0\tilde{h^c} - \tilde{u}\delta_{21}^-\tilde{h^c} - \tilde{d}\delta_{22}^+\tilde{u^c} + \tilde{u}\delta_{22}^0\tilde{u^c}) \\
& +A_d(\tilde{u}\phi_{L1}^-\tilde{d^c} - \tilde{d}\phi_{L1}^0\tilde{d^c}) + A_n(\tilde{n}\phi_{R1}^-\tilde{e^c} - \tilde{n}\phi_{R1}^0\tilde{n^c}) + A_h(\tilde{h}\phi_{R2}^0\tilde{h^c} - \tilde{h}\phi_{R2}^+\tilde{u^c}) && (3.3) \\
& +A_{N1}(\tilde{N}\eta_{L2}^0\tilde{\nu} - \tilde{N}\eta_{L2}^+\tilde{e}) + A_{N2}(\tilde{N}\eta_{R1}^-\tilde{e^c} - \tilde{N}\eta_{R1}^0\tilde{n^c})
\end{aligned}
$$

3.1.2 F-Term

From eq. (2.9),

$$V_F = \sum_i |\frac{\partial W}{\partial \Phi_i}|^2 \tag{3.4}$$

where Φ_i are the scalar superfields, explicitly

$$
\begin{aligned}
V_F &= |-\tilde{d}Y_u - \mu_L\phi^0_{L2} + f_1(-\delta^-_{21}\phi^+_{R2} + \delta^0_{22}\phi^0_{R2}) + f_5\eta^-_{L1}\chi^+_2 + f_9\eta^0_{L2}\chi^0_3|^2 \\
&+ |-\mu_L\phi^0_{L1} + f_2(-\delta^+_{11}\phi^-_{R1} + \delta^0_{11}\phi^0_{R1}) - f_7\eta^+_{L2}\chi^-_1 + f_{10}\eta^0_{L1}\chi^0_3|^2 \\
&+ |\tilde{d}^c\tilde{u}Y_u + \mu_L\phi^+_{L2} + f_1(\delta^0_{21}\phi^+_{R2} - \delta^+_{22}\phi^0_{R2}) - f_5\eta^0_{L1}\chi^+_2 - f_9\eta^+_{L2}\chi^0_3|^2 \\
&+ |\mu_L\phi^-_{L1} + f_2(\delta^0_{12}\phi^-_{R1} - \delta^-_{11}\phi^0_{R1}) + f_7\eta^0_{L2}\chi^-_1 - f_{10}\eta^-_{L1}\chi^0_3|^2 \\
&+ |-\tilde{n}\tilde{n}^cY_n + f_2(-\delta^-_{11}\phi^+_{L2} + \delta^0_{11}\phi^0_{L2}) - \ \mu_R\phi^0_{R2} + f_6\eta^-_{R1}\chi^+_2|^2 \\
&+ |\tilde{h}^c\tilde{h}Y_h + f_1(-\delta^+_{22}\phi^-_{L1} + \delta^0_{22}\phi^0_{L1}) - \ \mu_R\phi^0_{R1} - f_8\eta^+_{R2}\chi^-_1|^2 \\
&+ |\tilde{e}^c\tilde{n}Y_n + f_2(\delta^0_{12}\phi^+_{L2} - \delta^+_{11}\phi^0_{L2}) + \ \mu_R\phi^+_{R2} - f_6\eta^0_{R1}\chi^+_2|^2 \\
&+ |-\tilde{h}\tilde{u}^cY_h + f_1(\delta^0_{21}\phi^-_{L1} - \delta^-_{21}\phi^0_{L1}) + \ \mu_R\phi^-_{R1} + f_8\eta^0_{R2}\chi^-_1|^2 \\
&+ |e\tilde{e}^cY_e + f_3\eta^-_{L1}\eta^+_{R2} - \delta^0_{22}\mu_\Delta + f_2\phi^0_{L2}\phi^0_{R1}|^2 \\
&+ |\tilde{u}\tilde{u}^cY_u + f_4\eta^+_{L2}\eta^-_{R1} - \delta^0_{11}\mu_\Delta + f_1\phi^0_{L1}\phi^0_{R2}|^2 \\
&+ |-\tilde{e}^c\tilde{\nu}Y_e - f_3\eta^0_{L1}\eta^+_{R2} + \delta^+_{22}\mu_\Delta - f_2\phi^+_{L2}\phi^0_{R1}|^2 \\
&+ |-\tilde{d}\tilde{u}^cY_u - f_4\eta^0_{L2}\eta^-_{R1} + \delta^-_{11}\mu_\Delta - f_1\phi^-_{L1}\phi^0_{R2}|^2 \\
&+ |-\tilde{h}^c\tilde{u}Y_u - f_4\eta^+_{L2}\eta^0_{R1} + \delta^+_{11}\mu_\Delta - f_1\phi^0_{L1}\phi^+_{R2}|^2 \\
&+ |-\tilde{e}\tilde{n}^cY_e - f_3\eta^-_{L1}\eta^0_{R2} + \delta^-_{21}\mu_\Delta - f_2\phi^0_{L2}\phi^-_{R1}|^2 \\
&+ |\tilde{d}\tilde{h}^cY_u + f_4\eta^0_{L2}\eta^0_{R1} - \delta^0_{12}\mu_\Delta + f_1\phi^-_{L1}\phi^+_{R2}|^2 \\
&+ |\tilde{n}^c\tilde{\nu}Y_e + f_3\eta^0_{L1}\eta^0_{R2} - \delta^0_{21}\mu_\Delta + f_2\phi^+_{L2}\phi^-_{R1}|^2 \\
&+ |f_3(-\delta^-_{11}\eta^+_{R2} + \delta^0_{12}\eta^0_{R2}) - \ \eta^0_{L2}\mu_{L2} - f_5\phi^-_{L1}\chi^+_2 + f_{10}\phi^0_{L2}\chi^0_3|^2 \\
&+ |\tilde{N}\tilde{\nu}Y_{N1} + f_4(-\delta^+_{22}\eta^-_{R1} + \delta^0_{21}\eta^0_{R1}) - \ \eta^0_{L1}\mu_{L2} + f_7\phi^+_{L2}\chi^-_1 + f_9\phi^0_{L1}\chi^0_3|^2 \\
&+ |f_3(\delta^0_{11}\eta^+_{R2} - \delta^+_{11}\eta^0_{R2}) + \ \eta^+_{L2}\mu_{L2} + f_5\phi^0_{L1}\chi^+_2 - f_{10}\phi^+_{L2}\chi^0_3|^2 \\
&+ |-\tilde{e}\tilde{N}Y_{N1} + f_4(\delta^0_{22}\eta^-_{R1} - \delta^-_{21}\eta^0_{R1}) + \ \eta^-_{L1}\mu_{L2} - f_7\phi^0_{L2}\chi^-_1 - f_9\phi^-_{L1}\chi^0_3|^2 \\
&+ |-\tilde{N}\tilde{n}^cY_{N2} + f_4(-\delta^-_{21}\eta^+_{L2} + \delta^0_{21}\eta^0_{L2}) - \ \eta^0_{R2}\mu_{R2} - f_6\phi^-_{R1}\chi^+_2|^2 \\
&+ |f_3(-\delta^+_{11}\eta^-_{L1} + \delta^0_{12}\eta^0_{L1}) - \ \eta^0_{R1}\mu_{R2} + f_8\phi^+_{R2}\chi^-_1|^2 \\
&+ |\tilde{e}^c\tilde{N}Y_{N2} + f_4(\delta^0_{22}\eta^+_{L2} - \delta^+_{22}\eta^0_{L2}) + \ \eta^+_{R2}\mu_{R2} + f_6\phi^0_{R1}\chi^+_2|^2 \\
&+ |f_3(\delta^0_{11}\eta^-_{L1} - \delta^-_{11}\eta^0_{L1}) + \ \eta^-_{R1}\mu_{R2} - f_8\phi^0_{R2}\chi^-_1|^2 \\
&+ |f_7(\eta^0_{L2}\phi^+_{L2} - \eta^+_{L2}\phi^0_{L2}) + f_8(\eta^0_{R2}\phi^+_{R2} - \eta^+_{R2}\phi^0_{R2}) - \ \mu_{s12}\chi^+_2|^2 \\
&+ |f_5(-\eta^0_{L1}\phi^-_{L1} + \eta^-_{L1}\phi^0_{L1}) + f_6(-\eta^0_{R1}\phi^-_{R1} + \eta^-_{R1}\phi^0_{R1}) - \ \mu_{s12}\chi^-_1|^2 \\
&+ |f_9(-\eta^+_{L2}\phi^-_{L1} + \eta^0_{L2}\phi^0_{L1}) + f_{10}(-\eta^-_{L1}\phi^+_{L2} + \eta^0_{L1}\phi^0_{L2}) - 2\mu_{s3}\chi^0_3|^2
\end{aligned}
\tag{3.5}
$$

$$
\begin{aligned}
+ \quad & |Y_e(\tilde{e}^c\delta^0_{11} - \tilde{n}^c\delta^+_{11}) - \tilde{N}Y_{N1}\eta^+_{L2}|^2 + |Y_e(-\tilde{\nu}\delta^-_{11} + \tilde{e}\delta^0_{11}) + \tilde{N}Y_{N2}\eta^-_{R1} + \tilde{n}Y_n\phi^-_{R1}|^2 \\
+ \quad & |Y_e(-\tilde{e}^c\delta^-_{11} + \tilde{n}^c\delta^0_{12}) + \tilde{N}Y_{N1}\eta^0_{L2}|^2 + |Y_e(-\tilde{e}\delta^+_{11} + \tilde{\nu}\delta^0_{12}) - \tilde{N}Y_{N2}\eta^0_{R1} - \tilde{n}Y_n\phi^0_{R1}|^2 \\
+ \quad & |Y_u(\tilde{h}^c\delta^0_{21} - \tilde{u}^c\delta^+_{22}) - \tilde{d}^cY_u\phi^0_{L1}|^2 + |Y_u(-\tilde{u}\delta^-_{21} + \tilde{d}\delta^0_{21}) + \tilde{h}Y_h\phi^0_{R2}|^2 \\
+ \quad & |Y_u(-\tilde{h}^c\delta^-_{21} + \tilde{u}^c\delta^0_{22}) + \tilde{d}^cY_u\phi^-_{L1}|^2 + |Y_u(\tilde{u}\phi^-_{L1} - \tilde{d}\phi^0_{L1})|^2 + |Y_n(\tilde{e}^c\phi^-_{R1} - \tilde{n}^c\phi^0_{R1})|^2 \\
+ \quad & |Y_h(-\tilde{u}^c\phi^+_{R2} + \tilde{h}^c\phi^0_{R2})|^2 + |Y_u(-\tilde{d}\delta^+_{22} + \tilde{u}\delta^0_{22}) - \tilde{h}Y_h\phi^+_{R2}|^2 \\
+ \quad & |Y_{N1}(-\tilde{e}\eta^+_{L2} + \tilde{\nu}\eta^0_{L2}) + Y_{N2}(\tilde{e}^c\eta^-_{R1} - \tilde{n}^c\eta^0_{R1})|^2
\end{aligned}
$$

3.1.3 D-Term

From eq. (2.9)

$$V_D = \sum_j \frac{g_j^2}{2} \sum_a |\Phi_i T^j_a \Phi_i|^2 \tag{3.6}$$

Where T^j_a are the generators for the Group (i.e. Y for U(1), $\frac{\sigma_a}{2}$ for SU(2))

$$
\begin{aligned}
V_D = \quad & \tfrac{g_1^2}{8} \quad |-\Phi^\dagger_{L1}\Phi_{L1} + \Phi^\dagger_{L2}\Phi_{L2} - \Phi^\dagger_{R1}\Phi_{R1} + \Phi^\dagger_{R2}\Phi_{R2} & (3.7) \\
& \quad -\eta^\dagger_{L1}\eta_{L1} + \eta^\dagger_{L2}\eta_{L2} - \eta^\dagger_{R1}\eta_{R1} + \eta^\dagger_{R2}\eta_{R2} - 2s^\dagger_1 s_1 + 2s^\dagger_2 s_2 \\
& \tilde{\psi}^\dagger \quad -\tilde{\psi}^\dagger\tilde{\psi} + \tilde{\psi^c}^\dagger\tilde{\psi^c} + \frac{1}{3}\tilde{Q}^\dagger\tilde{Q} - \frac{1}{3}\tilde{Q^c}^\dagger\tilde{Q^c} - \frac{2}{3}\tilde{h}^\dagger\tilde{h} + \frac{2}{3}\tilde{d^c}^\dagger\tilde{d^c}|^2 & (3.8) \\
+ \quad & \tfrac{g_L^2}{8} \quad \sum_a |\Phi^\dagger_{L1}\sigma_a\Phi_{L1} + \Phi^\dagger_{L2}\sigma_a\Phi_{L2} + Tr\left[\Delta^\dagger_1\sigma_a\Delta_1\right] + Tr\left[\Delta^\dagger_2\sigma_a\Delta_2\right] \\
& \quad +\eta^\dagger_{L1}\sigma_a\eta_{L1} + \eta^\dagger_{L2}\sigma_a\eta_{L2} + \tilde{\psi}^\dagger\sigma_a\tilde{\psi} + \tilde{Q}^\dagger\sigma_a\tilde{Q}|^2 \\
+ \quad & \tfrac{g_R^2}{8} \quad \sum_a |\Phi^\dagger_{R1}\sigma_a\Phi_{R1} + \Phi^\dagger_{R2}\sigma_a\Phi_{R2} + Tr\left[\Delta^\dagger_1\sigma_a\Delta_1\right] + Tr\left[\Delta^\dagger_2\sigma_a\Delta_2\right] \\
& \quad +\eta^\dagger_{R1}\sigma_a\eta_{R1} + \eta^\dagger_{R2}\sigma_a\eta_{R2} + \tilde{\psi^c}^\dagger\sigma_a\tilde{\psi^c} + \tilde{Q^c}\sigma_a\tilde{Q^c}|^2
\end{aligned}
$$

3.2 Symmetry Breaking

The vevs are $\langle\phi^0_{L1}\rangle = v_{L1}, \langle\phi^0_{L2}\rangle = v_{L2}, \langle\phi^0_{R1}\rangle = v_{R1}, \langle\phi^0_{R2}\rangle = v_{R2}, \langle\delta^0_{11}\rangle = u_1, \langle\delta^0_{22}\rangle = u_4$

The potential minimum is given by,

$$
\begin{aligned}
V_0 = \; & \frac{1}{8}(-8m_{\Delta 1}u_1^2 - 16B_D u_1 u_4 - 8m_{\Delta 2}u_4^2 - 8m_{L1}v_{L1}^2 + 8f_1^2 u_4^2 v_{L1}^2 + g_1^2 v_{L1}^4 \\
& -16B_L v_{L1}v_{L2} - 8m_{L2}v_{L2}^2 + 8f_2^2 u_1^2 v_{L2}^2 - 2g_1^2 v_{L1}^2 v_{L2}^2 + g1^2 v_{L2}^4 + 16A_2 u_1 v_{L2} v_{R1} \\
& -8m_{R1}v_{R1}^2 + 8f_2^2 u_1^2 v_{R1}^2 + 2g1^2 v_{L1}^2 v_{R1}^2 + 8f_2^2 v_{L2}^2 v_{R1}^2 - 2g1^2 v_{L2}^2 v_{R1}^2 + g1^2 v_{R1}^4 \\
& +16A_1 u_4 v_{L1} v_{R2} - 16B_R v_{R1} v_{R2} - 8m_{R2}v_{R2}^2 + 8f_1^2 u_4^2 v_{R2}^2 + 8f_1^2 v_{L1}^2 v_{R2}^2 - 2g_1^2 v_{L1}^2 v_{R2}^2 \\
& +2g_1^2 v_{L2}^2 v_{R2}^2 - 2g_1^2 v_{R1}^2 v_{R2}^2 + g_1^2 v_{R2}^4 + g_L^2(2u_1^4 + 2u_4^4 + (v_{L1}^2 - v_{L2}^2)^2 + (v_{R1}^2 - v_{R2}^2)^2 \\
& +2u_4^2(-v_{L1}^2 + v_{L2}^2 - v_{R1}^2 + v_{R2}^2) - 2u_1^2(2u_4^2 - v_{L1}^2 + v_{L2}^2 - v_{R1}^2 + v_{R2}^2)) \\
& -16f_2 u_4 v_{L2} v_{R1}\mu_\Delta - 16f_1 u_1 v_{L1} v_{R2}\mu_\Delta + 8u_1^2\mu_\Delta^2 + 8u_4^2\mu_\Delta^2 - 16f_2 u_1 v_{L1} v_{R1}\mu_L \\
& -16f_1 u_4 v_{L2} v_{R2}\mu_L + 8v_{L1}^2\mu_L^2 + 8v_{L2}^2\mu_L^2 - 16(f_1 u_4 v_{L1} v_{R1} + f_2 u_1 v_{L2} v_{R2})\mu_R + 8(v_{R1}^2 + v_{R2}^2)\mu_R^2)
\end{aligned}
$$

Using the conditions of minimum of the potential

$$
\begin{aligned}
m_{L1}^2 = \; & \mu_L^2 + \frac{g_L^2}{4}(u_1^2 - u_4^2 + v_{L1}^2 - v_{L2}^2) + \frac{g_1^2}{4}(v_{L1}^2 - v_{L2}^2 + v_{R1}^2 - v_{R2}^2) + f_1^2(u_4^2 + v_{R2}^2) \\
+ \; & \frac{A_1 u_4 v_{R2} - B_L v_{L2} - f_2 u_1 v_{R1}\mu_L + f_1(-u_1 v_{R2}\mu_\Delta - u_4 v_{R1}\mu_R)}{v_{L1}} \\
m_{L2}^2 = \; & \mu_L^2 + \frac{g_L^2}{4}(-u_1^2 + u_4^2 - v_{L1}^2 + v_{L2}^2) + \frac{g_1^2}{4}(-v_{L1}^2 + v_{L2}^2 - v_{R1}^2 + v_{R2}^2) + f_2^2(u_1^2 + v_{R1}^2) \\
+ \; & \frac{A_2 u_1 v_{R1} - B_L v_{L1} - f_1 u_4 v_{R2}\mu_L - f_2(u_4 v_{R1}\mu_D + u_1 v_{R2}\mu_R)}{v_{L2}} \\
m_{R1}^2 = \; & \mu_R^2 + \frac{g_R^2}{4}(u_1^2 - u_4^2 + v_{R1}^2 - v_{R2}^2) + \frac{g_1^2}{4}(v_{L1}^2 - v_{L2}^2 + v_{R1}^2 - v_{R2}^2) + f_2^2(u_1^2 + v_{L2}^2) \\
+ \; & \frac{A u_1 v_{L2} - B_R v_{R2} - f_1 u_4 v_{L1}\mu_R - f_2(u_4 v_{L2}\mu_D + u_1 v_{L1}\mu_L)}{v_{R1}} \\
m_{R2}^2 = \; & \mu_R^2 + \frac{g_R^2}{4}(-u_1^2 + u_4^2 - v_{R1}^2 + v_{R2}^2) + \frac{g_1^2}{4}(-v_{L1}^2 + v_{L2}^2 - v_{R1}^2 + v_{R2}^2) + f_1^2(u_4^2 + v_{L1}^2) \\
+ \; & \frac{A_1 u_4 v_{L1} - B_R v_{R1} - f_2 u_1 v_{L2}\mu_R - f_1(u_1 v_{L1}\mu_D + u_4 v_{L2}\mu_L)}{v_{R2}} \\
m_{\Delta 1}^2 = \; & \mu_D^2 + \frac{g_L^2}{4}(u_1^2 - u_4^2 + v_{L1}^2 - v_{L2}^2) + \frac{g_R^2}{4}(u_1^2 - u_4^2 + v_{R1}^2 - v_{R2}^2) + f_2^2(v_{L2}^2 + v_{R1}^2) \\
+ \; & \frac{A_2 v_{L2} v_{R1} - B_\Delta u_4 - f_1 v_{L1} v_{R2}\mu_D - f_2(v_{L1} v_{R1}\mu_L + v_{L2} v_{R2}\mu_R)}{u_1} \\
m_{\Delta 2}^2 \; & = \mu_D^2 + \frac{g_L^2}{4}(-u_1^2 + u_4^2 - v_{L1}^2 + v_{L2}^2) + \frac{g_R^2}{4}(-u_1^2 + u_4^2 - v_{R1}^2 + v_{R2}^2) + f_1^2(v_{L1}^2 + v_{R2}^2) \\
+ \; & \frac{A_1 v_{L1} v_{R2} - B_\Delta u_1 - f_2 v_{L2} v_{R1}\mu_D - f_1(4v_{L2} v_{R2}\mu_L + v_{L1} v_{R1}\mu_R)}{u_4}
\end{aligned}
$$

3.3 Higgs masses

The masses of all of the neutral Higgs are written in appendix B, needless to say the matrices are non trivial, but we are able to see that the following basis are decoupled from each other.

1. $\Re(\phi_{L1}), \Re(\phi_{L2}), \Re(\phi_{R1}), \Re(\phi_{R2}), \Re(\delta_{11}), \Re(\delta_{22})$
2. $\Im(\phi_{L1}), \Im(\phi_{L2}), \Im(\phi_{R1}), \Im(\phi_{R2}), \Im(\delta_{11}), \Im(\delta_{22})$
3. $\Re(\eta_{L1}), \Re(\eta_{L2}), \Re(s_3)$
4. $\Im(\eta_{L1}), \Im(\eta_{L2}), \Im(s_3)$
5. $\Re(\delta_{12}), \Re(\delta_{21})$
6. $\Im(\delta_{12}), \Im(\delta_{21})$
7. $\Re(\eta_{R1}), \Re(\eta_{R2})$
8. $\Im(\eta_{R1}), \Im(\eta_{R2})$

For the charged Higgs the decoupled basis are

1. $\phi^{\pm}_{L1}, \phi^{\pm}_{L2}, \delta^{\pm}_{11}, \delta^{\pm}_{22}$
2. $\phi^{\pm}_{R1}, \phi^{\pm}_{R2}, \delta^{\pm}_{12}, \delta^{\pm}_{21}$
3. $\eta^{\pm}_{L1}, \eta^{\pm}_{L2}, \eta^{\pm}_{R1}, \eta^{\pm}_{R2}, s^{\pm}_1, s^{\pm}_2$

Chapter 4

Gauge Sector

4.1 Lagrangian

The terms in the Lagrangian that include interactions with the gauge Bosons are

$$\mathcal{L}_{gauge} = (D_\mu \Phi)^\dagger)(D^\mu \Phi) + \overline{\Psi} \imath \gamma^\mu D_\mu \Psi - \frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} \tag{4.1}$$

Where $D_\mu = \partial_\mu - \imath(g/2)\sigma \cdot W_\mu - \imath g' Y B_\mu$, $F^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + \epsilon^{ijk} W^j_\mu W^k_\nu$ and $G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$.

4.2 Mass of the Gauge particles

The masses of the gauge Bosons can be calculated from the first term in $\mathcal{L}_{gauge}$ Explicitly,

$$\begin{aligned}
\mathcal{L} &= Tr|\left[\partial_\mu \Delta_1 - \frac{\imath g_L}{\sqrt{2}} \sigma_a W^a_{\mu L} \Delta_1 + \Delta_1 \frac{\imath g_R}{\sqrt{2}} \sigma_a W^a_{\mu R}\right]|^2 \\
&+ Tr|\left[\partial_\mu \Delta_2 - \frac{\imath g_L}{\sqrt{2}} \sigma_a W^a_{\mu L} \Delta_2 + \Delta_2 \frac{\imath g_R}{\sqrt{2}} \sigma_a W^a_{\mu R}\right]|^2 \\
&+ |\left(\partial_\mu - \frac{\imath g_L}{\sqrt{2}} \sigma_a W^a_{\mu L} - \frac{\imath g_1}{\sqrt{2}} B_\mu\right) \Phi_{L1}|^2 \\
&+ |\left(\partial_\mu - \frac{\imath g_L}{\sqrt{2}} \sigma_a W^a_{\mu L} + \frac{\imath g_1}{\sqrt{2}} B_\mu\right) \Phi_{L2}|^2 \\
&+ |\left(\partial_\mu - \frac{\imath g_R}{\sqrt{2}} \sigma_a W^a_{\mu R} - \frac{\imath g_1}{2} B_\mu\right) \Phi_{R1}|^2 \\
&+ |\left(\partial_\mu - \frac{\imath g_R}{\sqrt{2}} \sigma_a W^a_{\mu R} + \frac{\imath g_1}{\sqrt{2}} B_\mu\right) \Phi_{R2}|^2
\end{aligned} \tag{4.2}$$

The notation is obvious g_1 is the gauge coupling for $U(1)$, while g_L and g_R are the corresponding couplings for $SU(2)_L$ and $SU(2)_R$. Is important to keep in mind that giving to operation order of matrices, the bidoublets Δ_1 and Δ_2 operate to the left of W_R.

With the expectation values,

$$\begin{aligned}
\langle\Delta_1\rangle &= \begin{pmatrix} u_1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \langle\Delta_2\rangle = \begin{pmatrix} 0 & 0 \\ 0 & u_4 \end{pmatrix}, \\
\langle\phi_{L1}\rangle &= \begin{pmatrix} v_{L1} \\ 0 \end{pmatrix}, \quad \langle\phi_{L2}\rangle = \begin{pmatrix} 0 \\ v_{L2} \end{pmatrix}, \quad \langle\phi_{R1\rangle} = \begin{pmatrix} v_{R1} \\ 0 \end{pmatrix}, \quad \langle\phi_{R2}\rangle = \begin{pmatrix} 0 \\ v_{R2} \end{pmatrix}
\end{aligned} \tag{4.3}$$

we can write the Lagragian, explicitly,

$$\begin{aligned}
&\mathcal{L}_{Gauge-Mass} = \\
&Tr|-\frac{\imath g_L}{\sqrt{2}}\begin{pmatrix} W_L^3 & \sqrt{2}W_L^+ \\ \sqrt{2}W_L^- & -W_L^3 \end{pmatrix}\begin{pmatrix} u_1 & 0 \\ 0 & 0 \end{pmatrix} - \frac{\imath g_R}{\sqrt{2}}\begin{pmatrix} u_1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} W_R^3 & \sqrt{2}W_R^+ \\ \sqrt{2}W_R^- & -W_R^3 \end{pmatrix}|^2 \\
&+Tr|-\frac{\imath g_L}{\sqrt{2}}\begin{pmatrix} W_L^3 & \sqrt{2}W_L^+ \\ \sqrt{2}W_L^- & -W_L^3 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & u_4 \end{pmatrix} - \frac{\imath g_R}{\sqrt{2}}\begin{pmatrix} 0 & 0 \\ 0 & u_4 \end{pmatrix}\begin{pmatrix} W_R^3 & \sqrt{2}W_R^+ \\ \sqrt{2}W_R^- & -W_R^3 \end{pmatrix}|^2 \\
&+|-\frac{\imath g_L}{\sqrt{2}}\begin{pmatrix} W_L^3 & \sqrt{2}W_L^+ \\ \sqrt{2}W_L^- & -W_L^3 \end{pmatrix}\begin{pmatrix} v_{L1} \\ 0 \end{pmatrix} - \frac{\imath g_1}{\sqrt{2}}\begin{pmatrix} B & 0 \\ 0 & B \end{pmatrix}\begin{pmatrix} v_{L1} \\ 0 \end{pmatrix}|^2 \\
&+|-\frac{\imath g_L}{\sqrt{2}}\begin{pmatrix} W_L^3 & \sqrt{2}W_L^+ \\ \sqrt{2}W_L^- & -W_L^3 \end{pmatrix}\begin{pmatrix} 0 \\ v_{L2} \end{pmatrix} + \frac{\imath g_1}{\sqrt{2}}\begin{pmatrix} B & 0 \\ 0 & B \end{pmatrix}\begin{pmatrix} 0 \\ v_{L2} \end{pmatrix}|^2 \\
&+|-\frac{\imath g_R}{\sqrt{2}}\begin{pmatrix} W_R^3 & \sqrt{2}W_R^+ \\ \sqrt{2}W_R^- & -W_R^3 \end{pmatrix}\begin{pmatrix} v_{R1} \\ 0 \end{pmatrix} - \frac{\imath g_1}{\sqrt{2}}\begin{pmatrix} B & 0 \\ 0 & B \end{pmatrix}\begin{pmatrix} v_{R1} \\ 0 \end{pmatrix}|^2 \\
&+|-\frac{\imath g_R}{\sqrt{2}}\begin{pmatrix} W_R^3 & \sqrt{2}W_R^+ \\ \sqrt{2}W_R^- & -W_R^3 \end{pmatrix}\begin{pmatrix} 0 \\ v_{R2} \end{pmatrix} + \frac{\imath g_1}{\sqrt{2}}\begin{pmatrix} B & 0 \\ 0 & B \end{pmatrix}\begin{pmatrix} 0 \\ v_{R2} \end{pmatrix}|^2
\end{aligned} \tag{4.4}$$

$$
\begin{aligned}
= \; & \frac{u_1^2}{2}[g_L^2((W_L^3)^2 + 2W_L^- W_L^+) + g_L g_R (W_L^3 W_R^3) + g_R^2((W_R^3)^2 + 2W_R^- W_R^+) \\
+ \; & \frac{u_4^2}{2}[g_L^2((W_L^3)^2 + 2W_L^- W_L^+) + g_L g_R (W_L^3 W_R^3) + g_R^2((W_R^3)^2 + 2W_R^- W_R^+) \\
+ \; & \frac{v_{L1}^2}{2}[g_L^2((W_L^3)^2 + 2W_L^- W_L^+) + g_L g_1 (W_L^3 B) + g_1^2((B)^2) \\
+ \; & \frac{v_{L2}^2}{2}[g_L^2((W_L^3)^2 + 2W_L^- W_L^+) + g_L g_1 (W_L^3 B) + g_1^2((B)^2) \\
+ \; & \frac{v_{R1}^2}{2}[g_R^2((W_R^3)^2 + 2W_R^- W_R^+) + g_R g_1 (W_R^3 B) + g_1^2((B)^2) \\
+ \; & \frac{v_{R2}^2}{2}[g_R^2((W_R^3)^2 + 2W_R^- W_R^+) + g_R g_1 (W_R^3 B) + g_1^2((B)^2)
\end{aligned}
$$

We will write the mass Lagrangian in matrix notation for that we will use the bases, $G = \{W_L^0, W_R^0, B\}$ and $W^\pm = \{W_L^\pm, W_R^\pm\}$, for the neutral and charge Bosons

$$
\mathcal{L}_{Gauge-Mass} = G M_G^2 G + W^+ M_W^2 W^- \tag{4.5}
$$

The mass matrices are

$$
M_W^2 = \begin{pmatrix} \frac{g_L^2}{2}(u^2 + v_L^2) & 0 \\ 0 & \frac{g_R^2}{2}(u^2 + v_R^2) \end{pmatrix} \quad , \quad M_G^2 = \begin{pmatrix} \frac{g_L^2}{2}(u^2 + v_L^2) & \frac{g_L g_R}{2} u^2 & \frac{g_1 g_L}{2} v_L^2 \\ \frac{g_L g_R}{2} u^2 & \frac{g_R^2}{2}(u^2 + v_R^2) & \frac{g_1 g_R}{2} v_R^2 \\ \frac{g_1 g_L}{2} v_L^2 & \frac{g_1 g_R}{2} v_R^2 & \frac{g_1^2}{2}(v_L^2 + v_R^2) \end{pmatrix} \tag{4.6}
$$

Where we have defined the sum square of the vevs $v_L^2 = v_{L1}^2 + v_{L2}^2$, $v_R^2 = v_{R1}^2 + v_{R2}^2$ and $u = u_1^2 + u_4^2$. The mass for the neutral charged Bosons is already diagonal and represents physical states. For the neutral Bosons, when need to have a rotation to a physical basis A, Z, Z'.

Using the transformation $G = U^{-1} G'$

$$
\begin{pmatrix} W_{\mu L}^0 \\ W_{\mu R}^0 \\ B_\mu \end{pmatrix} = \begin{pmatrix} s_L & c_L & 0 \\ s_R & -\frac{s_L s_R}{c_L} & \frac{\sqrt{c_L^2 - s_R^2}}{c_L} \\ \sqrt{c_L^2 - s_R^2} & -\frac{s_L}{c_L}\sqrt{c_L^2 - s_R^2} & -\frac{s_R}{c_L} \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \end{pmatrix} \tag{4.7}
$$

Where $s_L = \sin\theta_L$, $s_R = \sin\theta_R$ and $c_L = \cos\theta_L$, are related to the couplings as $g_L = e/s_L$, $g_R = e/s_R$ and $g_1 = e/\sqrt{c_L^2 - s_R^2}$.

The mass matrix after the transformation is,

$$M'^2 = UM_G^2U^{-1} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{g_L^2(u^2+v_L^2)}{2c_L^2} & \frac{g_Lg_R((s_R^2-c_L^2)u^2+s_R^2v_L^2)}{4c_L^2\sqrt{c_L^2-s_R^2}} \\ 0 & \frac{g_Lg_R^2((s_R^2-c_L^2)u^2+s_R^2v_L^2)}{4c_L^2\sqrt{c_L^2-s_R^2}} & -\frac{g_R^2((s_R^2-c_L^2)^2u^2+s_R^4v_L^2+c_L^4v_R^2)}{2c_L^2(s_R^2-c_L^2)} \end{pmatrix} \tag{4.8}$$

The mass of the photon is zero, and the Z Boson is mixed with a new Z' Boson. Since experimentally this mixture has not been found [37]. We'll assume no Z-Z' mixing, that can be accomplished by requiring the quantity $M_{ZZ'}/M_{Z'}^2$ to be very small. Or for simplicity, the off diagonal ($M_{ZZ'}$) terms should be equal to zero, this assumption leads to the the condition:

$$u^2 = \left(\frac{s_R^2}{c_L^2 - s_R^2}\right) v_L^2 \tag{4.9}$$

Using the no Z-Z' mixing condition, the masses of the all the gauge Bosons can be calculated.

$$\begin{aligned} M_Z^2 &= \frac{g_L^2}{2c_L^2}(u^2 + v_L^2) = \frac{g_L^2 v_L^2}{2(c_L^2 - s_R^2)} && (4.10) \\ M_{W_L}^2 &= \frac{g_L^2}{2}(u^2 + v_L^2) = \frac{g_L^2 c_L^2 v_L^2}{2(c_L^2 - s_R^2)} = M_Z^2 c_L^2 && (4.11) \\ M_{Z'}^2 &= -\frac{g_R^2((s_R^2 - c_L^2)^2u^2 + s_R^4 v_L^2 + c_L^4 v_R^2)}{2c_L^2(s_R^2 - c_L^2)} && (4.12) \\ &= M_Z^2 s_L^2 \left(1 + \left(\frac{c_L}{s_R}\right)^2 \left(\frac{v_R}{v_L}\right)^2\right) \\ M_{W_R}^2 &= \frac{g_R^2}{2}(u^2 + v_R^2) = M_Z^2 s_L^2 \left(1 + \left(\frac{c_L^2 - s_R^2}{s_R^2}\right)\left(\frac{v_R}{v_L}\right)^2\right) && (4.13) \end{aligned}$$

The mass of the Z and W_L Bosons are part of the SM and have been experimentally measured (CITE), while searches for Z' and W_R have put bounds on the their possible masses.

4.3 Gauge Bosons interactions with Fermions

The interaction between a generic Fermion doublet $X = (X_1, X_2)$ and the gauge Bosons is given by

$$
\begin{aligned}
\mathcal{L} &= \imath X^\dagger \gamma^\mu D_\mu X \qquad (4.14)\\
&= (\overline{X}_{1i}, \overline{X}_{2i}) \begin{pmatrix} \imath\partial_\mu + \frac{1}{2} g_i W^0_{\mu i} + g_1 Y_X B_\mu & \frac{1}{\sqrt{2}}(g_i W^+_{\mu i}) \\ \frac{1}{\sqrt{2}}(g_i W^-_{\mu i}) & \imath\partial_\mu - \frac{1}{2} g_i W^0_{\mu i} + g_1 Y_X B_\mu \end{pmatrix} \begin{pmatrix} X_{1i} \\ X_{2i} \end{pmatrix}\\
&= \overline{X_1}\gamma^\mu(\imath\partial_\mu + \frac{g_i}{2} W^0_{\mu i} + g_1 Y_X B_\mu) X_1 + \overline{X_2}\gamma^\mu(\imath\partial_\mu - \frac{g_i}{2} W^0_{\mu i} + g_1 Y_X B_\mu) X_2\\
&\quad + \frac{1}{\sqrt{2}} \overline{X_1}\gamma^\mu (g_i W^+_{\mu i}) X_2 + \frac{1}{\sqrt{2}} \overline{X_2}\gamma^\mu (g_i W^+_{\mu i}) X_1
\end{aligned}
$$

The chirality of the Fermions is given by $i = L, R$. Using the transformations in eq.(4.7), we find the interactions between Fermions and the physical neutral gauge Bosons.

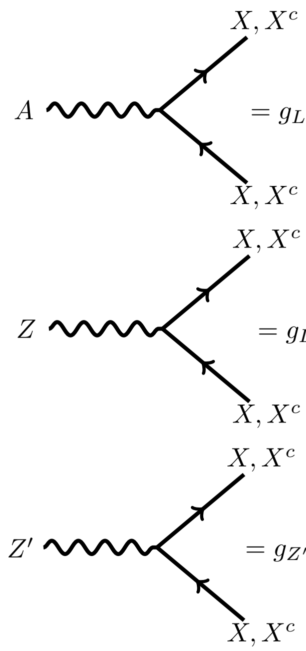
$$
\begin{aligned}
\mathcal{L} = (\overline{X}\gamma^\mu X) \quad & [\quad A_\mu g_L s_L \left[I_{3L} + I_{3R} + Y_X \right] \qquad (4.15)\\
+ \quad & [\quad Z_\mu g_L \left[c_L I_{3L} - \frac{s_R^2}{c_L} I_{3R} - \frac{s_R^2}{c_L} Y_X \right]]\\
+ \quad & [\quad Z'_\mu \frac{g_R}{c_L\sqrt{c_L^2 - s_R^2}} \left[I_{3R}(c_L^2 - s_R^2) - s_R^2 Y_X \right]]
\end{aligned}
$$

We define the currents

$$
\begin{aligned}
J_{W^+_{(L/R)}} &= \frac{g_{(L/R)}}{\sqrt{2}} (\overline{X_1}\gamma^\mu X_2) \qquad (4.16)\\
J_A &= eQ(\overline{X}\gamma^\mu X) = g_L s_L Q(\overline{X}\gamma^\mu X)\\
J_Z &= \frac{g_L}{c_L}(I_{3L} - s_L^2 Q)(\overline{X}\gamma^\mu X) \qquad (4.17)\\
J_{Z'} &= g_{Z'} \left[(c_L^2 - s_R^2) I_{3R} - s_R^2 Y_X \right] (\overline{X}\gamma^\mu X) \qquad (4.18)\\
&= g_{Z'}(c_L^2 I_{3R} + s_R^2 I_{3L} - s_R^2 Q)(\overline{X}\gamma^\mu X)
\end{aligned}
$$

Where $g_{Z'} = \frac{e}{c_L s_R \sqrt{c_L^2 - s_R^2}}$ and $Y_X = Q - I_{3L} - I_{3R}$.

A quick examination of the currents shows that $W^\pm$ connect two Fermions of the same doublet, while A, Z and Z' interact with a Fermion and antifermion of the same generation. Using these currents we can calculate the couplings for some of the relevant particles of the model as seen in table 4.1, and the Feynman rules in Fig. 4.1.



$$= g_L s_L \left[I_{3L} + I_{3R} + Y_X\right] \gamma^\mu \tfrac{(1\pm\gamma_5)}{2}$$

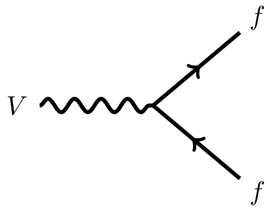
$$= g_L c_L \left[I_{3L} - \left(\tfrac{s_R}{c_L}\right)^2 (I_{3R} - Y_X)\right] \gamma^\mu \tfrac{(1\pm\gamma_5)}{2}$$

$$= g_{Z'} \left[I_{3R}(c_L^2 - s_R^2) - s_R^2 Y_X\right] \gamma^\mu \tfrac{(1\pm\gamma_5)}{2}$$

Figure 4.1: Feynman rules for interactions of Fermions and Bosons. For X we use $\frac{1-\gamma_5}{2}$ and for X^c we use $\frac{1+\gamma_5}{2}$

Fermion	I_{3L}	I_{3R}	Y_X	Coupling to $A_\mu(g_L)$	Coupling to $Z_\mu(g_L)$	Coupling to $Z'_\mu(g_{Z'})$
u	1/2	0	1/6	$\frac{2s_L}{3}$	$\frac{c_L}{2} + \frac{s_R^2}{6c_L}$	$-\frac{s_R^2}{6}$
u^c	0	-1/2	-1/6	-$\frac{2s_L}{3}$	$\frac{s_R^2}{3c_L}$	$-\frac{c_L^2}{2} + \frac{2s_R^2}{3}$
d	-1/2	0	1/6	-$\frac{s_L}{3}$	$-\frac{c_L}{2} + \frac{s_R^2}{3c_L}$	$-\frac{s_R^2}{6}$
d^c	0	0	1/3	$\frac{s_L}{3}$	$\frac{s_R^2}{3c_L}$	$-\frac{s_R^2}{3}$
h	0	0	-1/3	$-\frac{s_L}{3}$	-$\frac{s_R^2}{3c_L}$	$\frac{s_R^2}{3}$
h^c	0	1/2	-1/6	$\frac{s_L}{3}$	$-\frac{2s_R^2}{3c_L}$	$\frac{c_L^2}{2} - \frac{s_R^2}{3}$
e	-1/2	0	-1/2	$-s_L$	$-\frac{c_L}{2} - \frac{s_R^2}{2c_L}$	$\frac{s_R^2}{2}$
e^c	0	1/2	1/2	s_L	0	$\frac{c_L^2}{2} - s_R^2$
n	0	0	0	0	0	0
n^c	0	-1/2	1/2	0	$\frac{s_R^2}{2c_L}$	-$\frac{c_L^2}{2}$
ν	1/2	0	-1/2	0	$\frac{c_L}{2} - \frac{s_R^2}{2c_L}$	$\frac{s_R^2}{2}$
$\tilde{\eta}^0_{R1}$	0	1/2	-1/2	0	-$\frac{s_R^2}{c_L}$	$\frac{c_L^2}{2}$
$\tilde{\eta}^0_{R2}$	0	-1/2	1/2	0	$\frac{s_R^2}{c_L}$	-$\frac{c_L^2}{2}$

Table 4.1: Fermion couplings to gauge Bosons



$$= g\gamma^{\mu}(c_v - c_a\gamma_5)$$

Figure 4.2: V-A Fermion-Boson couplings

We can rewrite the current in the usual V-A structure, with the new couplings shown in table 4.2

$$\begin{aligned} A\overline{X}\gamma^{\mu}X + B\overline{X}^{c}\gamma^{\mu}X^{c} &= A\overline{X}\left(\frac{1+\gamma^5}{2}\right)\gamma^{\mu}\left(\frac{1-\gamma^5}{2}\right)X + B\overline{X^{c}}\left(\frac{1+\gamma^5}{2}\right)\gamma^{\mu}\left(\frac{1-\gamma^5}{2}\right)X^{c} \\ &= A\overline{X}\gamma^{\mu}\left(\frac{1-\gamma^5}{2}\right)X + B\overline{X^{c}}\gamma^{\mu}\left(\frac{1-\gamma^5}{2}\right)X^{c} \\ &= A\overline{X}\gamma^{\mu}\left(\frac{1-\gamma^5}{2}\right)X - B\overline{X}\gamma^{\mu}\left(\frac{1+\gamma^5}{2}\right)X \\ &= \overline{X}\gamma^{\mu}\left(\frac{A+B}{2} - \frac{A-B}{2}\gamma^5\right)X \\ &= \overline{X}\gamma^{\mu}\left(c_v - c_a\gamma^5\right)X \end{aligned} \tag{4.19}$$

Where we have used the properties of charged conjugated fields $\overline{X^c}\gamma^{\mu}X^c = -\overline{X}\gamma^{\mu}X$ and $\overline{X^c}\gamma^{\mu}\gamma_5X^c = \overline{X}\gamma^{\mu}\gamma_5X$

Fermion	c_a^A	c_v^Z	c_a^Z	$c_v^{Z'}$	$c_a^{Z'}$
u	$\frac{2s_L}{3}$	$\frac{c_L}{4} + \frac{s_R^2}{4c_L}$	$\frac{c_L}{4} - \frac{s_R^2}{12c_L}$	$\frac{s_R^2-c_L^2}{4}$	$\frac{-5s_R^2+3c_L^2}{12}$
d	$-\frac{s_L}{3}$	$-\frac{c_L}{4} + \frac{s_R^2}{4c_L}$	$-\frac{c_L}{4} - \frac{s_R^2}{12c_L}$	$\frac{-3s_R^2}{12}$	$\frac{s_R^2}{12}$
h	$-\frac{s_L}{3}$	$-\frac{s_R^2}{2c_L}$	$\frac{s_R^2}{6c_L}$	$\frac{c_L^2}{4}$	$\frac{4s_R^2-3c_L^2}{12}$
e	$-s_L$	$-\frac{c_L}{4} - \frac{s_R^2}{4c_L}$	$-\frac{c_L}{4} - \frac{s_R^2}{4c_L}$	$\frac{-s_R^2+c_L^2}{4}$	$\frac{3s_R^2-c_L^2}{4}$
n	0	$\frac{s_R^2}{2c_L}$	$\frac{s_R^2}{2c_L}$	$\frac{-c_L^2}{4}$	$\frac{c_L^2}{4}$
ν	0	$\frac{c_L}{4} - \frac{s_R^2}{4c_L}$	$\frac{c_L}{4} - \frac{s_R^2}{4c_L}$	$\frac{s_R^2}{4}$	$\frac{s_R^2}{4}$
$\tilde{\eta}^0$	0	$-\frac{s_R^2}{2c_L}$	$\frac{s_R^2}{2c_L}$	$\frac{c_L^2}{4}$	$\frac{c_L^2}{4}$

Table 4.2: V-A couplings to gauge Bosons

For A_μ and Z_μ, $g = g_L = e/s_L$. For Z'_μ, $g = g_{Z'} = e/(c_L s_R\sqrt{c_L^2 - s_R^2})$

4.4 Bound on the Z' Mass

We want to evaluate the bound on the mass of Z' in our model from LHC data. The data is shown in the left side of Fig. 4.3, the figure is from ATLAS [38], with $E_{CM} = 8$ TeV and integrated luminosity of 20

fb^{-1}, where the bound was obtained for producing Z' and subsequent decays to $e^{\pm}e^{\mp}$ for some popular Z' models. For example, The bound on SSM (the phenomenological Z' model with SM coupling), the Z' mass has been cross-checked to be around 2.8 TeV.

We use eq. (4.18) to find the current that Z' couples with $e^{\pm}e^{\mp}$, and assume the unification requirements of the model(we will examine in chapter 7), $g_L = g_R$ which implies, $\sin\theta_R = \sin\theta_L \equiv \sin\theta_W$.

We use event generator `CalcHEP` [39] for calculating the cross-section and use `CTEQ6L` parton distribution function [40]. Cuts on the electron $p_T > 40$ GeV and pseudorapidity $|\eta| < 2.47$ have been employed to obtain the signal in our model. On the right hand side of Fig. 4.3 we show our model cross-sections in blue and the bound from LHC data in red, as seen in the LHS of the figure. We obtain the bound on the mass of Z', $M_{Z'} = 2.045$ TeV $\simeq 2$ TeV.

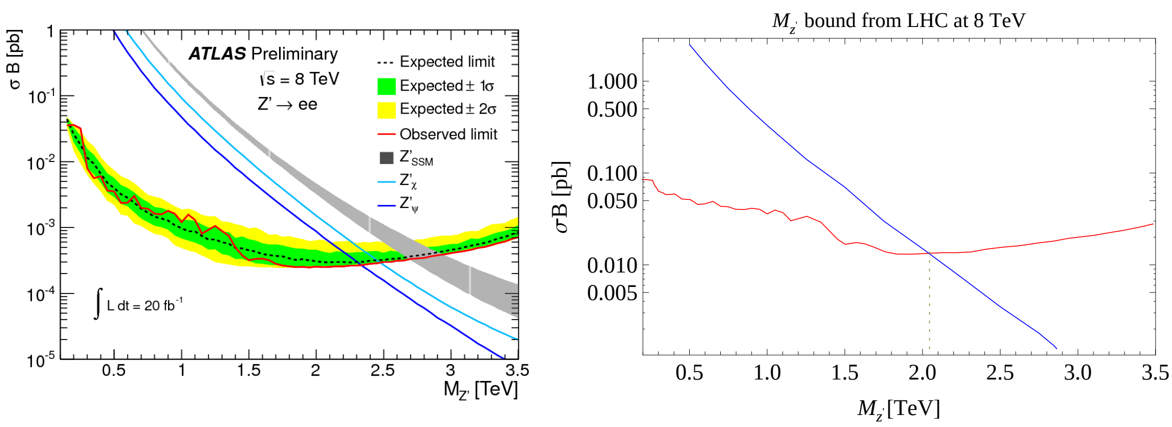


Figure 4.3: LHS: Bound on different Z' masses at LHC from ATLAS with $E_{CM} = 8$ TeV and integrated luminosity of 20 fb^{-1}. RHS: The limit is exploited to determine the bound on the Z' mass of this model.

Defining the ratio between the right and left vev,

$$r = \frac{v_R}{v_L} \tag{4.20}$$

And using the gauge coupling the unification condition, eqs. (4.10-4.13) for the masses of the gauge Bosons can be rewritten,

$$
\begin{aligned}
M_Z &= \frac{g_L v_L}{\sqrt{2(1-2\sin^2\theta_W)}} \qquad (4.21)\\
M_{Z'} &= M_Z\sqrt{\sin^2\theta_W + r^2\cos^2\theta_W}\\
M_{W_R} &= M_Z\sqrt{\sin^2\theta_W + r^2(\cos^2\theta_W - \sin^2\theta_W)}
\end{aligned}
$$

In Fig. 4.4, we show the linear dependence of the Z' and W_R mass on the ratio of Higgs vacuum expectation values r following eq. 4.22. We note that mass of Z' is bigger than W_R for for all values of $M_Z' \geq 30$ GeV. The bound on $M_Z' \geq 2$ TeV from LHC eventually put a bound on $r \geq 25$ as shown.

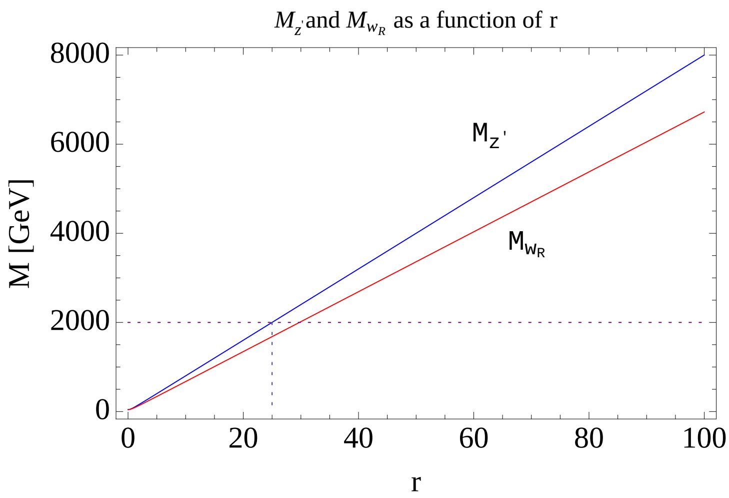


Figure 4.4: Linear dependence of $M_{Z'}$ (Blue) and M_{W_R} (Red) on the ratio of Higgs vevs r as defined in eq. 4.20. A horizontal dotted line indicates the bound from LHC on Z' mass at 2 TeV.

4.4.1 Numerical values for the vevs

After the calculations in this chapter, we are in liberty to assign numerical values to the vevs in the model by using the no Z-Z' condition, eq.(4.9), the mass of the Z, eq.(4.10) (SM vev is related to our model by $v_{SM} = \sqrt{u^2 + v_L^2}$), and the bound on the higgs ratio r , eq. (4.20). In Chapter 7 will be seen that to have GUT scale unification, the condition $g_L = g_R$ ($s_R^2 = s_L^2 \equiv \sin\theta_W \simeq 0.23$) is needed. The values are:

$$
\begin{aligned}
v_L &= \frac{\sqrt{(1-2\sin^2\theta_W)}}{\sqrt{1-\sin^2\theta_W}} v_{SM} \simeq 0.837 * v_{SM} \simeq 145 \text{ GeV} \qquad (4.22)\\
u &= \sqrt{\left(\frac{\sin^2\theta_W}{1-2\sin^2\theta_W}\right)} v_L \simeq 0.653 * v_L \simeq 95 \text{ GeV}\\
v_R &\geq r * v_L = 25 * v_L \simeq 3625 \text{ GeV}
\end{aligned}
$$

Chapter 5

Neutralino and Chargino sector

5.1 Introduction

The neutralinos are a linear combination of the neutral Higgsinos and the gauginos, the first ones being the superpartner of the neutral Higgs and the second ones being superpartners of the gauge Bosons, neutralinos are then neutral Fermions. We can have a quick count; for every SU(2) or U(1) gauge symmetry on the model there will be one neutralino (called "wino" and "bino" respectively) and for every neutral Higgs there will be another neutralino. In the simplest case (The MSSM) there will be 4 neutralinos (2 gauginos plus 2 higgsinos). In the eLRSUSY, then there will be 3 neutral gauginos and 13 neutral Higgsinos, giving a total of 16 neutralinos. As it will be shown in this chapter the Higgsinos and the gauginos will have non trivial mixing. The charginos are a linear combination of the charged Higgsinos and the gauginos. There will not be a chargino for a $U(1)$ since the B Boson is neutral, but there will be two charged winos for each $SU(2)$ (a positive and a negative).

Given the tools, computational time and energy that has been used on the MSSM, we will like to check if eLRSUSY an extension of the MSSM, in section 5.3 we will address this question.

In particular we are interested in the lightest neutralino; is a DM candidate in our model and will be examined in part III of this thesis.

5.2 Neutralino massses

The soft breaking term will give the gauginos SUSY masses, from $\mathcal{L}_{scalar-gaugino-fermion} = \imath\sqrt{2}g\phi_i^*\lambda_a\psi_i$ we can extract, after symmetry breaking, the interactions of the neutralinos that will give mixing between the gauginos and the Higgsinos, while term $\mathcal{L}_{fermion-scalar} - = \frac{\partial^2 W}{\partial\phi_i\partial\phi_j}\Psi_i\Psi_j$ will give mixing between different Higgsinos that will be proportional to the μ and f_i parameters.

The neutral gauginos/Higgsinos will have the following bases and matrices basis

$$
\begin{aligned}
\left(\tilde{\Psi_{N1}}^{0}\right)^{T} &= \{\tilde{B}, \tilde{W_L}, \tilde{\phi_{L1}}, \tilde{\phi_{L2}}, \tilde{W_R}, \tilde{\delta_{11}}, \tilde{\delta_{22}}, \tilde{\phi_{R1}}, \tilde{\phi_{R2}}\} \\
\left(\tilde{\Psi_{N2}}^{0}\right)^{T} &= \{\tilde{\eta^0_{L1}}, \tilde{\eta^0_{L2}}, \tilde{s^0_3}\} \\
\left(\tilde{\Psi_{N3}}^{0}\right)^{T} &= \{\tilde{\eta^0_{R1}}, \tilde{\eta^0_{R2}}\} \\
\left(\tilde{\Psi_{N4}}^{0}\right)^{T} &= \{\tilde{\delta^0_{12}}, \tilde{\delta^0_{21}}\}
\end{aligned}
$$

$$
M_{N1} = \left(\begin{array}{cccc|ccccc}
M_B & 0 & -\frac{g_1 v_{L1}}{\sqrt{2}} & \frac{g_1 v_{L2}}{\sqrt{2}} & 0 & 0 & 0 & -\frac{g_1 v_{R1}}{\sqrt{2}} & \frac{g_1 v_{R2}}{\sqrt{2}} \\
0 & M_L & \frac{g_L v_{L1}}{\sqrt{2}} & -\frac{g_L v_{L2}}{\sqrt{2}} & 0 & -\frac{g_L u_1}{\sqrt{2}} & \frac{g_L u_4}{\sqrt{2}} & 0 & 0 \\
-\frac{g_1 v_{L1}}{\sqrt{2}} & \frac{g_L v_{L1}}{\sqrt{2}} & 0 & -\mu_L & 0 & 0 & \frac{f_1 v_{R2}}{2} & 0 & \frac{f_1 u_4}{2} \\
\frac{g_1 v_{L2}}{\sqrt{2}} & -\frac{g_L v_{L2}}{\sqrt{2}} & -\mu_L & 0 & 0 & \frac{f_2 v_{R1}}{2} & 0 & \frac{f_2 u_1}{2} & 0 \\
\hline
0 & 0 & 0 & 0 & M_R & -\frac{g_R u_1}{\sqrt{2}} & \frac{g_R u_4}{\sqrt{2}} & \frac{g_R v_{R1}}{\sqrt{2}} & -\frac{g_R v_{R2}}{\sqrt{2}} \\
0 & -\frac{g_L u_1}{\sqrt{2}} & 0 & \frac{f_2 v_{R1}}{2} & -\frac{g_R u_1}{\sqrt{2}} & 0 & -\mu_\Delta & \frac{f_2 v_{L2}}{2} & 0 \\
0 & \frac{g_R u_4}{\sqrt{2}} & \frac{f_1 v_{R2}}{2} & 0 & \frac{g_R u_4}{\sqrt{2}} & -\mu_\Delta & 0 & 0 & \frac{f_1 v_{L1}}{2} \\
-\frac{g_1 v_{R1}}{\sqrt{2}} & 0 & 0 & \frac{f_2 u_1}{2} & \frac{g_R v_{R1}}{\sqrt{2}} & \frac{f_2 v_{L2}}{2} & 0 & 0 & -\mu_R \\
\frac{g_1 v_{R2}}{\sqrt{2}} & 0 & \frac{f_1 u_4}{2} & 0 & -\frac{g_R v_{R2}}{\sqrt{2}} & 0 & \frac{f_1 v_{L1}}{2} & -\mu_R & 0
\end{array}\right) \tag{5.1}
$$

$$
M_{N2} = \begin{pmatrix} 0 & -\mu_{L2} & f_{10} v_{L2} \\ -\mu_{L2} & 0 & f_9 v_{L1} \\ f_{10} v_{L2} & f_9 v_{L1} & -\mu_{s3} \end{pmatrix}, \quad M_{N3} = \begin{pmatrix} 0 & -\mu_{R2} \\ -\mu_{R2} & 0 \end{pmatrix}, \quad M_{N4} = \begin{pmatrix} 0 & -\mu_\Delta \\ -\mu_\Delta & 0 \end{pmatrix} \tag{5.2}
$$

The horizontal and vertical lines in M_{N1} are there to make the distinction that the upper left 4×4 matrix looks like the MSSM neutralino mass matrix. Although, it's important to keep in mind , that while, $\tilde{W^0_L}$ is the same as the MSSM Wino (i.e, the $SU(2)_L$ gaugino), $\tilde{B}$ is not the MSSM Bino. In this base $\tilde{B}$ is the superpartner of the $SU(2)_R \times U(1)_X$ Gauge boson , and not just $U(1)_Y$. Also the vevs v_{L1} and v_{L2} are not the same as in the MSSM, as explain in chapter 4.

5.3 Is eLRSUSY an extension of the MSSM?

We would like to write matrix (5.1) in a basis that would allow it to be written as

$$M_n = \left(\begin{array}{c|c} M_{MSSM} & G \\ (4\times 4) & (4\times 5) \\ \hline G & M_{heavy} \\ (5\times 4) & (5\times 5) \end{array}\right) \tag{5.3}$$

Where M_{MSSM} the neutralino mass matrix of the MSSM, M_{heavy} contains the masses of the neutralinos of the "extended" part, and G is a matrix with small parameters (compared to the ones in M_{heavy}), this will allow us to claim that the matrix M_{MSSM} is decoupled from M_{heavy} (up to a small perturbation given by the parameters in G).

The method consist in two parts, first we will rotate the basis to a completely off diagonal Higgsino matrix, secondly we will rotate the basis of the bino and the right wino, in this new basis we will have a $U(1)$ bino that is the same as the MSSM bino.

5.3.1 Higgsinos and μ matrix

In the MSSM, if we only look at the higgsino mass matrix, it has a structure that looks like

$$M_\mu = \left(\begin{array}{c|c} 0 & -\mu \\ \hline -\mu^T & 0 \end{array}\right) \tag{5.4}$$

If eLRSUSY is an extension of the MSSM, then it would be required to have "μ mass matrix" to looks like this matrix. First, we will write the Higgsino mass matrix of (5.1). The basis will be $(\Psi_{\tilde{H}})^T = \{\phi^{\tilde{0}}_{L1}, \phi^{\tilde{0}}_{L2}, \delta^{\tilde{0}}_{11}, \delta^{\tilde{0}}_{22}\phi^{\tilde{0}}_{R1}, \phi^{\tilde{0}}_{R2}\}$,

$$M_{\tilde{H}} = \left(\begin{array}{cc|cc|cc} 0 & -\mu_L & 0 & \frac{f_1 v_{R2}}{2} & 0 & \frac{f_1 u_4}{2} \\ -\mu_L & 0 & \frac{f_2 v_{R1}}{2} & 0 & \frac{f_2 u_1}{2} & 0 \\ \hline 0 & \frac{f_2 v_{R1}}{2} & 0 & -\mu_\Delta & \frac{f_2 v_{L2}}{2} & 0 \\ \frac{f_1 v_{R2}}{2} & 0 & -\mu_\Delta & 0 & 0 & \frac{f_1 v_{L1}}{2} \\ \hline 0 & \frac{f_2 u_1}{2} & \frac{f_2 v_{L2}}{2} & 0 & 0 & -\mu_R \\ \frac{f_1 u_4}{2} & 0 & 0 & \frac{f_1 v_{L1}}{2} & -\mu_R & 0 \end{array}\right) \tag{5.5}$$

We rearrange the fields $\{\phi^{\tilde{0}}_{L1}, \phi^{\tilde{0}}_{L2}, \delta^{\tilde{0}}_{11}, \delta^{\tilde{0}}_{22}\phi^{\tilde{0}}_{R1}, \phi^{\tilde{0}}_{R2}\} \to \{\phi^{\tilde{0}}_{L1}, \delta^{\tilde{0}}_{11}, \phi^{\tilde{0}}_{L2}, \delta^{\tilde{0}}_{22}\phi^{\tilde{0}}_{R1}, \phi^{\tilde{0}}_{R2}\}$, the matrix changes to,

$$
M'_{\tilde{H}} = \left(\begin{array}{cc|cc|cc}
0 & 0 & -\mu_L & \frac{f_1 v_{R2}}{2} & 0 & \frac{f_1 u_4}{2} \\
0 & 0 & \frac{f_2 v_{R1}}{2} & -\mu_\Delta & \frac{f_2 v_{L2}}{2} & \\
\hline
-\mu_L & \frac{f_2 v_{R1}}{2} & 0 & 0 & \frac{f_2 u_4}{2} & 0 \\
\frac{f_1 v_{R2}}{2} & -\mu_\Delta & 0 & 0 & 0 & \frac{f_1 v_{L1}}{2} \\
\hline
0 & \frac{f_2 v_{L2}}{2} & \frac{f_2 u_1}{2} & 0 & 0 & -\mu_R \\
\frac{f_1 u_4}{2} & 0 & 0 & \frac{f_1 v_{L1}}{2} & -\mu_R & 0
\end{array}\right) \tag{5.6}
$$

μ_R is much bigger than the vevs v_{L1}, v_{L2}, u_1 and u_4, this tells us the right Higgsinos are already decoupled from this matrix (up to a small perturbation); The right Higgsinos already have the structure we are looking for. In the basis $(\Psi''_{\tilde{H}})^T = \{\tilde{\phi^0_{L1}}, \tilde{\delta^0_{11}}, \tilde{\phi^0_{L2}}, \tilde{\delta^0_{11}}\}$, the 4×4 remaining matrix is

$$
M''_{\tilde{H}} = \left(\begin{array}{cc|cc}
0 & 0 & -\mu_L & \frac{f_1 v_{R2}}{2} \\
0 & 0 & \frac{f_2 v_{R1}}{2} & -\mu_\Delta \\
\hline
-\mu_L & \frac{f_2 v_{R1}}{2} & 0 & 0 \\
\frac{f_1 v_{R2}}{2} & -\mu_\Delta & 0 & 0
\end{array}\right) \tag{5.7}
$$

Matrix (5.7) already has the structure of matrix (5.4); A linear combinations of $\tilde{\phi^0_{L1}}$ with $\tilde{\delta^0_{11}}$, and $\tilde{\phi^0_{L2}}$ with $\tilde{\delta^0_{11}}$, will give the answer we are looking for. To find this linear combination will be necessary to rotate the 2×2 non zero matrix above, to a new base where the diagonal terms are zero. The rotated non diagonal terms will be then the desired μ parameter.
Explicilty the "μ Lagrangian" for matrix (5.7) is

$$
\mathcal{L}_\mu = \tilde{\Phi_1} \mu \tilde{\Phi_2} + \tilde{\Phi_2} \mu^T \tilde{\Phi_1} \tag{5.8}
$$

Where, $\tilde{\Phi_1} = \{\tilde{\phi^0_{L1}}, \tilde{\delta^0_{11}}\}$ and $\tilde{\Phi_2} = \{\tilde{\phi^0_{L2}}, \tilde{\delta^0_{11}}\}$.
To be able to make the necessary rotation to an off diagonal matrix, we need two unitary matrices (since the μ is not a symmetric matrix).
Explicitly

$$\tilde{\Phi_1}' = U_1\tilde{\Phi_1} \rightarrow \begin{pmatrix} \phi_{11}^{\tilde{0}} \\ \phi_{12}^{\tilde{0}} \end{pmatrix} = \begin{pmatrix} \cos[\theta_1] & \sin[\theta_1] \\ -\sin[\theta_1] & \cos[\theta_1] \end{pmatrix} \begin{pmatrix} \phi_{L1}^{\tilde{0}} \\ \delta_{11}^{\tilde{0}} \end{pmatrix} \tag{5.9}$$

$$\tilde{\Phi_2}' = U_2\tilde{\Phi_2} \rightarrow \begin{pmatrix} \phi_{21}^{\tilde{0}} \\ \phi_{22}^{\tilde{0}} \end{pmatrix} = \begin{pmatrix} \cos[\theta_2] & \sin[\theta_2] \\ -\sin[\theta_2] & \cos[\theta_2] \end{pmatrix} \begin{pmatrix} \phi_{L2}^{\tilde{0}} \\ \delta_{22}^{\tilde{0}} \end{pmatrix} \tag{5.10}$$

and (with $x = f_1 v_{R2}/2$, $y = f_2 v_{R1}/2$)

$$\mu' = U_1 \mu U_2^{-1} \rightarrow \begin{pmatrix} 0 & -\mu_1 \\ -\mu_2 & 0 \end{pmatrix} = \begin{pmatrix} \cos[\theta_1] & -\sin[\theta_1] \\ \sin[\theta_1] & \cos[\theta_1] \end{pmatrix} \begin{pmatrix} -\mu_L & x \\ y & -\mu_\Delta \end{pmatrix} \begin{pmatrix} \cos[\theta_2] & \sin[\theta_2] \\ -\sin[\theta_2] & \cos[\theta_2] \end{pmatrix} \tag{5.11}$$

The diagonal terms give the equations

$$s_2(xc_1 - s_1\mu_\Delta + c_2(ys_1 - c_1\mu_L) = 0 \tag{5.12}$$

$$-c_2(xs_1 + c_1\mu_\Delta - s_2(yc_1 + s_1\mu_L) = 0 \tag{5.13}$$

The angles θ_1 and θ_2 are given by the solution to the second order equations (with $t \equiv \tan[\theta]$)

$$t_1^2 - \left(\frac{x^2 - y^2 + \mu_L^2 - \mu_\Delta^2}{x\mu_\Delta + y\mu_L}\right) t_1 - 1 = 0 \tag{5.14}$$

$$t_2^2 + \left(\frac{x^2 - y^2 + \mu_\Delta^2 - \mu_L^2}{x\mu_L + y\mu_\Delta}\right) t_2 - 1 = 0 \tag{5.15}$$

With these solutions the rotated μ parameters are

$$-\mu_1 = \frac{(x + \mu_L t_2) - t_1(yt_2 + \mu_\Delta)}{\sqrt{(1 + t_1^2)(1 + t_2^2)}} \tag{5.16}$$

$$-\mu_2 = \frac{(y - \mu_\Delta t_2) - t_1(xt_2 - \mu_L)}{\sqrt{(1 + t_1^2)(1 + t_2^2)}} \tag{5.17}$$

After the rotations matrix (5.5) will have the desired shape (5.3)

$$M_{\tilde{H}} = \left(\begin{array}{cc|cc} 0 & 0 & 0 & -\mu_1 \\ 0 & 0 & -\mu_2 & 0 \\ \hline 0 & -\mu_2 & 0 & 0 \\ -\mu_1 & 0 & 0 & 0 \end{array}\right) \tag{5.18}$$

5.3.2 MSSM Bino

In matrix (5.1) the MSSM bino $\tilde{B}_{MSSM}$ should be a linear combination of $\tilde{B}$ and $\tilde{W}_R^0$.

Using the inverse transformations (5.9) and (5.10) we can rewrite the original 9×9 mass matrix (5.1) in the base $\{\tilde{B}, \tilde{W}_L, \tilde{\phi_{11}}, \tilde{\phi_{22}}, \tilde{\phi_{12}}, \tilde{\phi_{21}}, \tilde{W}_R, \tilde{\phi_{R1}}, \tilde{\phi_{R2}}\}$

$$M_n = \left(\begin{array}{cccc|ccccc} M_B & 0 & M_{B11} & M_{B22} & M_{B12} & M_{B21} & 0 & M_{BR1} & M_{BR2} \\ 0 & M_L & M_{L11} & M_{L22} & M_{L12} & M_{L21} & 0 & 0 & 0 \\ M_{B11} & M_{L11} & 0 & -\mu_1 & 0 & 0 & M_{R11} & 0 & 0 \\ M_{B22} & M_{L22} & -\mu_1 & 0 & 0 & 0 & M_{R22} & 0 & 0 \\ \hline M_{B12} & M_{L12} & 0 & 0 & 0 & -\mu_2 & M_{R12} & M_{11R1} & M_{11R2} \\ M_{B21} & M_{L21} & 0 & 0 & -\mu_2 & 0 & M_{R21} & 0 & 0 \\ 0 & 0 & M_{R11} & M_{R22} & M_{RD1} & M_{R21} & M_R & M_{RR1} & M_{RR2} \\ M_{BR1} & 0 & 0 & 0 & M_{RR1} & 0 & M_{RR1} & 0 & -\mu_R \\ M_{BR2} & 0 & 0 & 0 & M_{RR2} & 0 & M_{RR2} & -\mu_R & 0 \end{array}\right) \tag{5.19}$$

where,

$$\begin{aligned}
M_{B11} &= -\frac{g_1 v_{L1} c_1}{\sqrt{2}},\ M_{B222} = -\frac{g_1 v_{L2} s_2}{\sqrt{2}},\ M_{B12=}\frac{g_1 v_{L1} s_1}{\sqrt{2}}, \\
M_{B21} &= \frac{g_1 v_{L2} c_2}{\sqrt{2}},\ M_{BR1} = -\frac{g_1 v_{R1}}{\sqrt{2}},\ M_{BR2} = \frac{g_1 v_{R2}}{\sqrt{2}} \\
M_{L11} &= \frac{g_L(v_{L1}c_1 - u_1 s_1)}{\sqrt{2}},\ M_{L22} = \frac{g_L(v_{L2}s_2 + u_4 c_2)}{\sqrt{2}}, \\
M_{L12} &= \frac{g_L(-v_{L1}s_1 - u_1 c_1)}{\sqrt{2}},\ M_{L21} = \frac{g_L(v_{L2}c_2 + u_4 s_2)}{\sqrt{2}} \\
M_{R11} &= -\frac{g_R u_1 s_1}{\sqrt{2}},\ M_{R22} = \frac{g_R u_4 c_2}{\sqrt{2}},\ M_{R12} = -\frac{g_R u_1 c_1}{\sqrt{2}}, M_{R21} = \frac{g_R u_4 s_2}{\sqrt{2}} \\
M_{11R1} &= \frac{g_R v_{R1}}{\sqrt{2}},\ M_{11R2} - \frac{g_R v_{R2}}{\sqrt{2}},\ M_{RR1} = \frac{g_R v_{R1}}{\sqrt{2}},\ M_{RR2} = -\frac{g_R v_{R2}}{\sqrt{2}}
\end{aligned}$$

The off diagonal matrix G (as stated in eq (5.3)) still has large parameters (i.e proportional to v_R) given by the interactions between the bino and the right Higginos. Rotations in the fields are necessary. The mass matrix for the "heavy" components is, in the basis $(\Psi_{heavy})^T = \{\tilde{B}, \tilde{W}^0_R, \tilde{\phi}^0_{R1}, \tilde{\phi}^0_{R2}\}$,

$$M_{heavy} = \left(\begin{array}{cc|cc} M_B & 0 & -\frac{g_1 v_{R1}}{\sqrt{2}} & \frac{g_1 v_{R2}}{\sqrt{2}} \\ 0 & M_R & \frac{g_R v_{R1}}{\sqrt{2}} & -\frac{g_R v_{R2}}{\sqrt{2}} \\ \hline -\frac{g_1 v_{R1}}{\sqrt{2}} & \frac{g_R v_{R1}}{\sqrt{2}} & 0 & -\mu_R \\ \frac{g_1 v_{R2}}{\sqrt{2}} & -\frac{g_R v_{R2}}{\sqrt{2}} & -\mu_R & 0 \end{array}\right) \tag{5.20}$$

Making a rotation of the gauginos, by defining two new fields $\tilde{B_Y}$ and $\tilde{W_2}$

$$\begin{pmatrix} \tilde{B_Y} \\ \tilde{W_2} \end{pmatrix} = \frac{1}{\sqrt{g_1^2 + g_R^2}} \begin{pmatrix} g_R & g_1 \\ g_1 & -g_R \end{pmatrix} \begin{pmatrix} \tilde{B} \\ \tilde{W_R} \end{pmatrix} \tag{5.21}$$

Matrix (5.20) can be rewritten in the new base $\{\tilde{B}, \tilde{W}^0_R, \tilde{\phi}^0_{R1}, \tilde{\phi}^0_{R2}\} \rightarrow \{\tilde{B_Y}, \tilde{W}^0_2, \tilde{\phi}^0_{R1}, \tilde{\phi}^0_{R2}\}$

$$M'_{heavy} = \left(\begin{array}{c|ccc} \frac{g_1^2 M_B + g_R^2 M_R}{g_1^2 + g_R^2} & \frac{g_1 g_R (M_B - M_R)}{g_1^2 + g_R^2} & 0 & 0 \\ \hline \frac{g_1 g_R (M_B - M_R)}{g_1^2 + g_R^2} & \frac{g_R^2 M_B + g_1^2 M_R}{g_1^2 + g_R^2} & -\frac{\sqrt{g_1^2+g_R^2}}{\sqrt{2}} v_{R1} & \frac{\sqrt{g_1^2+g_R^2}}{\sqrt{2}} v_{R2} \\ 0 & -\frac{\sqrt{g_1^2+g_R^2}}{\sqrt{2}} v_{R1} & 0 & -\mu_R \\ 0 & \frac{\sqrt{g_1^2+g_R^2}}{\sqrt{2}} v_{R2} & -\mu_R & 0 \end{array}\right) \tag{5.22}$$

The bino ($\tilde{B_Y}$) will be decoupled from this sector if the off diagonal terms are small compared to the diagonal term. This can easily be accomplish when M_B and M_R have close values. In the particular case $M_B = M_R = M_Y$ then $\tilde{B_Y}$ gets completely decoupled from the "heavy" sector.

$$M_{heavy} = \left(\begin{array}{c|ccc} M_Y & 0 & 0 & 0 \\ \hline 0 & M_Y & -\frac{\sqrt{g_1^2+g_R^2}}{\sqrt{2}} v_{R1} & \frac{\sqrt{g_1^2+g_R^2}}{\sqrt{2}} v_{R2} \\ 0 & -\frac{\sqrt{g_1^2+g_R^2}}{\sqrt{2}} v_{R1} & 0 & -\mu_R \\ 0 & \frac{\sqrt{g_1^2+g_R^2}}{\sqrt{2}} v_{R2} & -\mu_R & 0 \end{array}\right) \tag{5.23}$$

5.3.3 Not MSSM Neutralinos

Once matrix (5.1) has been rotated to look like (5.3) is possible to check if the neutralino sector of eLRSUSY is an extension of the MSSM. The 4×4 upper left matrix is now

$$M_{light} = \begin{pmatrix} M_Y & 0 & -\frac{g_1 g_R (v_{L1} c_1 + u_1 s_1)}{\sqrt{2}\sqrt{g_1^2+g_R^2}} & \frac{g_1 g_R (u_4 c_2 - v_{L2} s_2)}{\sqrt{2}\sqrt{g_1^2+g_R^2}} \\ 0 & M_L & \frac{g_L (v_{L1} c_1 - u_1 s_1)}{\sqrt{2}} & \frac{g_L (v_{L2} s_2 + u_4 c_2)}{\sqrt{2}} \\ -\frac{g_1 g_R (v_{L1} c_1 + u_1 s_1)}{\sqrt{2}\sqrt{g_1^2+g_R^2}} & \frac{g_L (v_{L1} c_1 - u_1 s_1)}{\sqrt{2}} & 0 & -\mu_1 \\ \frac{g_1 g_R (u_4 c_2 - v_{L2} s_2)}{\sqrt{2}\sqrt{g_1^2+g_R^2}} & \frac{g_L (v_{L2} s_2 + u_4 c_2)}{\sqrt{2}} & -\mu_1 & 0 \end{pmatrix} \quad (5.24)$$

Which clearly has NOT the same entries as the neutralino mass matrix of the MSSM.

$$M_{MSSM} = \begin{pmatrix} M_1 & 0 & -M_Z * s_W * c_\beta & M_Z * s_W * s_\beta \\ 0 & M_2 & M_Z * c_W * c_\beta & -M_Z * c_W * s_\beta \\ -M_Z * s_W * c_\beta & M_Z * c_W * c_\beta & 0 & -\mu \\ M_Z * s_W * s_\beta & -M_Z * c_W * s_\beta & -\mu & 0 \end{pmatrix} \quad (5.25)$$

5.4 Numerical Analysis

The 9×9 neutralino matrix (5.1) is to big to be possible to diagonalize in terms of the parameters, nevertheless, if values are given to all the parameters, it is possible to get numerical values to the mass eigenvalues and eigenvectors. The objective is still the same as in the last section, we will like to have results that can be examined using the same calculations as in the MSSM.

From eq. (4.22), we can define the ratio between v_{SM} and v_L, $R = v_L/v_{SM} = 0.837$, also we define the ratio $\tan\beta_L = v_{L2}/v_{L1}$. We rewrite the left vevs as,

$$\begin{aligned} v_{L1} &= R * c_{\beta L} * v_{SM} = \left(\frac{\sqrt{2}}{g_L}\right) R * M_Z * c_{\beta L} * c_W \\ v_{L2} &= R * s_{\beta L} * v_{SM} = \left(\frac{\sqrt{2}}{g_L}\right) R * M_Z * s_{\beta L} * c_W \end{aligned} \quad (5.26)$$

This means that in the basis $\left\{\tilde{B}_Y, \tilde{W}, \tilde{\phi}_{L1}, \tilde{\phi}_{L2}\right\}$, (where $\tilde{W} = \frac{\tilde{W}_L}{R}$ and $\tilde{B}_Y$ as defined in (5.21)), the 4×4 upper left matrix in (5.1) will be

$$M_{MSSM} = \begin{pmatrix} M_Y & 0 & -M_Z * s_W * c_{\beta L} & M_Z * s_W * s_{\beta L} \\ 0 & \frac{M_L}{R^2} & M_Z * c_W * c_{\beta L} & -M_Z * c_W * s_{\beta L} \\ -M_Z * s_W * c_{\beta L} & M_Z * c_W * c_{\beta L} & 0 & -\mu_L \\ M_Z * s_W * s_{\beta L} & -M_Z * c_W * s_{\beta L} & -\mu_L & 0 \end{pmatrix} \tag{5.27}$$

This is exactly the MSSM neutralino mass matrix (5.25), where $M_Y = M_1, \mu_L = \mu, \beta_L = \beta$,and $\frac{M_L}{R^2} \simeq 1.43 M_L = M_2$ (Where M_1, M_2,μ and $\tan\beta$ the MSSM parameters).

With $\tilde{\chi}^0$ the mass eigenstates. The neutralino matrix gets diagonalized via

$$\tilde{\chi}_i^0 = N_{ij} \tilde{\Psi}_j^0 \tag{5.28}$$

We will define the fraction of a neutralino that have an MSSM component f_{iM} and the fraction that is not f_{iNM}.

$$f_{iM} = \sum_{j=1}^{4} (N_{ij})^2 \tag{5.29}$$

$$f_{iNM} = \sum_{j=5}^{9} (N_{ij})^2 \tag{5.30}$$

As long as the lightest of this eigenstates has $f_{1M} \simeq 1$, and a mass smaller (or degenerate) than the lightest chargino, then this neutralino can be considered to be the same as the MSSM LSP (with the only difference that the input for M_2 be modified by a factor of 1.43 with respect to eLRSUSY), and we are able to use the same results (and computational simulations).

Figure 5.1 has some plots showing MSSM fractions for different random values in the parameter space, for all the 9 neutralinos in ascending order for the mass. The lightest neutralino is plotted in black and a dash line to make it easier differentiate it from the other neutralinos. From these plots we can observe that in fact there is parameter space where the lightest neutralino is 100% MSSM.

For the numerical analysis we also defined the ratios $\tan\beta_R = v_{R2}/v_{R1}$ and $\tan\beta_\Delta = u_4/u_1$.

Note that from the keeping the Yukawas in eq (2.13) with a value $Y^2 \leq 4\pi$. will lead to the condition

$$\tan\beta_i \leq \sqrt{\frac{4\pi v_i^2}{M^2} - 1} \tag{5.31}$$

where we use $M = m_d, m_e, m_n$ for v_L, u, v_R respectively. Given the smallness of $m_e$and m_d compared to the value of the vev then $tan\beta_L$ and $tan\beta_\Delta$ are not by the perturbation of the Yukawa interaction, but for a value of $m_n \sim (1/3)v_R \sim 1$TeV then $\tan\beta_R \lesssim 10$.

5.5 Chargino mass matrix

We can extract then chargino mass matrices in the same manner we did for the neutralinos. In the bases

$$\begin{aligned}
(\Psi_1^+)^T &= \{\tilde{W}_L^+, \tilde{W}_R^+, \tilde{\phi}_{L2}^+, \tilde{\delta}_{22}^+, \tilde{\phi}_{R2}^+, \tilde{\delta}_{12}^+\} \\
(\Psi_1^-)^T &= \{\tilde{W}_L^-, \tilde{W}_R^-, \tilde{\phi}_{L1}^-, \tilde{\delta}_{11}^-, \tilde{\phi}_{R1}^-, \tilde{\delta}_{21}^-\} \\
(\Psi_2^+)^T &= \{\tilde{\eta}_{L2}^+, \tilde{\eta}_{R2}^+, \tilde{s_2}^+\} \\
(\Psi_2^-)^T &= \{\tilde{\eta}_{L1}^-, \tilde{\eta}_{R1}^-, \tilde{s_1}^-\}
\end{aligned}$$

the matrices are

$$M_{1\chi^\pm} = \begin{pmatrix} M_L & 0 & \frac{g_L v_{L2}}{2} & \frac{g_L u_4}{2} & 0 & 0 \\ 0 & M_R & 0 & \frac{g_R u_4}{2} & \frac{g_R v_{R2}}{2} & 0 \\ \frac{g_L v_{L1}}{2} & 0 & \mu_L & -f_1 v_{R2} & 0 & 0 \\ \frac{g_L u_1}{2} & \frac{g_R u_1}{2} & -f_2 v_{R1} & \mu_\Delta & 0 & 0 \\ 0 & \frac{g_R v_{R1}}{2} & 0 & 0 & \mu_R & -f_2 v_{L2} \\ 0 & 0 & 0 & 0 & -f_1 v_{L1} & \mu_\Delta \end{pmatrix} \tag{5.32}$$

$$M_{2\chi^\pm} = \begin{pmatrix} \mu_{L2} & f_3 u_1 & f_5 v_{L1} \\ f_4 u_4 & \mu_{R2} & f_6 v_{R1} \\ -f_7 v_{L2} & -f_8 v_{R2} & \mu_{s12} \end{pmatrix} \tag{5.33}$$

The chargino mass matrix is not symmetric a and will require two unitary matrices to be diagonalized.

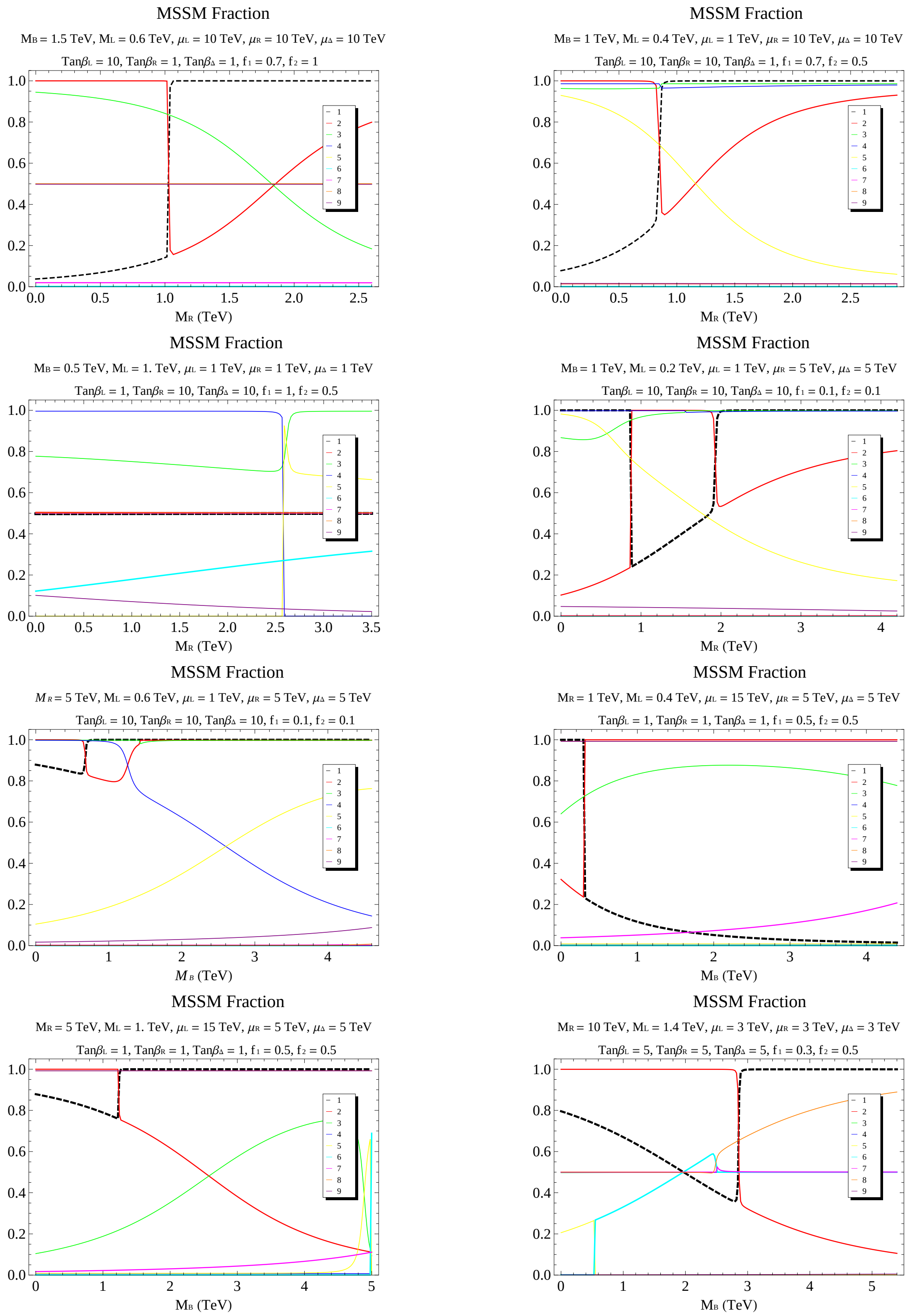


Figure 5.1: Examples of neutralino MSSM fractions

Part II

Theoretical Implications of eLRSUSY

Chapter 6

Radiative Neutrino Mass

One loop neutrino masses have been studied in detail [17–32, 41–43]. In our model we will has the correct particle content that will allow it to follow the Scotogenic-Ma model [10]. Starting with the Lagrangian for a massive Dirac particle

$$\mathcal{L} = \overline{\psi}(\imath \not{D} - m)\psi - \delta m_0 \overline{\psi}\psi \tag{6.1}$$

Where $\delta m_0 = m - m_0 = \Sigma(p^2)$, is the one loop correction to the mass, as seen in Fig. 6

$$\imath \Sigma_{ijR}(p^2) = -\int \frac{d^4k}{(2\pi)^4} h_{ik} \frac{(\not{k} + M_k)}{k^2 - M_k^2} h_{jk} \frac{1}{(p-k)^2 - m_R^2} \tag{6.2}$$

This integral can be solved exactly, first by noting that the part proportional to $\not{k}$ is zero, secondly we set $p^2 = 0$,since the bare mass of the neutrino will not depend on its momentum.

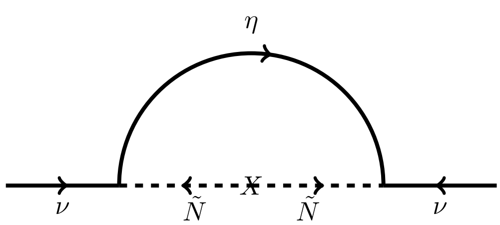


Figure 6.1: 1 loop neutrino mass

$$\imath\Sigma_{ijR} = M_k \int \frac{d^4k}{(2\pi)^4} h_{ik} \frac{1}{k^2 - M_k^2} h_{jk} \frac{1}{k^2 - m_R^2} \tag{6.3}$$

$$= \frac{h_{\alpha K} h_{\beta K} M_K}{16\pi^2} \left[\frac{m_R^2}{m_R^2 - M_K^2} \ln\left(\frac{m_R^2}{M_K^2}\right) + \frac{2}{\epsilon} \right] \tag{6.4}$$

This integral diverges, since $\Sigma \to \infty$ as $\epsilon \to 0$, never the less we still need to concider the contribution of the η_I. Given the the result is complex, then $\Sigma_{ij} = \Sigma_{ijR} - \Sigma_{ijR}$, The infinities will cancel, and we achieved a mass given by,

$$(M_\nu)_{\alpha\beta} = \sum_K \frac{h_{\alpha K} h_{\beta K} M_K}{16\pi^2} \quad \left[\frac{m_R^2}{m_R^2 - M_K^2} \ln\left(\frac{m_R^2}{M_K^2}\right) - \frac{m_I^2}{m_I^2 - M_K^2} \ln\left(\frac{m_I^2}{M_K^2}\right) \right] \tag{6.5}$$

Note that for this mechanism to work a Majorana mass for N. should exist which violates lepton number by two units $\Delta L = 2$.

In our model the radiative neutrino mass will have two contributions seen in Fig. 6.2. We have to take into account that η_{L2}^0 and $\tilde{\eta}_{L2}^0$ are not mass eigenstates, the neutrino mass can be calculated in the similar manner as shown in eq. (6.4) with the generalization,

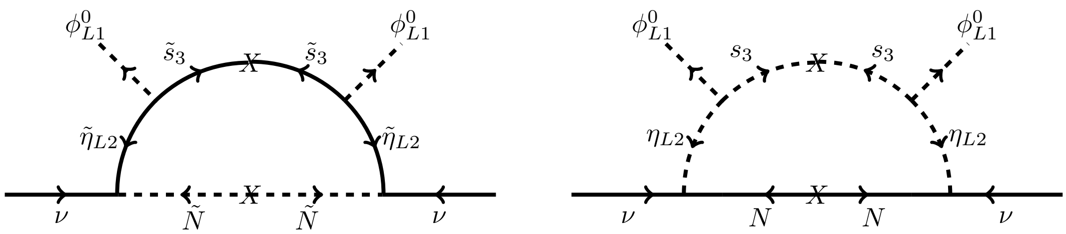


Figure 6.2: Scotogenic neutrino mass

$$(M_\nu)_{\alpha\beta} = \sum_i \left\{ \frac{h_{\alpha i} h_{\beta i} M_{Ni}}{16\pi^2} \sum_j \left[(U_R)_{1j} \frac{m_{Rj}^2}{m_{Rj}^2 - M_{Ni}^2} \ln\left(\frac{m_{Rj}^2}{M_{Ni}^2}\right) - (U_I)_{1j} \frac{m_{Ij}^2}{m_{Ij}^2 - M_{Ni}^2} \ln\left(\frac{m_{Ij}^2}{M_{Ni}^2}\right) \right] \right\}$$

Where U_R (U_I) is the unitary matrix that makes $m_R(m_I)$ mass eigenstates (This matrices have been worked in chapters 3 and 5) and h is the parameter for the interactions $\nu N \eta_{L2}^0$ and $\nu \tilde{N} \eta_{\tilde{L}2}{}^0$.

A simple method to find an approximate solution can be achived by first noticing that it is required that $m_R \neq m_I$ or M_ν will be zero, in a non-SUSY theory this is accomplished by a λ_5 term (i.e $(\Phi^\dagger\eta)^2 + (\eta^\dagger\Phi)^2$). Then $m_R^2 - m_I^2 = 2\lambda_5 v^2$, with v the vev of the scalar introduced with the external lines.

We will make the approximations $M_K^2 \gg m_0^2 \gg 2\lambda_5 v^2$, where $m_0^2 = (m_R^2 + m_I^2)/2$. The first one just stated that N_K is very heavy, the second that the masses of η_R and η_I are very split.
The mass of the neutrinos is reduced to,

$$(M_\nu)_{\alpha\beta} = \frac{-\lambda_5 v^2}{8\pi^2} \sum_K \frac{h_{\alpha K} h_{\beta K}}{M_K} \quad \left[1 + \ln\left(\frac{m_0^2}{M_K^2}\right)\right] \tag{6.6}$$

In the next chapter it will be seen that the requirement for gauge coupling unification makes the mass of s_3 very heavy ($\sim 10^5$ GeV), this will allow us to use of an effective potential; reducing the propagator by means of the exchange $\lambda_5 \to \left(\frac{f}{M_S}\right)^2$, with f being the coupling for $\phi^0_{L1}\eta^0_{L2}s^0_3$, and in general could be it breaking term.

$$(M_\nu)_{\alpha\beta} = \frac{-f^2 v_{L1}^2}{8\pi^2 M_S^2} \sum_K \frac{h_{\alpha K} h_{\beta K}}{M_K} \quad \left[1 + \ln\left(\frac{m_0^2}{M_K^2}\right)\right]$$

In Fig. 6 we can observe the right diagram needs the A term $\phi^0_{L1}\eta^0_{L2}s_3$ twice and the B term $s_3 s_3$ once whereas the one on the left requires only the B term $\tilde{N}\tilde{N}$ once. We expect thus the latter diagram to be much more important. We estimate its contribution to be given by

$$m_\nu \simeq \frac{h^2 v_{L1}^2 f^2}{16\pi^2 M_N^3}, \tag{6.7}$$

where h is the diagonal Yukawa coupling, f is the supersymmetry breaking B term, and $M_3 \simeq M_N$ has been assumed. Using $v_{L1} \simeq 100$ GeV, $f^2 \simeq 1$ TeV^2, and $M_N \simeq 10^5$ GeV, and $h^2 \simeq 10^{-3}$, we find $m_\nu \simeq 0.1$ eV.

Chapter 7

Unification

7.1 RGE at 1 loop

The 1 loop evolution of the gauge couplings is given by,

$$\frac{d\alpha_i}{d\ln Q^2} = b_i \frac{\alpha_i^2}{4\pi} \tag{7.1}$$

With $\alpha_i = g_i^2/4\pi$ being the gauge coupling constant and b_i the coefficient of the β function, whereb_1 is the coefficient for U(1), and b_N is for an SU(N) symmetry. The solution to this equation gives the one loop RGE equations,

$$\frac{1}{\alpha_i(M_U)} = \frac{1}{\alpha_i(M_Z)} - \frac{b_i}{4\pi} \ln\left(\frac{M_U^2}{M_Z^2}\right) \tag{7.2}$$

We conciser that there is a larger symmetry at for which unification of the gauge coupling constants exists at a given scale M_U, then GUT unification means,

$$\alpha(M_U) \equiv \alpha_U = \alpha_3(M_U) = \alpha_2(M_U) = N * \alpha_1(M_U) \tag{7.3}$$

Where the factor N comes for the normalization

$$\frac{Tr\left[Y_F^2\right]}{Tr\left[T^3T^3\right]} = N \tag{7.4}$$

It's value depends on the GUT symmetry, being the most common SU(5) and SO(10), with values for N of 5/3 and 2/3 respectively.
The experimental values of the gauge coupling constants are [44],

$$\begin{aligned} sin^2\theta_W(M_Z) &= 0.23116 \pm 5.9\times 10^{-3} \\ \alpha_{em}^{-1}(M_Z) &= 127.953 \pm 0.049 \\ \alpha_s(M_Z) &= 0.1184 \pm 2.3\times 10^{-5} \end{aligned} \tag{7.5}$$

In the SM the Weinberg angle is defined as $\tan\theta_w = g'/g$, where g' being the $U(1)_Y$ coupling and g the coupling for $SU(2)_L$. Or making the connection with the EM constant, is possible to write, $\frac{1}{e} = \frac{1}{g'} + \frac{1}{g}$. And is easy to see that at the EM breaking scale, the gauge couplings are related by,

$$\begin{aligned} \alpha_L &= \frac{\alpha_{em}}{\sin^2\theta_W} \\ \alpha_Y &= \frac{\alpha_{em}}{\cos\theta_W^2} \end{aligned} \tag{7.6}$$

In the simple case where the model has only two scales(i.e $M_Z$and M_U), using (7.2) explicitly, is possible to calculate mass scale for unification M_U,

$$\ln\left(\frac{M_U}{M_Z}\right) = \frac{2\pi}{b_1 - b_2}\left(\frac{1}{\alpha_1(M_Z)} - \frac{1}{\alpha_2(M_Z)}\right) \tag{7.7}$$

It's also possible to combine the 3 RGE equations by eliminating the M_U dependance, then there will be a mandatory correlation between the coupling constants,

$$\frac{1}{\alpha_3(M_Z)} = \left(1 + \frac{b_3 - b_2}{b_2 - b_1}\right)\frac{1}{\alpha_2(M_Z)} - \left(\frac{b_3 - b_2}{b_2 - b_1}\right)\frac{1}{\alpha_1(M_Z)} \tag{7.8}$$

This simplification shows an important characteristic of this unifying models, clearly is the difference between the b_i coefficients will lead to unification, and not the individual values.

7.1.1 b_i coefficients

Explicitly the value of the coefficients b_i will depend on the particle content by,

$$b = \frac{-11}{3} C_2(G) + \frac{2}{3} \sum_f T_f(R) + \frac{1}{3} \sum_s T_s(R) \tag{7.9}$$

$C_2(G)$ is the quadratic Casimir with a value of N for SU(N) and zero for U(1), $T(R)$ is the Dynkin index has a value of 1/2 for SU(N), Y^2 for U(1) and N for the adjoint of an SU(N). The subindex f(s) stands for fermion(scalar). For a non-SUSY theory the coefficients can be written as,

$$\begin{aligned} N * b_1 &= \frac{2}{3} \sum_f Y_f^2 + \frac{1}{3} \sum_s Y_s^2 \\ b_N &= -\frac{11N}{3} + \frac{n_f}{3} + \frac{n_s}{6} \end{aligned} \tag{7.10}$$

Y is the hypercharge, n the number of particles in the SU(N) group, n_A is the number of fermions in the adjoint.

For the SUSY theories there will be 2 gauginos for SU(2) and 3 gluinos for SU(3), also for every scalar there will be a fermion superpartner and vice versa, eqs.(7.9) will be rewritten as,

$$b = -3C_2(G) + \sum_f T_f(R) + \sum_s T_s(R) \tag{7.11}$$

or explicitly,

$$\begin{aligned} N * b_1 &= \sum Y_f^2 + \sum_s Y_s^2 \\ b_N &= -3N + \frac{n_f}{2} + \frac{n_s}{2} \end{aligned} \tag{7.12}$$

7.2 Running constants in the SM

First, let's remember than in the SM the particle assignments are given by table 7.1

Field	Doublet?	3 Colors?	3 Flavors?	U(1) Particles	SU(2) Particles	SU(3) Particles
Q	✓	✓	✓	$2\times3\times3=18$	$3\times3=9$	$2\times3=6$
u^c		✓	✓	$1\times3\times3=9$	0	$1\times3=3$
d^c		✓	✓	$1\times3\times3=9$	0	$1\times3=3$
ψ_L	✓		✓	$2\times1\times3=6$	$1\times3=3$	0
e^c			✓	$1\times1\times3=3$	0	0
H_1	✓			$2\times1\times1=2$	1	0
H_2	✓			$2\times1\times1=2$	1	0

Table 7.2: SM and 2HSM, particles with each symmetry

Field	$SU(3)$	$SU(2)$	$U(1)$
$Q=\begin{pmatrix}u\\d\end{pmatrix}$	3	2	1/6
u^c	3	1	-2/3
d^c	3	1	1/3
$\psi_L=\begin{pmatrix}\nu\\e\end{pmatrix}$	1	2	-1/2
e^c	1	1	1
$H=\begin{pmatrix}\phi^0\\\phi^-\end{pmatrix}$	1	2	1/2

Table 7.1: SM particle content

In table 7.2 we show the particle content per each of the gauge symmetries (Note that $H_2$is not a part of the SM, it was included in the last line to avoid writing the table again in the next section).
Assuming an $SU(5)$ GUT symmetry, the coefficients are,

$$\begin{aligned}(5/3)b_Y &= \frac{2}{3}\left[18\left(\frac{1}{6}\right)^2+9\left(-\frac{2}{3}\right)^2+9\left(\frac{1}{3}\right)^2+6\left(-\frac{1}{2}\right)^2+3\,(1)^2\right]+\frac{1}{3}\left[2\left(\frac{1}{2}\right)^2\right]=\frac{41}{6} \quad (7.13)\\ b_L &= -\frac{11}{3}2+\frac{1}{3}\,(9+3)+\frac{1}{6}(1)=-\frac{19}{6}\\ b_s &= -\frac{11}{3}3+\frac{6+3+3}{3}=-7\end{aligned}$$

Eqs. (7.2) become,

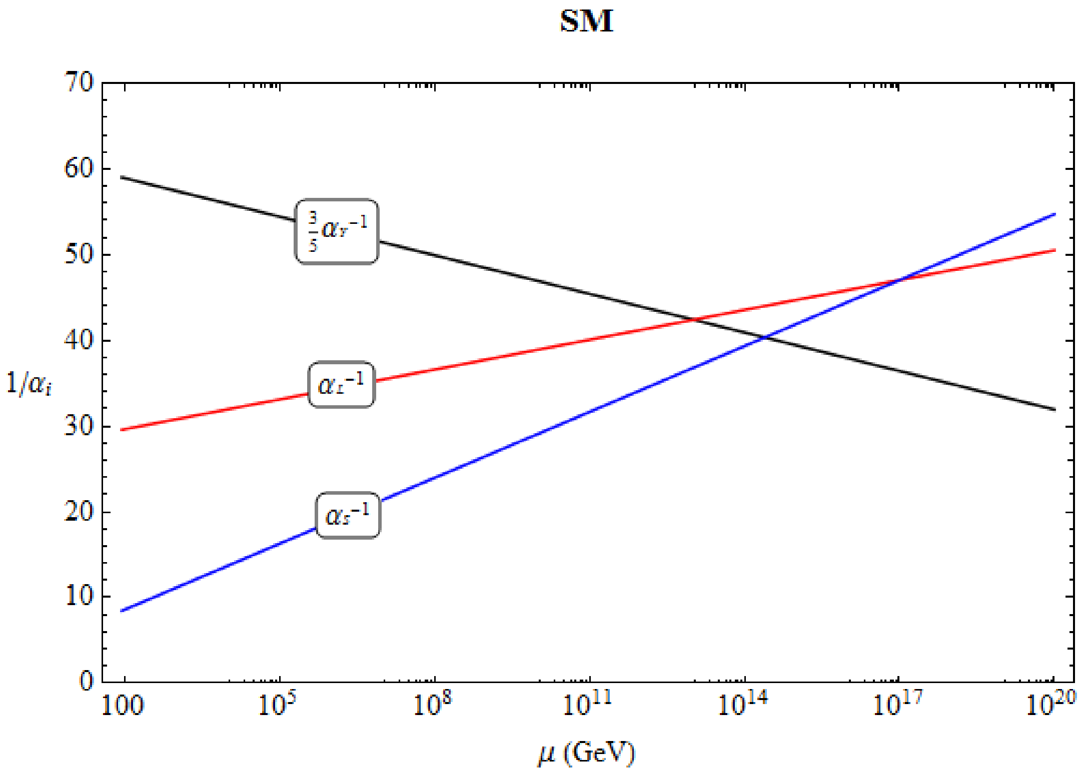


Figure 7.1: SM Running constants

$$\left(\frac{3}{5}\right)\frac{1}{\alpha_Y(M_U)} = \left(\frac{3}{5}\right)\frac{1}{\alpha_Y(M_Z)} - \frac{41}{20\pi}\ln\left(\frac{M_U}{M_Z}\right) \tag{7.14}$$
$$\frac{1}{\alpha_L(M_U)} = \frac{1}{\alpha_L(M_Z)} + \frac{19}{12\pi}\ln\left(\frac{M_U}{M_Z}\right)$$
$$\frac{1}{\alpha_s(M_U)} = \frac{1}{\alpha_s(M_Z)} + \frac{7}{2\pi}\ln\left(\frac{M_U}{M_Z}\right)$$

From eq. (7.8) then,

$$\frac{1}{\alpha_s(M_Z)} = \left(\frac{333}{218}\right)\frac{1}{\alpha_L(M_Z)} - \left(\frac{115}{218}\right)\frac{1}{\alpha_Y(M_Z)} \simeq 15 \tag{7.15}$$

This disagrees with the experimental value of $\alpha_s(M_Z) \simeq 1/8.5$

7.3 2HSM (2 Higgs SM)

In 2HSM, the particle content is almost the same as the SM with the difference that there is one more Higgs doublet. As seen in table 7.3.

Field	$SU(3)$	$SU(2)$	$U(1)$
Q	3	2	1/6
u^c	3	1	-2/3
d^c	3	1	1/3
ψ_L	1	2	-1/2
e^c	1	1	1
H_1	1	2	1/2
H_2	1	2	-1/2

Table 7.3: 2HSM particle content

Using table 7.2, we can see that the coefficients change to,

$$\begin{aligned} (5/3)b_Y &= \frac{2}{3}\left[18\left(\frac{1}{6}\right)^2+9\left(-\frac{2}{3}\right)^2+9\left(\frac{1}{3}\right)^2+6\left(-\frac{1}{2}\right)^2+3\,(1)^2\right] \\ &+ \frac{1}{3}\left[2\left(\frac{1}{2}\right)^2+2\left(\frac{1}{2}\right)^2\right]=7 \\ b_L &= -\frac{11}{3}2+\frac{1}{3}\,(9+3)+\frac{1}{6}(1+1)=-3 \\ b_s &= -\frac{11}{3}3+\frac{6+3+3}{3}=-7 \end{aligned} \tag{7.16}$$

with the 1 loop RGEs,

$$\left(\frac{3}{5}\right)\frac{1}{\alpha_Y(M_U)} = \left(\frac{3}{5}\right)\frac{1}{\alpha_Y(M_Z)}-\frac{7}{2\pi}\ln\left(\frac{M_U}{M_Z}\right) \tag{7.17}$$

$$\frac{1}{\alpha_L(M_U)} = \frac{1}{\alpha_L(M_Z)}+\frac{3}{2\pi}\ln\left(\frac{M_U}{M_Z}\right) \tag{7.18}$$

$$\frac{1}{\alpha_s(M_U)} = \frac{1}{\alpha_s(M_Z)}+\frac{7}{2\pi}\ln\left(\frac{M_U}{M_Z}\right) \tag{7.19}$$

But once again we can see in Fig. 7.2, GUT unification is not achieved.

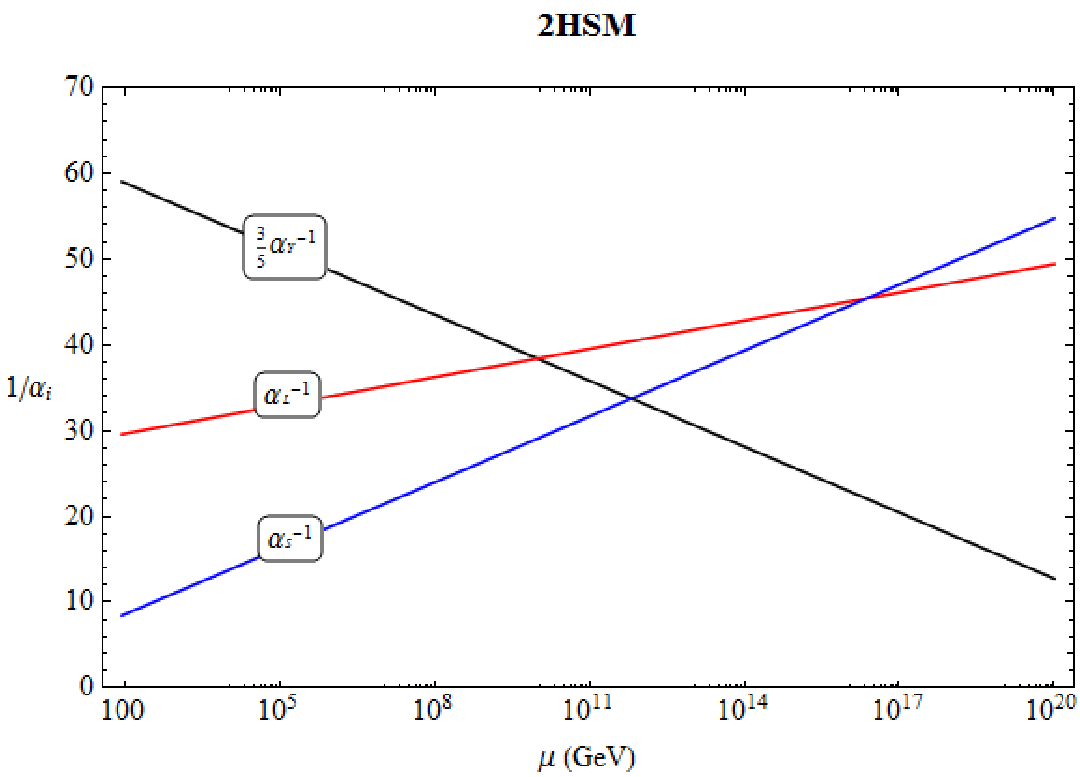


Figure 7.2: 2HSM Running constants

7.4 MSSM

In supersymmetric theories there will be one more scale (i.e the energy where SUSY breaks M_S). The particle content is the same as in table 7.3, with the difference that the fields are supefields. Using eq. (7.12) for the MSSM above M_S,

$$\begin{aligned}(5/3)b_Y &= \left[18\left(\frac{1}{6}\right)^2+9\left(-\frac{2}{3}\right)^2+9\left(\frac{1}{3}\right)^2+6\left(-\frac{1}{2}\right)^2+3\,(1)^2\right]+\left[2\left(\frac{1}{2}\right)^2+2\left(\frac{1}{2}\right)^2\right]=11 \quad (7.20)\\ b_L &= -3(2)+\frac{1}{2}(9+3)+\frac{1}{2}(1+1)=1\\ b_s &= -3(3)+\frac{6+3+3}{2}=-3\end{aligned}$$

The 1 Loop RGEs

$$\begin{aligned}\left(\frac{3}{5}\right)\frac{1}{\alpha_Y(M_U)} &= \left(\frac{3}{5}\right)\frac{1}{\alpha_Y(M_Z)}-\frac{41}{20\pi}\ln\left(\frac{M_S}{M_Z}\right)-\frac{33}{10\pi}\ln\left(\frac{M_U}{M_S}\right) \quad (7.21)\\ \frac{1}{\alpha_L(M_U)} &= \frac{1}{\alpha_L(M_Z)}+\frac{19}{12\pi}\ln\left(\frac{M_S}{M_Z}\right)-\frac{1}{2\pi}\ln\left(\frac{M_U}{M_S}\right)\\ \frac{1}{\alpha_s(M_U)} &= \frac{1}{\alpha_s(M_Z)}+\frac{7}{2\pi}\ln\left(\frac{M_S}{M_Z}\right)+\frac{3}{2\pi}\ln\left(\frac{M_U}{M_S}\right)\end{aligned}$$

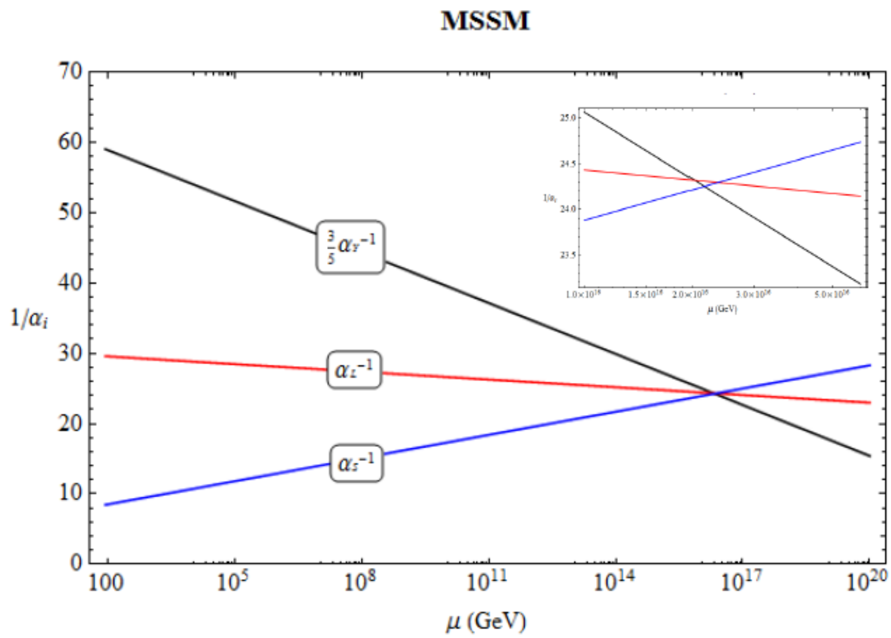


Figure 7.3: MSSM Running constant

Solving the equations we get a solution with $M_U \sim 3. \times 10^{-16}$GeV, $M_S \sim M_Z$ GeV and $\alpha_3(M_U) = \alpha_2(M_U) = (3/5)\alpha_1(M_U) \equiv \alpha_U \sim 0.04245$.

For the MSSM eq. (7.8) takes the form,

$$\frac{1}{\alpha_3(M_Z)} = \left(\frac{12}{7}\right)\frac{1}{\alpha_2(M_Z)} - \left(\frac{5}{7}\right)\frac{1}{\alpha_1(M_Z)} \simeq 8.5 \tag{7.22}$$

This is very close to the experimental value of α_s, which implies that to a good approximation the MSSM gauge couplings unify atM_U. In Fig. 7.3, we can see the evolution of the couplings. In the upper left corner there is a "zoom" of the plot at the unification scale, it shows that the unification is nor perfect and there is a small difference, which can be reduced by using 2 Loop RGEs [46], and/or also if there is an extra scale.

7.4.1 MSSM + 1 Higgs doublet

The MSSM is simplest supersymmetric extension to the SM, but since 2 Higgs doublets are required, it makes sense to write the 1 loop RGEs with two Higgs doublets even below the SUSY breaking scale.

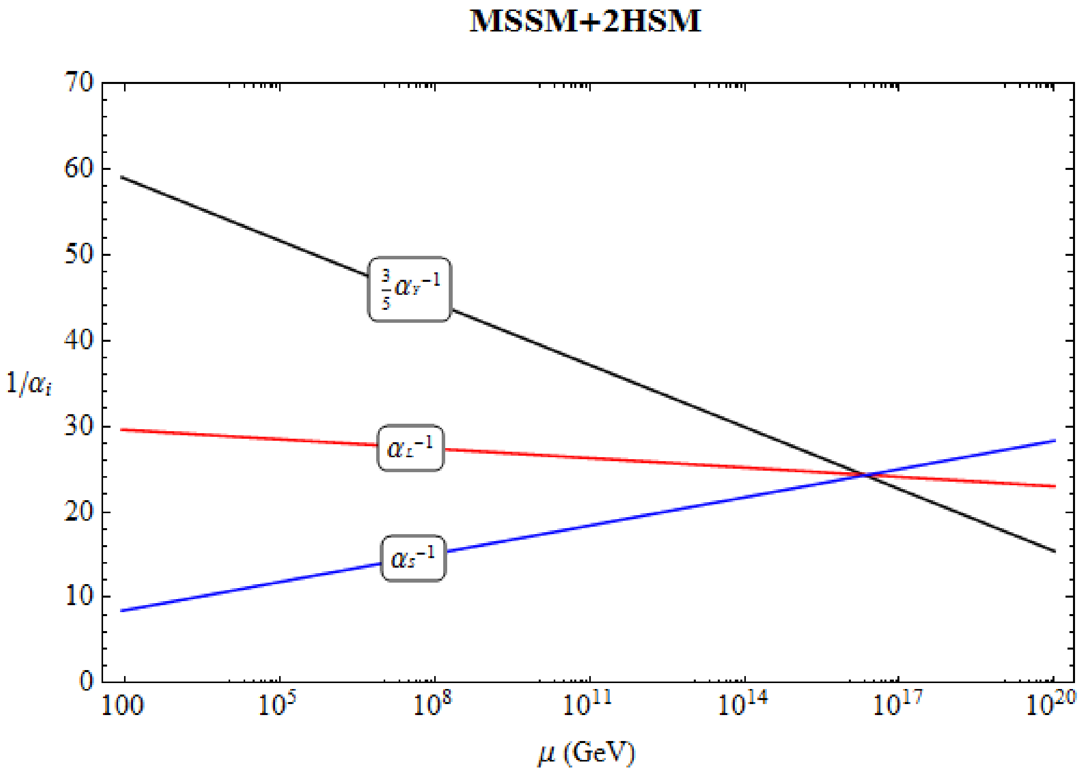


Figure 7.4: MSSM+2HDM

$$\frac{1}{\alpha_U} = \frac{1}{\alpha_1(M_Z)} - \frac{7}{2\pi}\ln\left(\frac{M_S}{M_Z}\right) - \frac{33}{10\pi}\ln\left(\frac{M_U}{M_S}\right) \tag{7.23}$$
$$\frac{1}{\alpha_U} = \frac{1}{\alpha_2(M_Z)} + \frac{3}{2\pi}\ln\left(\frac{M_S}{M_Z}\right) - \frac{1}{2\pi}\ln\left(\frac{M_U}{M_S}\right)$$
$$\frac{1}{\alpha_U} = \frac{1}{\alpha_3(M_Z)} + \frac{7}{2\pi}\ln\left(\frac{M_S}{M_Z}\right) + \frac{3}{2\pi}\ln\left(\frac{M_U}{M_S}\right)$$

Given the M_S is so small, then the plot is the same as MSSM and the solutions are going to be extremely similar $\alpha_U = 0.0413734$, $M_U \sim 3. \times 10^{16}$ GeV , $M_S \sim M_Z$

7.5 eLRSUSY

Let's assume that our model is a consequence of an $SO(10)$ supersymetric model that breaks at a M_U scale into $SU(3)_s \times SU(2)_L \times SU(2)_R \times U(1)_x$, same as in reference [45]. At M_S SUSY gets broken and at M_R it breaks into the SM with $SU(3)_s \times SU(2)_L \times U(1)_Y$. Also we can assume the s_i singlets are much heavier than the rest of the particles with a mass$\sim M_X$.

Field	Doublet?	N_c	N_f	#	$U(1)_X$	$SU(2)_L$,$SU(2)_R$	$SU(3)_c$
Q, Q^c	✓	3	3	2	$2\times2\times3\times3=36$	$1\times3\times3=9$	$2\times2\times3=12$
d^c ,h		3	3	2	$2\times1\times3\times3=18$	0	$2\times1\times3=6$
ψ, ψ^c	✓		3	2	$2\times2\times1\times3=12$	$1\times1\times3=3$	0
ν^c,n			3	2	$2\times1\times1\times3=6$	0	0
Φ_i,η_i,	✓			8	$8\times2\times1\times1=16$	$4\times1\times1=4$	0
Δ_i	✓(×2)			2 (×2)	$4\times2\times1\times1=36$	$4\times1\times1=4$	0
s_1, s_2				2	$2\times1\times1\times1=2$	0	0
s_3				1	$1\times1\times1\times1=1$	0	0

Table 7.4: eLRSUSY, particles with each symmetry

Table 7.4 shows the counting of the particles per each gauge symmetry.

Note that the above M_R the gauge symmetries $SU(2)_L$ and $SU(2)_R$ are unified with the coupling α_{LR}, and given particle content of $SU(2)_L$and $SU(2)_R$is exactly the same, their gauge constants will run the same after the symmetry breaking (i.e $\alpha_L(M_R)=\alpha_R(M_R)$). Also, $\alpha_x^{-1}(M_R)=\alpha_Y^{-1}(M_R)+\alpha_L^{-1}(M_R)$ (i.e the coupling α_Y doesn't exist above M_R and becomes α_x)

The $b_i$coefficients are (below M_X)

$$\begin{aligned}(2/3)b_x &= 36\left(\frac{1}{6}\right)^2+18\left(\frac{1}{3}\right)^2+12\left(\frac{1}{2}\right)^2+16\left(\frac{1}{2}\right)^2=10 \\ b_{LR} &= -3(2)+\frac{1}{2}(9+3+4+4)=4 \\ b_s &= -3(3)+\frac{1}{2}(12+6)=0\end{aligned} \tag{7.24}$$

And above M_X the coefficient $b_1$get modified so

$$(2/3)b_x=36\left(\frac{1}{6}\right)^2+18\left(\frac{1}{3}\right)^2+12\left(\frac{1}{2}\right)^2+16\left(\frac{1}{2}\right)^2+2\,(1)^2=12 \tag{7.25}$$

Eqs. 7.2 are below M_R

$$\begin{aligned}\left(\frac{3}{5}\right)\frac{1}{\alpha_Y(M_R)} &= \left(\frac{3}{5}\right)\frac{1}{\alpha_Y(M_Z)}-\frac{7}{2\pi}\ln\left(\frac{M_R}{M_Z}\right) \\ \frac{1}{\alpha_L(M_R)} &= \frac{1}{\alpha_2(M_Z)}+\frac{3}{2\pi}\ln\left(\frac{M_R}{M_Z}\right) \\ \frac{1}{\alpha_s(M_R)} &= \frac{1}{\alpha_s(M_Z)}+\frac{7}{2\pi}\ln\left(\frac{M_R}{M_Z}\right)\end{aligned} \tag{7.26}$$

Above M_R

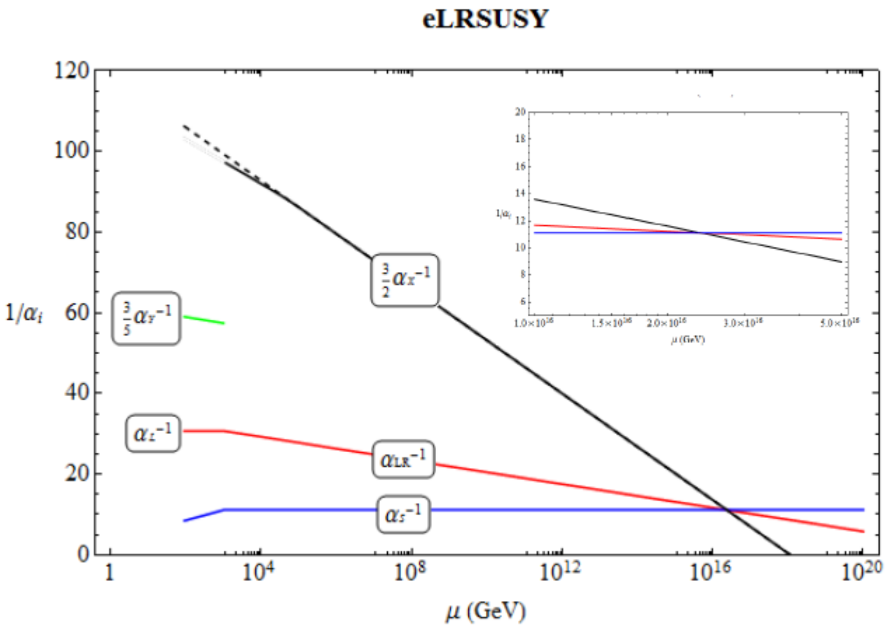


Figure 7.5: Running constants in eLRSUSY

$$
\begin{aligned}
\left(\frac{3}{2}\right)\frac{1}{\alpha_x(M_U)} &= \left(\frac{3}{2}\right)\frac{1}{\alpha_x(M_R)} - \frac{15}{2\pi}\ln\left(\frac{M_X}{M_R}\right) - \frac{18}{2\pi}\ln\left(\frac{M_U}{M_X}\right) \\
\frac{1}{\alpha_{LR}(M_U)} &= \frac{1}{\alpha_L(M_R)} - \frac{4}{2\pi}\ln\left(\frac{M_U}{M_R}\right) \\
\frac{1}{\alpha_s(M_U)} &= \frac{1}{\alpha_s(M_R)}
\end{aligned} \tag{7.27}
$$

These equations can be combined

$$
\ln\left(\frac{M_U}{M_Z}\right) = \frac{\pi}{2}\left(\frac{1}{\alpha_2(M_Z)} - \frac{1}{\alpha_3(M_Z)}\right) \tag{7.28}
$$

$$
\ln\left(\frac{M_R^7}{M_X^3 M_Z^4}\right) = \pi\left(\frac{3}{\alpha_2(M_Z)} - \frac{12}{\alpha_2(M_Z)} - \frac{7}{\alpha_3(M_Z)}\right) \tag{7.29}
$$

If $M_R = M_X$ then eqs. (7.28) and (7.29) would be exactly the same as the solutions for section 7.4.1 These equations have the solutions $M_U \sim 2\times 10^{16}$, and $(M_R^7/M_X^3)^{1/4} = 53.28$ GeV, and $M_R \sim 1$TeV. Plot 7.5shows the evolution of the constants, with a dashed line included to make clear the change of behavior of α_x at $M_X \sim 50$ TeV

Part III

Multi-partite Dark Matter in eLRSUSY

Chapter 8

Pair Annihilation of $\tilde{\eta}_R$ and n

For simplicity we will assume the following scenario scenario, the masses m_χ, m_n, m_η of the three stable dark-matter particles $\tilde{\chi}_1^0, n, \tilde{\eta}_R^0$ to be arranged in ascending order. Now, $\tilde{\eta}_R^0$ has $I_{3L} = 0$, so it couples only to Z'. Hence the annihilation of $\tilde{\eta}_R^0 \bar{\tilde{\eta}}_R^0$ to Z' to particles with masses less than m_η will determine its relic abundance. Once $\tilde{\eta}_R^0$ freezes out, we need to consider the interactions of n. Again n has $I_{3L} = 0$, so it couples to Z', and also has the interaction $\bar{e}n^c W_R^-$. Hence the annihilation of $n\bar{n}$ occurs through Z' to particles with mass less than m_n as well as to e^+e^- through $W_R^\pm$ exchange. This will determine the relic abundance of n. After n freezes out, the remaining particles are presumably those of the MSSM, and the annihilation of $\tilde{\chi}_1^0 \tilde{\chi}_1^0$ will determine its relic abundance.

The annihilation cross-sections for DM $\tilde{\eta}_R^0$ to the SM particles eventually goes through s-channel diagram exchanging Z', while the cross-section for DM n has an additional piece through a t-channel diagram to $e_R^\pm$ through W_R exchange.

The total abundance will be a sum of the three DM components, i.e.

$$\Omega_{DM_{tot}} h^2 = \Omega_\eta h^2 + \Omega_n h^2 + \Omega_{\chi_1^0} h^2 \tag{8.1}$$

The strategy then is clear, we will separate the study of the relic abundance into two parts, first we will calculate the scattering of n and $\tilde{\eta}_R^0$ into SM particles using the Feynman diagrams shown in fig. 8.1 and 8.1. Secondly, we will study the neutralino $\tilde{\chi}_1^0$ assuming it is the same as the MSSM neutralino as explained in section 5.4.

8.1 Calculation of the scattering for n and $\tilde{\eta}_R^0$.

For a $2 \to 2$ scattering process,

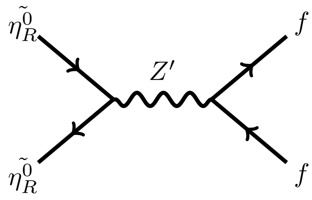


Figure 8.1: Feynman Diagram for $\tilde{\eta}_R^0$ annihilation to SM.

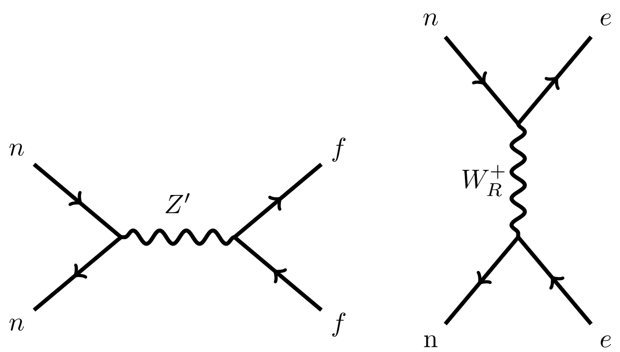


Figure 8.2: Feynman Diagram for n annihilation to SM.

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{|\mathcal{M}|^2}{(E_1+E_2)^2} \frac{|\overrightarrow{p_f}|}{|\overrightarrow{p_i}|} \tag{8.2}$$

Using E as the center of mass energy $E = E_1 = E_2$, M the mass of the initial particles and m the mass of the final particles, then

$$\sigma = 4\pi \left(\frac{1}{8\pi}\right)^2 \frac{|\mathcal{M}|^2}{(2E)^2} \frac{\sqrt{E^2 - m^2}}{\sqrt{E^2 - M^2}} \tag{8.3}$$

Assuming the final particles are lighter than the initial ones and a non relativistic limit $v \to 0$, then $E = \gamma M = \frac{M}{\sqrt{1-v^2}} \simeq M$, and $E, M \gg m$,

$$\sigma \simeq \frac{1}{64\pi} \frac{|\mathcal{M}|^2}{vM^2} \sqrt{1 - \left(\frac{m}{M}\right)^2} \simeq \frac{1}{64\pi} \frac{|\mathcal{M}|^2}{vM^2} \tag{8.4}$$

In general then $\sigma v = a + bv^2 + \mathcal{O}(v^4)$. Where a would be the biggest contribution given the non relativistic nature.

In the rest of this section the square of the amplitude $\mathcal{M}$, and the scattering will be calculated explicitly.

8.1.1 s-Chanel

As shown in figures (8.1) and (8.2), both the scotino n and the Higgsino $\tilde{\eta}_R$, will annihilate via a Z'. For this calculation we will use the scotino, but we should keep in mid that the calculation will be the same for the Higgsino $\tilde{\eta}^0_R$.

For $\overline{n}n \to Z' \to \overline{f}f$ the amplitude for the s-channel is given by

$$\imath\mathcal{M} = g^2_{Z'} * \overline{\tilde{\eta_R}}\gamma^\mu(c_{v1} - c_{a1}\gamma^5)\tilde{\eta_R}\left(\frac{g_{\mu\nu} - k_\mu k_\nu/M^2_{z'}}{k^2 - M^2_{Z'}}\right)\overline{f}\gamma^\nu(c_{v2} - c_{a2}\gamma^5)f \tag{8.5}$$

Note that we can separate the amplitude in two terms

$$\begin{aligned}
\imath\mathcal{M}_1 &= \left(\frac{g^2_{Z'}}{k^2 - M^2_{Z'}}\right)\overline{n}\gamma^\mu(c_{v1} - c_{a1}\gamma^5)n(g_{\mu\nu})\overline{f}\gamma^\nu(c_{v2} - c_{a2}\gamma^5)f \qquad (8.6)\\
\imath\mathcal{M}_2 &= \left(\frac{g^2_{Z'}}{M^2_{Z'}(k^2 - M^2_{Z'})}\right)\overline{n}\gamma^\mu(c_{v1} - c_{a1}\gamma^5)n\left(-k_\mu k_\nu\right)\overline{f}\gamma^\nu(c_{v2} - c_{a2}\gamma^5)f
\end{aligned}$$

The amplitude square will be given then by

$$|\mathcal{M}|^2 = |\mathcal{M}_1 + \mathcal{M}_2|^2 = |\mathcal{M}_1|^2 + |\mathcal{M}_2|^2 + \mathcal{M}_1\mathcal{M}_2^* + \mathcal{M}_1^*\mathcal{M}_2 \tag{8.7}$$

Using Dirac's equations $(\gamma^\mu p_\mu - m)u = 0$, $(\gamma^\mu p_\mu + m)v = 0$ the fact that fields anticommute, and $\Gamma^\alpha = \gamma^\alpha(c_v - c_a\gamma^5)$, $\Gamma^5 = (c_v - c_a\gamma^5)$ we can write these terms explicitly,

$$\imath\mathcal{M}_1 \sim \overline{n}(p_2)\Gamma^\alpha n(p_1)g_{\alpha\beta}\overline{f}(q_2)\Gamma^\beta f(q_1) \tag{8.8}$$

$$\begin{aligned}
\imath\mathcal{M}_2 &\sim \overline{n}(p_2)\gamma^\alpha\Gamma^5_1 n(p_1)k_\alpha k_\beta\overline{f}(q_2)\gamma^\beta\Gamma^5_2 f(q_1) \qquad (8.9)\\
&= \overline{n}(p_2)\not{k}\Gamma^5_1 n(p_1)\overline{f}(q_2)\not{k}\Gamma^5_2 f(q_1)\\
&= (\overline{n}(p_2)\not{p}_1 n(p_1)\Gamma^5_1 + \overline{n}(p_2)\not{p}_2 n(p_1)\Gamma^5_1)(\overline{f}(q_2)\not{q}_1 f(q_1)\Gamma^5_2 + \overline{f}(q_2)\not{q}_2 f(p_1)\Gamma^5_2)\\
&= (2M)(2m)(\overline{n}(p_2)\Gamma^5_1 n(p_1)\overline{f}(q_2)\Gamma^5_2 f(p_1))
\end{aligned}$$

Notice that $\mathcal{M}_1$ and $\mathcal{M}_2$ have the same structure.

Calculating the squares,

$$
\begin{aligned}
|\mathcal{M}_1|^2 &\sim \left[\overline{n}_a(p_2)\Gamma^{\alpha}_{1ab}n_b(p_1)g_{\alpha\beta}\overline{f_c}(q_2)\Gamma^{\beta}_{2cd}f_d(q_1)\right]\left[\overline{f}_e(q_1)\Gamma^{\mu}_{2ef}f_f(q_4)g_{\mu\nu}\overline{n}_g(p_1)\Gamma^{\nu}_{1gh}n_h(p_2)\right] \\
&= (-1)^{14}g_{\alpha\beta}g_{\mu\nu}\left[\Gamma^{\alpha}_{1ab}n_b(p_1)\overline{n}_g(p_1)\Gamma^{\nu}_{1gh}n_h(p_2)\overline{n}_a(p_2)\right] \qquad (8.10) \\
&\times \left[\Gamma^{\beta}_{2cd}f_d(q_1)\overline{f}_e(q_1)\Gamma^{\mu}_{2ef}f_f(q_2)\overline{f}_c(q_2)\right] \\
&= Tr\left[\Gamma^{\alpha}_1(\not{p}_1 + M)\Gamma_{\mu 1}(\not{p}_2 - M)\right] \times Tr\left[\Gamma_{\alpha 2}(\not{q}_1 + m)\Gamma^{\mu}_2(\not{q}_2 - m)\right]
\end{aligned}
$$

$$
\begin{aligned}
|\mathcal{M}_2|^2 &\sim (16M^2m^2)\left[\overline{n}_a(p_2)\Gamma^5_{1ab}n_b(p_1)\overline{f_c}(q_2)\Gamma^5_{2cd}f_d(q_1)\right]\left[\overline{f}_e(q_1)\Gamma^5_{2ef}f_f(q_4)\overline{n}_g(p_1)\Gamma^5_{1gh}n_h(p_2)\right] \\
&= (-1)^{14}(16M^2m^2)\left[\Gamma^5_{1ab}n_b(p_1)\overline{n}_g(p_1)\Gamma^5_{1gh}n_h(p_2)\overline{n}_a(p_2)\right] \qquad (8.11) \\
&\times \left[\Gamma^5_{2cd}f_d(q_1)\overline{f}_e(q_1)\Gamma^5_{2ef}f_f(q_2)\overline{f}_c(q_2)\right] \\
&= (16M^2m^2)Tr\left[\Gamma^5_1(\not{p}_1 + M)\Gamma^5_1(\not{p}_2 - M)\right] \times Tr\left[\Gamma^5_2(\not{q}_1 + m)\Gamma^5_2(\not{q}_2 - m)\right]
\end{aligned}
$$

And the cross terms,

$$
\begin{aligned}
\mathcal{M}_1\mathcal{M}_2^* &\sim (4Mm)\left[\overline{n}_a(p_2)\Gamma^{\alpha}_{1ab}n_b(p_1)g_{\alpha\beta}\overline{f_c}(q_2)\Gamma^{\beta}_{2cd}f_d(q_1)\right]\left[\overline{f}_e(q_1)\Gamma^5_{2ef}f_f(q_4)\overline{n}_g(p_1)\Gamma^5_{1gh}n_h(p_2)\right] \\
&= (-1)^{14}(4Mm)g_{\alpha\beta}\left[\Gamma^{\alpha}_{1ab}n_b(p_1)\overline{n}_g(p_1)\Gamma^5_{1gh}n_h(p_2)\overline{n}_a(p_2)\right] \qquad (8.12) \\
&\times \left[\Gamma^{\beta}_{2cd}f_d(q_1)\overline{f}_e(q_1)\Gamma^5_{2ef}f_f(q_2)\overline{f}_c(q_2)\right] \\
&= (4Mm)Tr\left[\Gamma^{\alpha}_1(\not{p}_1 + M)\Gamma^5_1(\not{p}_2 - M)\right] \times Tr\left[\Gamma_{\alpha 2}(\not{q}_1 + m)\Gamma^5_2(\not{q}_2 - m)\right] \\
&\sim \mathcal{M}_2\mathcal{M}_1^*
\end{aligned}
$$

We can find the mass dependence of the amplitude by choosing a center of mass coordinate system for the four-momentums, and doing some simple algebra, with a non relativistic approximation,

$$
\begin{aligned}
p_1 &= (E, 0, 0, p) \qquad (8.13) \\
p_2 &= (E, 0, 0, -p) \\
q_1 &= (E, q\sin\theta, 0, q\cos\theta) \\
q_1 &= (E, -q\sin\theta, 0, -q\cos\theta) \\
k &= p_1 + p_2 = (2E, 0, 0, 0)
\end{aligned}
$$

$$
\begin{aligned}
E &= \gamma M_\eta = \frac{M_n}{\sqrt{1-v^2}} \simeq M_n \qquad (8.14)\\
p_1 \cdot p_2 &= E^2 + p^2 = E^2 + (E^2 - M_n^2) = 2E^2 - M_n^2 \simeq M_n^2\\
q_1 \cdot q_2 &= E^2 + q^2 = E^2 + (E^2 - m_f^2) = 2E^2 - m_f^2 \simeq 2M_n^2\\
k^2 &= 4E^2 = 4\gamma^2 M_\eta^2 = 4\frac{M_n^2}{1-v^2} = 4M_n^2(1+v^2+...) \simeq 4M_{\eta n}^2\\
k \cdot p_1 &= k \cdot p_2 = k \cdot q_1 = k \cdot q_2 = 2E^2 \simeq 2M_n^2
\end{aligned}
$$

Using FeynCalc [48] we can calculate the traces, the total amplitude will be given by,

$$
\begin{aligned}
|\mathcal{M}|^2 &= \left(\frac{64 g_{Z'}^4 M_n^4}{(4M_n^2 - M_{Z'}^2)^2}\right)\Big[2c_{v1}^2(c_{v2}^2 + c_{a2}^2) \qquad (8.15)\\
&+ \left(\frac{m_f}{M_n}\right)^2 \left[c_{v1}^2(c_{v2}^2 - c_{a2}^2) + c_{a2}^2(c_{a1}^2 - c_{v1}^2)\right]\\
&+ 8\left(\frac{m_f}{M_{Z'}}\right)^2 \Big[\left(\frac{M_n}{M_{Z'}}\right)^2 (2c_{a1}^2(c_{a2}^2 - c_{v2}^2)) + \left(\frac{m_f}{M_{Z'}}\right)^2 (2c_{a1}^2 c_{v2}^2) + c_{a1}^2 c_{a2}^2\Big]\Big]
\end{aligned}
$$

All contributions with $m_f \neq 0$ come from $\mathcal{M}_2$ as expected.

From table 4.2 we can extract the coupling of the scotino and Z' (since is the same as for $\tilde{\eta}_R^0$ the results remain valid for both particles). Using $c_{v1} = c_{a1} = \frac{c_L^2}{4}$, $g_{Z'}^2 = g_R^2/c_L^2(c_L^2 - s_R^2)$ and taking the average over the initial spins (a factor of is 1/4 added for 2 particles with 2 spins), we get,

$$
\begin{aligned}
\langle|\mathcal{M}|^2\rangle &= \frac{1}{4}\sum_{spins} |\mathcal{M}|^2 \qquad (8.16)\\
&\simeq \left(\frac{2g_R^4 M_n^4 (c_{v2}^2 + c_{a2}^2)}{(4M_n^2 - M_{Z'}^2)^2 (c_L^2 - s_R^2)^2}\right)\Bigg[1 + m_f^2\Bigg[\left(\frac{1}{2M_n^2}\right)\left(\frac{c_{v2}^2 - c_{a2}^2}{c_{v2}^2 + c_{a2}^2}\right)\\
&+ 8\Big[\left(\frac{M_n}{M_{Z'}^2}\right)^2 \left(\frac{c_{a2}^2 - c_{v2}^2}{c_{v2}^2 + c_{a2}^2}\right) + \left(\frac{m_f}{M_{Z'}^2}\right)^2 \left(\frac{c_{v2}^2}{c_{v2}^2 + c_{a2}^2}\right) + \left(\frac{1}{2M_{Z'}^2}\right)\left(\frac{c_{a2}^2}{c_{v2}^2 + c_{a2}^2}\right)\Big]\Bigg]\Bigg] \qquad (8.17)
\end{aligned}
$$

For scattering into SM particles, we will use the approximation $m_f \simeq 0$ this means that we will use a prefactor of 9 for the up and down quarks (3 colors $\times$ 3 families) and 3 for leptons and neutrinos (3 families),

For $n + n \to SM + SM$, the scattering cross section times the relative speed will be then,

$$
\langle \sigma v \rangle_{SM} \simeq \frac{g_R^4}{64\pi}\frac{M_n^2}{(4M_n^2 - M_{Z'}^2)^2}\frac{(10s_R^4 - 9s_R^2 c_L^2 + 3c_L^4)}{(c_L^2 - s_R^2)^2} \qquad (8.18)
$$

With the unification condition $g_R = g_L$ then (Using $g_L^2 \simeq 0.427$ and $\cos^2\theta_W \simeq .77$),

$$\langle \sigma v \rangle_{SM} \simeq \frac{g_L^4}{64\pi} \frac{M_n^2}{(4M_n^2 - M_{Z'}^2)^2} \frac{(10 - 29c_W^2 + 22c_W^4)}{(2c_W^2 - 1)^2} \simeq \frac{(2.22 \times 10^{-3})M_n^2}{(4M_n^2 - M_{Z'}^2)^2} \tag{8.19}$$

8.1.2 t-channel

Only the scotino will have t-channel scattering into SM Fermions as seen in fig. 8.1, even more given that this interaction is via the $SU(2)_R$ charged Boson, the end result can only be the partner of the soctiono in a $SU(2)_R$ doublet (i.e. e^c). This interaction then will only have right handed helicity and be given by,

$$\imath\mathcal{M} = \frac{g_R^2}{2} * \overline{n}\gamma^\mu \left(\frac{1+\gamma^5}{2}\right) e \left(\frac{g_{\mu\nu} - t_\mu t_\nu / M_{W_R}^2}{t^2 - M_{W_R}^2}\right) \overline{e}\gamma^\nu \left(\frac{1+\gamma^5}{2}\right) n \tag{8.20}$$

Following the same procedure as in 8.1.1 by separating the amplitude in two,

$$\imath\mathcal{M}_1 = \left(\frac{g_R^2}{8(t^2 - M_{W_R}^2)}\right) * \overline{\tilde{n}}\gamma^\mu \left(1+\gamma^5\right) f \left(g_{\mu\nu}\right) \overline{f}\gamma^\nu \left(1+\gamma^5\right) n \tag{8.21}$$

$$\begin{aligned} \imath\mathcal{M}_2 &= \left(\frac{g_R^2}{8M_{W_R}^2(t^2 - M_{W_R}^2)}\right) * \overline{\tilde{n}}\gamma^\mu \left(1+\gamma^5\right) f \left(-t_\mu t_\nu\right) \overline{f}\gamma^\nu \left(1+\gamma^5\right) n \\ &= \left(\frac{g_R^2(M_n + m_f)^2}{8M_{W_R}^2(t^2 - M_{W_R}^2)}\right) * \overline{\tilde{n}} \left(1+\gamma^5\right) f\overline{f} \left(1+\gamma^5\right) n \end{aligned} \tag{8.22}$$

Using FeynCalc the square of the amplitudes (and the cross terms) are,

$$\begin{aligned} |\mathcal{M}_1|^2 &\sim \left[\overline{n}_a(p_1)\Gamma^\alpha_{1ab} f_b(q_1) g_{\alpha\beta} \overline{f_c}(q_2)\Gamma^\beta_{2cd} n_d(p_2)\right]\left[\overline{n}_e(p_2)\Gamma^\mu_{2ef} f_f(q_2) g_{\mu\nu} \overline{f}_g(q_1)\Gamma^\nu_{1gh} n_h(p_1)\right] \\ &= (-1)^{14} g_{\alpha\beta} g_{\mu\nu} \left[\Gamma^\alpha_{1ab} f_b(q_1)\overline{f}_g(q_1)\Gamma^\nu_{1gh} n_h(p_1)\overline{n}_a(p_1)\right] \\ &\times \left[\Gamma^\beta_{2cd} n_d(p_2)\overline{n}_e(p_2)\Gamma^\mu_{2ef} f_f(q_2)\overline{f}_c(q_2)\right] \\ &= Tr\left[\Gamma^\alpha_1(\not{q}_1 + m_f)\Gamma^\nu_1(\not{p}_1 + M_n)\right] \times Tr\left[\Gamma_{\alpha 2}(\not{p}_2 - M_n)\Gamma_{\nu 2}(\not{q}_2 - m_f)\right] \\ &\simeq \left(\frac{g_R^4}{64(t^2 - M_{W_R}^2)^2}\right) 256(p1\cdot q2)(p2\cdot q1) \\ &\simeq \left(\frac{4g_R^4 M_n^4}{(M_n^2 + M_{W_R}^2 - m_f^2)^2}\right) \end{aligned} \tag{8.23}$$

$$
\begin{aligned}
|\mathcal{M}_2|^2 &= \left(\frac{g_R^2(M_n+m_f)^2}{8M_{W_R}^2(t^2-M_{W_R}^2)}\right)^2 \\
&* \; Tr\big[(1+\gamma^5)(\not{q}_1+m_f)(1+\gamma^5)(\not{p}_1+M_n)\big]Tr\big[(1+\gamma^5)(\not{p}_2-M_n)(1+\gamma^5)(\not{q}_2-m_f)\big] \\
&= \left(\frac{g_R^4(M_n+m_f)^4M_n^2m_f^2}{M_{W_R}^4(M_n^2+M_{W_R}^2-m_f^2)^2}\right)
\end{aligned} \tag{8.24}
$$

$$
\begin{aligned}
\mathcal{M}_1\mathcal{M}_2^* &= \left(\frac{g_R^4(M_n+m_f)^2}{64M_{W_R}^2(t^2-M_{W_R}^2)^2}\right) \\
&* \; Tr\big[\gamma^\mu(1+\gamma^5)(\not{q}_1+m_f)(1+\gamma^5)(\not{p}_1+M_n)\big]Tr\big[\gamma_\mu(1+\gamma^5)(\not{p}_2-M_n)(1+\gamma^5)(\not{q}_2-m_f)\big] \\
&\simeq \left(-\frac{g_R^4(M_n+m_f)^2M_n^3m_f}{M_{W_R}^2(M_n^2+M_{W_R}^2-m_f^2)^2}\right) \simeq \mathcal{M}_2\mathcal{M}_1^*
\end{aligned} \tag{8.25}
$$

The total amplitude for the t-channel is then given by,

$$
\frac{1}{4}\sum_{spins}|\mathcal{M}|^2 = \tfrac{1}{4}\left(\frac{4g_R^4M_n^4}{(M_n^2+M_{W_R}^2-m_f^2)^2}\right) \tag{8.26}
$$

$$
\left[1-\frac{1}{2}\left(\frac{m_f}{M_n}\right)\left(\frac{M_n+m_f}{M_{W_R}}\right)^2+\frac{1}{4}\left(\frac{m_f}{M_n}\right)^2\left(\frac{M_n+m_f}{M_{W_R}}\right)^4\right] \tag{8.27}
$$

The only interaction in this channel is between W_R^+, n and e. we include a prefactor of 3(3 families)

$$
\langle\sigma v\rangle_{SM} \simeq \left(\frac{3}{4}\right)\left(\frac{1}{64\pi M_n^2}\right)\left(\frac{4g_R^4M_n^4}{(M_n^2+M_{W_R}^2)^2}\right) = \left(\frac{3g_R^4M_n^2}{64\pi(M_n^2+M_{W_R}^2)^2}\right) \simeq \frac{0.00272M_n^2}{(M_n^2+M_{W_R}^2)^2} \tag{8.28}
$$

8.1.3 Cross terms s-t channels

There is a cross term between the s-channel and the t-channel

$$
\mathcal{M}_s\mathcal{M}_t^* = \mathcal{M}_{s1}\mathcal{M}_{t1}^* + \mathcal{M}_{s1}\mathcal{M}_{t2}^* + \mathcal{M}_{s2}\mathcal{M}_{t1}^* + \mathcal{M}_{s2}\mathcal{M}_{t2}^* \tag{8.29}
$$

Using the definitions

$\Gamma_1^\alpha = (c_L^2/4)\gamma^\alpha(1-\gamma^5)$, $\Gamma_2^\alpha = \gamma^\alpha(c_v - c_a\gamma^5)$, $\Gamma_3^\alpha = \Gamma_4^\alpha = \gamma^\alpha(1+\gamma^5)$,

$\Gamma_1^5 = (c_L^2/4)(1-\gamma^5)$, $\Gamma_2^5 = (c_v - c_a\gamma^5)$, $\Gamma_3^5 = \Gamma_4^5 = (1+\gamma^5)$

We can write these terms explicitly,

$$
\begin{aligned}
&\mathcal{M}_{s1}\mathcal{M}^*_{t1} && (8.30)\\
\sim\quad & \left[\overline{n}_a(p_1)\Gamma^{\alpha}_{1ab}n_b(p_2)g_{\alpha\beta}\overline{f_c}(q_2)\Gamma^{\beta}_{2cd}f_d(q_1)\right]\left[\overline{n}_e(p_2)\Gamma^{\mu}_{3ef}f_f(q_2)g_{\mu\nu}\overline{f}_g(q_1)\Gamma^{\nu}_{4gh}n_h(p_1)\right]\\
\sim\quad & (-1)^{11}g_{\alpha\beta}g_{\mu\nu}Tr\left[\Gamma^{\alpha}_1(\not{p}_2-M_n)\Gamma^{\mu}_3(\not{q}_2-m_f)\Gamma^{\beta}_2(\not{q}_1+m_f)\Gamma^{\nu}_4(\not{p}_1+M_n)\right]\\
\sim\quad & 16c_L^2M_n^2(2m_f^2(c_a+c_v)+(c_v-c_a)q_1\cdot q_2)\simeq 16c_L^2(m_f^2M_n^2(3c_a+c_v)-2M_n^4(c_a-c_v))
\end{aligned}
$$

$$
\begin{aligned}
&\mathcal{M}_{s1}\mathcal{M}^*_{t2} && (8.31)\\
\sim\quad & (M_n+m_f)^2\left[\overline{n}_a(p_1)\Gamma^{\alpha}_{1ab}n_b(p_2)g_{\alpha\beta}\overline{f_c}(q_2)\Gamma^{\beta}_{2cd}f_d(q_1)\right]\left[\overline{n}_e(p_2)\Gamma^{5}_{3ef}f_f(q_2)\overline{f}_g(q_1)\Gamma^{5}_{4gh}n_h(p_1)\right]\\
\sim\quad & (-1)^{11}(M_n+m_f)^2g_{\alpha\beta}Tr\left[\Gamma^{\alpha}_1(\not{p}_2-M_n)\Gamma^{5}_3(\not{q}_2-m_f)\Gamma^{\beta}_2(\not{q}_1+m_f)\Gamma^{5}_4(\not{p}_1+M_n)\right]\\
\sim\quad & -8c_L^2(M_n+m_f)^2m_fM_n((c_a+c_v)p_2\cdot q_1+2(c_v-c_a)p_2\cdot q_2)\\
\simeq\quad & 8c_L^2(M_n+m_f)^2m_fM_n^3(c_a-3c_v)
\end{aligned}
$$

$$
\begin{aligned}
&\mathcal{M}_{s2}\mathcal{M}^*_{t1} && (8.32)\\
\sim\quad & 4M_nm_f\left[\overline{n}_a(p_1)\Gamma^{5}_{1ab}n_b(p_2)\overline{f_c}(q_2)\Gamma^{5}_{2cd}f_d(q_1)\right]\left[\overline{n}_e(p_2)\Gamma^{\mu}_{3ef}f_f(q_2)g_{\mu\nu}\overline{f}_g(q_1)\Gamma^{\nu}_{4gh}n_h(p_1)\right]\\
\sim\quad & (-1)^{11}4M_nm_fg_{\mu\nu}Tr\left[\Gamma^{5}_1(\not{p}_2-M_n)\Gamma^{\mu}_3(\not{q}_2-m_f)\Gamma^{5}_2(\not{q}_1+m_f)\Gamma^{\nu}_4(\not{p}_1+M_n)\right]\\
\sim\quad & -32c_L^2m_f^2M_n^2((c_a-c_v)p_1\cdot q_1+(c_a+c_v)p_1\cdot q_2)\\
\simeq\quad & -64c_L^2m_f^2M_n^4c_a
\end{aligned}
$$

$$
\begin{aligned}
&\mathcal{M}_{s2}\mathcal{M}^*_{t2} && (8.33)\\
\sim\quad & 4M_nm_f(M_n+m_f)^2\left[\overline{n}_a(p_1)\Gamma^{5}_{1ab}n_b(p_2)\overline{f_c}(q_2)\Gamma^{5}_{2cd}f_d(q_1)\right]\left[\overline{n}_e(p_2)\Gamma^{5}_{3ef}f_f(q_2)\overline{f}_g(q_1)\Gamma^{5}_{4gh}n_h(p_1)\right]\\
\sim\quad & (-1)^{11}4M_nm_f(M_n+m_f)^2Tr\left[\Gamma^{5}_1(\not{p}_2-M_n)\Gamma^{5}_3(\not{q}_2-m_f)\Gamma^{5}_2(\not{q}_1+m_f)\Gamma^{5}_4(\not{p}_1+M_n)\right]\\
\sim\quad & -16c_L^2M_nm_f(M_n+m_f)^2(p_1\cdot p_2(m_f^2(c_a-c_v)+(c_a+c_v)q_1\cdot q_2)-(c_a+c_v)(p_1\cdot q_2p_2\cdot q_1-p_1\cdot q_1p_2\cdot q_2))\\
\simeq\quad & 32c_L^2M_nm_f(M_n+m_f)^2(-M_n^4(c_a+c_v)+m_f^2M_n^2c_v)
\end{aligned}
$$

$$
\begin{aligned}
& \mathcal{M}_{t1}\mathcal{M}^*_{s1} \qquad (8.34) \\
\sim\ & \left[\overline{n}_a(p_1)\Gamma^{\nu}_{4ab}f_b(q_1)g_{\nu\mu}\overline{f_c}(q_2)\Gamma^{\mu}_{3cd}n_d(p_2)\right]\left[\overline{f}_e(q_1)\Gamma^{\beta}_{2ef}f_f(q_2)g_{\beta\alpha}\overline{n}_g(p_2)\Gamma^{\alpha}_{1gh}n_h(p_1)\right] \\
\sim\ & (-1)^{11}g_{\beta\alpha}g_{\nu\mu}Tr\left[\Gamma^{\nu}_4(\not{q}_1+m_f)\Gamma^{\beta}_2(\not{q}_2-m_f)\Gamma^{\mu}_3(\not{p}_2-M_n)\Gamma^{\alpha}_1(\not{p}_1+M_n)\right] \\
\sim\ & 16c_L^2M_n^2(2m_f^2(c_a+c_v)+(c_v-c_a)q_1\cdot q_2)) \\
\simeq\ & 16c_L^2(m_f^2M_n^2(3c_a+c_v)-2M_n^4(c_a-c_v))
\end{aligned}
$$

$$
\begin{aligned}
& \mathcal{M}_{t1}\mathcal{M}^*_{s2} \qquad (8.35) \\
\sim\ & 4M_nm_f\left[\overline{n}_a(p_1)\Gamma^{\nu}_{4ab}f_b(q_1)g_{\nu\mu}\overline{f_c}(q_2)\Gamma^{\mu}_{3cd}n_d(p_2)\right]\left[\overline{f}_e(q_1)\Gamma^{5}_{2ef}f_f(q_2)\overline{n}_g(p_2)\Gamma^{5}_{1gh}n_h(p_1)\right] \\
\sim\ & (-1)^{11}4M_nm_fg_{\nu\mu}Tr\left[\Gamma^{\nu}_4(\not{q}_1+m_f)\Gamma^{5}_2(\not{q}_2-m_f)\Gamma^{\mu}_3(\not{p}_2-M_n)\Gamma^{5}_1(\not{p}_1+M_n)\right] \\
\sim\ & -32c_L^2M_n^2m_f^2((c_a+c_v)p_2\cdot q_1+(c_a-c_v)p_2\cdot q_2) \\
\simeq\ & -64c_L^2m_f^2M_n^4c_a
\end{aligned}
$$

$$
\begin{aligned}
& \mathcal{M}_{t2}\mathcal{M}^*_{s1} \qquad (8.36) \\
\sim\ & (M_n+m_f)^2\left[\overline{n}_a(p_1)\Gamma^{5}_{4ab}f_b(q_1)\overline{f_c}(q_2)\Gamma^{5}_{3cd}n_d(p_2)\right]\left[\overline{f}_e(q_1)\Gamma^{\beta}_{2ef}f_f(q_2)\overline{n}_g(p_2)\Gamma^{\alpha}_{1gh}n_h(p_1)\right] \\
\sim\ & (-1)^{11}(M_n+m_f)^2g_{\beta\alpha}Tr\left[\Gamma^{5}_4(\not{q}_1+m_f)\Gamma^{\beta}_2(\not{q}_2-m_f)\Gamma^{5}_3(\not{p}_2-M_n)\Gamma^{\alpha}_1(\not{p}_1+M_n)\right] \\
\sim\ & -8c_L^2(M_n+m_f)^2m_fM_n((c_a+c_v)p_1\cdot q_2-2(c_a-c_v)p_1q_1) \\
\simeq\ & 8c_L^2(M_n+m_f)^2m_fM_n^3(c_a-3c_v)
\end{aligned}
$$

$$
\begin{aligned}
& \mathcal{M}_{t2}\mathcal{M}^*_{s2} \qquad (8.37) \\
\sim\ & 4M_nm_f(M_n+m_f)^2\left[\overline{n}_a(p_1)\Gamma^{5}_{4ab}f_b(q_1)\overline{f_c}(q_2)\Gamma^{5}_{3cd}n_d(p_2)\right]\left[\overline{f}_e(q_1)\Gamma^{5}_{2ef}f_f(q_2)\overline{n}_g(p_2)\Gamma^{5}_{1gh}n_h(p_1)\right] \\
\sim\ & (-1)^{11}4M_nm_f(M_n+m_f)^2Tr\left[\Gamma^{5}_4(\not{q}_1+m_f)\Gamma^{5}_2(\not{q}_2-m_f)\Gamma^{5}_3(\not{p}_2-M_n)\Gamma^{5}_1(\not{p}_1+M_n)\right] \\
\sim\ & -16c_L^2M_nm_f(M_n+m_f)^2(p_1\cdot p_2(m_f^2(c_a-c_v)+(c_a+c_v)q_1\cdot q_2))-(c_a+c_v)(p_1\cdot q_2p_2\cdot q_1-p_1\cdot q_1p_2\cdot q_2)) \\
\simeq\ & 32c_L^2M_nm_f(M_n+m_f)^2(-M_n^4(c_a+c_v)+m_f^2M_n^2c_v)
\end{aligned}
$$

The total amplitude for this cross terms is then,

$$\mathcal{M}_s\mathcal{M}_t^* + \mathcal{M}_t\mathcal{M}_s^* \simeq \left(\frac{g_R^4 M_n^4}{(c_L^2 - s_R^2)(M_n^2 + M_{W_R}^2 - m_f^2)(4M_n^2 - M_{Z'}^2)}\right) \times \left[8(c_a - c_v) - 2\left(\frac{m_f}{M_n}\right)\left(\frac{M_n + m_f}{M_{W_R}}\right)^2 (c_a - 3c_v) - 4\left(\frac{m_f}{M_n}\right)\left(\frac{M_n + m_f}{M_{Z'}M_{W_R}}\right)^2 \left(m_f^2(3c_a + c_v) - 2M_n^2(c_a - c_v)\right) - 4\left(\frac{m_f}{M_n}\right)^2 (3c_a + c_v)\right] \tag{8.38}$$

when $m_f \simeq 0$

$$\frac{1}{4}\sum_{spin} \mathcal{M}_s\mathcal{M}_t^* + \mathcal{M}_t\mathcal{M}_s^* \simeq \left(\frac{2(c_a - c_v) g_R^4 M_n^4}{(c_L^2 - s_R^2)(M_n^2 + M_{W_R}^2 - m_f^2)(4M_n^2 - M_{Z'}^2)}\right) \tag{8.39}$$

for scattering into SM.

$$\langle \sigma v \rangle_{SM} = \frac{g_R^4 M_n^2}{64\pi}\left[\frac{3(c_L^2 - 2s_R^2)}{(4M_n^2 - M_{Z'}^2)(M_n^2 + M_{W_R}^2)(c_L^2 - s_R^2)}\right] \simeq \frac{0.00156 M_n^2}{(4M_n^2 - M_{Z'}^2)(M_n^2 + M_{W_R}^2)} \tag{8.40}$$

8.2 Relic Density

As a reminder, the annihilation cross-sections for DM $\tilde{\eta}_R^0$ to SM particles goes through s-channel diagram exchanging Z', while n has an additional piece through a t-channel diagram to $e_R^{\pm}$ through $W_R^{\pm}$ exchange. The Feynman diagrams are shown in Figs. 8.1 and 8.2.

The expressions for thermally averaged cross-section ($\langle \sigma v \rangle$) for these two DM components annihilating to SM are indicated in eq. (8.41) and eq. (8.43).

$$\langle \sigma v \rangle_\eta \simeq \frac{g_L^4 m_{\eta_{R^0}}^2}{64\pi}\left[\frac{10 s_W^4 - 9 s_W^2 c_W^2 + 3 c_W^4}{(4m_{\eta_{R^0}}^2 - M_{Z'}^2)^2 (c_W^2 - s_W^2)^2}\right] \tag{8.41}$$

$$\langle \sigma v \rangle_n \simeq \frac{g_L^4 m_n^2}{64\pi}\left[\frac{10 s_W^4 - 9 s_W^2 c_W^2 + 3 c_W^4}{(4m_n^2 - M_{Z'}^2)^2 (c_W^2 - s_W^2)^2} + \frac{3}{(m_n^2 + M_{W_R}^2)^2} + \frac{3(c_W^2 - 2s_W^2)}{(4m_n^2 - M_{Z'}^2)(m_n^2 + M_{W_R}^2)(c_W^2 - s_W^2)}\right] \tag{8.42}$$

With the unification condition, $g_R^2 = g_L^2 \simeq 0.427$ and $\sin^2\theta_W = 0.23$, numerically, we obtain:

$$\langle \sigma v \rangle_\eta \simeq \frac{0.00222 m_{\tilde{\eta}_R^0}{}^2}{4m_{\tilde{\eta}_R^0}{}^2 - M_Z'^2} \tag{8.43}$$

$$\langle \sigma v \rangle_n \simeq \frac{0.00222 m_n{}^2}{4m_n{}^2 - M_Z'^2} + \frac{0.00272 m_n{}^2}{m_n{}^2 + M_{W_R}{}^2} + \frac{0.00156 m_n{}^2}{(4m_n{}^2 - M_Z'^2)(m_n{}^2 + M_{W_R}{}^2)} \tag{8.44}$$

If we assume the decoupling of $\tilde{\eta}_R^0$, n and $\tilde{\chi}_1^0$ from the hot soup of SM particles are independent of interactions with each other, relic density for each DM component can be approximated as

$$\Omega_i h^2 \simeq \frac{0.1pb}{\langle \sigma v \rangle_i} \tag{8.45}$$

The total abundance will be a sum of the three DM components, i.e.

$$\Omega_{DM_{tot}} h^2 = \Omega_\eta h^2 + \Omega_n h^2 + \Omega_{\chi_1^0} h^2 \tag{8.46}$$

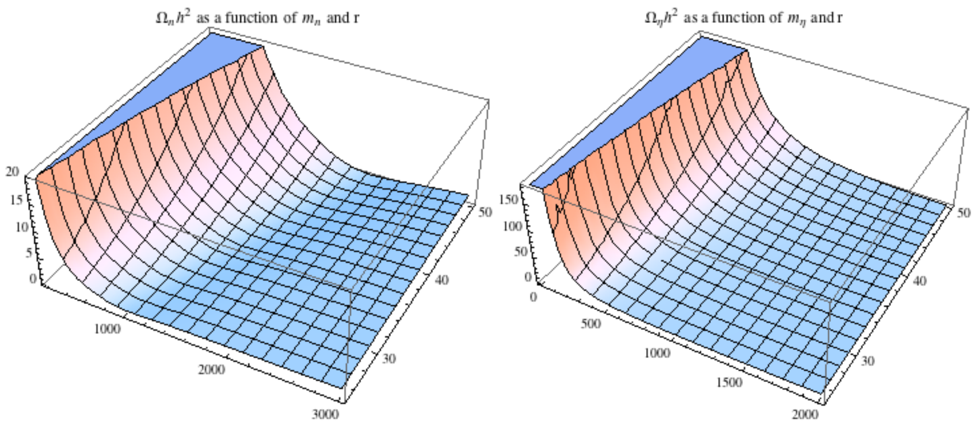


Figure 8.3: A 3-dim plot showing Ωh^2 (z-axis) dependence on mass (x-axis) and r (y-axis). LHS: n and RHS: $\tilde{\eta}_R^0$

With this assumption, we evaluate relic abundance for each of the DM component and look for the parameter space where they add up to the constraint from WMAP [49] [1].

$$0.094 < \Omega_{DM_{tot}} h^2 < 0.130 \tag{8.47}$$

In Fig. 8.3, we show a 3-dimensional plot with Ωh^2 along z-axis, DM mass m along x-axis and the ratio

[1]PLANCK [50] data essentially indicates a very similar range, though more stringent, almost indistinguishable from WMAP in present context.

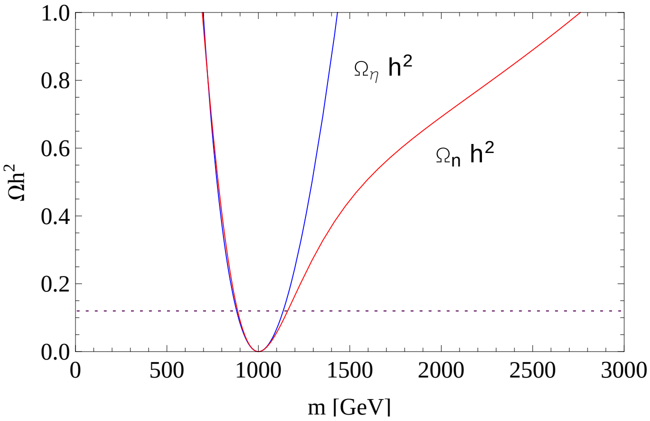


Figure 8.4: Ωh^2 dependence for DM n and $\tilde{\eta}_R^0$ on mass for r =25.

of Higgs vevs r defined in eq. (4.20) along y-axis for the DM component n on LHS and $\tilde{\eta}_R^0$ on RHS. We use eq. (8.43), eq. (8.44) and eq. (8.45) to draw them. Both of the DMs show similar behavior. A cut along the r-axis at 25, shows the dependence of Ωh^2 on DM mass m which is shown in Fig 8.4. The difference in n and $\tilde{\eta}_R^0$ annihilation is clear from here.

In the three component DM framework, we study a scenario where the two components n and $\tilde{\eta}_R^0$ dominate in relic abundance leaving a very tiny space for neutralino $\tilde{\chi}_1^0$. We will discuss neutralino DM shortly. For example, we focus on a region of parameter space, where,

$$\Omega_\eta h^2 + \Omega_n h^2 = 0.1 \tag{8.48}$$

In such a case, if we assume in addition that each of the components contribute equally, then we end up getting Fig. 8.2. This indicates that we obtain two possible masses for a given value of r and Ωh^2 and the difference in n and $\tilde{\eta}_R^0$ annihilation doesn't matter in the range of r and Ωh^2 we are interested. This is shown in the top panel of Fig. 8.2, for n (left) and $\tilde{\eta}_R^0$ (right). They look exactly the same, where DM mass is plotted with r. In the bottom panel, we show the case when one of the components contribute fully to relic abundance with $\Omega_i h^2 = 0.1$. In that case both allowed region of r and m_{DM} shrinks significantly.

Eq. (8.48) is appropriately depicted in Fig. 8.6 for different Z' masses. They represent as three circles (The circular shape is understandable from looking at Fig. 8.4) in m_n and $m_{\tilde{\eta}_R^0}$ plane for $M_Z' = 2$, 3 and 4 TeV around $m_n = m_{\tilde{\eta}_R^0} = M_Z'/2$. The reason is simple to understand; the s-channel diagram is proportional to $\sim 1/(4m_n^2 - M_{Z'}^2)$, it will have a resonance region that contributes greatly for relic abundance. We highlight the case for $M_Z' = 2$ TeV in the RHS of Fig. 8.6. The whole region in green becomes allowed when we have the condition $\Omega_\eta h^2 + \Omega_n h^2 \leq 0.12$ (i.e. the contour shrinks for smaller

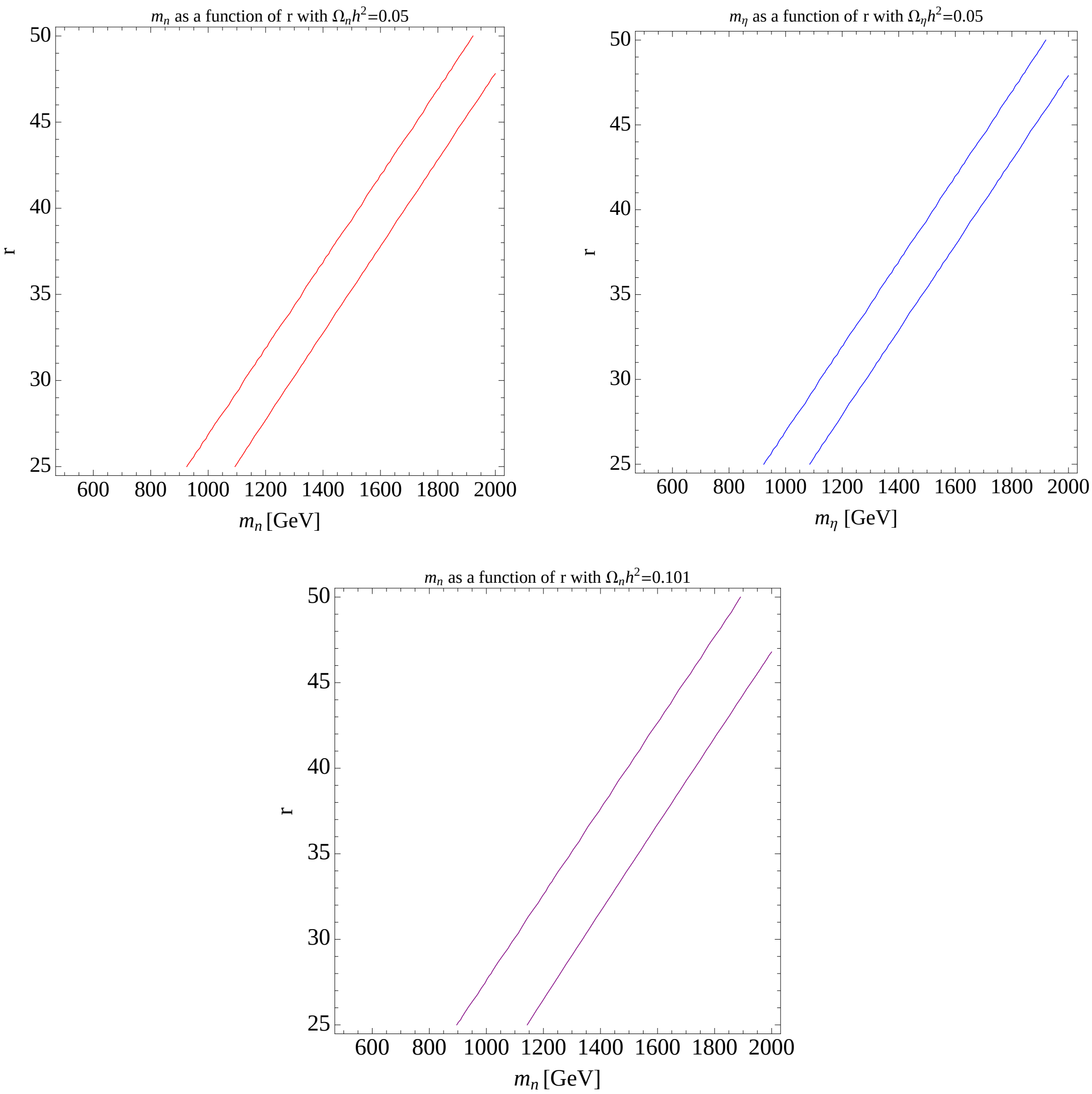


Figure 8.5: Top: $m - r$ dependence when each of the DM component n (left) and $\tilde{\eta}_R^0$ (right) DM contributing equally to the relic density with $\Omega_i h^2$ =0.05. Bottom: When one component dominates, i.e. $\Omega_i h^2$ =0.1. Masses are in GeVs.

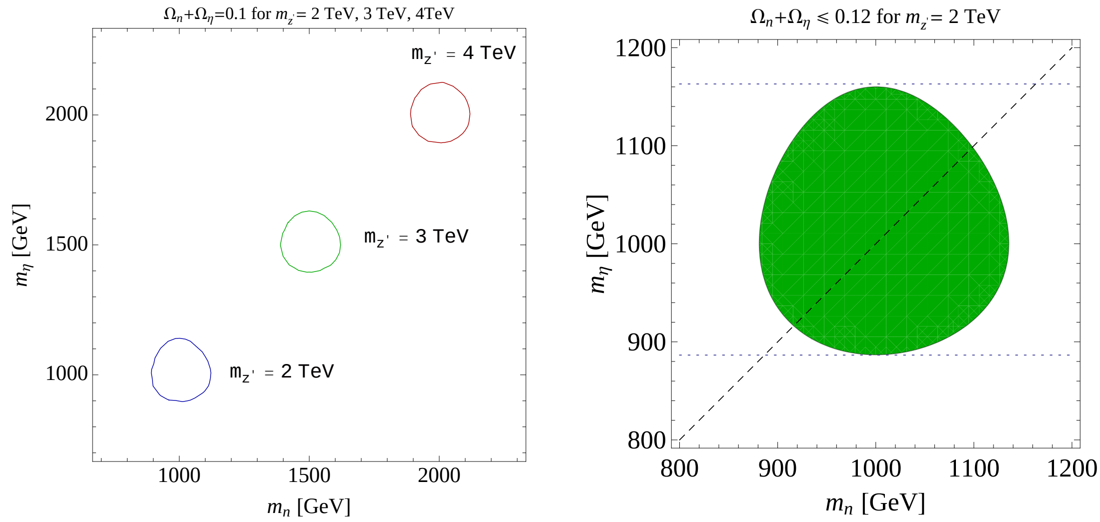


Figure 8.6: c
ontours and region for $\Omega_n h^2 + \Omega_\eta h^2$]LHS: A plot showing m_n-m_η [GeV] contours for $\Omega_n h^2 + \Omega_\eta h^2$=0.1 for $M_Z' = 2$, 3, 4 TeV. RHS: Region of the m_n-m_η [GeV] parameter space when $\Omega_n h^2 + \Omega_\eta h^2$ $\leq$0.12 with M_Z'= 2 TeV.

abundance). We also note that, if we adhere to the assumption made initially that $m_\eta \geq m_n$, then only half of the circle above the diagonal line is allowed for relic abundance. Given that the plot is close to a perfect circle, $\Omega_n h^2 \gtrsim \Omega_\eta h^2$ in this limit.

Chapter 9

Direct Detection of $\tilde{\eta}_R$ and n

9.1 Direct Detection of n and $\tilde{\eta}_R^0$

Direct detection of n and $\tilde{\eta}_R^0$ takes place through t-channel Z' interaction with quarks. The Feynman graph is shown in Fig. 9.1. Due to only this contribution, the spin-independent (SI) cross-section is very small.

We use `MicrOMEGAs` [51] to calculate the effective SI nucleon scattering cross-section. The parton-level interaction is converted to the nucleon level by using effective nucleon ${f_q}^N$ $(N = p, n)$ couplings defined as [51]

$$\langle N|m_q\bar{\psi}_q\psi_q|N\rangle = {f_q}^N M_N \, , \tag{9.1}$$

where M_N is the nucleon mass and we use the default form factors in [51] as $f_u^p = 0.033$, $f_d^p = 0.023$, $f_s^p = 0.26$, for the proton; $f_u^n = 0.042$, $f_d^n = 0.018$, $f_s^n = 0.26$ for the neutron; while for the heavy quarks the f_q^N are generated by gluon exchange with the nucleon and are given by

$$f_Q^N = \frac{2}{27}\left(1 - \sum_{q=u,d,s} f_q^N\right) \qquad Q = c, t, b. \tag{9.2}$$

Then, the nucleon scattering can be written as (with A_q the amplitude),

$$\sigma^{SI} = \frac{4}{\pi}\left(\frac{m_\chi m_n}{m_\chi + m_n}\right)^2 f_N^2 \tag{9.3}$$

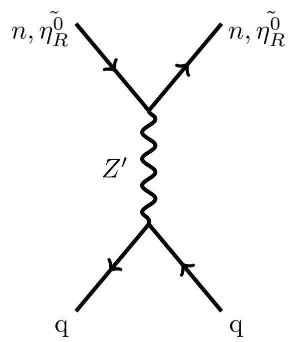


Figure 9.1: Diagram for scattering with quarks for direct detection.

$$f_N = m_N \left(\sum_{q=u,d,s} f_q^N \frac{A_q}{m_q} + \frac{2}{27} \left(1 - \sum_{q=c,b,t} f_q^N \right) \sum_{q=u,d,s} \frac{A_q}{m_q} \right) \tag{9.4}$$

We calculate the amplitude using

$$\imath\mathcal{M} = g_{Z'}^2 * \overline{\tilde{\eta_R}} \gamma^\mu (c_{v1} - c_{a1}\gamma^5) \tilde{\eta_R} \left(\frac{g_{\mu\nu} - t_\mu t_\nu / M_{z'}^2}{t^2 - M_{Z'}^2} \right) \overline{q} \gamma^\nu (c_{v2} - c_{a2}\gamma^5) q \tag{9.5}$$

$$\begin{aligned} t &= p_1 - q_1 = (0, q\sin\theta, 0, p + q\sin\theta) \qquad (9.6) \\ t^2 &= -q^2 \sin^2\theta - p^2 - q^2\cos^2\theta - pq\cos\theta = -(p^2 + q^2 + pq\cos\theta) \\ &= -2(E^2 - M_{\tilde{\eta}}^2)(1 + \cos\theta) \simeq 0 \end{aligned}$$

The amplitude has the same form as the one for the s-channel for relic abundance with the only difference being the denominator of the propagator.

Taking $m_f << M_{\tilde{\eta}}, M_{Z'}$

$$|\mathcal{M}|^2 = \left(\frac{64 g_{Z'}^4 M_{\tilde{\eta}}^4}{M_{Z'}^4} \right) \left[2c_{v1}^2 (c_{v2}^2 + c_{a2}^2) \right] \tag{9.7}$$

Looking at table 4.2 for u, $c_v = \frac{s_R^2 - c_L^2}{4}$, $c_a = \frac{-5s_R^2 + 3c_L^2}{12}$, and for d, $c_v = \frac{-s_R^2}{4}$, $c_a = \frac{s_R^2}{12}$, and for $\tilde{\eta}$, $c_v = c_L^2/4$

$$|\mathcal{M}_u|^2 = \left(\frac{g_{Z'}^4 M_{\tilde{\eta}}^4}{9 M_{Z'}^4} \right) \left[c_L^4 (9c_L^4 - 24 c_L^2 s_R^2 + 17 s_R^4) \right] \tag{9.8}$$

$$|\mathcal{M}_d|^2 = \left(\frac{g_{Z'}^4 M_{\tilde{\eta}}^4}{9M_{Z'}^4}\right)\left[17c_L^4 s_R^4\right] \tag{9.9}$$

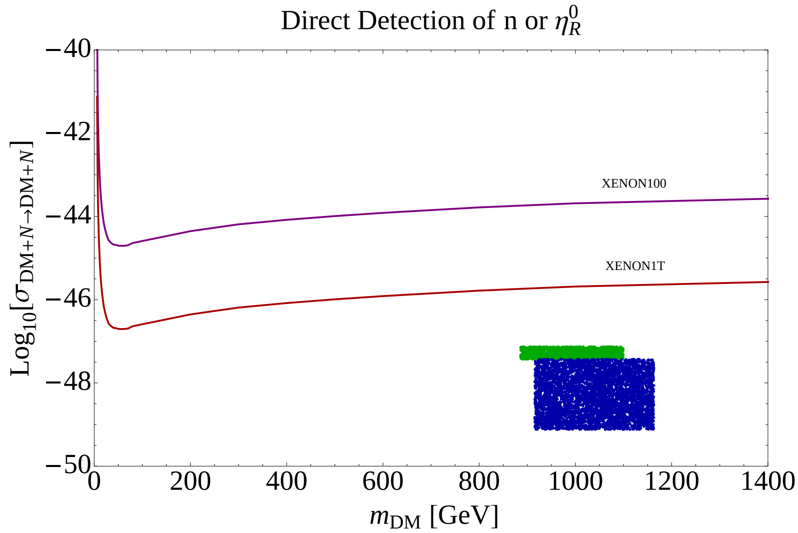


Figure 9.2: Direct detection constraint for DM n or $\tilde{\eta}_R^0$. The upper curve is for XENON100 and the lower one is for XENON1T. Points in the blue box represents spin-independent effective nucleon cross-section for n or $\tilde{\eta}_R^0$ contributing from 1-90 % (bottom to top).

9.2 Results

The results are shown in Fig. 9.2. The bounds from XENON100 (above) and XENON1T (below) are shown in two continuous lines in purple and red respectively. Any points above the XENON100 lines will be discarded by the direct search experiments. In Fig. 9.2, points in blue shows the results of SI direct detection cross-section for $\tilde{\eta}_R^0$ with M_Z'= 2 TeV and those in green represent n within the allowed mass range to obtain correct relic density; m_n between 866-1100 GeV and for m_η between 915-1163 GeV. Although n and $\tilde{\eta}_R^0$ have same quark interaction as in Fig. 9.1 and have same direct detection cross-section, given the mass hierarchy $m_\eta \geq m_n$, n contributes more than $\tilde{\eta}_R^0$ to the dark matter density. Due to multi-component nature of the dark matter, the effective direct detection cross-section is multiplied by the fraction of DM-density for each component $\frac{n_{DM}}{n_{tot}}$ with the nucleon cross-section σ_N (assuming that all of the DMs are accessible to the detector).

$$\sigma_{N eff} = \frac{n_{DM}}{n_{tot}}\sigma_N \simeq \frac{\Omega_{DM}h^2}{\Omega_{tot}h^2}\sigma_N \tag{9.10}$$

The thickness of the direct detection cross-section essentially comes from the fraction $\frac{n_{DM}}{n_{tot}}$, which has been varied between 1-45 % for $\tilde{\eta}_R^0$ (in blue) and 45-90% for n (in green). Hence, points at the bottom of the blue box constitute only 1% while those at the top in green constitute 90% of the total DM. The unequal thickness in blue and green box is due to the logarithmic scale of the effective cross-section.
The direct detection cross-section also doesn't depend on DM mass, while it depends on the Z' mass very much. With higher M_Z' they go down even below to make it harder for direct search. Possibility of early discovery of these DMs in near-future experiments seems to be small, although they are surely allowed by the exclusion limits set by XENON.

Chapter 10

Relic Abundance and Direct detection of Wino type of Neutralino $\tilde{\chi}_1^0$

10.1 MSSM neutralino $\tilde{\chi}^0$

Let us now discuss the lightest neutralino ($\tilde{\chi}_1^0$) as the third DM candidates in this three-component DM set up. The neutralino sector in this extended LR SUSY model is non-trivial and constitutes of three gauginos (M_B, M_L, M_R) and thirteen Higgsinos. Seven out of them, which are superpartners of the scalar fields that do not have a vev, do not mix with the gauginos or the rest of the Higgsinos. This yields to a nine dimensional neutralino mass matrix.

For simplicity, we take a limit where the neutralino DM is predominantly a wino. In this limit, the neutralino of this model can easily mimic minimal supersymmetric Standard Model (MSSM) neutralino, with $M_Y = M_1, \mu_L = \mu, \beta_L = \beta$, and $1.43 M_L = M_2$. This is explicitly shown in Chapter 5. In Fig. 10.1, we show as an example, that when μ_L (x-axis) is larger than M_L (which we set at 0.7 TeV), fraction of bino and Higgsino components in lightest neutralino, in black thick line goes to almost zero; giving rise to a wino DM with the red line reaching 1. We also show that the lightest chargino (in blue, called LC) becomes degenerate with the lightest neutralino (in green) and both have mass around 600 GeV in this particular point in parameter space. This degeneracy is a very well known feature of wino dominated neutralino in MSSM. Note that in order to achieve this limit in this model, we kept $M_R \simeq M_B$ and other

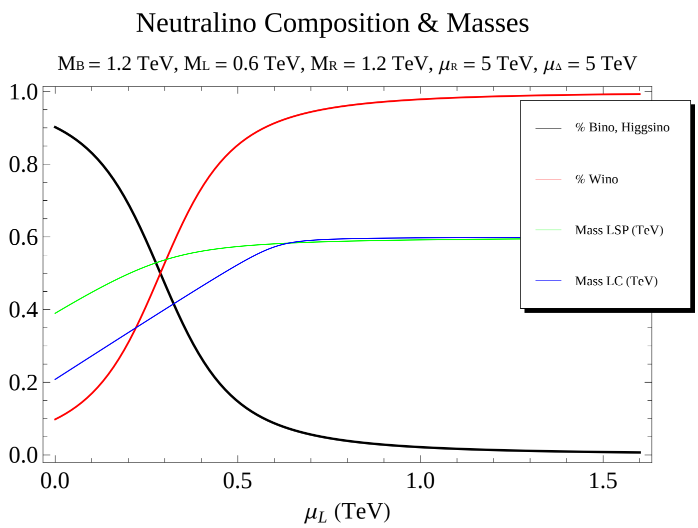


Figure 10.1: Plot showing a limit when the lightest neutralino becomes predominantly a wino and the first chargino becomes degenerate with LSP.

non-MSSM parts heavy, $\mu_R, \mu_\Delta = 5$ TeV.

It is also known that when lightest chargino is degenerate with neutralino DM, co-annihilation occurs [52], making $\tilde{\chi}_1^0$ annihilation cross-section much larger to yield very small abundance. In table 10.2 extracted verbatim from [53], shows all possible scattering interactions relevant for relic abundance including coannihilations with degenerate lightest neutralino and chargino, (H_1 and H_2 are the CP-even higgs Bosons, H_3 is the CP-odd Higgs), from this table is clear the reason we have chosen an MSSM neutralino (where computer simulations can be used) there are 162 possible diagrams. The degeneracy has been crafted in different ways [54–58] to make wino a viable DM candidate by having moduli decay in anomaly mediated SUSY breaking [55] or by non-thermal productions [58] etc. Wino DM has been studied also to justify PAMELA data [59]. However, the under-abundance works perfectly fine for us with the other two components to make up. Of course, other regions of neutralino DM parameter space where it is an admixture of Higgsino-wino-bino that yields under-abundance is also allowed for the model. We show a sample scan of wino dominated neutralino for relic density and direct detection. The MSSM parameter space scanned here: M_1 between 800-1200 GeV, $M_2 \simeq 1.43 M_L$, between 200-775 GeV, and the Higgsino parameter μ between 600-1000 GeV (with $\mu, M_1 > M_2$). In Fig 10.3, we show that the neutralino-DM under abundance for $\Omega_{\chi_1^0} h^2$ is not larger than 0.02 if we keep $m_{\tilde{\chi}_1^0} \leq 800$ GeV (This is following the assumption that neutralino is the lightest of the three DMs and the limit can be increased for higher Z' mass). The neutralino DM constitutes only 1%-20% of the total DM density making eq. (8.48) a good benchmark. Note that the scan yields a pre-dominantly wino, but with some Higgsino component in it. We use MicrOMEGAs [51] to evaluate relic abundance and direct detection cross-sections for neutralino

Initial state	Final state	channel (intermediate)
$\chi_i^0\chi_j^0$	$H_1H_1,\ H_1H_2,\ H_2H_2,\ H_3H_3$	$t(\chi_k^0),\ u(\chi_k^0),\ s(H_{1,2})$
	$H_1H_3,\ H_2H_3$	$t(\chi_k^0),\ u(\chi_k^0),\ s(H_3),\ s(Z^0)$
	H^-H^+	$t(\chi_e^+),\ u(\chi_e^+),\ s(H_{1,2}),\ s(Z^0)$
	$Z^0H_1,\ Z^0H_2$	$t(\chi_k^0),\ u(\chi_k^0),\ s(H_3),\ s(Z^0)$
	Z^0H_3	$t(\chi_k^0),\ u(\chi_k^0),\ s(H_{1,2})$
	$W^-H^+,\ W^+H^-$	$t(\chi_e^+),\ u(\chi_e^+),\ s(H_{1,2,3})$
	Z^0Z^0	$t(\chi_k^0),\ u(\chi_k^0),\ s(H_{1,2})$
	W^-W^+	$t(\chi_e^+),\ u(\chi_e^+),\ s(H_{1,2}),\ s(Z^0)$
	$f\bar{f}$	$t(\tilde{f}_{L,R}),\ u(\tilde{f}_{L,R}),\ s(H_{1,2,3}),\ s(Z^0)$
$\chi_c^+\chi_i^0$	$H^+H_1,\ H^+H_2$	$t(\chi_k^0),\ u(\chi_e^+),\ s(H^+),\ s(W^+)$
	H^+H_3	$t(\chi_k^0),\ u(\chi_e^+),\ s(W^+)$
	$W^+H_1,\ W^+H_2$	$t(\chi_k^0),\ u(\chi_e^+),\ s(H^+),\ s(W^+)$
	W^+H_3	$t(\chi_k^0),\ u(\chi_e^+),\ s(H^+)$
	H^+Z^0	$t(\chi_k^0),\ u(\chi_e^+),\ s(H^+)$
	γH^+	$t(\chi_c^+),\ s(H^+)$
	W^+Z^0	$t(\chi_k^0),\ u(\chi_e^+),\ s(W^+)$
	γW^+	$t(\chi_c^+),\ s(W^+)$
	$u\bar{d}$	$t(\tilde{d}_{L,R}),\ u(\tilde{u}_{L,R}),\ s(H^+),\ s(W^+)$
	$\nu\bar{\ell}$	$t(\tilde{\ell}_{L,R}),\ u(\tilde{\nu}_L),\ s(H^+),\ s(W^+)$
$\chi_c^+\chi_d^-$	$H_1H_1,\ H_1H_2,\ H_2H_2,\ H_3H_3$	$t(\chi_e^+),\ u(\chi_e^+),\ s(H_{1,2})$
	$H_1H_3,\ H_2H_3$	$t(\chi_e^+),\ u(\chi_e^+),\ s(H_3),\ s(Z^0)$
	H^+H^-	$t(\chi_k^0),\ s(H_{1,2}),\ s(Z^0,\gamma)$
	$Z^0H_1,\ Z^0H_2$	$t(\chi_e^+),\ u(\chi_e^+),\ s(H_3),\ s(Z^0)$
	Z^0H_3	$t(\chi_e^+),\ u(\chi_e^+),\ s(H_{1,2})$
	$H^+W^-,\ W^+H^-$	$t(\chi_e^+),\ s(H_{1,2,3})$
	Z^0Z^0	$t(\chi_e^+),\ u(\chi_e^+),\ s(H_{1,2})$
	W^+W^-	$t(\chi_k^0),\ s(H_{1,2}),\ s(Z^0,\gamma)$
	$\gamma\gamma$ (only for $c = d$)	$t(\chi_c^+),\ u(\chi_c^+)$
	$Z^0\gamma$	$t(\chi_d^+),\ u(\chi_c^+)$
	$u\bar{u}$	$t(\tilde{d}_{L,R}),\ s(H_{1,2,3}),\ s(Z^0,\gamma)$
	$\nu\bar{\nu}$	$t(\tilde{\ell}_{L,R}),\ s(Z^0)$
	$\bar{d}d$	$t(\tilde{u}_{L,R}),\ s(H_{1,2,3}),\ s(Z^0,\gamma)$
	$\bar{\ell}\ell$	$t(\tilde{\nu}_L),\ s(H_{1,2,3}),\ s(Z^0,\gamma)$
$\chi_c^+\chi_d^+$	H^+H^+	$t(\chi_k^0),\ u(\chi_k^0)$
	H^+W^+	$t(\chi_k^0),\ u(\chi_k^0)$
	W^+W^+	$t(\chi_k^0),\ u(\chi_k^0)$

Figure 10.2: MSSM Neutralino and Chargino scattering (including co-annihilation)

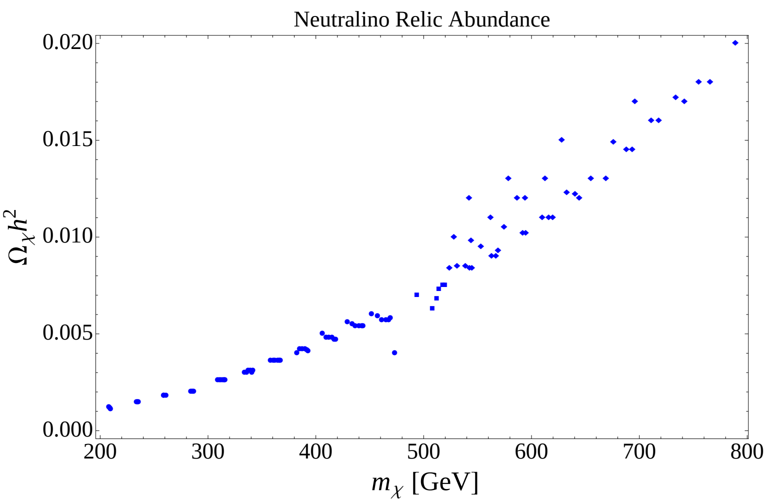


Figure 10.3: Relic Abundance of lightest neutralino as a function of mass when it is predominantly a wino. The scanned parameter space here ranges M_1: (800-1200) GeV, M_2: (200-775) GeV, μ: (600-1000) GeV with $\mu, M_1 > M_2$.

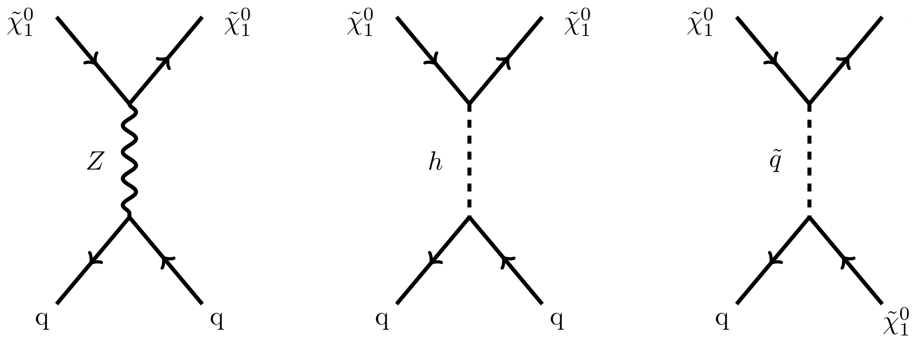


Figure 10.4: Diagram for lightest neutralino $\tilde{\chi}_1^0$ scattering with quarks for direct detection.

DM which mimics MSSM in the parameter space mentioned above. The direct detection cross-section for neutralino goes through t-channel processes as in Fig. 10.4. The squark contribution is negligible as they are heavy $\simeq 2$ TeV. Also, for pure wino, there is no Higgs channel and the Z-channel contributes more to spin- dependent cross-section. Hence, having some Higgsino fraction in the neutralino enhance direct detection. in Fig. 10.5, we see that the neutralino can be accessible to direct detection experiments in near future with points close to XENON100 and XENON1T limit. Points in blue have relic abundance contribution with more than 10% and they have a early detection possibility while points in green have relic density less than 10% and direct detection for them may be delayed depending on the mass and composition. While higher order calculations for direct detection of purely wino DM has been studied [60] to boost direct detection, we are not using them, since we are exploiting a small Higgsino fraction in the neutralino, that increases direct detection while having co-annihilations to yield under-abundance.

The mass range and the wino content in neutralino studied here is consistent with the indirect detection

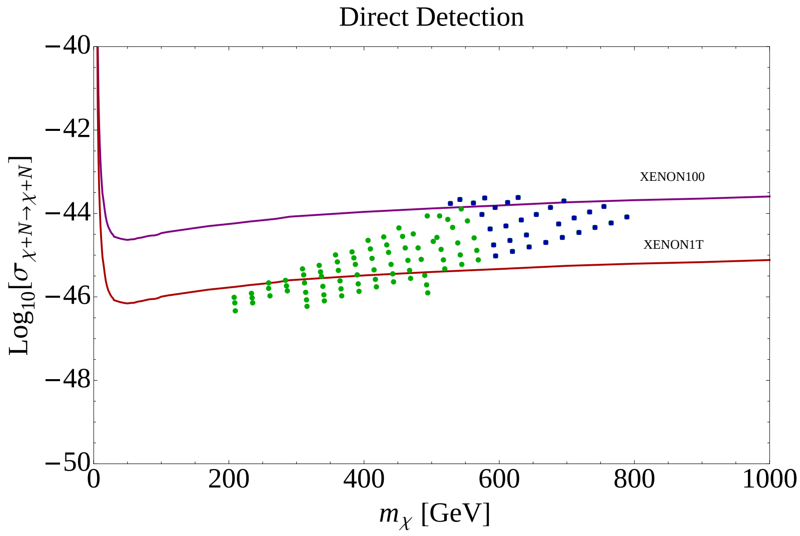


Figure 10.5: Direct detection constraint on neutralino DM mass when it is predominantly wino. The upper curve is for XENON100 and the lower one is for XENON1T. Points in blue have relic abundance more than 10% and those in green have less. Scanned parameter space ranges: M_1: (800-1200) GeV, M_2: (200-775) GeV, μ: (600-1000) GeV with $\mu, M_1 > M_2$.

constraints from Fermi Gamma-Ray space telescope or the High Energy Spectroscopic System (H.E.S.S.) [61].

We also note that the MSSM parameter space scan performed here, doesn't correspond to a specific high-scale SUSY breaking pattern. So, the bounds on the chargino or neutralino masses obtained from LHC [44], which mostly assumes some specific high-scale pattern like minimal Supergravity (mSUGRA) [62], are not applicable here.

Part IV

Conclusions, Bibliography and Appendixes

Chapter 11

Conclusions

11.1 Example

We show an example of indirect and direct detection for one point in the parameter space, we will use the central value for relic density of $\Omega_{tot}h^2 = 0.112$. Using the MSSM parameters $M_1 = 1200$ GeV, $M_2 = 625$ GeV, $\mu = 1000$ GeV, $\tan\beta = 10$, we find the mass for the lightest neutralino to be $m_\chi = 644.3$ GeV, with it we find the relic density $\Omega_\chi h^2 = 0.012$, then we choose the relic abundance for the other to DM candidates to be $\Omega_\eta h^2 = 0.03$ and $\Omega_n h^2 = 0.07$, which can be accomplished with the masses $m_\eta = 939$ GeV and $m_n = 912$ GeV .

With the parameters chosen the MSSM neutralino has the composition

$$\chi_1^0 = 0.009\tilde{B} - 0.986\tilde{W} + 0.138\tilde{H}_1 - 0.096\tilde{H}_2 \tag{11.1}$$

This corresponds to to a neutralino composed of 97.2% wino, 1.9% Higgsino1 and 0.9% Higgsino2. With this small percentage of Higgsino, the direct detection of the neutralino es enhanced.

Channels which contribute to scattering more than 1% are shown in table 11.1,

Contribution	Initial	final
9%	$\chi_1^0\chi_1^0$	W^+W^-
9%	$\chi_1^0\chi_1^+$	$\bar{s}c$
9%	$\chi_1^0\chi_1^+$	$u\bar{d}$
9%	$\chi_1^+\chi_1^+$	W^+W^+
8%	$\chi_1^0\chi_1^+$	$t\bar{b}$
7%	$\chi_1^0\chi_1^+$	ZW^+
5%	$\chi_1^+\chi_1^-$	ZZ
5%	$\chi_1^+\chi_1^-$	W^+W^-
3%	$\chi_1^0\chi_1^+$	$\mu\nu_\mu$
3%	$\chi_1^0\chi_1^+$	$\bar{e}\nu_e$
3%	$\chi_1^0\chi_1^+$	nlL
3%	$\chi_1^+\chi_1^-$	AZ
2%	$\chi_1^+\chi_1^-$	$c\bar{c}$
2%	$\chi_1^+\chi_1^-$	$u\bar{u}$
2%	$\chi_1^+\chi_1^-$	$s\bar{s}$
2%	$\chi_1^+\chi_1^-$	$d\bar{d}$
2%	$\chi_1^0\chi_1^+$	AW^+
2%	$\chi_1^+\chi_1^-$	$t\bar{t}$
2%	$\chi_1^+\chi_1^-$	$b\bar{b}$

Table 11.1: Scattering contributions ($> 1\%$)

We show a plot of the three components and their direct detection scatting in Fig. 11.1, where the percentages correspond to their contribution to relic abundance.

11.2 More work to be done

In extended LR SUSY model three DM components can co-exist together: the lightest neutralino $\tilde{\chi}_1^0$, the lightest scotino n, and the exotic $\tilde{\eta}_R^0$ Higgsino. We show that in the limit of wino dominated $\tilde{\chi}_1^0$, thanks to the co-annihilation with chargino to yield under-abundance, the other two components contribute heavily to relic abundance, with masses m_n and m_η around 1 TeV that corresponds to the resonance annihilation with $m_{DM} \simeq M_Z'/2$. We found a bound on Z' from LHC to be at 2.045 TeV. With this value of M_Z', the direct detection cross-section for n and $\tilde{\eta}_R^0$ is calculated to lie between $10^{-47} - 10^{-49}$ (cm^2)(depending on

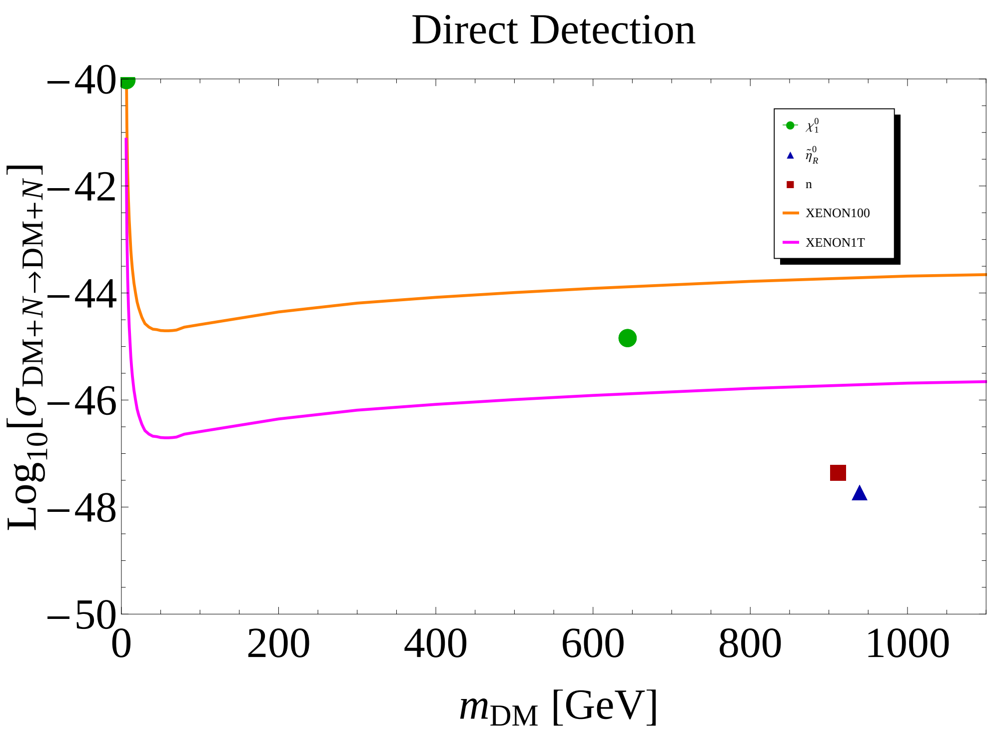


Figure 11.1: Example of direct detection for three component DM

the fraction in which it contributes to total DM density). This is at least an order of magnitude smaller than XENON1T detection limits. Nevertheless, in such a multicomponent set up, a large wino dominated neutralino region becomes allowed without much complications while still obeying the existing limits and constraints; with appropriate parameters, $\tilde{\chi}_1^0$ does lie within the direct detection limits.

It is worthy to mention that the situation studied in this article is a simplification in the thermal history of three component DM set-up. Interaction of DM components (between n and $\tilde{\eta}_R^0$, which have been neglected given the specific mass hierarchy), can make the general situation more complicated and one needs to solve the coupled Boltzman equations corresponding to n, $\tilde{\eta}_R^0$ and $\tilde{\chi}_1^0$ to study the exact decoupling of each DM component depending on their relative masses and coupling strength.

The rich particle spectrum of this model with the right handed sector, makes it very likely to have interesting collider signatures at LHC by producing these new excitations. They also open up new decay channels that may alter the final state event rates in the lepton or jet-rich final states with missing energy. This can serve as a distinctive feature of this model from MSSM and change the bounds on sparticle masses at LHC. We plan to elaborate on this in a future work, where also, is possible to study a more complicated neutralino.

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Appendix A

Explicit Lagrangian Terms

$$
\begin{aligned}
\mathcal{L}_{G\phi} = |D_\mu \phi|^2 \quad &= \quad Tr|\left[\partial_\mu \Delta_1 - \frac{\imath g_L}{\sqrt{2}} \sigma_a W^a_{\mu L} \Delta_1 + \Delta_1 \frac{\imath g_R}{\sqrt{2}} \sigma_a W^a_{\mu R}\right]|^2 \\
&+ \quad Tr|\left[\partial_\mu \Delta_2 - \frac{\imath g_L}{\sqrt{2}} \sigma_a W^a_{\mu L} \Delta_2 + \Delta_2 \frac{\imath g_R}{\sqrt{2}} \sigma_a W^a_{\mu R}\right]|^2 \\
&+ \quad |\left(\partial_\mu - \frac{\imath g_L}{\sqrt{2}} \sigma_a W^a_{\mu L} + \frac{\imath g_1}{\sqrt{2}} Y_{L1} B_\mu\right) \Phi_{L1}|^2 \\
&+ \quad |\left(\partial_\mu - \frac{\imath g_L}{\sqrt{2}} \sigma_a W^a_{\mu L} + \frac{\imath g_1}{\sqrt{2}} Y_{L2} B_\mu\right) \Phi_{L2}|^2 \\
&+ \quad |\left(\partial_\mu - \frac{\imath g_R}{\sqrt{2}} \sigma_a W^a_{\mu R} + \frac{\imath g_1}{\sqrt{2}} Y_{R1} B_\mu\right) \Phi_{R1}|^2 \\
&+ \quad |\left(\partial_\mu - \frac{\imath g_R}{\sqrt{2}} \sigma_a W^a_{\mu R} + \frac{\imath g_1}{\sqrt{2}} Y_{R2} B_\mu\right) \Phi_{R2}|^2 \\
&+ \quad |\left(\partial_\mu - \frac{\imath g_L}{\sqrt{2}} \sigma_a W^a_{\mu L} + \frac{\imath g_1}{\sqrt{2}} Y_{L1} B_\mu\right) \eta_1|^2 \\
&+ \quad |\left(\partial_\mu - \frac{\imath g_L}{\sqrt{2}} \sigma_a W^a_{\mu L} + \frac{\imath g_1}{\sqrt{2}} Y_{L2} B_\mu\right) \eta_{L2}|^2 \\
&+ \quad |\left(\partial_\mu - \frac{\imath g_R}{\sqrt{2}} \sigma_a W^a_{\mu R} + \frac{\imath g_1}{\sqrt{2}} Y_{R1} B_\mu\right) \eta_{R1}|^2 \\
&+ \quad |\left(\partial_\mu - \frac{\imath g_R}{\sqrt{2}} \sigma_a W^a_{\mu R} + \frac{\imath g_1}{\sqrt{2}} Y_{R2} B_\mu\right) \eta_{R2}|^2 \\
&+ \quad |\left(\partial_\mu - \frac{\imath g_L}{\sqrt{2}} \sigma_a W^a_{\mu L} + \frac{\imath g_1}{\sqrt{2}} Y_{L1} B_\mu\right) \tilde{Q}|^2 \\
&+ \quad |\left(\partial_\mu - \frac{\imath g_L}{\sqrt{2}} \sigma_a W^a_{\mu L} + \frac{\imath g_1}{\sqrt{2}} Y_{L2} B_\mu\right) \tilde{\Psi}|^2 \\
&+ \quad |\left(\partial_\mu - \frac{\imath g_R}{\sqrt{2}} \sigma_a W^a_{\mu R} + \frac{\imath g_1}{\sqrt{2}} Y_{R1} B_\mu\right) \tilde{Q^c}|^2 \\
&+ \quad |\left(\partial_\mu - \frac{\imath g_R}{\sqrt{2}} \sigma_a W^a_{\mu R} + \frac{\imath g_1}{\sqrt{2}} Y_{R2} B_\mu\right) \tilde{\Psi^c}|^2
\end{aligned}
\tag{A.1}
$$

$$\mathcal{L}_{FFG} = \imath\overline{\psi}\sigma^{\mu}D_{\mu}\psi \quad = \quad Tr\left[\tilde{\Delta_1}^{\dagger}\overline{\sigma}_{\mu}\left[\partial_{\mu} - \frac{\imath g_L}{\sqrt{2}}\sigma_a W^a_{\mu L} - \frac{\imath g_R}{\sqrt{2}}\sigma_a W^a_{\mu R}\right]\tilde{\Delta}_1\right] \tag{A.2}$$
$$+ \quad Tr\left[\tilde{\Delta_2}^{\dagger}\overline{\sigma}_{\mu}\left[\partial_{\mu} - \frac{\imath g_L}{\sqrt{2}}\sigma_a W^a_{\mu L} - \frac{\imath g_R}{\sqrt{2}}\sigma_a W^a_{\mu R}\right]\tilde{\Delta}_2\right]$$
$$+ \quad \tilde{\Phi_{L1}}^{\dagger}\overline{\sigma}_{\mu}\left[\partial_{\mu} - \frac{\imath g_L}{\sqrt{2}}\sigma_a W^a_{\mu L} - \imath g_1 Y_{L1}B_{\mu}\right]\tilde{\Phi_{L1}}$$
$$+ \quad \tilde{\Phi_{L2}}^{\dagger}\overline{\sigma}_{\mu}\left[\partial_{\mu} - \frac{\imath g_L}{\sqrt{2}}\sigma_a W^a_{\mu L} - \imath g_1 Y_{L2}B_{\mu}\right]\tilde{\Phi_{L2}}$$
$$+ \quad \tilde{\Phi_{R1}}^{\dagger}\overline{\sigma}_{\mu}\left[\partial_{\mu} - \frac{\imath g_R}{\sqrt{2}}\sigma_a W^a_{\mu R} - \imath g_1 Y_{R1}B_{\mu}\right]\tilde{\Phi_{R1}}$$
$$+ \quad \tilde{\Phi_{R2}}^{\dagger}\overline{\sigma}_{\mu}\left[\partial_{\mu} - \frac{\imath g_R}{\sqrt{2}}\sigma_a W^a_{\mu R} - \imath g_1 Y_{R2}B_{\mu}\right]\tilde{\Phi_{R2}}$$

$$\mathcal{L}_{F\phi\lambda} \quad = \quad \imath\sqrt{2}g\phi^*_i\lambda_a\psi_i \tag{A.3}$$
$$= \quad \frac{\imath}{\sqrt{2}}Tr\left[\Delta^{\dagger}_1(g_L\sigma_a\lambda^a_L + g_R\sigma_a\lambda^a_R)\tilde{\Delta}_1\right] + \frac{\imath}{\sqrt{2}}Tr\left[\Delta^{\dagger}_2(g_L\sigma_a\lambda^a_L + g_R\sigma_a\lambda^a_R)\tilde{\Delta}_2\right]$$
$$+ \quad \frac{\imath}{\sqrt{2}}\left(\Phi^{\dagger}_{L1}(g_L\sigma_a\lambda^a_L + g_1Y_{L1}\lambda_1)\tilde{\Phi}_{L1}\right) + \frac{\imath}{\sqrt{2}}\left(\Phi^{\dagger}_{L2}(g_L\sigma_a\lambda^a_L + g_1Y_{L2}\lambda_1)\tilde{\Phi}_{L2}\right)$$
$$+ \quad \frac{\imath}{\sqrt{2}}\left(\Phi^{\dagger}_{R1}(g_R\sigma_a\lambda^a_R + g_1Y_{R1}\lambda_1)\tilde{\Phi}_{R1}\right) + \frac{\imath}{\sqrt{2}}\left(\Phi^{\dagger}_{R2}(g_R\sigma_a\lambda^a_R + g_1Y_{R2}\lambda_1)\tilde{\Phi}_{R2}\right)$$
$$+ \quad \frac{\imath}{\sqrt{2}}\left(\eta^{\dagger}_{L1}(g_L\sigma_a\lambda^a_L + g_1Y_{L1}\lambda_1)\tilde{\eta}_{L1}\right) + \frac{\imath}{\sqrt{2}}\left(\eta^{\dagger}_{L2}(g_L\sigma_a\lambda^a_L + g_1Y_{L2}\lambda_1)\tilde{\eta}_{L2}\right)$$
$$+ \quad \frac{\imath}{\sqrt{2}}\left(\eta^{\dagger}_{R1}(g_R\sigma_a\lambda^a_R + g_1Y_{R1}\lambda_1)\tilde{\eta}_{R1}\right) + \frac{\imath}{\sqrt{2}}\left(\eta^{\dagger}_{R2}(g_R\sigma_a\lambda^a_R + g_1Y_{R2}\lambda_1)\tilde{\eta}_{R2}\right)$$
$$+ \quad \frac{\imath}{\sqrt{2}}\left(s^{\dagger}_1(g_1Y_1\lambda_1)\tilde{s}_1\right) + \frac{\imath}{\sqrt{2}}\left(s^{\dagger}_2(g_1Y_2\lambda_1)\tilde{s}_2\right)$$
$$+ \quad \imath\tilde{Q}^{\dagger}\left[\frac{g_L}{\sqrt{2}}\sigma_a\lambda^a_L + \frac{g_1}{\sqrt{2}}Y_Q\lambda_1\right]Q + \imath\tilde{\Psi}^{\dagger}\left[\frac{g_L}{\sqrt{2}}\sigma_a\lambda^a_L + \frac{g_1}{\sqrt{2}}Y_{\Psi}\lambda_1\right]\Psi$$
$$+ \quad \imath\tilde{Q^c}^{\dagger}\left[\frac{g_R}{\sqrt{2}}\sigma_a\lambda^a_R + \frac{g_1}{\sqrt{2}}Y_{Q^c}\lambda_1\right]Q^c + \imath\tilde{\Psi}^{c\dagger}\left[\frac{g_R}{\sqrt{2}}\sigma_a\lambda^a_R + \frac{g_1}{\sqrt{2}}Y_{\Psi^c}\lambda_1\right]\Psi^c + h.c.$$

With

$$D_{\mu} \quad = \quad \partial_{\mu} - \frac{\imath g_L}{\sqrt{2}}\epsilon_L\sigma_a W^a_{\mu L} - \frac{\imath g_R}{\sqrt{2}}\epsilon_R\sigma_a W^a_{\mu R} - \imath g_1 Y_j B_{\mu} \tag{A.4}$$

Note that there is one exception in the covariant derivative D_{μ} when it operates over the bidoublets Δ_i.

$$D_{\mu}\Delta \quad = \quad \partial_{\mu}\Delta - \frac{\imath g_L}{\sqrt{2}}\sigma_a W^a_{\mu L}\Delta - \frac{\imath g_R}{\sqrt{2}}\Delta\sigma_a W^a_{\mu R} \tag{A.5}$$

Where $\epsilon_L = 1$ If the field transforms under $SU(2)_L$, $\epsilon_R = 1$ If the field transforms under $SU(2)_R$, and Y_j is the $U(1)_Y$ hypercharge of each field.

There is no s_3 term in $\mathcal{L}_{F\phi\lambda}$ since $Y_{s3} = 0$.

Appendix B

Masses of Higgs Particles

B.1 Masses of Neutral Higgs

$$M_{Re[\phi_{R1}]Re[\phi_{R1}]} = 1/4(-4m_{R1}^2 + 4f_2^2(u_1^2 + v_{L2}^2) + g_1^2(v_{L1}^2 - v_{L2}^2 + 3v_{R1}^2 - v_{R2}^2) \quad \text{(B.1)}$$

$$+g_R^2(u_1^2 - u_4^2 + 9v_{R1}^2 + 3v_{R2}^2) + 4\mu_R^2)$$

$$M_{Re[\phi_{R1}]Re[\phi_{R2}]} = -2B_R - (g_1^2 - 3g_R^2)v_{R1}v_{R2}$$

$$M_{Re[\phi_{R2}]Re[\phi_{R2}]} = 1/4(-4m_{R2}^2 + 4f_1^2(u_4^2 + v_{L1}^2) - g_1^2(v_{L1}^2 - v_{L2}^2 + v_{R1}^2 - 3v_{R2}^2)$$

$$+g_R^2(u_1^2 - u_4^2 + 3v_{R1}^2 + 9v_{R2}^2) + 4\mu_R^2)$$

$$M_{Re[\phi_{R1}]Re[\phi_{L1}]} = g_1^2 v_{L1}v_{R1} - 2(f_2 u_1 \mu_L + f_1 u_4 \mu_R)$$

$$M_{Re[\phi_{R2}]Re[\phi_{L1}]} = 2A_1 u_4 + 4f_1^2 v_{L1}v_{R2} - g_1^2 v_{L1}v_{R2} - 2f_1 u_1 \mu_\Delta$$

$$M_{Re[\phi_{L1}]Re[\phi_{L1}]} = 1/4(-4m_{L1}^2 + g_L^2(u_1^2 - u_4^2 + 9v_{L1}^2 + 3v_{L2}^2)$$

$$+g_1^2(3v_{L1}^2 - v_{L2}^2 + v_{R1}^2 - v_{R2}^2) + 4f_1^2(u_4^2 + v_{R2}^2) + 4\mu_L^2)$$

$$M_{Re[\phi_{R1}]Re[\phi_{L2}]} = 2A_2 u_1 + 4f_2^2 v_{L2}v_{R1} - g_1^2 v_{L2}v_{R1} - 2f_2 u_4 \mu_\Delta$$

$$M_{Re[\phi_{R2}]Re[\phi_{L2}]} = g_1^2 v_{L2}v_{R2} - 2(f_1 u_4 \mu_L + f_2 u_1 \mu_R)$$

$$M_{Re[\phi_{L1}]Re[\phi_{L2}]} = -2B_L - (g_1^2 - 3g_L^2)v_{L1}v_{L2}$$

$$M_{Re[\phi_{L2}]Re[\phi_{L2}]} = 1/4(-4m_{L2}^2 + g_L^2(u_1^2 - u_4^2 + 3v_{L1}^2 + 9v_{L2}^2)$$

$$-g_1^2(v_{L1}^2 - 3v_{L2}^2 + v_{R1}^2 - v_{R2}^2) + 4f_2^2(u_1^2 + v_{R1}^2) + 4\mu_L^2)$$

$$M_{Re[\phi_{R1}]Re[\delta_{11}]} = 2A_2 v_{L2} + (4f_2^2 + g_R^2)u_1 v_{R1} - 2f_2 v_{L1}\mu_L$$

$$M_{Re[\phi_{R2}]Re[\delta_{11}]} = g_R^2 u_1 v_{R2} - 2(f_1 v_{L1}\mu_\Delta + f_2 v_{L2}\mu_R)$$

$$M_{Re[\phi_{L1}]Re[\delta_{11}]} = g_L^2 u_1 v_{L1} - 2(f_1 v_{R2}\mu_\Delta + f_2 v_{R1}\mu_L)$$

$$M_{Re[\phi_{L2}]Re[\delta_{11}]} = (4f_2^2 + g_L^2)u_1 v_{L2} + 2A_2 v_{R1} - 2f_2 v_{R2}\mu_R$$

$$M_{Re[\delta_{11}]Re[\delta_{11}]} = 1/4(-4m_{\Delta 1}^2 + g_L^2(3u_1^2 - u_4^2 + v_{L1}^2 + v_{L2}^2)$$

$$+g_R^2(3u_1^2 - u_4^2 + v_{R1}^2 + v_{R2}^2) + 4(f_2^2(v_{L2}^2 + v_{R1}^2) + \mu_\Delta^2))$$

$$M_{Re[\phi_{R1}]Re[\delta_{22}]} = -g_R^2 u_4 v_{R1} - 2f_2 v_{L2}\mu_\Delta - 2f_1 v_{L1}\mu_R$$

$$M_{Re[\phi_{R2}]Re[\delta_{22}]} = 2A_1 v_{L1} + 4f_1^2 u_4 v_{R2} - g_R^2 u_4 v_{R2} - 2f_1 v_{L2}\mu_L$$

$$M_{Re[\phi_{L1}]Re[\delta_{22}]} = 4f_1^2 u_4 v_{L1} - g_L^2 u_4 v_{L1} + 2A_1 v_{R2} - 2f_1 v_{R1}\mu_R$$

$$M_{Re[\phi_{L2}]Re[\delta_{22}]} = -g_L^2 u_4 v_{L2} - 2f_2 v_{R1}\mu_\Delta - 2f_1 v_{R2}\mu_L$$

$$M_{Re[\delta_{11}]Re[\delta_{22}]} = -2B_\Delta - (g_L^2 + g_R^2)u_1 u_4$$

$$M_{Re[\delta_{22}]Re[\delta_{22}]} = 1/4(-4m_{\Delta 2}^2 - g_L^2(u_1^2 - 3u_4^2 + v_{L1}^2 + v_{L2}^2)$$

$$-g_R^2(u_1^2 - 3u_4^2 + v_{R1}^2 + v_{R2}^2) + 4(f_1^2(v_{L1}^2 + v_{R2}^2) + \mu_\Delta^2))$$

$$M_{Im[\phi_{R1}]Im[\phi_{R1}]} = 1/4(-4m_{R1}^2 + 4f_2^2(u_1^2 + v_{L2}^2) + g_1^2(v_{L1}^2 - v_{L2}^2 + v_{R1}^2 - v_{R2}^2)$$

$$+g_R^2(u_1^2 - u_4^2 + 3(v_{R1}^2 + v_{R2}^2)) + 4\mu_R^2)$$

$$M_{Im[\phi_{R1}]Im[\phi_{R2}]} = 2B_R$$

$$M_{Im[\phi_{R2}]Im[\phi_{R2}]} = 1/4(-4m_{R2}^2 + 4f_1^2(u_4^2 + v_{L1}^2) + g_1^2(-v_{L1}^2 + v_{L2}^2 - v_{R1}^2 + v_{R2}^2)$$

$$+g_R^2(u_1^2 - u_4^2 + 3(v_{R1}^2 + v_{R2}^2)) + 4\mu_R^2)$$

$$M_{Im[\phi_{R1}]Im[\phi_{L1}]} = -2(f_2 u_1 \mu_L + f_1 u_4 \mu_R)$$

$$M_{Im[\phi_{R2}]Im[\phi_{L1}]} = -2A_1 u_4 + 2f_1 u_1 \mu_\Delta$$

$$M_{Im[\phi_{L1}]Im[\phi_{L1}]} = 1/4(-4m_{L1}^2 + g_L^2(u_1^2 - u_4^2 + 3(v_{L1}^2 + v_{L2}^2))$$

$$+g_1^2(v_{L1}^2 - v_{L2}^2 + v_{R1}^2 - v_{R2}^2) + 4f_1^2(u_4^2 + v_{R2}^2) + 4\mu_L^2)$$

$$M_{Im[\phi_{R1}]Im[\phi_{L2}]} = -2A_2 u_1 + 2f_2 u_4 \mu_\Delta$$

$$M_{Im[\phi_{R2}]Im[\phi_{L2}]} = -2(f_1 u_4 \mu_L + f_2 u_1 \mu_R)$$

$$M_{Im[\phi_{L1}]Im[\phi_{L2}]} = 2B_L$$

$$M_{Im[\phi_{L2}]Im[\phi_{L2}]} = 1/4(-4m_{L2}^2 + g_L^2(u_1^2 - u_4^2 + 3(v_{L1}^2 + v_{L2}^2))$$

$$+g_1^2(-v_{L1}^2 + v_{L2}^2 - v_{R1}^2 + v_{R2}^2) + 4f_2^2(u_1^2 + v_{R1}^2) + 4\mu_L^2)$$

$$M_{Im[\phi_{R1}]Im[\delta_{11}]} = -2A_2 v_{L2} + 2f_2 v_{L1}\mu_L$$

$$M_{Im[\phi_{R2}]Im[\delta_{11}]} = -2(f_1 v_{L1}\mu_\Delta + f_2 v_{L2}\mu_R)$$

$$M_{Im[\phi_{L1}]Im[\delta_{11}]} = -2(f_1 v_{R2}\mu_\Delta + f_2 v_{R1}\mu_L)$$

$$M_{Im[\phi_{L2}]Im[\delta_{11}]} = -2A_2 v_{R1} + 2f_2 v_{R2}\mu_R$$

$$M_{Im[\delta_{11}]Im[\delta_{11}]} = 1/4(-4m_{\Delta 1}^2 + g_L^2(u_1^2 - u_4^2 + v_{L1}^2 + v_{L2}^2)$$

$$+g_R^2(u_1^2 - u_4^2 + v_{R1}^2 + v_{R2}^2) + 4(f_2^2(v_{L2}^2 + v_{R1}^2) + \mu_\Delta^2))$$

$M_{Im[\phi_{R1}]Im[\delta_{22}]} = -2(f_2 v_{L2}\mu_\Delta + f_1 v_{L1}\mu_R)$

$M_{Im[\phi_{R2}]Im[\delta_{22}]} = -2A_1 v_{L1} + 2f_1 v_{L2}\mu_L$

$M_{Im[\phi_{L1}]Im[\delta_{22}]} = -2A_1 v_{R2} + 2f_1 v_{R1}\mu_R$

$M_{Im[\phi_{L2}]Im[\delta_{22}]} = -2(f_2 v_{R1}\mu_\Delta + f_1 v_{R2}\mu_L)$

$M_{Im[\delta_{11}]Im[\delta_{22}]} = 2B_\Delta$

$M_{Im[\delta_{22}]Im[\delta_{22}]} = 1/4(-4m_{\Delta 2}^2 - g_L^2(u_1^2 - u_4^2 + v_{L1}^2 + v_{L2}^2)$

$-g_R^2(u_1^2 - u_4^2 + v_{R1}^2 + v_{R2}^2) + 4(f_1^2(v_{L1}^2 + v_{R2}^2) + \mu_\Delta^2))$

$M_{Re[\delta_{12}]Re[\delta_{12}]} = 1/4(-4m_{\Delta 1}^2 - g_L^2(u_1^2 - u_4^2 + v_{L1}^2 + v_{L2}^2) - g_R^2(u_1^2 - u_4^2 + v_{R1}^2 + v_{R2}^2) + 4\mu_\Delta^2)$

$M_{Re[\delta_{12}]Re[\delta_{21}]} = -2B_\Delta$

$M_{Re[\delta_{21}]Re[\delta_{21}]} = 1/2(-4m_{\Delta 2}^2 + g_L^2(u_1^2 - u_4^2 + v_{L1}^2 + v_{L2}^2) + g_R^2(u_1^2 - u_4^2 + v_{R1}^2 + v_{R2}^2) + 4\mu_\Delta^2)$

$M_{Im[\delta_{12}]Im[\delta_{12}]} = 1/2(-4m_{\Delta 1}^2 - g_L^2(u_1^2 - u_4^2 + v_{L1}^2 + v_{L2}^2) - g_R^2(u_1^2 - u_4^2 + v_{R1}^2 + v_{R2}^2) + 4\mu_\Delta^2)$

$M_{Im[\delta_{12}]Im[\delta_{21}]} = 2B_\Delta$

$M_{Im[\delta_{21}]Im[\delta_{21}]} = 1/4(-4m_{\Delta 2}^2 + g_L^2(u_1^2 - u_4^2 + v_{L1}^2 + v_{L2}^2) + g_R^2(u_1^2 - u_4^2 + v_{R1}^2 + v_{R2}^2) + 4\mu_\Delta^2)$

$M_{Re[\eta_{L1}]Re[\eta_{L1}]} = 1/4(-4m_{L3} + g_L^2(u_1^2 - u_4^2 + 3(v_{L1}^2 + v_{L2}^2))$

$+g_1^2(v_{L1}^2 - v_{L2}^2 + v_{R1}^2 - v_{R2}^2) + 4(f_{10}^2 v_{L2}^2 + \mu_{L2}^2))$

$M_{Re[\eta_{L1}]Re[\eta_{L2}]} = -2B_L 2 + 2f_{10}f_9 v_{L1}v_{L2}$

$M_{Re[\eta_{L2}]Re[\eta_{L2}]} = 1/4(-4m_{L4} + 4f_9^2 v_{L1}^2 + g_L^2(u_1^2 - u_4^2 + 3(v_{L1}^2 + v_{L2}^2))$

$+g_1^2(-v_{L1}^2 + v_{L2}^2 - v_{R1}^2 + v_{R2}^2) + 4\mu_{L2}^2)$

$M_{Re[\eta_{L1}]Re[s_3]} = 2(A_{10}v_{L2} - f_9 v_{L1}\mu_{L2} + f_{10}(f_2 u_1 v_{R1} - v_{L1}\mu_L - 2v_{L2}\mu_{s3}))$

$M_{Re[\eta_{L2}]Re[s_3]} = 2(f_1 f_9 u_4 v_{R2} - v_{L2}(f_9\mu_L + f_{10}\mu_{L2}) + v_{L1}(A_9 - 2f_9\mu_{s3}))$

$M_{Re[s_3]Re[s_3]} = -4B_{s3} - 2m_{s3} + 2f_9^2 v_{L1}^2 + 2f_{10}^2 v_{L2}^2 + 8\mu_{s3}^2$

$M_{Im[\eta_{L1}]Im[\eta_{L1}]} = 1/4(-4m_{L3} + g_L^2(u_1^2 - u_4^2 + 3(v_{L1}^2 + v_{L2}^2))$

$+g_1^2(v_{L1}^2 - v_{L2}^2 + v_{R1}^2 - v_{R2}^2) + 4(f_{10}^2 v_{L2}^2 + \mu_{L2}^2))$

$M_{Im[\eta_{L1}]Im[\eta_{L2}]} = 2(B_L 2 + f_{10}f_9 v_{L1}v_{L2})$

$M_{Im[\eta_{L2}]Im[\eta_{L2}]} = 1/2(-4m_{L4} + 4f_9^2 v_{L1}^2 + g_L^2(u_1^2 - u_4^2 + 3(v_{L1}^2 + v_{L2}^2))$

$+g_1^2(-v_{L1}^2 + v_{L2}^2 - v_{R1}^2 + v_{R2}^2) + 4\mu_{L2}^2)$

$M_{Im[\eta_{L1}]Im[s_3]} = -2(A_{10}v_{L2} + f_9 v_{L1}\mu_{L2} + f_{10}(f_2 u_1 v_{R1} - v_{L1}\mu_L + 2v_{L2}\mu_{s3}))$

$M_{Im[\eta_{L2}]Im[s_3]} = -2(A_9 v_{L1} + f_1 f_9 u_4 v_{R2} - f_9 v_{L2}\mu_L + f_{10}v_{L2}\mu_{L2} + 2f_9 v_{L1}\mu_{s3})$

$M_{Im[s_3]Im[s_3]} = 2(2B_{s3} - m_{s3} + f_9^2 v_{L1}^2 + f_{10}^2 v_{L2}^2 + 4\mu_{s3}^2)$

$$M_{Re[\eta_{R1}]Re[\eta_{R1}]} = 1/2(-4m_{R3} + g_1^2(v_{L1}^2 - v_{L2}^2 + v_{R1}^2 - v_{R2}^2)$$
$$+g_R^2(u_1^2 - u_4^2 + 3(v_{R1}^2 + v_{R2}^2)) + 4\mu_{R2}^2)$$
$$M_{Re[\eta_{R1}]Re[\eta_{R2}]} = -2B_{R2}$$
$$M_{Re[\eta_{R2}]Re[\eta_{R2}]} = 1/4(-4m_{R4} + g_1^2(-v_{L1}^2 + v_{L2}^2 - v_{R1}^2 + v_{R2}^2)$$
$$+g_R^2(u_1^2 - u_4^2 + 3(v_{R1}^2 + v_{R2}^2)) + 4\mu_{R2}^2)$$
$$M_{Im[\eta_{R1}]Im[\eta_{R1}]} = 1/4(-4m_{R3} + g_1^2(v_{L1}^2 - v_{L2}^2 + v_{R1}^2 - v_{R2}^2)$$
$$+g_R^2(u_1^2 - u_4^2 + 3(v_{R1}^2 + v_{R2}^2)) + 4\mu_{R2}^2)$$
$$M_{Im[\eta_{R1}]Im[\eta_{R2}]} = 2B_{R2}$$
$$M_{Im[\eta_{R2}]Im[\eta_{R2}]} = 1/4(-4m_{R4} + g_1^2(-v_{L1}^2 + v_{L2}^2 - v_{R1}^2 + v_{R2}^2)$$
$$+g_R^2(u_1^2 - u_4^2 + 3(v_{R1}^2 + v_{R2}^2)) + 4\mu_{R2}^2)$$

B.2 Masses of the charged scalars

$$M_{\phi_{R1}^-\phi_{R1}^+} = 1/4(-4m_{R1}^2 + g_1^2(v_{L1}^2 - v_{L2}^2 + v_{R1}^2 - v_{R2}^2) \tag{B.2}$$
$$+g_R^2(u_1^2 - u_4^2 + 3(v_{R1}^2 + v_{R2}^2)) + 4(f_2^2 v_{L2}^2 + \mu_R^2))$$
$$M_{\phi_{R1}^-\phi_{R2}^+} = B_R$$
$$M_{\phi_{R2}^-\phi_{R2}^+} = 1/4(-4m_{R2}^2 + 4f_1^2 v_{L1}^2 + g_1^2(-v_{L1}^2 + v_{L2}^2 - v_{R1}^2 + v_{R2}^2)$$
$$+g_R^2(u_1^2 - u_4^2 + 3(v_{R1}^2 + v_{R2}^2)) + 4\mu_R^2)$$
$$M_{\phi_{R1}^-\delta_{12}^+} = -A_2 v_{L2} + f_2(-f_2 u_1 v_{R1} + v_{L1}\mu_L)$$
$$M_{\phi_{R2}^-\delta_{12}^+} = -f_1 v_{L1}\mu_\Delta - f_2 v_{L2}\mu_R$$
$$M_{\delta_{12}^-\delta_{12}^+} = 1/4(-4m_{\Delta 1}^2 + g_L^2(u_1^2 - u_4^2 + v_{L1}^2 + v_{L2}^2) + g_R^2(u_1^2 - u_4^2 + v_{R1}^2 + v_{R2}^2)$$
$$+4(f_2^2 v_{L2}^2 + \mu_\Delta^2))$$
$$M_{\phi_{R1}^-\delta_{21}^+} = -f_2 v_{L2}\mu_\Delta - f_1 v_{L1}\mu_R$$
$$M_{\phi_{R2}^-\delta_{21}^+} = -A_1 v_{L1} + f_1(-f_1 u_4 v_{R2} + v_{L2}\mu_L)$$
$$M_{\delta_{12}^-\delta_{21}^+} = B_\Delta$$
$$M_{\delta_{21}^-\delta_{21}^+} = 1/4(-4m_{\Delta 2}^2 - g_L^2(u_1^2 - u_4^2 + v_{L1}^2 + v_{L2}^2) - g_R^2(u_1^2 - u_4^2 + v_{R1}^2 + v_{R2}^2)$$
$$+4(f_1^2 v_{L1}^2 + \mu_\Delta^2))$$

$$M_{\eta_{L1}^-\eta_{L1}^+} = -m_{L3}^2 + f_3^2 u_1^2 + 1/4(4f_5^2 v_{L1}^2 + g_L^2(u_1^2 - u_4^2 + 3(v_{L1}^2 + v_{L2}^2))$$
$$+g_1^2(v_{L1}^2 - v_{L2}^2 + v_{R1}^2 - v_{R2}^2)) + \mu_{L2}^2$$
$$M_{\eta_{L1}^-\eta_{L2}^+} = B_{L2}$$
$$M_{\eta_{L2}^-\eta_{L2}^+} = 1/4(-4m_{L4}^2 + 4f_4^2 u_4^2 + g_L^2(u_1^2 - u_4^2 + 3(v_{L1}^2 + v_{L2}^2))$$
$$+g_1^2(-v_{L1}^2 + v_{L2}^2 - v_{R1}^2 + v_{R2}^2) + 4(f_7^2 v_{L2}^2 + \mu_{L2}^2))$$
$$M_{\eta_{L1}^-\eta_{R1}^+} = f_5 f_6 v_{L1} v_{R1} + f_4 u_4 \mu_{L2} + f_3 u_1 \mu_{R2}$$
$$M_{\eta_{L2}^-\eta_{R1}^+} = A_4 u_4 + f_1 f_4 v_{L1} v_{R2} - f_4 u_1 \mu_\Delta$$
$$M_{\eta_{R1}^-\eta_{R1}^+} = 1/4(-4m_{R3}^2 + 4f_4^2 u_4^2 + g_1^2(v_{L1}^2 - v_{L2}^2 + v_{R1}^2 - v_{R2}^2)$$
$$+g_R^2(u_1^2 - u_4^2 + 3(v_{R1}^2 + v_{R2}^2)) + 4(f_6^2 v_{R1}^2 + \mu_{R2}^2))$$
$$M_{\eta_{L1}^-\eta_{R2}^+} = A_3 u_1 + f_2 f_3 v_{L2} v_{R1} - f_3 u_4 \mu_\Delta$$
$$M_{\eta_{L2}^-\eta_{R2}^+} = f_7 f_8 v_{L2} v_{R2} + f_3 u_1 \mu_{L2} + f_4 u_4 \mu_{R2}$$
$$M_{\eta_{R1}^-\eta_{R2}^+} = B_{R2}$$
$$M_{\eta_{R2}^-\eta_{R2}^+} = -m_{R4}^2 + f_3^2 u_1^2 + 1/4(4f_8^2 v_{R2}^2 + g_1^2(-v_{L1}^2 + v_{L2}^2 - v_{R1}^2 + v_{R2}^2)$$
$$+g_R^2(u_1^2 - u_4^2 + 3(v_{R1}^2 + v_{R2}^2))) + \mu_{R2}^2$$
$$M_{\eta_{L1}^- s_1^+} = -f_3 f_8 u_1 v_{R2} - f_7 v_{L2} \mu_{L2} - f_5 v_{L1} \mu_{s12}$$
$$M_{\eta_{L2}^- s_1^+} = -A_7 v_{L2} - f_2 f_7 u_1 v_{R1} + f_7 v_{L1} \mu_L$$
$$M_{\eta_{R1}^- s_1^+} = -f_4 f_7 u_4 v_{L2} - f_8 v_{R2} \mu_{R2} - f_6 v_{R1} \mu_{s12}$$
$$M_{\eta_{R2}^- s_1^+} = -f_1 f_8 u_4 v_{L1} - A_8 v_{R2} + f_8 v_{R1} \mu_R$$
$$M_{s_1^- s_1^+} = 1/2(-2m_{s1}^2 + g_1^2(v_{L1}^2 - v_{L2}^2 + v_{R1}^2 - v_{R2}^2) + 2(f_7^2 v_{L2}^2 + f_8^2 v_{R2}^2 + \mu_{s12}^2))$$
$$M_{\eta_{L1}^- s_2^+} = A_5 v_{L1} + f_1 f_5 u_4 v_{R2} - f_5 v_{L2} \mu_L$$
$$M_{\eta_{L2}^- s_2^+} = f_4 f_6 u_4 v_{R1} + f_5 v_{L1} \mu_{L2} + f_7 v_{L2} \mu_{s12}$$
$$M_{\eta_{R1}^- s_2^+} = f_2 f_6 u_1 v_{L2} + A_6 v_{R1} - f_6 v_{R2} \mu_R$$
$$M_{\eta_{R2}^- s_2^+} = f_3 f_5 u_1 v_{L1} + f_6 v_{R1} \mu_{R2} + f_8 v_{R2} \mu_{s12}$$
$$M_{s_1^- s_2^+} = -B_{s12}$$
$$M_{s_2^- s_2^+} = -m_{s2}^2 + f_5^2 v_{L1}^2 + f_6^2 v_{R1}^2 + 1/2 g_1^2(-v_{L1}^2 + v_{L2}^2 - v_{R1}^2 + v_{R2}^2) + \mu_{s12}^2$$

Appendix C

Exotic scalar as Dark Matter candidate

A possible Dark Matter candidate is the exotic scalar η_R. It will contribute to the relic abundance via only one channel, $\eta_R + \eta_R \rightarrow Z^{'} \rightarrow SM + SM$

The amplitude for the annihilation of into fermions is given by,

$$\imath\mathcal{M} = g_{z'}^2 * \left(\frac{C_L^2 - 2S_R^2}{2}\right) \overline{\eta_R^0}(p_1^\mu - p_2^\mu)\eta_R^0 \left(\frac{g_{\mu\nu} - k_\mu k_\nu / M_{z'}^2}{k^2 - M_{Z'}^2}\right) \overline{f}\gamma^\nu(c_v - c_a\gamma^5)f \tag{C.1}$$

Notice that $(p_1 - p_2) \cdot k = 0$, the term proportional to $k_\mu k_\nu$ does not contribute to the amplitude.

$$|\mathcal{M}|^2 = \frac{g^4}{(k^2 - M_{Z'}^2)^2} \left[\overline{\eta_R^0}(p_1^\mu - p_2^\mu)\eta_R^0\right]^2 \left[\overline{f}\gamma^\mu(c_v - c_a\gamma^5)f\right]^2 \tag{C.2}$$

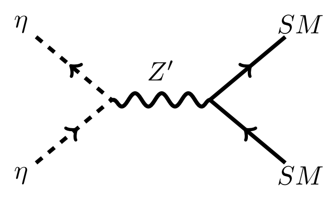

Figure C.1: Exotic scalar η_R scattering via Z' into DM

$$
\begin{aligned}
\left[\overline{f}\gamma^\mu(c_v - c_a\gamma^5)f\right]^2 &= \gamma^\mu(c_v - c_a\gamma^5)(\sum f(q_1)\overline{f}(q_1))\gamma^\nu(c_v - c_a\gamma^5)(\sum f(q_2)\overline{f}(q_2)) \\
&= Tr\left[\gamma^\mu(c_v - c_a\gamma^5)(\not{q}_1 + m_f)\gamma^\nu(c_v - c_a\gamma^5)(\not{q}_2 - m_f))\right] \\
&= (4(c_v^2 + c_a^2)(-q_1 \cdot q_2 g^{\mu\nu} + q_1^\nu q_2^\mu + q_1^\mu q_2^\nu) \\
&- 8\imath c_v c_a(q_{1\lambda}q_{2\sigma})\epsilon^{\mu\nu\lambda\sigma} - 4m_f^2(c_v^2 - c_a^2)g^{\mu\nu})
\end{aligned}
$$

$$
\begin{aligned}
\left[\overline{\eta_R^0}(p_1^\mu - p_2^\mu)\eta_R^0\right]^2 &= (\sum \eta_R^0(p_1)\overline{\eta_R^0}(p_1))(\sum \eta_R^0(p_2)\overline{\eta_R^0}(p_2))(p_{1\mu} - p_{2\mu})(p_{1\nu} - p_{2\nu}) \\
&= (p_{1\mu} - p_{2\mu})(p_{1\nu} - p_{2\nu})
\end{aligned}
$$

Some algebra

$$
\begin{aligned}
p_1 &= (E, 0, 0, p) \\
p_2 &= (E, 0, 0, -p) \\
q_1 &= (E, q\sin\theta, 0, q\cos\theta) \\
q_1 &= (E, -q\sin\theta, 0, -q\cos\theta) \\
k &= p_1 + p_2 = (2E, 0, 0, 0) \\
p_1 \cdot p_2 &= E^2 + p^2 = E^2 + (E^2 - M_\eta^2) = 2E^2 - M_\eta^2 \simeq M_\eta^2 \\
q_1 \cdot q_2 &= E^2 + q^2 = E^2 + (E^2 - M_f^2) = 2E^2 - m_f^2 \simeq 2M_\eta^2 \\
k^2 &= 4E^2 = 4\gamma^2 M_\eta^2 = 4\frac{M_\eta^2}{1 - v^2} = 4M_\eta^2(1 + v^2 + ...) \simeq 4M_\eta^2 \\
k \cdot p_1 &= k \cdot p_2 = k \cdot q_1 = k \cdot q_2 = 2E^2 \simeq 2M_\eta^2
\end{aligned} \tag{C.3}
$$

$$
\begin{aligned}
&(p_{1\mu} - p_{2\mu})(p_{1\nu} - p_{2\nu})g^{\mu\nu} = (p_1 - p_2)^2 = p_1^2 + p_2^2 - 2p_1 \cdot p_2 \\
= \;& 2M_\eta^2 - 2(2E^2 - M_\eta^2) = 4(M_\eta^2 - E^2) = 4M_\eta^2(1 - \gamma^2) \\
= \;& 4M_\eta^2\left(\frac{-v^2}{1 - v^2}\right) = -M_\eta^2 v^2(1 + v^2 + v^4 + ...) \simeq -M_\eta^2 v^2
\end{aligned} \tag{C.4}
$$

$$p_{1\mu}p_{1\nu}q_{1\lambda}q_{2\sigma}\epsilon^{\mu\nu\lambda\sigma} = p_{2\mu}p_{2\nu}q_{1\lambda}q_{2\sigma}\epsilon^{\mu\nu\lambda\sigma} = 0 \tag{C.5}$$

$$\begin{aligned} & p_{1\mu}p_{2\nu}q_{1\lambda}q_{2\sigma}\epsilon^{\mu\nu\lambda\sigma} + p_{2\mu}p_{1\nu}q_{1\lambda}q_{2\sigma}\epsilon^{\mu\nu\lambda\sigma} \\ = \; & p_{1\mu}p_{2\nu}q_{1\lambda}q_{2\sigma}\epsilon^{\mu\nu\lambda\sigma} + p_{1\nu}p_{2\mu}q_{1\lambda}q_{2\sigma}\epsilon^{\mu\nu\lambda\sigma} \\ = \; & p_{1\mu}p_{2\nu}q_{1\lambda}q_{2\sigma}\epsilon^{\mu\nu\lambda\sigma} - p_{1\nu}p_{2\mu}q_{1\lambda}q_{2\sigma}\epsilon^{\nu\mu\lambda\sigma} = 0 \end{aligned} \tag{C.6}$$

$$\begin{aligned} & (p_{1\mu} - p_{2\mu})(p_{1\nu} - p_{2\nu})(q_1^\mu q_2^\nu + q_1^\nu q_2^\mu) \\ = \; & 2\left[(p_1 - p_2)\cdot q_1\right]\left[(p_1 - p_2)\cdot q_2\right] = 4p^2q^2\cos^2\theta \\ \simeq \; & 4M_\eta^4 v^2 \cos^2\theta \end{aligned} \tag{C.7}$$

the only non zero terms in the amplitude are the ones proportional to $g^{\mu\nu}$

$$|\mathcal{M}|^2 \simeq \frac{4g^4M_\eta^4v^2}{(4M_\eta^2 - M_{Z'}^2)^2}\left(2(c_v^2 + c_a^2)(1 + 2\cos^2\theta) + (c_v^2 - c_a^2)\left(\frac{m_f^2}{M_\eta^2}\right)\right) \tag{C.8}$$

The amplitude is proportional to v^2. The term proportional to v^0 is explicitly zero even without the approximations that has been done. η_R is not a good candidate for non relativistic DM.