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DISCONTINUITY STRESSES IN BEAMS ON ELASTIC FOUNDATIONS AND CYLINDRICAL SHELLS SUBJECTED TO AXISYMMETRICAL LOADING

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NOTATION

A, B, F, G	= constants of integration;
C	= carry over factor;
$D = Eh^3/12(1-\mu^2)$	= flexural rigidity of cylinder per unit width;
DF	= distribution factor;
$e = 2.718$	= Naperian base;
E	= Young's modulus;
FEM	= fixed-end moment;
h	= cylinder wall thickness;
I	= moment inertia of beam;
k	= stiffness influence coefficient;
k_f	= foundation stiffness;
K	= stiffness factor;
L	= length of beam or cylinder segment;
M	= moment per unit width of cylinder; beam moment;
p	= radial loading per unit area for cylinder; load per unit length for beam;
r	= cylinder radius;
T	= temperature;
V	= shear per unit width for cylinder; beam shear;
x	= longitudinal distance along beam or cylinder from discontinuity point;
w	= radial displacement of cylinder wall; beam deflection;
w_p, w'_p, w''_p, w'''_p	= particular solution to differential equation and its derivatives;
	= coefficient of thermal expansion;
θ	= slope of cylinder wall relative to axis; beam slope;
μ	= Poisson's ratio;
$\beta = \sqrt[4]{\frac{3(1-\mu^2)}{r^2h^2}}$	= for cylinder; and
$\beta = \sqrt[4]{\frac{k_f}{4EI}}$	= for beam.

SYNOPSIS

A method of analysis is presented for obtaining discontinuity stresses in beams on elastic foundations and cylindrical shells subjected to axi-symmetrical loading. Also presented are the solutions of numerous basic cases which provide the necessary numerical constants required in the proposed method of analysis. Several examples are included.

INTRODUCTION

The deflection of a beam on an elastic foundation and the radial displacement of a longitudinal strip along a cylinder subjected to axi-symmetrical loading are given by[1]

$$E\,I\,\frac{d^4w}{dx^4} + k_f\,w = p(x) \tag{1a}$$

and

$$D\,\frac{d^4w}{dx^4} + \frac{Eh}{r^2}\,w = p(x) \tag{1b}$$

Using the notation

$$\beta\ (\text{beam}) = \sqrt[4]{\frac{k_f}{4\,E\,I}} \tag{2a}$$

and

$$\beta\ (\text{cylinder}) = \sqrt[4]{\frac{3(1-\mu^2)}{r^2h^2}} \tag{2b}$$

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[1] "Strength of Materials," by S. Timoshenko, Part II, D. Van Nostrand Co., New York, 1956.

the solution of Eqs. 1 for any segment of beam or cylinder having constant section is given by

$$w(x) = e^{\beta x} (A \cos \beta x + B \sin \beta x) + e^{-\beta x} (F \cos \beta x + G \sin \beta x) + w_p(x) \quad (3)$$

where $w_p(x)$ is the particular solution to the non-homogeneous equation which results from the loading $p(x)$. By successive differentiation of Eq. 3, the slope, moment, and shear relationships for the case of a beam are expressed as

$$\theta(x) = \beta e^{\beta x} \left[(B + A) \cos \beta x + (B - A) \sin \beta x\right] + \beta e^{-\beta x} \left[(G - F) \cos \beta x + (-G - F) \sin \beta x\right] + w_p'(x) \quad (4)$$

$$M(x) = E I \beta^2 e^{\beta x} \left[2 B \cos \beta x - 2 A \sin \beta x\right] + E I \beta^2 e^{-\beta x} \left[-2 G \cos \beta x + 2 F \sin \beta x\right] + E I w_p''(x) \quad (5)$$

and

$$V(x) = E I \beta^3 e^{\beta x} \left[(2 B - 2 A) \cos \beta x + (-2 A - 2 B) \sin \beta x\right] + E I \beta^3 e^{-\beta x} \left[(2 F + 2 G) \cos \beta x + (2 G - 2 F) \sin \beta x\right] + E I w_p'''(x) \quad (6)$$

For the case of a cylinder, Eqs. 4, 5, and 6 are identical except that E I is replaced by D. The constants of integration A, B, F, and G can be adjusted to satisfy any set of boundary conditions.

If one is concerned only with a single segment of beam or a uniform cylinder having continuous loading, the four constants of integration are obtained by solving four simultaneous equations which satisfy the prescribed boundary conditions. However, if discontinuities which divide the structure into n segments, exist in either the loading or the flexural rigidity (E I, D), it becomes necessary to write a separate expression for the radial displacements in each segment between points of discontinuity.

This results in 4 n unknown constants of integration which requires writing 4 n equations to satisfy continuity between segments and the prescribed boundary conditions of the system. Considerable work is involved in setting up and solving such equations; therefore, it is the purpose of this paper to present the complete solution to numerous basic cases involving a uniform beam on an elastic foundation having continuous loadings, along with a method of analysis which makes use of these data in solving the more general discontinuous case described above.

METHOD OF ANALYSIS

The method of analysis described herein, for obtaining discontinuity stresses in beams or cylinders, makes use of an iterative procedure, similar to that introduced by H. Cross, for continuous beams and frames.[2] This iterative procedure, commonly known as moment distribution, consists of first obtaining fixed-end moments in each element resulting from the application of applied loads or resulting from some prescribed joint displacement. While the resulting deformations are compatible, the joints are not in static-moment equilibrium; therefore, it is necessary to allow the joints to rotate one by one until each joint in the structure is in static equilibrium. While the rate of convergence of this procedure is not within the scope of this investigation, the examples included in this paper would indicate that, generally, a more rapid rate of convergence is obtained when applying the method to beams on elastic foundations and to cylinders, than when applying it to continuous beams and frames.

To define the changes in the end moments during this iteration, one must predetermine the following quantities: (1) stiffness factor, K, of each element defined as that moment required at the end of an element to produce a unit rotation of that end, (2) carry-over factor, C, for each element defined as the moment produced at one end of an element resulting from placing a unit moment at the opposite end, and (3) distribution factors, DF, at each joint definedas the fractional parts of an externally applied joint moment carried by each element entering the joint.

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[2] "Analysis of Continuous Frames by Distributing Fixed-End Moments" by H. Cross, Proceedings, ASCE, 1930.

The general method of analysis presented herein for determining discontinuity stresses makes use of the above iterative procedure along with the use of the principle of superposition. This general method is outlined in the subsequent steps.

Step No. 1.

(a) The beam or cylinder is considered as externally restrained against radial displacements and rotations at all points of discontinuity except at the free ends. Locations of concentrated loadings, such as those produced by stiffening rings around a cylinder, are considered as discontinuity points and are, therefore, restrained in the manner described above.

Distributed loadings which, in the case of cylinders, may be due to temperature changes or internal pressures are placed on the beam or cylinder and the fixed-end shears and moments are determined.

(b) The external-moment restraint at each discontinuity point is now removed by the iterative procedure described previously which results in a balanced set of end moments at each discontinuity point.

(c) Knowing the end moments found in Step 1(b), one must now determine the end shears for each segment. This step is best illustrated by considering a single beam or cylinder segment A B as shown in Fig. 1. For the case of a cylinder, this segment is considered as a flat plate on an elastic foundation where the foundation stiffness is equivalent to the hoop stiffness provided by the cylinder. With the end moments M_{Ao} and M_{Bo} for segment A B in Fig 1(a) known, the unknown end shears V_{Ao} and V_{Bo} are determined by separating the total loading on the segment into three separate, but equivalent, loadings as shown in Figs. 1(b), 1(c), and 1(d). This separation is necessary due to the fact the elastic foundation forces on the segment in Fig 1(a) are unknown.

The fixed-end moments and shears for the segment in Fig. 1(b) are those obtained in Step 1(a).

The moments M''_A and M''_B in Figs. 1(b) and 1(c), respectively, are now determined to satisfy the conditions that

$$M''_A = M_{Ao} + FEM_A \tag{7}$$

and

$$M''_B = M_{Bo} + FEM_B \tag{7}$$

The corresponding end shears are determined next for these cases; thus, the total end shears (Fig. 1(a)) can be determined by superposition as follows:

$$\begin{aligned} V_{Ao} &= V'_A - V''_A + V'''_A \\ V_{Bo} &= V'_B - V''_B + V'''_B \end{aligned} \tag{8}$$

(d) Force equilibrium considerations at the restrained discontinuity points will now yield the reaction forces necessary, at these points, to prevent the normal displacements.

Step No. 2. A complete set of stiffness-influence coefficients are now obtained as follows:

(a) Restrain all discontinuity points as in Step 1(a) and produce a unit normal displacement at one of the discontinuity points without allowing any joint rotations.

(b) Remove the external moment restraints for each discontinuity by iteration in the same manner as in Step 1(b). This results in known end moments for each segment of the beam or cylinder, for example, M_{AA} and M_{BA} in Fig. 2.

(c) The end shears are determined, for this case, in a manner similar to that in Step 1(c), as illustrated in Fig. 2. Using superposition, the algebraic sum of the moments and shears in Figs. 2(b), (c), and (d) must equal the moments and shears, respectively, in Fig. 2(a). For those segments not adjacent to the point of unit displacement, the fixed-end moments (FEM) will be zero.

(d) The normal forces required at all discontinuity points are now calculated from the known end shears in a manner similar to that in Step 1(d). These forces are the stiffness-influence coefficients.

(e) Steps 2(a) through 2(d) must be repeated for a unit displacement at each of the remaining discontinuities, in order that the complete set of influence coefficients be known. These coefficients, when written

in matrix form, produce a symmetrical matrix about the diagonal due to Maxwell's law of reciprocal deflections.

Step No. 3. Using the information determined in Steps 1 and 2, one can write a separate superposition equation for each discontinuity point, expressing the corresponding applied normal force at each point in terms of the unknown, normal displacements of all discontinuity points. These simultaneous equations, when solved, will yield the magnitude of the normal displacements of all discontinuity points in the beam or cylinder.

Step No. 4.

(a) The end shears and moments, for each case solved in Step 2 involving a unit displacement at a particular point, are multiplied by the displacement of that particular point as found in Step 3. Using superposition, all resulting end moments and shears obtained in this manner are added algebraically to those found in Step 1; thus, the final end moments and shears for each segment of the beam or cylinder are known.

(b) One can now write the general equations for the normal displacement, slope, moment, and shear along each segment using Eqs. 3, 4, 5, and 6, respectively. The constants of integration A, B, F, and G appearing in these equations are determined by superposition, using the final end moments and shears found in Step 4(a) along with the basic integration constants, found in Tables 10 and 11, which were determined for the unit loading conditions shown at the head of that table.

To illustrate this step, consider a beam segment AB as shown in Fig. 3. Consider also that the end moments M_A and M_B and that the shears V_A and V_B are the final values determined in Step 4(a). The constants of integration A, B, F, and G for this segment may now be expressed in terms of these known end moments and shears and the appropriate constants in Tables 10 and 11 as follows:

$$\begin{Bmatrix} A \\ B \\ F \\ G \end{Bmatrix} = \begin{bmatrix} A_1 & A_2 & A_3 & A_4 \\ B_1 & B_2 & B_3 & B_4 \\ F_1 & F_2 & F_3 & F_4 \\ G_1 & G_2 & G_3 & G_4 \end{bmatrix} \begin{Bmatrix} M_A \\ V_A L \\ M_B \\ V_B L \end{Bmatrix} \frac{L^2}{E I} \qquad (9)$$

For the positive loading shown in segment A B in Fig. 3, the particular solution $w_p(x)$ appearing in Eq. 3 and its derivatives may be expressed as follows:

$$w_p(x) = -\frac{1}{k_f} \left[(1 - \frac{x}{L}) p_A + \frac{x}{L} p_B \right] \tag{10a}$$

$$w_p'(x) = -\frac{1}{k_f L} \left[(p_B - p_A) \right] \tag{10b}$$

and

$$w_p''(x) = w_p'''(x) = 0 \tag{10c}$$

In the case of a cylinder, D is substituted for E I in Eqs. 9, and E H/r^2 is substituted for k_f in Eqs. 10. Having the constants of integration as defined by Eqs. 9 and the particular solution to the differential equation with its derivatives as given by Eqs. 10, the normal displacements, slopes, moments, and shears are completely defined in the interval A B as used in this illustration. The trigometric relations shown in Table 12, which appear in Eqs. 3, 4, 5, and 6, have been tabulated to simplify the use of these equations.

The general method of analysis, as outlined in Steps 1-4, can be carried out quite rapidly using the basic data tabulated in Tables 1 through 9. All curves have been plotted over a range of β L between 0 to 10. Also shown in these tables are the approximate relationships for obtaining the ordinates for values of β L greater than 10.

The stiffness and carry-over factors to be used in the iteration procedure are given in Tables 1, 2, and 3. The fixed-end moments and shears for the case of a unit displacement, as required in Steps 2(a) and 2(e), are presented in Tables 3, 4, and 5. The fixed-end moments and shears required in Step 1(a) for uniform and triangular loadings are given in Tables 5 through 9.

Three problems have been completely solved in the following section to illustrate the generality of the method of analysis. Example 1

illustrates its use on a combined footing, Example 2 illustrates its use when stresses in a cylinder are produced by axisymmetrical temperature changes, and Example 3 illustrates its use for the case of axisymmetrical loading on a cylinder. The general method presented herein can also account for partial restraint against both normal displacement and rotation at various points along the beam or cylinder.

SAMPLE PROBLEMS

Example 1. A combined footing 26 ft. long, 48 in. wide, and 24 in. deep is loaded by two concentrated loades, as shown in Fig. 4. The foundation stiffness k_f is assumed constant. This is required to determine the foundation pressures and the bending-moment diagram for the footing.

Step No. 1 - The fixed-end moments and shears are zero.

Step No. 2 - The stiffness coefficients are determined as shown in Fig. 5.

Step No. 3 - The superposition equations are

$$0.947\ Y_B + 0.070\ Y_C = -\frac{160}{40}\ (0.0289)$$

and

$$0.070\ Y_B + 0.339\ Y_C = -\frac{80}{48}\ (0.0289)$$

from which $Y_B = -0.0928$ in. and $Y_C = - -.1225$ in.

Step No. 4 - The final end moments and shears are shown in Fig. 6. For segment B C Eqs. 9 become

$$\begin{Bmatrix} A \\ B \\ F \\ G \end{Bmatrix} = \begin{bmatrix} 0.00774 & 0.000667 & -0.0118 & 0.00261 \\ -0.000763 & -0.000324 & -0.000107 & -0.00248 \\ 0.0909 & 0.0386 & -0.0116 & 0.00756 \\ -0.0894 & -0.000324 & -0.000107 & -0.00247 \end{bmatrix} \begin{Bmatrix} 0.823 \\ -5.33 \\ 0.0303 \\ 4.07 \end{Bmatrix}$$

which leads to A = 0.0073 in., B = -0.0090 in., F = -0.101 in., and G = -0.0819 in.

Step No. 5 - The general equations for moment and deflection are shown in Fig. 7.

Example 2. The long cylindrical shell shown in Fig. 8 is held at a temperature T_1 in region A B and at a temperature T_2 in region C D. It is desirable to calculate the unit stresses in the transition region B C for a linear temperature change over this region. For purposes of analysis, stresses produced by the axisymmetrical loading, as shown, which are equivalent to those stresses produced by the temperature ΔT across region B C are obtained.

Step No. 1 - In Fig. 9, joints B and C are supported.

Step No. 2 - To determine the stiffness coefficients, the effect of a unit displacement at B is shown in Fig. 10. The effect of a unit displacement of C is shown in Fig. 11; the solution is similar to that for a unit displacement of B.

Step No. 3 - The superposition equations are

$$-9.13\ D\ Y_B + 0.42\ D\ Y_C = -0.186\ p$$

$$0.42\ D\ Y_B - 9.13\ D\ Y_C = -0.1640\ p$$

from which $Y_B = 0.0285p/D$ and $Y_C = 0.181\ p/I$.

Step No. 4 - The final end moments and shears are shown in Fig. 12. Since this problem is completely anti-symmetrical, $M_B = -M_C$ and $V_B = V_C$, therefore, the foregoing values should be adjusted as follows: $M_B = 0.052\ p$, $V_B = -0.014\ p$, $M_C = -0.052\ p$, and $V_C = -0.014\ p$.

For section B C ($\beta L = 2.08$), Eq. 9 becomes (Step 4(b))

$$\begin{Bmatrix} A \\ B \\ F \\ G \end{Bmatrix} = \begin{bmatrix} 0.00273 & 0.00173 & -0.0206 & 0.00382 \\ -0.00327 & -0.00144 & 0.00641 & -0.00649 \\ 0.1248 & 0.0602 & -0.0334 & 0.01680 \\ -0.1189 & -0.00144 & 0.00641 & -0.00649 \end{bmatrix} \begin{Bmatrix} 0.052 \\ -0.014\ L_{BC} \\ -0.052 \\ -0.014\ L_{BC} \end{Bmatrix}$$

Find Maximum Stress at B and C. - Let ΔT represent 200 F, α equal 6.5×10^{-6} in. per in., h be 0.100 in, E equal 30×10^{6} psi, and r represent 15 in. These lead to

$$p = \frac{2h\ E\ \Delta T}{r} = 260 \text{ psi}$$

$$M_B = -M_C = (0.052)\ (260) = 13.5 \text{ lb-in. per in.}$$

and

$$\sigma = \frac{6\ M}{h^2} 8100 \text{ psi.}$$

Example 3. The cylinder shown in Fig. 13 has a recess between locations B and C, and is loaded axially on the recess collar at location C. In obtaining radial displacements, slopes, moments, and shears, a longitudinal strip of unit width is removed and placed on an elastic foundation whose stiffness equals the hoop stiffness of the cylinder. In this solution, the axis of all segments of the strip are assumed to coincide; therefore, it is necessary to load the strip with concentrated moments at points B and C in order to account for the eccentricities of the axial loads at these points in the cylinder. Once the loading on the unit strip is established, as shown in Fig. 13, the solution is carried out in a manner similar to Example 1.

Step No. 1 - In Fig. 14 joints B and C are supported.

Step No. 2 - To determine the stiffness coefficients, a unit displacement of B is shown in Fig. 15. In Fig. 16 a unit displacement of C is shown.

Step No. 3 - The superposition equations are

$$21.1\ D_o\ Y_B - 0.48\ D_o\ Y_C = -0.0339\ p$$

and

$$-0.47\ D_o\ Y_B + 20.3\ D_o\ Y_C = -0.060\ p$$

from which $Y_B = -0.00167\ p/D_o$ and $Y_C = -0.00299p/D_o$.

Step No. 4 - The final end moments and shears are shown in Fig. 17.

FIGURES

In all figures and tables the flexural rigidity term E I (for beams) should be replaced by the flexural rigidity D when applying to plates.

ADDITIONAL REFERENCES

1. "Beams on Elastic Foundation," by M. Hetinyi. The Univ. of Michigan Press, Ann Arbor, Michigan, 1946.

2. "Analysis of Finite Beams on Elastic Foundations," by D. C. Gazis. Proceedings, ASCE, No. ST 4, July, 1958.

3. "Influence Lines for Pressure Distribution Under a Finite Beam on Elastic Foundation," by K. C. Ray. Journal of the Amer. Concrete Inst. December, 1958.

4. "Analysis of Continuous Conical Shells of Rotational Symmetry by Method of Successive Approximation," by G. Oravas. Internatl. Assn. for Bridge and Structural Engrg. Publications, Vol. 14, p. 195, 1954.

5. "Solution of Problems in Beams on Elastic Foundations by a Method of Transformation," by R. S. Alwar, The Journal of the Institution of Engineers (India), Vol. 40, No. 12, Part I, August, 1960.

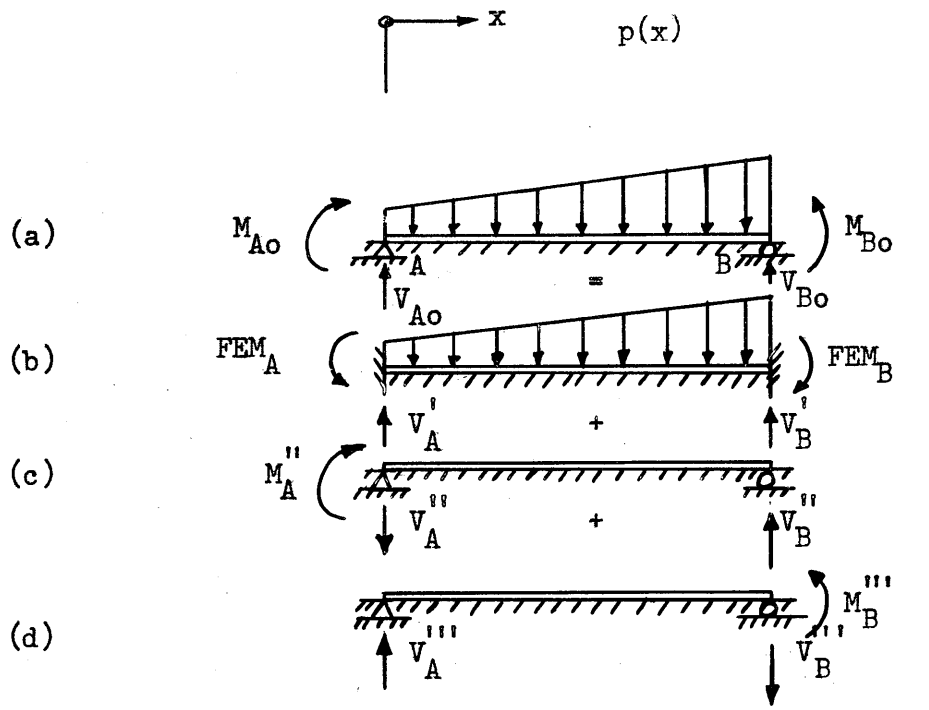


FIG. 1

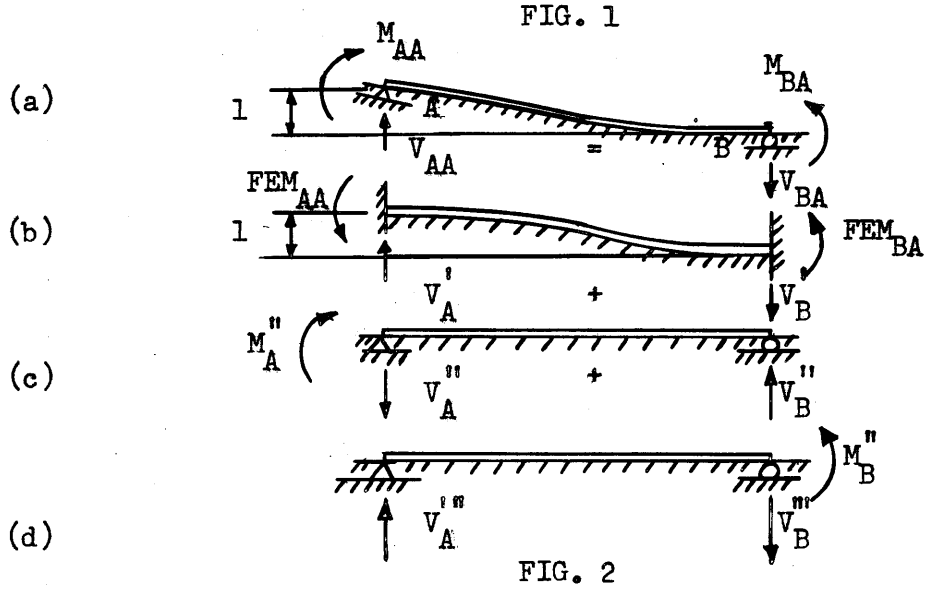


FIG. 2

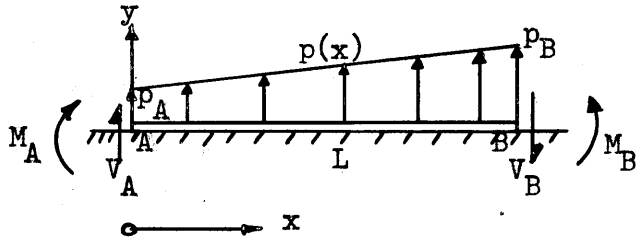
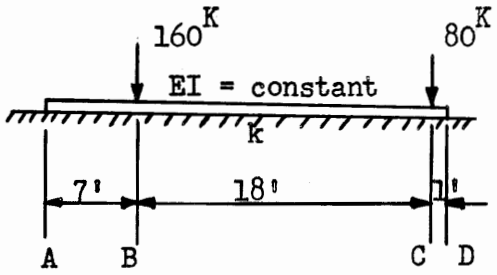
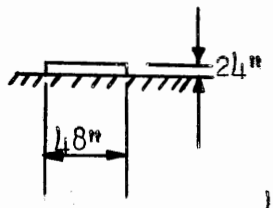


FIG. 3





$k = .200\ K/in^3$

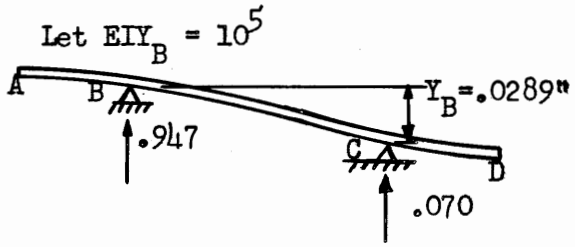
$I = 1152\ in^3$

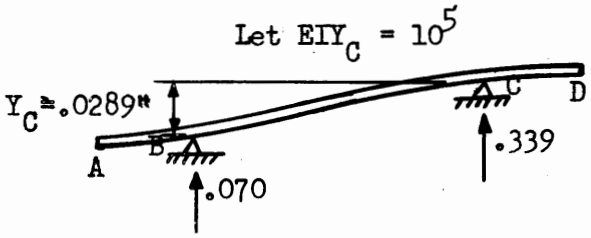
$E = 3000\ K/in^2$

$$\beta = \sqrt[4]{\frac{k_f}{4EI}} = 0.0110\ in^{-1}$$

	βL	K		K/EI	C		
BA	.925	.796	EI/L_{AB}	0.114	0	ΣK_B=0.392	ΣK_C=0.278
BC, CB	2.375	5.001	EI/L_{BC}	0.278	.258	DF_{BA}=0.709	DF_{CB}=0.998
CD	.132	.0005	EI/L_{CD}	0.0005	0	DF_{BC}=0.291	DF_{CD}=0.002

FIG. 4





.291	.709	.998	.002
0	.258	.258	0
-17.2	+24.9	+6.4	0
- 2.2	- 5.5	-6.4	
	- 1.7	-1.4	
+ 0.5	+ 1.2	+1.4	
	+ 0.4	+0.3	
- 0.1	- 0.3	-0.3	
	- 0.1	-0.1	
	+ 0.1	+0.1	
-19.0	+19.0	0	0
19.0	19.0 K"		
.470	.477 K	.070	0

D F
C

FEM

.291	.709	.998	.002
0	.258	.258	0
0	-6.4	-24.9	+.5
+1.9	+4.5	+24.4	0
	+6.3	+ 1.2	
-1.8	-4.5	- 1.2	0
	-0.3	- 1.2	
+0.1	+0.2	+ 1.2	0
0	0	- .5	+.5
0	.5		.5
	.070	.277	.062

FIG. 5

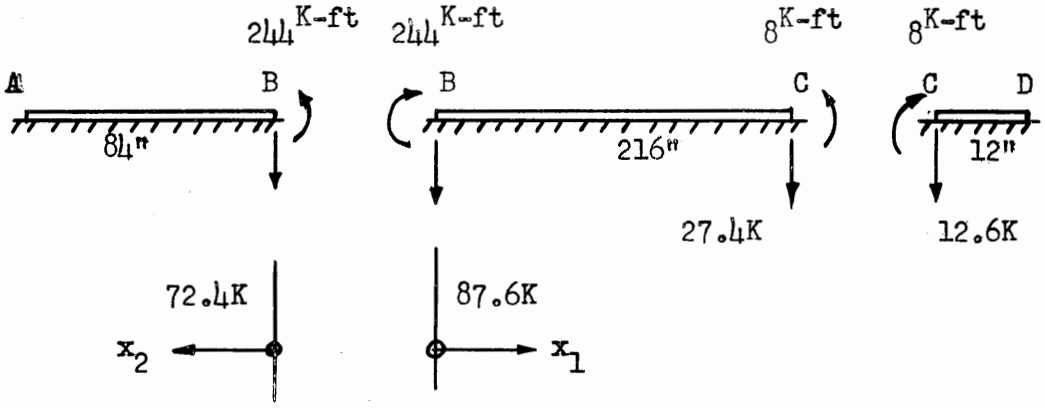


FIG. 6

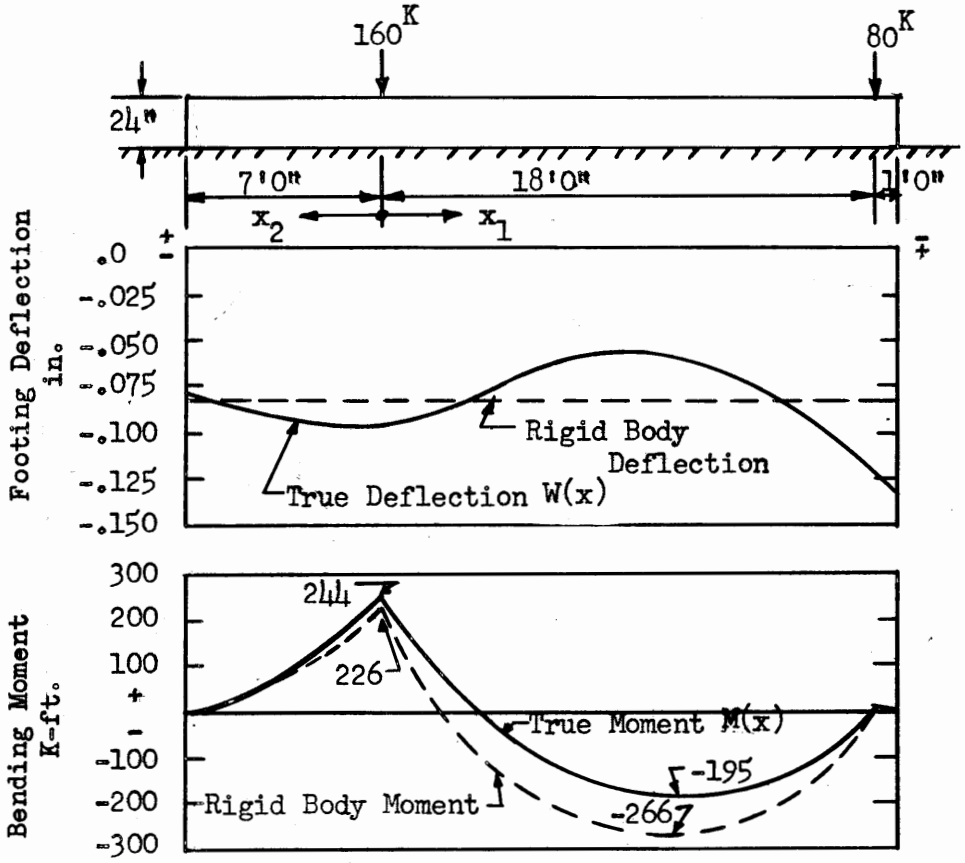


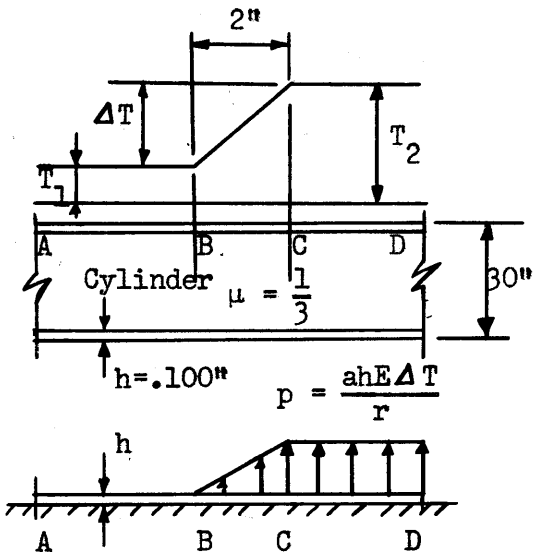
FIG. 7

$$W(x_1)=.0073e^{\beta x_1}\cos\beta x_1-.009e^{\beta x_1}\sin\beta x_1-.101e^{-\beta x_1}\cos\beta x_1-.082e^{-\beta x_1}\sin\beta x_1$$

$$W(x_2)=-.0099e^{\beta x_2}\cos\beta x_2-.0083e^{\beta x_2}\sin\beta\, x_2-.084e^{-\beta x_2}\cos\beta x_2-.0813e^{-\beta x_2}\sin\beta x_2$$

$$M(x_1)=-24.5e^{\beta x_1}\sin\beta x_1-30.2e^{\beta x_1}\cos\beta x_1-338e^{-\beta x_1}\sin\beta x_1+274e^{-\beta x_1}\cos\beta x_1$$

$$M(x_2)=33.2e^{\beta x_2}\sin\beta x_2-27.8e^{\beta x_2}\cos\beta x_2-281e^{-\beta x_2}\sin\beta x_2+272e^{-\beta x_2}\cos\beta x_2$$



$$\beta = \sqrt[4]{\frac{3(1-\mu^2)}{r^2 h^2}} = 1.042$$

	βL	K*	C
AB	Large	2.08D (Fig. 6)	-----
BC	2.08	2.32D (Fig. 4)	.330 (Fig. 4)
CD	Large	2.08D (Fig. 6)	-----

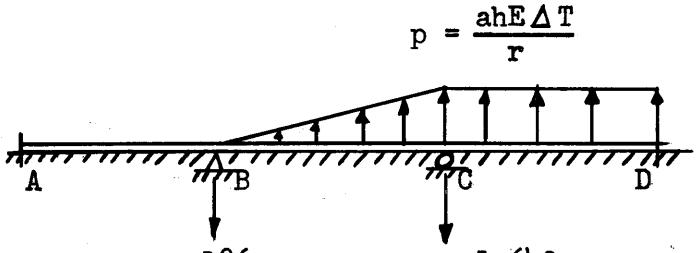
* For Large βL = 2 βD

$\Sigma K_B = 4.40$ $\Sigma K_C = 4.40$

$DF_{BA} = .473$ $DF_{CB} = .527$

$DF_{BC} = .527$ $DF_{CD} = .473$

FIG. 8



	.473	.527	.527	.473	
	0	.330	.330	0	
0	0	-.114p	+.180p	-.460p	
	+.054	+.060	+.148	+.132	
		+.049	+.020		
	-.023	-.026	-.010	-0.10	
		-.003	-.009		
	+.001	+.002	+.005	+.004	
		+.002	+.001		
	-.001	-.001	-.001		
0	+.031p	-.031p	+.334p	-.334p	0
	.031p →	← .031p	.334p →	← .334p	
	.032p ↓	↓ .154p	.811p ↓	↓ .829p	

Remarks

DF; Figs. 4, 6

C; Fig. 4

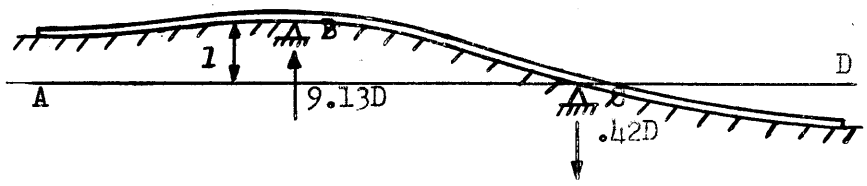
FEM; Figs. 11, 12 (step 1a)

Iteration Procedure (Step 1b)

End Moments

End Shears (Step 1c) (Figs. 5, 6, 11, 12)

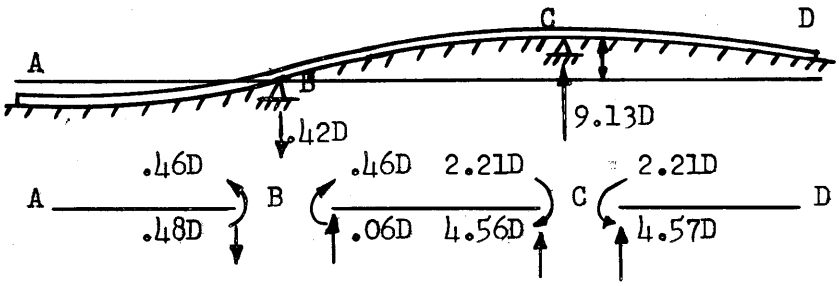
FIG. 9



.473	.527	.527	.473	DF
0	.330	.330	0	C
-2.18 D	+2.40 D	+1.00 D	0	FEM; Fig. 7, 9 (Step 2a)
-0.11	-0.11	- .53	-.47	
	- .17	- .04		Iteration Procedure (Step 2b)
+ .08	+ .09	+ .02	+.02	
	+ .01	+ .03		
	- .01	- .02	-.01	
-2.21 D	+2.21 D	+ .46 D	-.46 D	End Moments
2.21 D	2.21 D	.46 D	.46 D	End Shears (Step 2c) Figs. 5, 6, 7, 9
A ——— B	B ——— C		C ——— D	
4.57 D	4.56 D	.06 D	.48 D	

Remarks

Fig. 10



(Step 2e)

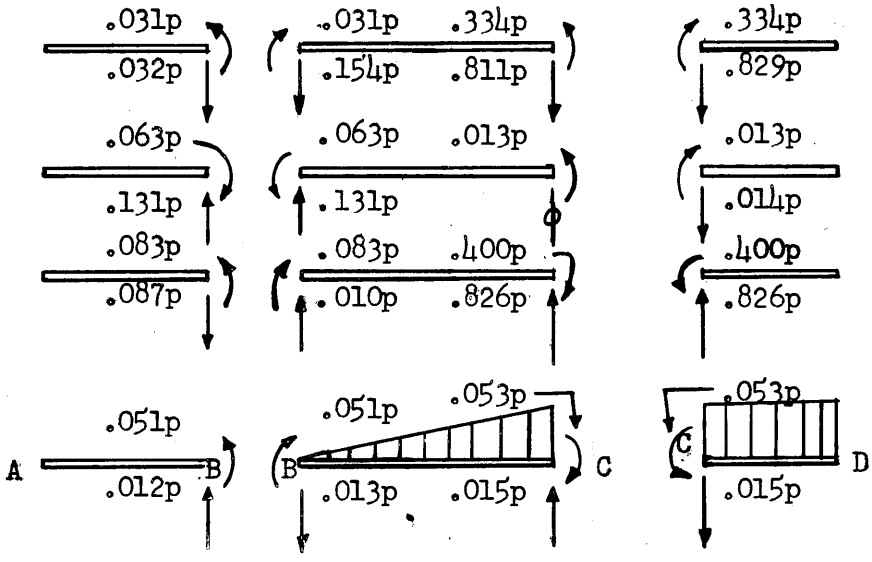
k_{BB}=+9.13D k_{CB}=-.42D

k_{BC}=- .42D k_{CC}=9.13D

$-9.13D\ Y_B + .42D\ Y_C = -.186p \qquad Y_B = +.0285\frac{p}{D}$

$+.42D\ Y_B - 9.13D\ Y_C = -1.640p \qquad Y_C = +.1810\frac{p}{D}$

FIG. 11



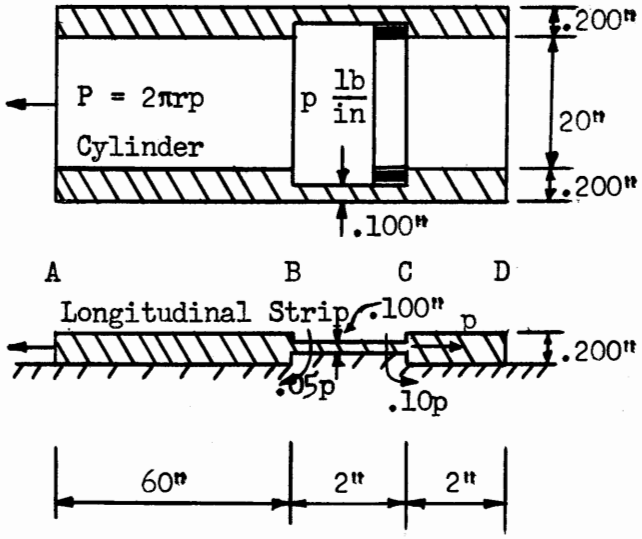
Step No. 1

.0285 $\frac{p}{D}$ (Step No. 2a)

.181 $\frac{p}{D}$ (Step No. 2b)

Final End Moments And Shears Equal Sum of Above Three Cases. (Step 4a)

FIG. 12



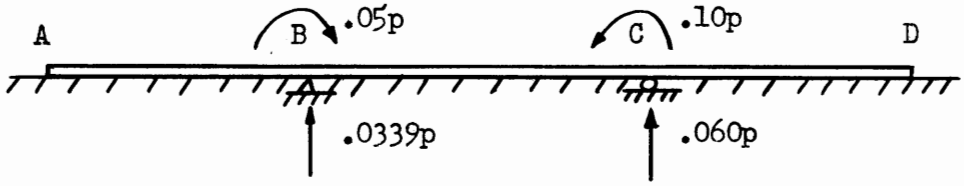
	t	L	r	β	βL	D	K	C
AB	.200	60	10	.901	54.00	$8D_o$	$14.40D_o$	-
BC	.100	2	10	1.275	2.55	D_o	$2.65D_o$	.21
CD	.200	2	10	.901	1.80	$8D_o$	$13.80D_o$	-

K_B=17.05 DF_{BA}=.845 DF_{BC}=.155

K_C=16.45 DF_{CB}=.161 DF_{CD}=.839

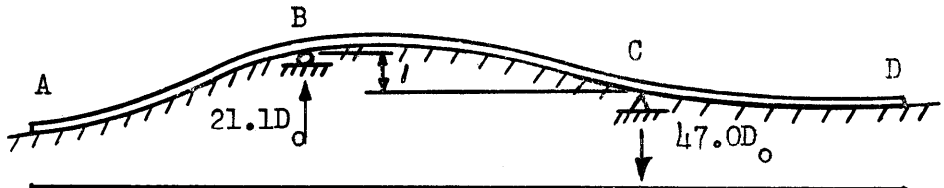
μ = 1/3

FIG. 13



	.845	.155	.161	.839	DF
	0	.21	.21	0	C
		.05p	-.10p		Applied Moment
0	-.0422p	-.0078p	+.0161p	+.0839p	0
		+.0034	-.0016		
	-.0029	-.0005	+.0003	+.0013	Iteration
		+.0001	-.0001		
—	-.0001	—	—	+.0001	—
0	-.0452p	-.0048p	+.0147p	+.0853p	0 End Moments
	.0452p	.0048p	.0147p	.0853p	
	.0407p	.0063p	.0186p	.079p	End Shears

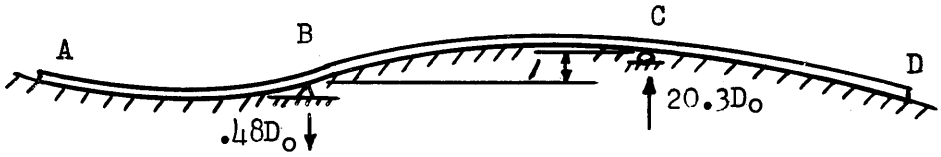
FIG. 14



	.845	.155	.161	.839		DF
	0	.21 →	← .21	0		C
	-12.95D$_o$	+3.30D$_o$	+.55D$_o$			FEM
	+ 8.15	+1.50	-.09	-.46		
		- .02	+.31			
	+ .02	0	-.05	-.26		
		- .01				
	+ .01					
0	- 4.77D$_o$	+4.77D$_o$	+0.72D$_o$	-.72D$_o$	0	End Moments
	4.77D$_o$	4.77D$_o$	0.72D$_o$	.72D$_o$		
	15.9D$_o$	5.15D$_o$	.20D$_o$	.67D$_o$		End Shears

Remarks

FIG. 15



	.845	.155	.161	.839		DF
	0	.21 →	← .21	0		C
		-.55D$_o$	-3.30D$_o$	+13.00D$_o$		FEM
	+.46	+.09	-1.55	- 8.15		
		-.33	+ .02			
	+.28	+.05	0	- .02		
			- .01			
			0	+ .01		
0	+.74D$_o$	-.74D$_o$	-4.82D$_o$	+ 4.82D$_o$	0	
	.74D$_o$	.74D$_o$	4.82D$_o$	4.82D$_o$		
	.66D$_o$	.18D$_o$	5.21D$_o$	15.1D$_o$		

Remarks

FIG. 16

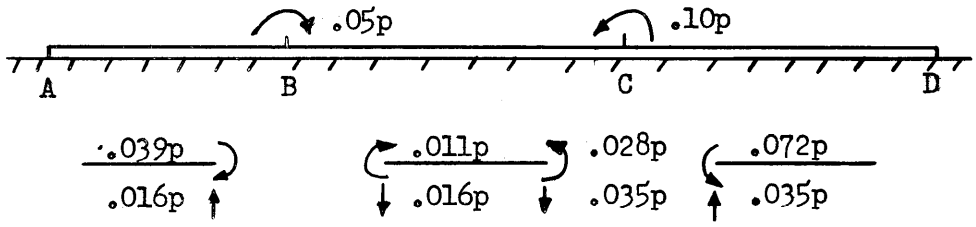


FIG. 17

TABLES

The location of the decimal points in all subsequent tables is given by subtracting 50 from the last two digits of each tabular value. If this difference is zero the decimal point is located in front of the first digit of the number. However, if the difference is +n, the decimal is moved n digits to the right and if the difference is -n, the decimal is moved n digits to the left. For example, the tabular value -155546 is equal to -.00001555 and the tabular value 358151 is equal to 3.581.

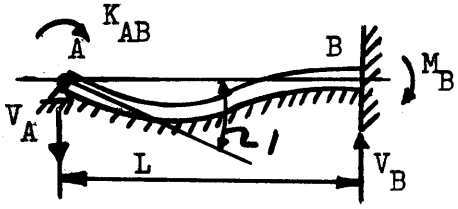


TABLE 1

$K_{AB} = m \frac{EI}{L}$ $\quad V_B = v_B \frac{K_{AB}}{L}$

$V_A = v_A \frac{K_{AB}}{L}$ $\quad M_B = c_{AB} K_{AB}$

For Large βL: $m = 2(\beta L)$ $\quad v_B = 0$

$v_A = \beta L$ $\quad c_{AB} = 0$

βL	m	c_{AB}	v_A	v_B
0.00	400051	500050	150051	150051
0.05	400051	500050	150051	150051
0.10	400051	500050	150051	150051
0.15	500051	500050	150051	149951
0.20	400051	499950	150051	149951
0.25	400051	499950	150051	149951
0.30	400051	499950	150051	149951
0.35	400051	499850	150051	149951
0.40	400051	499650	150051	149851
0.45	400051	499550	150151	149851
0.50	400251	499250	150251	149751
0.55	400351	498950	150351	149551
0.60	400451	498450	150451	149451
0.65	400651	497850	150651	149151
0.70	400951	497150	150951	148951
0.75	401251	496250	151151	148551
0.80	401551	495150	151551	148151
0.85	401951	493850	151951	147651
0.90	402451	492250	152451	147051
0.95	403051	490450	153051	146351
1.00	403751	488350	153751	145551
1.05	404551	485850	154551	144651
1.10	405551	483050	155451	143551
1.15	406551	479750	156451	142351
1.20	407751	476150	157651	140951
1.25	409151	472150	158951	139451
1.30	410651	467650	160451	137751
1.35	412351	462650	162051	135851
1.40	414251	457150	163851	133751
1.45	416351	451150	165751	131451
1.50	418651	444650	167951	129051
1.55	421151	437650	170251	126351
1.60	423951	430050	172751	123551
1.65	426851	421950	175451	120451
1.70	430151	413350	178351	117251
1.75	433551	404150	181351	113751

TABLE 1

βL	m	c_{AB}	v_A	v_B
1.80	437351	394550	184651	110151
1.85	441351	384450	188051	106351
1.90	445651	373950	191651	102451
1.95	450151	362950	195451	983050
2.00	455051	351550	199351	940650
2.05	460151	339850	203451	897150
2.10	465551	327850	207751	852550
2.15	471251	315650	212151	807050
2.20	477151	303150	216651	760950
2.25	483451	290550	221351	714450
2.30	489851	277650	226051	667650
2.35	496651	264950	230851	620750
2.40	503651	252150	235851	574050
2.45	510951	239350	240851	527650
2.50	518451	226650	245851	481750
2.55	526151	214050	251051	436550
2.60	534051	201650	256151	392150
2.65	542251	189450	261351	348850
2.70	550551	177450	266651	306650
2.75	559051	165750	271851	265750
2.80	567751	154350	277151	226250
2.85	576551	143250	282351	188250
2.90	585551	132550	287651	151850
2.95	594651	122250	292951	117050
3.00	603951	112250	298151	839849
3.10	622751	9356$\overline{49}$	308651	231749
3.20	641851	765149	319051	-304149
3.30	661351	611449	329451	-767649
3.40	680951	474349	339751	-115950
3.50	700751	353449	349951	-148350
3.60	720651	247949	360151	-174150
3.70	740651	157149	370251	-193950
3.80	760551	798$\overline{48}$	380251	-208150
3.90	780651	152348	390351	-217250
4.00	800651	-380$\overline{248}$	400351	-221850
4.10	820651	-808$\overline{648}$	410251	-222450
4.20	840651	-114549	420251	-219650
4.30	860651	-139949	430251	-213850
4.40	880551	-158249	440151	-205650
4.50	900551	-170449	450151	-195450
4.60	920551	-177149	460151	-183750
4.70	940451	-178049	470051	-170250
4.80	960451	-177549	480051	-157050
4.90	980351	-174249	490051	-143450
5.00	100052	-167249	500051	-129050

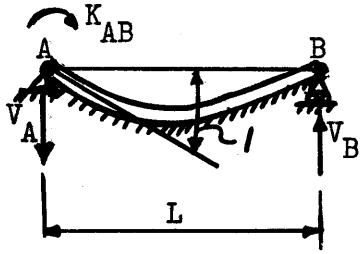
TABLE 1

βL	m	c_{AB}	v_A	v_B
5.10	102052	-158949	510051	-115050
5.20	104052	-149149	519951	-101350
5.30	106052	-138449	529951	-880249
5.40	108052	-127149	539951	-753949
5.50	110052	-115649	549951	-634649
5.60	112052	-104049	559951	-522749
5.70	114052	-926648	569951	-419049
5.80	116052	-820448	579951	-328449
5.90	118052	-715348	589951	-242949
6.00	120052	-613048	599951	-165349
6.10	122052	-525148	609951	-101149
6.20	124052	-441348	619951	-439248
6.30	126052	-358948	629951	909247
6.40	128052	-291548	639951	494248
6.50	130052	-229848	649951	833248
6.60	132052	-173148	659951	111349
6.70	134052	-125248	670051	133249
6.80	136052	-836847	680051	149549
6.90	138052	-518647	690051	157849
7.00	140052	-291247	700051	160149
7.10	142052	-529146	710051	162949
7.20	144052	345747	720051	177149
7.30	146052	484247	730051	174549
7.40	148052	499447	740051	159049
7.50	150052	657647	750051	154649
7.60	152052	763947	760051	155049
7.70	154052	121048	770051	174549
7.80	156052	211048	779951	611448
7.90	158052	302148	790051	287349
8.00	160052	870247	800051	109949
8.10	162052	147647	809951	476048
8.20	164052	871747	820051	931448
8.30	166052	809147	830051	796248
8.40	168052	702447	839951	734948
8.50	170052	535647	849951	559648
8.60	172052	491747	860051	327247
8.70	174052	402247	869951	442848
8.80	176052	349447	879951	314148
8.90	178052	304047	889951	217148
9.00	180052	417047	899951	342748
9.10	182052	272547	910051	125347
9.20	184052	200547	919951	454947
9.30	186052	907146	930051	-737847
9.40	188052	826946	940051	-758147
9.50	190052	746446	950051	-762547

TABLE 1

β L	m	c_{AB}	v_A	v_B
9.60	192052	666946	960051	-753547
9.70	194052	589746	970051	-733647
9.80	196052	515946	980051	-704747
9.90	198052	446146	990051	-668947
10.00	200052	380946	100052	-627947

TABLE 2



$$K_{AB} = m_A \frac{K_{AB}}{L}$$

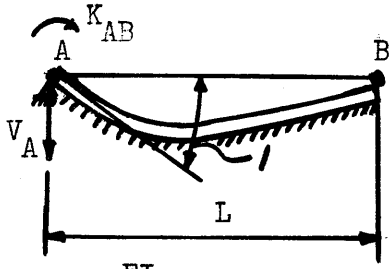
$$V_A = v_A \frac{K_{AB}}{L}$$

$$V_B = v_B \frac{K_{AB}}{L}$$

For Large βL: $m_A = 2(\beta L)$

$v_A = \beta L$

$v_B = 0$



$$K_{AB} = m \frac{EI}{L}$$

$$V_A = v \frac{Kab}{L}$$

For Large βL: $m_A = 2(\beta L)$

$v_A = \beta$

βL	m_A	v_A	v_B	m	v
0.00	300051	100051	100051	000000	150051
0.05	300051	100051	100051	833745	150051
0.10	300051	100051	999950	133347	150051
0.15	300051	100051	999950	674847	150051
0.20	300051	100051	999850	213248	150051
0.25	300051	100051	999650	520148	150051
0.30	300051	100051	999350	107749	150051
0.35	300151	100151	998850	199149	150051
0.40	300151	100251	998050	338649	150051
0.45	300351	100351	996850	539749	150151
0.50	300451	100551	995150	817249	150251
0.55	300651	100851	992950	118650	150351
0.60	300951	101151	989950	166050	150451
0.65	301351	101551	986250	225350	150651
0.70	301851	102151	981550	297750	150951
0.75	302351	102751	975750	383850	151151
0.80	303151	103551	968650	484050	151551
0.85	303951	104551	960250	598350	151951
0.90	304951	105651	950350	725950	152451
0.95	306151	107051	938850	865950	153051
1.00	307551	108551	925450	101651	153751
1.05	309051	110351	910150	117651	154551
1.10	310951	112351	892950	134251	155451
1.15	312951	114551	873550	151351	156451
1.20	315351	117051	852050	168651	157651
1.25	317951	119851	828350	185951	158951

TABLE 2

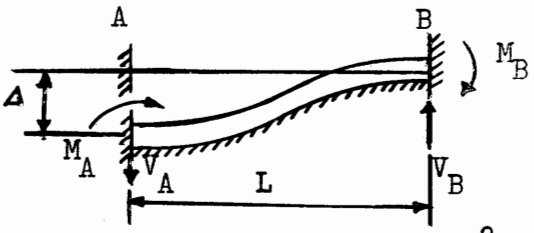
βL	m_A	v_A	v_B	m	v
1.30	320851	122951	802350	203051	160451
1.35	324151	126251	774250	219851	162051
1.40	327651	129851	744050	236251	163851
1.45	331551	133651	711750	252151	165751
1.50	335851	137851	677450	267451	167951
1.55	340551	142151	641450	282151	170251
1.60	345551	146751	603850	296351	172751
1.65	350851	151651	564850	309851	175451
1.70	356651	156651	524750	322751	178351
1.75	362751	161851	483650	335251	181351
1.80	369251	167151	441850	347251	184651
1.85	376151	172651	399750	358851	188051
1.90	383351	178351	357450	370051	191651
1.95	390851	184051	315250	380851	195451
2.00	398751	189751	273450	391551	199351
2.05	406951	195051	232350	401951	203451
2.10	415551	201451	191950	412151	207751
2.15	424251	207351	152750	422251	212151
2.20	433351	213151	114750	432151	216651
2.25	442651	219051	781349	442051	221351
2.30	452051	224851	430749	451951	226051
2.35	461751	230651	969448	461751	230851
2.40	471651	236351	-219349	471651	235851
2.45	481651	242051	-517149	481451	240851
2.50	491751	247651	-796149	491251	245851
2.55	502051	253251	-105550	501151	251051
2.60	512351	258751	-129550	511151	256151
2.65	522751	264251	-151650	521051	261351
2.70	533251	269651	-171750	531151	266651
2.75	543651	274951	-190050	541151	271851
2.80	554251	280251	-206350	551251	277151
2.85	564751	285551	-220850	561451	282351
2.90	575251	290751	-233550	571651	287651
2.95	585851	295851	-244550	581851	292951
3.00	596351	301051	-253950	592151	298151
3.10	617251	311151	-267950	612751	308651
3.20	638151	321151	-276150	633451	319051
3.30	658851	331151	-379250	654151	329451
3.40	679451	341051	-277750	674951	339751
3.50	699851	350951	-272350	695651	349951
3.60	720251	360751	-263650	716351	360151
3.70	740451	370651	-252150	736951	370251
3.80	760551	380451	-238550	757551	380251
3.90	780651	390351	-223250	778051	390351
4.00	800651	400251	-206650	798451	400351

TABLE 2

βL	m_A	v_A	v_B	m	v
4.10	820551	410151	-189250	818851	410251
4.20	840551	420051	-171550	839151	420251
4.30	860451	430051	-153650	859351	430251
4.40	880351	439951	-135950	879551	440151
4.50	900251	449951	-118750	899651	450151
4.60	920251	459951	-102150	919751	460151
4.70	940151	469951	-854749	939851	470051
4.80	960151	479951	-712949	959851	480051
4.90	980051	489951	-580849	979951	490051
5.00	100052	499951	-454949	999951	500051
5.10	102052	509951	-339849	101952	510051
5.20	103952	519951	-238049	103952	519951
5.30	105952	529951	-147149	105952	529951
5.40	107952	539951	-670748	107952	539951
5.50	109952	549951	182947	109952	549951
5.60	111952	559951	603948	111952	559951
5.70	113952	569951	1082949	113952	569951
5.80	115952	579951	1453949	115952	579951
5.90	117952	590051	1808849	117952	589951
6.00	119952	600051	2018849	119952	599951
6.10	121952	610051	2205849	121952	609951
6.20	123952	620051	2314449	123952	619951
6.30	125952	630051	2364449	125952	629951
6.40	127952	640051	2374449	127952	639951
6.50	129952	650051	2326449	129952	649951
6.60	131952	660051	2257449	131952	659951
6.70	134052	670051	2182449	133952	670051
6.80	136052	680051	2068449	135952	680051
6.90	138052	690051	1948449	137952	690051
7.00	140052	700051	1849449	139952	700051
7.10	142052	710051	1626449	141952	710051
7.20	144052	720051	1610449	143952	720051
7.30	146052	730051	1371449	145952	730051
7.40	148052	740051	1171449	147952	740051
7.50	150052	749951	9836448	149952	750051
7.60	152052	759951	8655448	151952	760051
7.70	154052	769951	6558448	153952	770051
7.80	156052	779951	6343448	155952	780051
7.90	158052	789951	6244448	157952	790051
8.00	160052	799951	5073448	159952	799951
8.10	162052	809951	4760448	161952	810051
8.20	164052	819951	4926448	163952	820051
8.30	166052	829951	2555448	166052	829951
8.40	168052	839951	4004448	167952	839951
8.50	170052	849951	2680448	170052	849951

TABLE 2

β L	m_A	v_A	v_B	m	v
8.60	172052	860051	881546	172052	860051
8.70	173952	869951	152147	173952	869951
8.80	175952	879951	257547	175952	879951
8.90	178052	890051	-442347	178052	890051
9.00	180052	900051	196447	180052	900051
9.10	182052	910051	-215548	181952	909951
9.20	184052	920051	-185248	183952	919951
9.30	186052	930051	-737847	186052	930051
9.40	188052	940051	-758147	188052	940051
9.50	190052	950051	-762547	189952	950051
9.60	192052	960051	-753547	191952	960051
9.70	194052	970051	-733647	193952	970051
9.80	196052	980051	-704747	195952	980051
9.90	198052	990051	-668947	198052	990051
10.00	200052	100052	-627947	200052	100052



For Large βL: $m_A = 2(\beta L)^2$

$v_A = 2(\beta L)$

TABLE 3

$$M_A = m_A \frac{EI}{L^2}\Delta \qquad M_B = m_B \frac{EI}{L^2}\Delta$$

$$V_A = v_A \frac{M_A}{L} \qquad V_B = v_B \frac{EI}{L^3}\Delta$$

βL	m_A	m_B	v_A	v_B
0.00	600051	600051	200051	120052
0.05	600051	600051	200051	120052
0.10	600051	600051	200051	120052
0.15	600051	599951	200051	119952
0.20	600051	599951	200051	119952
0.25	600051	599951	200051	119952
0.30	600051	599951	200051	119952
0.35	600351	599851	200251	119952
0.40	600551	599651	200451	119852
0.45	600851	599451	200751	119752
0.50	601351	599251	201151	119652
0.55	601951	598851	201651	119552
0.60	602751	598351	202251	119352
0.65	603751	597751	203151	119052
0.70	605051	597051	204251	118752
0.75	606651	596051	205551	118352
0.80	608551	594951	207151	117952
0.85	610951	593551	209051	117352
0.90	613651	591951	211351	116652
0.95	616951	589951	214051	115852
1.00	620851	587751	217151	114952
1.05	625251	585151	220651	113852
1.10	630451	582151	224651	112552
1.15	636251	578651	229151	111152
1.20	642951	574851	234151	109552
1.25	650451	570451	239751	107752
1.30	658851	565551	245851	105752
1.35	668251	560151	252451	103452
1.40	678751	554051	259651	100952
1.45	690351	547451	267351	982551
1.50	703051	540151	275651	952651
1.55	717051	532251	284351	919951
1.60	732351	523551	293551	884451
1.65	748951	514251	303251	846051
1.70	766051	504151	313251	804851
1.75	786451	493351	323651	760651

TABLE 3

βL	m_A	m_B	v_A	v_B
1.80	807451	481751	334351	713551
1.85	830051	469451	345351	663551
1.90	854151	456351	356651	610551
1.95	879951	442551	368051	554751
2.00	907351	428051	379551	496251
2.05	936451	412851	391251	435051
2.10	967151	396851	402951	371351
2.15	999651	380351	414651	305351
2.20	103352	363151	426351	237251
2.25	106952	345351	438051	167151
2.30	110752	327051	449751	954050
2.35	114652	308351	461251	222250
2.40	118752	289151	472751	-521050
2.45	123052	269551	484151	-127251
2.50	127452	249751	495351	-202951
2.55	132052	229651	506551	-278851
2.60	136852	209451	517551	-354551
2.65	141752	189151	528451	-429851
2.70	146752	168851	539351	-504251
2.75	151952	148551	549951	-577551
2.80	157352	128451	560551	-649351
2.85	162852	108551	571051	-719251
2.90	168452	889150	581451	-786851
2.95	174152	696050	591751	-852151
3.00	180052	507250	602051	-914551
3.10	192252	144350	622351	-102952
3.20	204852	-195250	642351	-113152
3.30	217852	-507750	662251	-121652
3.40	231352	-789850	682051	-128552
3.50	245252	-103951	701851	-133552
3.60	259552	-125551	721551	-136852
3.70	274152	-143651	741251	-138252
3.80	289252	-158351	760951	-137952
3.90	304652	-169651	780651	-136052
4.00	320452	-177651	800451	-132452
4.10	336652	-182551	820251	-127452
4.20	353252	-184651	840151	-121152
4.30	370252	-184051	860051	-113752
4.40	387652	-181051	879951	-105452
4.50	405352	-176051	899851	-963051
4.60	423552	-169151	919851	-866051
4.70	442052	-160851	939851	-765251
4.80	461052	-151151	959851	-662151
4.90	480452	-140551	979851	-558151
5.00	500152	-129251	999851	-455151

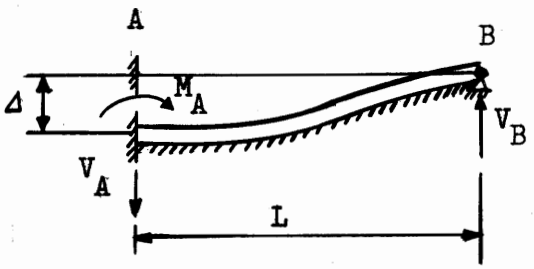
TABLE 3

βL	m_A	m_B	v_A	v_B
5.10	520352	-117451	101952	-354451
5.20	540952	-105451	103952	-257651
5.30	561852	-933150	105952	-164951
5.40	583252	-814950	107952	-789550
5.50	605052	-697850	109952	180049
5.60	627252	-584750	111952	754750
5.70	649852	-479450	113952	140351
5.80	672852	-381950	115952	195651
5.90	696252	-296150	117952	240251
6.00	720052	-196750	120052	293751
6.10	744252	-123550	122052	324851
6.20	768852	-522549	124052	352851
6.30	793852	105049	126052	372651
6.40	819252	601649	128052	381451
6.50	845052	107250	130052	391951
6.60	871252	145550	132052	392551
6.70	897852	177950	134052	389951
6.80	924852	202650	136052	380951
6.90	952252	221750	138052	368251
7.00	980052	234050	140052	351651
7.10	100853	242150	142052	333451
7.20	103653	247850	144052	315051
7.30	106553	244750	146052	288451
7.40	109553	235650	148052	261751
7.50	112553	229650	149952	235351
7.60	115553	229350	151952	217751
7.70	118553	203650	153952	178851
7.80	121653	196650	155952	162851
7.90	124853	200650	157952	160151
8.00	128053	176650	159952	122951
8.10	131253	144350	161952	822450
8.20	134453	120650	163952	624850
8.30	137753	101250	165952	379150
8.40	141153	692949	167952	200050
8.50	144553	827049	169952	580049
8.60	147953	514249	171952	-420049
8.70	151353	821849	173952	-123050
8.80	154853	101950	176052	-185050
8.90	158453	940049	177852	-239050
9.00	161953	660049	180052	-277050
9.10	165653	380049	181952	-298050
9.20	169253	204849	184052	-306750
9.30	172953	176549	186052	-291850
9.40	176753	149749	188052	-274750
9.50	180553	124549	190052	-255950

TABLE 3

βL	m_A	m_B	v_A	v_B
9.60	184353	100049	192052	-235050
9.70	188153	796448	194052	-215350
9.80	192053	600648	196052	-194250
9.90	196053	500048	197952	-176050
10.00	200053	267948	200052	-152350

TABLE 4



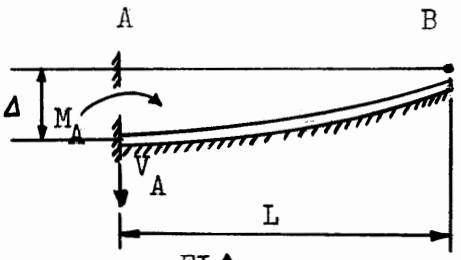
$$M_A = m_A \frac{EI\Delta}{L^2}$$

$$V_A = v_A \frac{M_A}{L}$$

$$V_B = v_B \frac{EI\Delta}{L^3}$$

For Large βL: $m_A = 2(\beta L)^2$

$v_A = 2(\beta L)$



$$M_A = m_A \frac{EI\Delta}{L^2}$$

$$V_A = v_A \frac{M_A}{L}$$

For Large βL: $v_A = 2\beta L$

$m_A = 2(\beta L)^2$

βL	m_A	v_A	v_B	m_A	v_A
0.00	300051	100051	300051	0000	200051
0.05	300051	100051	300051	125046	200051
0.10	300051	100051	300051	199947	200051
0.15	300051	100051	299951	101248	200051
0.20	300051	100051	299951	319848	200051
0.25	300151	100251	299751	780348	200151
0.30	300251	100451	299551	161649	200151
0.35	300551	100751	299151	298849	200351
0.40	300951	101351	298551	508249	200551
0.45	301451	102151	297751	810549	200751
0.50	302151	103351	296551	122750	201151
0.55	303151	104851	294951	178350	201651
0.60	304451	106851	292751	249850	202351
0.65	306151	109351	290051	339650	203251
0.70	308251	112451	286651	449250	204251
0.75	310851	116251	282451	580350	205651
0.80	313951	120851	277351	733550	207251
0.85	317751	126151	271251	909250	209151
0.90	322351	132351	263951	110651	211451
0.95	327651	139551	255351	132551	214051
1.00	333851	147551	245451	156351	217151
1.05	341051	156451	233851	181751	220751
1.10	349251	166251	220851	208651	224751
1.15	358651	176951	206051	236751	229251
1.20	369251	188351	189351	265851	234251
1.25	381151	200451	170651	295551	239751
1.30	394351	213151	149951	325751	245851
1.35	409151	226451	127151	356351	252551
1.40	425451	240051	102151	387151	259751
1.45	443351	253951	749350	418051	269451
1.50	462851	268051	454850	449151	275651

TABLE 4

βL	m_A	v_A	v_B	m_A	v_A
1.55	484151	282251	138250	480451	284451
1.60	507151	296351	-200450	511851	293651
1.65	531951	310451	-560450	543551	303251
1.70	558551	324351	-941050	575651	313351
1.75	587051	337951	-134051	608051	323751
1.80	617351	351351	-175851	641151	334451
1.85	649551	364451	-219351	674751	345451
1.90	683551	377251	-264151	709151	356651
1.95	719251	389751	-310251	744451	368051
2.00	756851	401851	-357251	780651	379651
2.05	796051	413651	-404951	817851	391251
2.10	837051	425151	-453151	856251	402951
2.15	879651	436351	-501451	895651	414651
2.20	923751	447251	-549551	936351	426451
2.25	969451	457951	-597151	978351	438151
2.30	101652	468451	-643951	102152	449751
2.35	106552	478651	-689651	106652	461351
2.40	111452	488751	-733951	111252	472851
2.45	116552	498751	-776451	115952	484151
2.50	121852	508551	-817051	120852	495451
2.55	127152	518251	-855351	125852	506651
2.60	132552	527851	-891151	130952	517651
2.65	138152	537451	-924251	136152	525851
2.70	143752	546951	-954451	141552	539351
2.75	149552	556451	-981451	147152	550051
2.80	155352	565851	-100552	152752	560651
2.85	161252	575351	-102552	158 52	571151
2.90	167252	584751	-104252	164452	581551
2.95	173352	594251	-105652	170452	591851
3.00	179552	603651	-106552	176552	602051
3.10	192052	622751	-107452	189152	622351
3.20	204952	641851	-106852	202152	642451
3.30	218152	661151	-104952	215552	662351
3.40	231752	680551	-101652	229252	682051
3.50	245552	700151	-972051	243452	701851
3.60	259852	719751	-915951	257952	721551
3.70	274452	739551	-851151	272852	741251
3.80	289352	759451	-777851	288052	760951
3.90	304652	779351	-698151	303652	780751
4.00	320452	799351	-613551	319652	800551
4.10	336552	819451	-525651	335952	820351
4.20	353052	839451	-436051	352652	840151
4.30	370052	859551	-346051	369752	860051
4.40	387352	879651	-257251	387152	879951
4.50	405052	899751	-170751	404952	899951

TABLE 4

β L	m_A	v_A	v_B	m_A	v_A
4.60	423252	919751	-877850	423152	919951
4.70	441852	939851	-933449	441852	939851
4.80	460752	959851	637050	460752	959851
4.90	480152	979951	130651	480152	979951
5.00	499952	999951	191151	499952	999951
5.10	520152	101952	244551	520152	102052
5.20	540752	104052	290751	540752	104052
5.30	561752	106052	329551	561752	106052
5.40	583152	108052	361151	583152	108052
5.50	604952	110052	385551	604952	110052
5.60	627152	112052	403351	627152	112052
5.70	649752	114052	413951	649752	114052
5.80	672752	116052	418551	672752	116052
5.90	696152	118052	416951	696152	118052
6.00	719952	120052	410851	719952	120052
6.10	744152	122052	402151	744152	122652
6.20	768752	124052	386351	768752	124052
6.30	793852	126052	368151	793752	126052
6.40	819252	128052	342851	819152	128052
6.50	845052	130052	320051	844952	130052
6.60	871252	132052	284051	871152	132052
6.70	897852	133952	231651	897752	134052
6.80	924852	135952	258251	924752	136052
6.90	952252	137952	214051	952152	138052
7.00	980052	139952	191651	979952	140052
7.10	100853	141952	163451	100853	142052
7.20	103653	143952	133751	103653	144052
7.30	106553	145952	118151	106553	146052
7.40	109553	147952	876350	109553	148052
7.50	112553	149952	626350	112453	150052
7.60	115553	151952	440750	115553	152052
7.70	118553	153952	289950	118553	154052
7.80	121653	155952	632149	121653	156052
7.90	124853	157952	-215150	124853	158052
8.00	127953	159952	-220650	127953	160052
8.10	131253	161952	-364250	131253	162052
8.20	134453	163952	-414250	134453	164052
8.30	137753	166052	-470950	137753	166052
8.40	141153	168052	-518350	141153	168052
8.50	144553	169952	-570150	144453	170052
8.60	147953	172052	-873950	147953	172052
8.70	151353	174052	-642550	151353	174052
8.80	154853	176052	-478150	154853	176052
8.90	158453	177952	-550950	158453	178052
9.00	162053	179952	-917850	161953	180052

TABLE 4

βL	m_A	v_A	v_B	m_A	v_A
9.10	165653	182052	-227750	165653	182052
9.20	169253	184052	346650	169253	184052
9.30	172953	185952	-342550	172953	186052
9.40	176753	187952	-474950	176753	188052
9.50	180453	190052	-328150	180553	190052
9.60	184353	192052	210550	184353	192052
9.70	188153	194052	-341951	188153	194052
9.80	192053	196052	154651	192053	196052
9.90	196053	197952	-173150	196053	198052
10.00	200053	200052	-152350	200053	200052

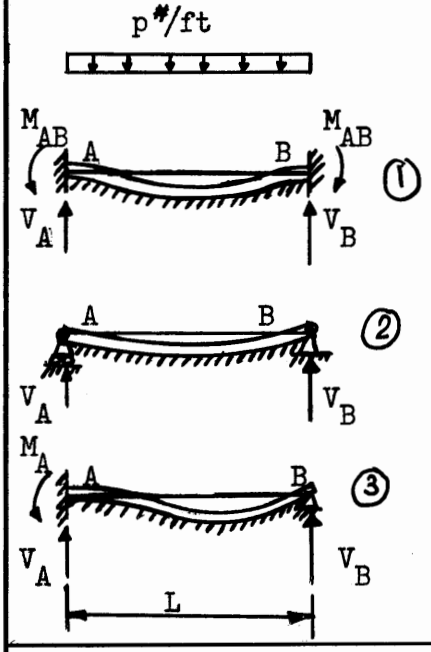


TABLE 5

For Large βL:

$$m_1 = \frac{1}{2(\beta L)^2}$$

$$m_A = \frac{1}{2(\beta L)^2}$$

$$v_A = v_1 = \frac{1}{\beta L}$$

$$v_B = v_2 = \frac{1}{2\beta L}$$

$$V_A = V_B = v_2 pL \quad (2)$$

$$M_{AB} = m_1 pL^2 = M_{BA}$$

$$V_A = v_1 pL = V_B \quad (1)$$

$$M_A = mpL^2$$

$$V_A = v_A pL$$

$$V_B = v_B pL \quad (3)$$

βL	m_1	v_1	v_2	m	v_A	v_B
0.00	833349	500050	500050	125050	625050	375050
0.05	833349	500050	500050	125050	625050	375050
0.10	833349	500050	499950	125050	625050	375050
0.15	833349	500050	499950	125050	625050	375050
0.20	833349	500050	499950	125050	625050	375050
0.25	833349	500050	499950	124950	624950	374950
0.30	833349	500050	499950	124950	624950	374950
0.35	833349	499950	499850	124950	624950	374950
0.40	833249	499950	499550	124950	624850	374950
0.45	833149	499850	499350	124950	624650	374750
0.50	832949	499850	498950	124850	624550	374650
0.55	832749	499750	498450	124850	624350	374550
0.60	832549	499650	497850	124750	624050	374350
0.65	832249	499550	497050	124650	623650	374050
0.70	831949	499350	496050	124550	623250	373750
0.75	831449	499150	494750	124450	622650	373450
0.80	830940	498850	493250	124250	621950	372950
0.85	830249	498550	491450	124050	621150	372350
0.90	829449	498150	489350	123750	620150	371750
0.95	828549	497750	486850	123450	619050	370950
1.00	827449	497250	483950	123150	617650	370050
1.05	826149	496650	480750	122750	616150	368950
1.10	824749	495950	476950	122350	614350	367750
1.15	823049	495250	472850	121750	612350	366450
1.20	821149	494350	468150	121250	610050	364850
1.25	819049	493350	463050	120550	607550	363150
1.30	816749	492250	457350	119850	604750	361250
1.35	814049	491050	451250	119050	601550	359050
1.40	811149	489650	444650	118150	598150	356750
1.45	807949	488150	437650	117250	594350	354150
1.50	804349	486450	430150	116250	590250	351350

TABLE 5

βL	m_1	v_1	v_2	m	v_A	v_B
1.55	800449	484650	422250	115050	585850	348350
1.60	796249	482650	413950	113850	581050	345150
1.65	791649	480550	405250	112550	575950	341650
1.70	786749	478250	396250	111150	570450	337950
1.75	781349	475750	387050	109750	564650	334050
1.80	775649	473050	377650	108150	558550	329850
1.85	769549	470250	368050	106550	552050	325550
1.90	763049	467150	358450	104850	545350	320950
1.95	756149	463950	348650	103050	538350	316150
2.00	748849	460550	338950	101250	531050	311250
2.05	741149	456950	329250	993049	523450	306150
2.10	733049	453150	319550	973449	515650	300850
2.15	724649	449250	310050	953349	507750	295550
2.20	715849	445150	300750	932849	499550	290050
2.25	706649	440850	291550	911949	491350	284450
2.30	697149	436350	282650	890749	482950	278750
2.35	687249	431750	273950	869349	474450	273050
2.40	677149	427050	265450	847849	465850	267350
2.45	666649	422150	257350	826249	457350	261550
2.50	655949	417150	249450	804649	448750	255850
2.55	645049	412050	241850	783149	440150	250150
2.60	633849	406750	234550	761649	431650	244450
2.65	622549	401450	227550	740449	423150	238750
2.70	611049	396050	220850	719449	414850	233150
2.75	599349	390650	214450	698749	406550	227650
2.80	587649	385150	208250	678349	398450	222250
2.85	575849	379550	202450	658349	390450	216950
2.90	563949	374050	196850	638649	382550	211750
2.95	552049	368450	191550	619449	374850	206750
3.00	540149	362850	186550	600749	367350	201750
3.10	516449	351650	177150	564749	352850	192250
3.20	492949	340650	168650	530649	339150	183350
3.30	469949	329850	161050	498649	326150	174950
3.40	447549	319250	154150	468749	314050	167150
3.50	425849	308950	147950	440849	302650	159950
3.60	404949	299050	142250	414949	291950	153250
3.70	384849	289550	137150	390949	282050	147050
3.80	365749	280450	132550	368649	272850	141350
3.90	347549	271750	128350	348049	264150	136050
4.00	330349	263450	124450	329049	256150	131250
4.10	314049	255650	120850	311449	248650	126750
4.20	298649	248150	117650	295249	241650	122650
4.30	284249	241150	114550	280249	235050	118850
4.40	270649	234550	111750	266349	228950	115450
4.50	257849	228250	109150	253449	223250	112150

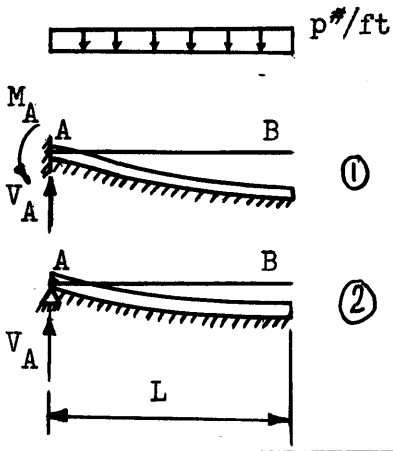
TABLE 5

βL	m_1	v_1	v_2	m	v_A	v_B
4.60	245949	222350	106750	241549	217850	109250
4.70	234749	216750	104450	230549	212750	106450
4.80	224249	211550	102250	220249	208050	103850
4.90	214449	206550	100250	210649	203450	101450
5.00	205249	201850	983249	201849	199250	992449
5.10	196649	197450	964849	193449	195150	971449
5.20	188549	193250	947249	185749	191250	951649
5.30	180949	189250	930349	178449	187650	932949
5.40	173849	185450	914149	171649	184150	915349
5.50	167249	181850	898649	165249	180750	898549
5.60	160949	178350	883549	159249	177550	882649
5.70	155049	175150	869049	153549	174450	867449
5.80	149449	171950	855049	148249	171450	852849
5.90	144249	168950	841449	143149	168650	838849
6.00	139249	166150	828249	138449	165850	825449
6.10	134549	163350	815349	133849	163250	812449
6.20	130149	160650	802949	129549	160650	799849
6.30	125949	158150	790749	125549	158150	787849
6.40	121949	155650	778949	121649	155750	776049
6.50	118149	153250	767449	117949	153350	764849
6.60	114549	150950	756249	114349	151150	753649
6.70	111149	148750	745349	111049	148950	742849
6.80	107849	146650	734649	107849	146750	732649
6.90	104749	144550	724249	104749	144650	722349
7.00	101749	142450	714149	101749	142650	712249
7.10	989549	140550	704249	989548	140650	702549
7.20	962248	138550	694649	962448	138750	693249
7.30	936148	136750	685249	936548	136850	684049
7.40	911048	134950	676049	911548	135050	674949
7.50	887048	133150	667149	887648	133250	666049
7.60	863948	131450	658349	864548	131550	657549
7.70	841848	129750	649849	842448	129850	649349
7.80	820448	128050	641549	821148	128150	641049
7.90	799948	126450	633449	800548	126550	633149
8.00	780248	124950	625449	780848	125050	625049
8.10	761148	123450	617749	761748	123450	617349
8.20	742848	121950	610149	743348	121950	609749
8.30	725148	120450	602849	725648	120550	602649
8.40	708048	119050	595649	708548	119050	595749
8.50	691548	117650	588549	691948	117650	588549
8.60	675648	116250	581649	676048	116350	581649
8.70	660248	114950	574949	660648	114950	574649
8.80	645448	113650	568449	645748	113650	568349
8.90	631048	112350	562049	631248	112350	561649
9.00	617148	111150	555749	617348	111150	555949

TABLE 5

βL	m_1	v_1	v_2	m	v_A	v_B
9.10	603748	109950	549649	603848	109950	549449
9.20	590648	108750	543649	590848	108750	543549
9.30	578048	107550	537749	578148	107550	537849
9.40	565848	106450	532049	565948	106450	531949
9.50	554048	105250	526349	554148	105250	526549
9.60	542548	104150	520849	542648	104150	521049
9.70	531448	103150	515549	531448	103150	514449
9.80	520648	102050	510249	520648	102050	511949
9.90	510148	101050	505049	510248	101050	500749
10.00	500048	100050	500049	500048	100050	500349

TABLE 6



$M_A = m_1 pL^2$

$V_A = v_1 pL$ ①

$V_A = v_2 pL$ ②

For Large β L:

$$m_1 = \frac{1}{2(\beta L)^2}$$

$$v_1 = \frac{1}{\beta L}$$

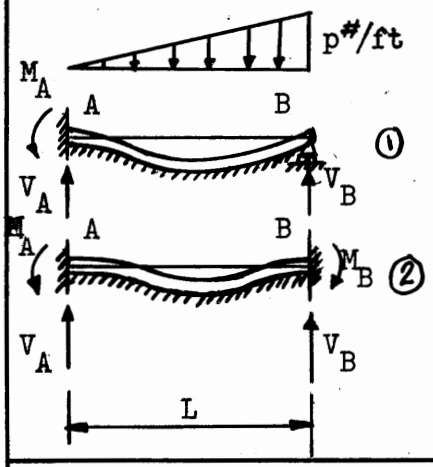
$$v_2 = \frac{1}{2\beta L}$$

βL	m_1	v_1	v_2	βL	m_1	v_1	v_2
0.00	50050	10051	250050				
0.05	50050	100051	250050	1.55	208050	591750	237450
0.10	499950	100051	250050	1.60	195250	573250	235950
0.15	499950	999850	250050	1.65	183350	555950	234250
0.20	499750	999650	250050	1.70	172250	539750	232450
0.25	499450	999250	250050	1.75	162050	524650	230650
0.30	498850	998350	250050	1.80	152650	510550	228650
0.35	497850	997050	249950	1.85	144050	497450	226550
0.40	496350	994950	249950	1.90	136050	485150	224350
0.45	494150	991950	249950	1.95	128750	473750	222150
0.50	491150	987750	249850	2.00	121950	462950	219750
0.55	487150	982250	249750	2.05	115750	452950	217350
0.60	482050	975150	249650	2.10	110050	443450	214750
0.65	475650	966250	249550	2.15	104750	434550	212250
0.70	467850	955450	249450	2.20	999349	426050	209550
0.75	458550	942550	249250	2.25	954349	418050	206850
0.80	447750	927550	249050	2.30	912649	410450	204150
0.85	435450	910550	248750	2.35	873949	403150	201350
0.90	421750	891550	248450	2.40	838049	396150	198550
0.95	406850	870850	248050	2.45	804449	389450	195750
1.00	390750	848550	247650	2.50	773149	383050	192850
1.05	373850	824950	247150	2.55	743849	376750	190050
1.10	356350	800550	246550	2.60	716349	370750	187250
1.15	338450	775650	245950	2.65	690449	364850	184450
1.20	320450	750550	245250	2.70	666049	359250	181650
1.25	302650	725550	244450	2.75	643049	353650	178850
1.30	285150	700950	243550	2.80	621349	348350	176150
1.35	268250	677150	242450	2.85	600749	343050	173450
1.40	251950	654150	241350	2.90	581149	337950	170750
1.45	236450	632250	240150	2.95	562649	332950	168150
1.50	221850	611350	238850	3.00	544949	328050	165550

TABLE 6

βL	m_1	v_1	v_2	βL	m_1	v_1	v_2
3.10	511949	318550	160550	6.60	114749	151550	757549
3.20	481849	309550	155750	6.70	111349	149250	746249
3.30	454349	300850	151250	6.80	108149	147050	735249
3.40	428949	292550	146850	6.90	105049	144950	724649
3.50	405549	284650	142750	7.00	102049	142850	714249
3.60	383949	277050	138750	7.10	991848	140850	704249
3.70	363949	269750	135050	7.20	964548	138850	694449
3.80	345349	262850	131450	7.30	938248	136950	684949
3.90	328149	256150	128150	7.40	913048	135150	675649
4.00	312149	249850	124950	7.50	888848	133350	666649
4.10	297249	243850	121850	7.60	865648	131550	657849
4.20	283349	238050	118950	7.70	843348	129850	649349
4.30	270349	232550	116150	7.80	821848	128250	641049
4.40	258249	227250	113550	7.90	801148	126550	632949
4.50	246949	222150	111050	8.00	781248	124950	624949
4.60	236249	217350	108650	8.10	762048	123450	617249
4.70	226349	212750	106350	8.20	743648	121950	609749
4.80	217049	208250	104150	8.30	725748	120450	602449
4.90	208249	204050	102050	8.40	708648	119050	595249
5.00	199949	199950	999649	8.50	692048	117650	588249
5.10	192249	196050	980149	8.60	676048	116250	581349
5.20	184949	192250	961349	8.70	660548	114950	574749
5.30	177949	188650	943249	8.80	645648	113650	568149
5.40	171449	185150	925849	8.90	631248	112350	561749
5.50	165249	181750	909049	9.00	617248	111150	555549
5.60	159449	178550	892749	9.10	603748	109850	549449
5.70	153849	175450	877149	9.20	590748	108650	543449
5.80	148649	172450	862049	9.30	578148	107550	537649
5.90	143649	169450	847449	9.40	565848	106350	531949
6.00	138849	166650	833349	9.50	554048	105250	526349
6.10	134349	163950	819649	9.60	542548	104150	520849
6.20	130049	161250	806449	9.70	531448	103050	515449
6.30	125949	158750	793649	9.80	520648	102050	510249
6.40	122049	156250	781249	9.90	510148	101050	505049
6.50	118349	153850	769249	10.00	499948	100050	500049

TABLE 7



$$M_A = m_{A2} pL^2 \quad V_A = v_{A2} pL \quad V_B = v_{B2} pL \quad \} ①$$

$$M_A = m_{A1} pL^2 \quad \} ②$$

$$M_B = m_{B1} pL^2 \quad V_A = v_{A1} pL \quad V_B = v_{B1} pL \quad \} ②$$

βL	m_{A2}	v_{A2}	v_{B2}	m_{A1}	m_{B1}	v_{A1}	v_{B1}
0.00	583349	225050	275050	333349	500049	150050	350050
0.05	583349	225050	275050	333349	500049	150050	350050
0.10	583349	225050	275050	333349	500049	150050	350050
0.15	583349	225050	275050	333349	500049	150050	350050
0.20	583349	225050	275050	333349	500049	150050	350050
0.25	583349	225050	275050	333349	500049	150050	350050
0.30	583349	225050	275050	333349	500049	150050	350050
0.35	583249	224950	275050	333349	499949	150050	350050
0.40	583149	224950	274950	333349	499949	150050	350050
0.45	582949	224850	274950	333249	499949	149950	349950
0.50	582749	224750	274850	333249	499849	149950	349950
0.55	582449	224650	274750	333149	499749	149950	349950
0.60	582049	224450	274650	333049	499649	149850	349850
0.65	581449	224250	274450	332849	499449	149850	349750
0.70	580849	224050	274350	332649	499249	149750	349650
0.75	580049	223750	274050	332449	499049	149550	349550
0.80	579149	223350	273850	332149	498749	149450	349450
0.85	577949	222950	273550	331849	498449	149350	349250
0.90	576549	222350	273150	331449	498049	149150	349050
0.95	574949	221750	272650	330949	497549	148950	348850
1.00	573149	221050	272150	330449	496949	148650	348550
1.05	570949	220150	271550	329849	496349	148350	348250
1.10	568449	219250	270850	329149	495549	148050	347950
1.15	565649	218150	270050	328349	494749	147650	347550
1.20	562549	216850	269250	327349	493749	147250	347050
1.25	558949	215450	268250	326349	492749	146850	346550
1.30	555049	213950	267150	325249	491549	146250	345950
1.35	550649	212250	265850	323949	490149	145650	345350
1.40	545849	210350	264550	322449	488649	145050	344650
1.45	540649	208350	263050	320949	487049	144350	343850
1.50	534949	206150	261450	319149	485149	143550	342950

TABLE 7

βL	m_{A2}	v_{A2}	v_{B2}	m_{A1}	m_{B1}	v_{A1}	v_{B1}
1.55	528749	203650	259750	317249	483149	142650	342050
1.60	522049	201150	257950	315249	481049	141650	341050
1.65	514949	198350	255950	312949	478649	140650	339850
1.70	507349	195350	253750	310549	476149	139550	338650
1.75	499349	192250	251550	307949	473449	138350	337350
1.80	490849	188950	249150	305149	470449	137050	335950
1.85	481849	185450	246650	302249	467349	135750	334550
1.90	472549	181750	243950	299049	464049	134250	332950
1.95	462849	178050	241250	295649	460449	132750	331250
2.00	452749	174050	238350	292149	456749	131150	329450
2.05	442249	170050	235450	288449	452749	129450	327550
2.10	431549	165850	232350	284449	448649	127650	325550
2.15	420549	161550	229250	280349	444249	125750	323550
2.20	409349	157250	226050	276049	439749	123750	321350
2.25	398049	152850	222850	271649	435049	121750	319050
2.30	386449	148350	219550	267049	430049	119650	316750
2.35	374849	143850	216150	262249	425049	117450	314350
2.40	363149	139350	212850	257349	419749	115250	311850
2.45	351449	134750	209450	252349	414349	112950	309250
2.50	339749	130250	206050	247149	408849	110550	306550
2.55	328149	125750	202650	241849	403149	108150	303850
2.60	316649	121250	199250	236449	397349	105750	301050
2.65	305149	116850	195950	231049	391449	103250	298250
2.70	293949	112550	192550	225549	385549	100750	295350
2.75	282849	108250	189350	219949	379449	981749	292450
2.80	271949	104050	186050	214349	373349	956249	289550
2.85	261249	999749	182850	208649	367149	930649	286550
2.90	250849	959749	179650	202949	360949	904949	283550
2.95	240649	920749	176550	197349	354749	879249	280450
3.00	230749	882949	173550	191649	348449	853749	277450
3.10	211849	810849	167650	180449	335949	803049	271350
3.20	194149	743549	162050	169449	323549	753349	265350
3.30	177649	681149	156750	158649	311349	705049	259250
3.40	162449	623649	151650	148249	299349	658449	253350
3.50	148349	571049	146950	138149	287649	613749	247550
3.60	135449	523149	142450	128649	276349	571249	241950
3.70	123649	479549	138150	119549	265349	531049	236450
3.80	112949	440149	134150	110949	254849	493249	231050
3.90	103149	404749	130450	102849	244749	457849	225950
4.00	943448	372849	126850	952348	235049	424949	220950
4.10	863348	344249	123450	881648	225849	394549	216150
4.20	791148	318649	120350	815948	217049	366349	211550
4.30	725848	295849	117350	755048	208649	340449	207150
4.40	667048	275549	114450	698848	200749	316749	202850
4.50	614048	257349	111850	646948	193149	295149	198750

TABLE 7

βL	m_{A2}	v_{A2}	v_{B2}	m_{A1}	m_{B1}	v_{A1}	v_{B1}
4.60	566348	241249	109250	599348	185949	275449	194850
4.70	523348	226849	106850	555548	179149	257449	191050
4.80	484648	214049	104550	515448	172749	241149	187450
4.90	449748	202549	102250	478748	166549	226449	183950
5.00	418248	192349	100150	445148	160749	213149	180550
5.10	389748	183149	981749	414448	155149	201049	177350
5.20	364048	174949	962449	386448	149949	190149	174150
5.30	340748	167549	943949	360848	144849	180349	171150
5.40	319648	160849	926149	337448	140149	171449	168250
5.50	300448	154749	909149	316148	135549	163349	165450
5.60	283048	149149	892749	296648	131249	156049	162750
5.70	267048	144049	876949	278848	127149	149449	160150
5.80	252448	139349	861749	262548	123249	143449	157650
5.90	239048	135049	847049	247648	119449	137949	155150
6.00	226748	130949	832849	233948	115849	132849	152850
6.10	215448	127149	819249	221348	112449	128249	150550
6.20	204948	123549	805949	209748	109149	124049	148250
6.30	195248	120149	793149	199048	106049	120049	146150
6.40	186248	116949	780849	189248	103049	116449	144050
6.50	177848	113849	768849	180148	100149	112949	141950
6.60	169948	110849	757149	171648	974248	109749	140050
6.70	162648	108049	745849	163848	947848	106749	138150
6.80	155748	105249	735049	156548	922548	103949	136250
6.90	149248	102649	724349	149648	898048	101149	134450
7.00	143148	100049	714049	143348	874648	986048	132650
7.10	137348	975948	703949	137348	852148	961448	130950
7.20	131948	951848	694249	131748	830548	937748	129250
7.30	126748	928548	684749	126448	809648	914948	127550
7.40	121848	905848	675549	121448	789648	892948	125950
7.50	117248	883748	666549	116748	770248	871748	124450
7.60	112848	862348	657749	112348	751648	851248	122950
7.70	108648	841548	649249	108048	733748	831348	121450
7.80	104648	821248	640949	104048	716348	812148	119950
7.90	100848	801548	632849	100348	699648	793348	118550
8.00	972147	782448	624949	966947	683548	775248	117150
8.10	937447	763948	617249	932547	667948	757548	115850
8.20	904347	745848	609749	899747	652848	740348	114550
8.30	872747	728348	602349	868547	638248	723648	113250
8.40	842547	711348	595249	838647	624148	707348	111950
8.50	813647	694948	588249	810147	610548	691548	110750
8.60	785947	678948	581349	782847	597348	676148	109550
8.70	759447	663448	574649	756747	584648	661248	108350
8.80	734147	648448	568149	731747	572248	646648	107150
8.90	709847	633848	561849	707747	560248	632548	106050
9.00	686647	619748	555549	684847	548648	618748	104950

TABLE 7

βL	m_{A2}	v_{A2}	v_{B2}	m_{A1}	m_{B1}	v_{A1}	v_{B1}
9.10	664347	606148	549449	662847	537448	605448	103850
9.20	643047	592848	543549	641747	526548	592448	102750
9.30	622547	580048	537649	621447	515948	579848	101750
9.40	602847	567648	531949	602047	505648	567548	100750
9.50	584047	555548	526249	583347	495648	555648	997249
9.60	565947	543948	520749	565447	486048	544148	987449
9.70	548647	532648	515549	548247	476648	532948	977749
9.80	531947	521648	510349	531647	467248	522048	968049
9.90	515947	511048	503949	515747	459148	511448	959049
10.00	500647	500748	499949	500547	450048	501248	950049

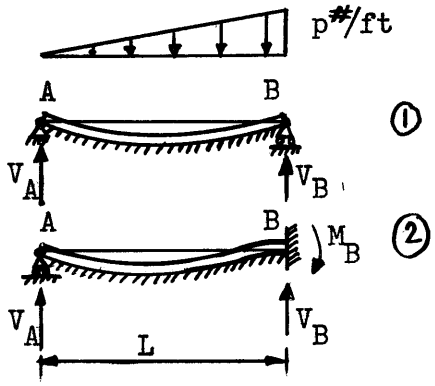


TABLE 8

$$\left.\begin{aligned} V_A &= v_{A2}pL \\ V_B &= v_{B2}pL \end{aligned}\right\} ①$$

$$\left.\begin{aligned} M_B &= m_B pL^2 \\ V_A &= v_{A1}pL \\ V_B &= v_{B1}pL \end{aligned}\right\} ②$$

β L	v_{A2}	v_{B2}	m_B	v_{A1}	v_{B1}
0.00	166750	333350	666749	100050	400050
0.05	166750	333350	666749	100050	400050
0.10	166750	333350	666749	100050	400050
0.15	166750	333350	666749	100050	400050
0.20	166650	333350	666749	100050	400050
0.25	166650	333350	666749	999949	400050
0.30	166650	333250	666549	999849	399950
0.35	166550	333250	666549	999649	399950
0.40	166450	333150	666449	999449	399950
0.45	166350	332950	666349	999149	399850
0.50	166150	332850	666149	998649	399750
0.55	165950	332550	665849	998049	399650
0.60	165650	332250	665549	997149	399550
0.65	165250	331850	665149	996149	399350
0.70	164750	331350	664649	994849	399150
0.75	164150	330650	664049	993149	398950
0.80	163350	329950	663249	991149	398650
0.85	162450	329050	662249	988749	398250
0.90	161450	327950	661149	985949	397850
0.95	160250	326650	659849	982549	397250
1.00	158750	325150	658349	978649	396650
1.05	157150	323550	656549	974149	395950
1.10	155350	321650	654549	969049	395150
1.15	153250	319550	652249	963149	394250
1.20	151050	317150	649649	956549	393250
1.25	148450	314550	646749	949149	392050
1.30	145750	311650	643549	940949	390750
1.35	142750	308550	640049	931849	389350
1.40	139550	305150	636049	921849	387750
1.45	136050	301550	631749	910949	386050
1.50	132350	297750	627149	899049	384150
1.55	128550	293650	622049	886149	382150
1.60	124450	289450	616549	872349	379950
1.65	120250	284950	610749	857549	377550
1.70	115850	280450	604549	841749	375050
1.75	111450	275650	597849	825049	372450

TABLE 8

βL	v_{A2}	v_{B2}	m_B	v_{A1}	v_{B1}
1.80	106850	270850	590849	807449	369650
1.85	102250	265850	583549	788949	366650
1.90	975349	260850	575849	769549	363550
1.95	928449	255850	567749	749449	360250
2.00	881649	250750	559449	728649	356950
2.05	835049	245750	550749	707149	353450
2.10	789149	240650	541849	685049	349850
2.15	743849	235650	532749	662449	346150
2.20	699549	230750	523449	639549	342350
2.25	656349	225950	513949	616249	338450
2.30	614349	221150	504249	592649	334550
2.35	573749	216550	494549	568949	330650
2.40	534549	212050	484649	545249	326550
2.45	496949	207650	474749	521449	322550
2.50	460849	203350	464849	497849	318450
2.55	426349	199150	454949	474449	314450
2.60	393549	195150	445049	451249	310350
2.65	362349	191250	435249	428349	306350
2.70	332749	187550	425549	405849	302250
2.75	304849	183950	415949	383849	298350
2.80	278449	180450	406449	362349	294350
2.85	253749	177050	397049	341349	290450
2.90	230449	173850	387849	321049	286550
2.95	208649	170750	378849	301249	282750
3.00	188249	167650	369949	282249	279050
3.10	151549	161950	352849	246149	271750
3.20	119949	156750	336549	212849	264750
3.30	928048	151750	321049	182449	258050
3.40	698148	147150	306349	154949	251650
3.50	504948	142850	292549	130149	245550
3.60	344348	138850	279549	108149	239650
3.70	212248	135050	267249	886248	234150
3.80	105048	131450	255749	715148	228750
3.90	195047	128150	244949	566148	223750
4.00	474447	124950	234749	437648	218850
4.10	985447	121850	225149	327648	214250
4.20	136248	118950	216149	234448	209750
4.30	162648	116250	207649	156348	205550
4.40	179748	113550	199649	917047	201450
4.50	189248	111050	192049	388947	197450
4.60	192648	108650	184949	351646	193750
4.70	191048	106350	178149	368847	190050
4.80	185748	104150	171749	624647	186650
4.90	177648	102050	165749	813747	183250
5.00	167448	999949	159949	946547	179950

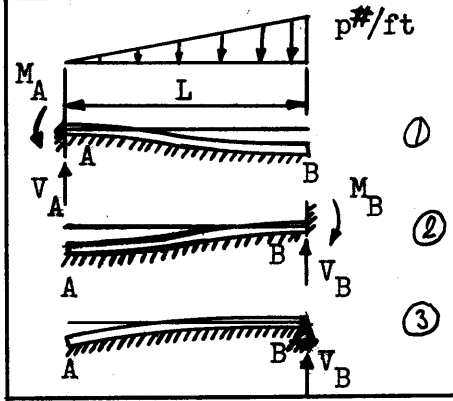
TABLE 8

βL	v_{A2}	v_{B2}	m_B	v_{A1}	v_{B1}
5.10	155848	980349	154549	103248	176850
5.20	143448	961549	149349	107848	173850
5.30	130548	943449	144349	109348	170850
5.40	117748	925949	139749	108348	168050
5.50	105048	909149	135249	105248	165250
5.60	928947	892849	130949	100748	162650
5.70	813247	877249	126849	950747	160050
5.80	704747	862049	123049	886547	157550
5.90	604247	847449	119249	817747	155150
6.00	512147	833349	115749	746447	152750
6.10	428547	819649	112349	674747	150450
6.20	353347	806449	109049	604147	148250
6.30	286547	793649	105949	535847	146150
6.40	227547	781249	102949	470647	144050
6.50	176147	769249	100149	409347	142050
6.60	131647	757549	973948	352347	140050
6.70	936146	746249	947548	299847	138150
6.80	614646	735249	922348	251947	136250
6.90	346547	724649	898048	208747	134450
7.00	126246	714249	874648	170147	132650
7.10	516245	704249	852148	135947	130950
7.20	192146	694449	830548	105947	129250
7.30	300146	684949	809748	799146	127650
7.40	380146	675649	789648	575246	126050
7.50	436046	666649	770348	384946	124450
7.60	471946	657849	751748	225246	122950
7.70	490946	649349	733748	932045	121450
7.80	496146	641049	716448	141945	119950
7.90	490346	632949	699748	997845	118550
8.00	475846	624949	683548	166246	117150
8.10	454746	617249	667948	216146	115850
8.20	428646	609749	652948	251746	114550
8.30	399246	602449	638348	275346	113250
8.40	367746	595249	624248	288746	111950
8.50	335246	588249	610648	293746	110750
8.60	302546	581349	597448	291946	109550
8.70	270246	574749	584648	284846	108450
8.80	239146	568149	572248	273446	107150
8.90	209446	561749	560348	258946	106050
9.00	181446	555549	548648	242246	104950
9.10	155446	549449	537448	224146	103850
9.20	131546	543449	526548	205146	102750
9.30	109746	537649	515948	185946	101750
9.40	901545	531949	505648	166846	100750
9.50	726445	526349	495648	148246	997249

TABLE 8

βL	v_{A2}	v_{B2}	m_B	v_{A1}	v_{B1}
9.60	571745	520849	486048	130446	987449
9.70	436345	515449	476648	113546	977749
9.80	319045	510249	467448	978045	968349
9.90	218745	505049	458648	832345	959049
10.0	133945	500049	450048	699045	950049

TABLE 9



$$M_A = m_A pL^2 \quad M_B = m_B pL^2 \quad \} ①$$
$$V_A = v_A pL \quad V_B = v_{B1} pL \quad \} ②$$
$$V_B = v_{B2} pL \quad \} ③$$

βL	m_A	v_A	m_B	v_{B1}	v_{B2}
0.00	333350	500050	166750	500050	250050
0.05	333350	500050	166750	500050	250050
0.10	333350	500050	166750	500050	250050
0.15	333350	499950	166650	500050	250050
0.20	333150	499750	166650	499950	250050
0.25	332950	499450	166550	499850	250050
0.30	332450	498850	166350	499650	250050
0.35	331750	497850	166050	499250	250050
0.40	330650	496350	165650	498650	249950
0.45	329050	494150	165050	497750	249950
0.50	326950	491150	164250	496550	249950
0.55	324050	487150	163150	495050	249850
0.60	320350	482050	161750	493050	249750
0.65	315650	475650	159950	490650	249650
0.70	309950	467850	157850	487650	249450
0.75	303250	458550	155250	484050	249250
0.80	295450	447750	152350	479850	249050
0.85	286550	435450	148950	475050	248750
0.90	276650	421750	145150	469750	248450
0.95	265750	406850	141050	463950	248050
1.00	254150	390750	136650	457750	247650
1.05	241950	373850	131950	451150	247150
1.10	229250	356350	127150	444250	246550
1.15	216250	338450	122150	437150	245950
1.20	203250	320450	117250	430050	245250
1.25	190350	302650	112250	422850	244450
1.30	177750	285150	107450	415850	243550
1.35	165550	268250	102750	408950	242450
1.40	153750	251950	981649	402250	241350
1.45	142650	236450	938349	395750	240150
1.50	132050	221850	897349	389550	238850
1.55	122250	208050	858649	383650	237450
1.60	113050	195250	822349	377950	235950
1.65	104550	183350	788349	372550	234250
1.70	966249	172250	756749	367450	232450
1.75	893649	162050	727249	362550	230650

TABLE 9

βL	m_A	v_A	m_B	v_{B1}	v_{B2}
1.80	826949	152650	699849	357850	228650
1.85	765849	144050	674349	353350	226550
1.90	709849	136050	650649	349150	224350
1.95	658549	128750	628649	344950	222150
2.00	611749	121950	608049	341050	219750
2.05	568949	115750	588849	337150	217350
2.10	529749	110050	570849	333350	214750
2.15	493949	104750	553949	329750	212250
2.20	461249	999349	538049	326150	209550
2.25	431249	954349	523149	322650	206850
2.30	403749	912649	508949	319150	204150
2.35	378549	873949	495449	315750	201350
2.40	355349	838049	482649	312350	198550
2.45	334049	804449	470449	309050	195750
2.50	314449	773149	458749	305650	192850
2.55	296349	743849	447449	302350	190050
2.60	279649	716349	436649	299150	187250
2.65	264149	690449	426249	295850	184450
2.70	249849	666049	416249	292650	181650
2.75	236549	643049	406549	289350	178850
2.80	224249	621349	397149	286150	176150
2.85	212749	600749	387949	282950	173450
2.90	202049	581149	379149	249850	170750
2.95	192049	562649	370549	276650	168150
3.00	182749	544949	362149	273550	165550
3.10	165849	511949	346049	267350	160550
3.20	151049	481849	330849	261350	155750
3.30	137949	454349	316349	255450	151250
3.40	126249	428949	302649	249650	146850
3.50	115849	405549	289649	244050	142750
3.60	106649	383949	277349	238650	138750
3.70	983048	363949	265649	233350	135050
3.80	908248	345349	254549	228250	131450
3.90	840748	328149	244049	223350	128150
4.00	779748	312149	234149	218650	124950
4.10	724448	297249	224749	214050	121850
4.20	674148	283349	215949	209650	118950
4.30	628348	270349	207549	205450	116150
4.40	586648	258249	199549	201450	113550
4.50	548448	246949	192049	197450	111050
4.60	513548	236249	184949	193750	108650
4.70	481548	226349	178149	190050	106350
4.80	452048	217049	171849	186550	104150
4.90	424948	208249	165749	183250	102050
5.00	399948	199949	159949	179950	999649

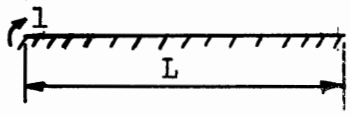
TABLE 9

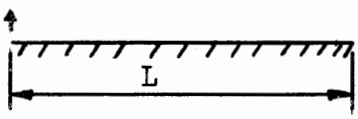
βL	m_A	v_A	m_B	v_{B1}	v_{B2}
5.10	376948	192249	154549	176850	980149
5.20	355548	184949	149349	173750	961349
5.30	335848	177949	144449	170850	943249
5.40	317548	171449	139749	168050	925849
5.50	300548	165249	135249	165250	908949
5.60	284748	159449	130949	162650	892749
5.70	269948	153849	126849	160050	877149
5.80	256248	148649	122949	157550	862049
5.90	243448	143649	119249	155150	847449
6.00	231448	138849	115749	152750	833349
6.10	220248	134349	112349	150450	819649
6.20	209748	130049	109049	148250	806449
6.30	199948	125949	105949	146150	793649
6.40	190748	122049	102949	144050	781249
6.50	182048	118349	100149	142050	769249
6.60	173948	114749	973948	140050	757549
6.70	166248	111349	947548	138150	746249
6.80	159048	108149	922248	136250	735249
6.90	152248	105049	897948	134450	724649
7.00	145748	102049	874648	132650	714249
7.10	139648	991848	852148	130950	704249
7.20	133948	964548	830548	129250	694449
7.30	128548	938248	809748	127650	684949
7.40	123348	913048	789648	126050	675649
7.50	118548	888848	770348	124450	666649
7.60	113948	865648	751748	122950	657849
7.70	109548	843348	733748	121450	649349
7.80	105348	821848	716448	119950	641049
7.90	101448	801148	699748	118550	632949
8.00	976547	781248	683548	117150	624949
8.10	940847	762048	667948	115850	617249
8.20	906847	743648	652948	114550	609749
8.30	874447	725748	638348	113250	602449
8.40	843547	708648	624248	111950	595249
8.50	814147	692048	610648	110750	588249
8.60	786047	676048	597448	109550	581349
8.70	759247	660548	584648	108350	574749
8.80	733747	645648	572248	107150	568149
8.90	709247	631248	560348	106050	561749
9.00	685847	617248	548648	104950	555549
9.10	663547	603748	537448	103850	549449
9.20	642147	590748	526548	102750	543449
9.30	621647	578148	515948	101750	537649
9.40	601947	565848	505648	100750	531949
9.50	583147	554048	495648	997249	526349

TABLE 9

βL	m_A	v_A	m_B	v_{B1}	v_{B2}
9.60	565147	542548	486048	987449	520849
9.70	547847	531448	476648	977749	515449
9.80	531247	520648	467448	968349	510249
9.90	515347	510148	458648	959049	505049
10.00	499947	499948	450048	950049	500049

TABLE 10





βL	A_1	B_1	F_1	G_1	A_2	B_2	F_2	G_2
0.00	-227957	-239957	251957	-239957	-112157	-119957	128157	-119957
0.05	-227957	-239957	251957	-239957	-112157	-119957	128157	-119957
0.10	-675155	-749855	825155	-750355	-326355	-373855	426355	-373855
0.15	-839654	-986654	113655	-988854	-398854	-490154	596354	-490154
0.20	-187554	-233854	281354	-235054	-875053	-115654	150054	-115654
0.25	-576353	-764453	960453	-772453	-264053	-376153	520153	-376153
0.30	-216353	-306253	401653	-311753	-972652	-149753	220753	-149753
0.35	-930652	-141053	193153	-145153	-410352	-685252	107753	-685252
0.40	-441552	-719152	102853	-750452	-190752	-347052	581452	-347052
0.45	-225352	-396252	591752	-420952	-952551	-189852	339252	-189852
0.50	-121652	-231952	362152	-251952	-502151	-110352	210352	-110352
0.55	-684951	-142452	232952	-159052	-275751	-672451	137052	-672451
0.60	-398751	-910551	156152	-104952	-156051	-426651	928651	-426651
0.65	-237951	-601551	108352	-719851	-901550	-279751	651351	-279751
0.70	-144551	-408551	774551	-510551	-527150	-188551	470251	-188551
0.75	-887650	-284051	568151	-372951	-309050	-130151	347951	-130151
0.80	-547550	-201451	426251	-279551	-179450	-916650	263051	-916650
0.85	-336550	-145351	326251	-214551	-101550	-657050	202571	-657050
0.90	-203850	-106451	254251	-168251	-542149	-478150	158851	-478150
0.95	-119850	-789750	201451	-134451	-255449	-352650	126351	-352650
1.00	-664649	-592550	161951	-109251	-829048	-263050	101851	-263050
1.05	-326349	-448950	131951	-902450	188848	-198250	830350	-198250
1.10	-114049	-343050	108851	-756250	765848	-150750	684850	-150750
1.15	166148	-264050	907850	-642150	106849	-115550	570550	-115550
1.20	939948	-204650	765750	-551850	120049	-891549	479650	-891549
1.25	136849	-159350	652450	-479350	122749	-692149	406750	-692149
1.30	157249	-124750	561050	-420650	119349	-540149	347550	-540149
1.35	163349	-979949	486750	-372350	112449	-423449	299150	-423449
1.40	160549	-772549	425750	-332450	103749	-333249	259250	-333249
1.45	152349	-610749	375250	-298950	942048	-263149	226050	-263149
1.50	141149	-483749	333150	-270650	846848	-208349	198350	-208349
1.55	128449	-383849	297750	-246550	754948	-165249	174950	-165249
1.60	115449	-304849	267850	-225850	668748	-131349	155050	-131349
1.65	102549	-242249	242450	-207950	589248	-104549	138150	-104549
1.70	902848	-192549	220550	-192350	516948	-831848	123650	-831848
1.75	788748	-153049	201750	-178650	451748	-662448	111150	-662448
1.80	684048	-121549	185550	-166550	393548	-527448	100250	-527448
1.85	589348	-963548	171350	-155750	341848	-419748	907849	-419748
1.90	504348	-763248	158850	-146150	296048	-333548	825349	-333548
1.95	428848	-603448	147850	-137550	255748	-264748	752849	-264748
2.00	362248	-476248	138150	-129850	220448	-209648	689049	-209648

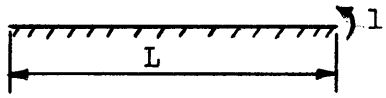
TABLE 10

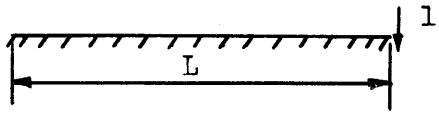
βL	A_1	B_1	F_1	G_1	A_2	B_2	F_2	G_2
2.05	303848	-375048	129550	-122750	189448	-165648	632449	-165648
2.10	252948	-294648	121850	-116350	162448	-130448	582249	-130448
2.15	208848	-230948	114950	-110850	138948	-102348	537449	-102348
2.20	170948	-180648	108650	-105150	118548	-798847	497449	-798847
2.25	138448	-141048	103050	-100250	100848	-620847	461549	-620847
2.30	110748	-110048	978249	-956249	855647	-479747	429149	-479747
2.35	873047	-857647	931349	-914049	724147	-368247	399949	-368247
2.40	676347	-669847	888249	-874849	610947	-280447	373449	-280447
2.45	512147	-525047	848649	-838249	513947	-211747	349449	-211747
2.50	376347	-414247	812049	-804149	430947	-158147	327549	-158147
2.55	264947	-330047	778249	-772249	360047	-116747	307549	-116747
2.60	174547	-266747	746749	-742349	299847	-848446	289249	-848446
2.65	102047	-219447	717449	-714249	248647	-606446	272449	-606446
2.70	447146	-184447	690049	-687749	205447	-424446	256949	-424446
2.75	266344	-158947	664349	-662749	169047	-289346	242749	-289346
2.80	-334746	-140347	640249	-639249	138347	-190846	229549	-190846
2.85	-582846	-162947	617549	-616849	112747	-120446	217449	-120446
2.90	-757246	-117247	596149	-595749	913346	-715445	206149	-715445
2.95	-872146	-110247	575949	-575649	735946	-389245	195649	-389245
3.00	-939046	-104947	556749	-556649	589246	-183845	185849	-183845
3.10	-965846	-973646	521349	-521349	369746	-118344	168249	-118344
3.20	-902046	-913146	489249	-489249	224346	-173444	152849	-173444
3.30	-789546	-851946	460149	-450049	130446	-944644	139349	-944644
3.40	-658046	-784146	434449	-433349	717045	-185545	127349	-185545
3.50	-526146	-709646	409149	-408949	364645	-262345	116749	-262345
3.60	-404546	-630546	386749	-386449	164845	-314045	107249	-314045
3.70	-299046	-550246	366049	-365849	608744	-339445	987848	-339445
3.80	-211546	-471546	347049	-346749	144344	-342045	911948	-342045
3.90	-141946	-397246	329449	-329149	362442	-327245	843648	-327245
4.00	-885545	-329146	313149	-312849	270443	-300645	781948	-300645
4.10	-492045	-268246	297949	-297749	117744	-267245	726048	-267245
4.20	-213845	-215346	283949	-283749	220644	-230845	675448	-230845
4.30	-276344	-170146	270849	-270649	307744	-194645	629348	-194645
4.40	884344	-132346	258549	-258449	367344	-160345	587348	-160345
4.50	152145	-101346	247149	-247049	398344	-129545	549048	-129545
4.60	179845	-764145	236549	-236449	403444	-102545	513948	-102545
4.70	182745	-566645	226849	-226449	388644	-797044	481848	-797044
4.80	170545	-413345	217149	-217149	359744	-607944	452348	-607944
4.90	149945	-296445	208349	-208349	322144	-455144	425148	-455044
5.00	125545	-208545	200149	-200049	280444	-334044	400148	-334044
5.10	101145	-144045	192349	-192249	238244	-240244	377048	-240244
5.20	780144	-975244	184949	-184949	197944	-169044	355748	-169044
5.30	583544	-649144	178049	-178049	160944	-115944	335948	-115944
5.40	408144	-422344	171549	-171549	128344	-773743	317648	-773743
5.50	275444	-275444	165349	-165349	100444	-499743	300548	-499743

TABLE 10

βL	A_1	B_1	F_1	G_1	A_2	B_2	F_2	G_2
5.60	171944	-179344	159449	-159449	770643	-310343	284748	-310343
5.70	920343	-118644	153949	-153949	580043	-183443	270048	-183443
5.80	327243	-825243	148649	-148649	428243	-101443	256348	-101443
5.90	-996341	-617743	143649	-143649	309543	-510542	243548	-510542
6.00	-264243	-525943	138949	-138949	218543	-222342	231548	-222742
6.10	-395043	-480643	134449	-134449	150443	-739641	220348	-739641
6.20	-494643	-453443	130149	-130149	100743	-122441	209848	-122841
6.30	-700843	-439243	126049	-126049	651642	-350939	200048	-350939
6.40	-426943	-423043	122149	-122149	404442	-146241	190748	-146241
6.50	-352543	-407243	118349	-118349	238542	-383041	182148	-383041
6.60	-292243	-379943	114849	-114849	132342	-605641	173948	-605641
6.70	-240143	-349243	111449	-111449	653541	-826841	166248	-826841
6.80	-182943	-313643	108149	-108149	273741	-978341	159048	-978341
6.90	-138843	-285443	105049	-105049	887040	-102542	152248	-102542
7.00	-922542	-246943	102049	-102049	115740	-104442	145848	-104442
7.10	-577542	-209342	991948	-991948	183739	-101042	139748	-101042
7.20	-432042	-176743	964548	-964548	250640	-936341	134048	-936341
7.30	-174242	-154643	938348	-938348	628040	-853141	128548	-853141
7.40	-245541	-125243	913148	-913148	782640	-688441	123448	-688441
7.50	281241	-913942	888948	-888948	109441	-600341	118548	-600341
7.60	102142	-775842	865748	-865748	134841	-514541	113948	-514541
7.70	105842	-528842	843348	-843348	152641	-433341	109548	-433341
7.80	-487441	-172042	821848	-821848	162841	-358341	105448	-358341
7.90	135942	-363642	801248	-801248	166141	-290841	101448	-290841
8.00	195942	-264842	781348	-781348	127141	-204441	976647	-204441
8.10	190342	-229642	762148	-762148	123441	-160641	940847	-160641
8.20	-205941	-411841	743648	-743648	116441	-123141	906847	-123141
8.30	-262041	-524041	725848	-725848	107141	-918140	874547	-918140
8.40	143842	-845941	708648	-708648	744040	-608240	843647	-608240
8.50	-166541	0000	692048	-692048	662040	-429940	814247	-429940
8.60	370741	0000	676048	-676048	577140	-289240	786147	-289240
8.70	-252941	-252941	660648	-660648	358340	-189040	759347	-189040
8.80	176841	0000	645748	-645748	302040	-117840	733747	-117840
8.90	255341	-127641	631248	-631248	249840	-653239	709347	-653239
9.00	0000	961440	617348	-617348	202840	-279939	685947	-279939
9.10	0000	-770840	603848	-603848	161640	-254138	663547	-254138
9.20	-686341	686340	590748	-590748	765439	-175039	642147	-175039
9.30	-229842	153241	578148	-578148	575739	-722138	621647	-722138
9.40	240242	240241	565948	-565948	421939	-104638	602047	-104638
9.50	-495342	0000	554048	-554048	300439	226438	583247	226438
9.60	0000	267641	542548	-542548	206939	366236	565147	366238
9.70	538641	269341	531448	-531448	136739	386238	547847	386238
9.80	0000	-251141	520648	-520648	857138	337638	531247	337638
9.90	0000	126741	510248	-510248	497938	256238	515347	256238
10.00	0000	-671940	500048	-500048	255138	165438	500047	165438

TABLE 11





βL	A_3	B_3	F_3	G_3	A_4	B_4	F_4	G_4
0.00	228057	240057	-252057	240057	-115957	-119957	123957	-119957
0.05	228057	240057	-252057	240057	-115957	-119957	123957	-119957
0.10	675155	750155	-825155	750155	-350055	-375055	400155	-375055
0.15	839554	987654	-113655	987654	-444454	-493854	543254	-493854
0.20	187554	234454	-281254	234454	-101654	-117254	132854	-117254
0.25	575953	767953	-959853	767953	-320053	-384053	448053	-384053
0.30	216053	308653	-401153	308553	-123453	-154353	185253	-154353
0.35	927452	142753	-192753	142753	-547252	-713852	880352	-713852
0.40	438852	731652	-102453	731652	-268452	-366052	463652	-366052
0.45	223052	405752	-588552	405852	-142152	-203052	264052	-203052
0.50	119552	239452	-359252	239452	-798851	-119852	159852	-119852
0.55	666351	148452	-230252	148452	-470951	-743751	101752	-743751
0.60	382051	959251	-153652	959251	-288451	-481051	673551	-481051
0.65	222851	641551	-106052	641551	-182351	-322051	461751	-322051
0.70	130851	441751	-752551	441751	-118351	-222051	325851	-222051
0.75	762850	311851	-547351	311851	-783450	-157051	235751	-157051
0.80	433250	224951	-406451	224951	-528050	-113551	174251	-113551
0.85	231350	165351	-307451	165351	-360750	-836150	131151	-836150
0.90	106950	123551	-236251	123551	-249050	-626550	100451	-626550
0.95	302749	935750	-184151	935750	-173250	-476550	779950	-476550
1.00	-164849	718250	-145351	718250	-120950	-367350	613850	-367350
1.05	-443149	557450	-115951	557450	-843949	-286550	488750	-286550
1.10	-600949	436950	-933950	436950	-586849	-225950	393150	-225950
1.15	-681649	345450	-759050	345450	-404049	-179850	319250	-179850
1.20	-713349	275250	-621750	275250	-273249	-144450	261450	-144450
1.25	-713949	220750	-512750	220750	-179249	-116850	215750	-116850
1.30	-695449	178050	-425550	178050	-111549	-952049	179250	-952049
1.35	-665549	144250	-355050	144250	-627448	-780749	149950	-780749
1.40	-629249	117450	-297750	117450	-276848	-643949	126050	-643949
1.45	-589949	958149	-250650	958149	-263347	-533849	106550	-533849
1.50	-549749	784049	-211850	784049	150648	-444649	904249	-444649
1.55	-509949	642649	-179550	642649	273548	-371849	770949	-371849
1.60	-471549	527249	-152650	527249	356448	-312149	659849	-312149
1.65	-434849	432449	-130050	432449	409848	-262849	566649	-262849
1.70	-400249	354349	-110950	354349	441648	-221949	488049	-221949
1.75	-367849	289749	-947149	289749	457348	-187949	421449	-187949
1.80	-337649	236049	-809749	236049	461348	-159349	364849	-159349
1.85	-309649	191449	-692349	191449	456848	-135449	316549	-135449
1.90	-283649	154149	-591949	154149	446348	-115249	275049	-115249
1.95	-259649	123049	-505649	123049	431548	-981148	239449	-981148
2.00	-237549	969248	-431349	969248	413848	-836048	208649	-836048

TABLE 11

βL	A_3	B_3	F_3	G_3	A_4	B_4	F_4	G_4
2.05	-217149	751048	-367349	751048	394348	-712648	182049	-712648
2.10	-198349	568448	-311949	568448	373748	-607448	158849	-607448
2.15	-180949	415548	-264049	415548	352648	-517448	138749	-517448
2.20	-165049	287748	-222549	287748	331548	-440448	121249	-440448
2.25	-150349	181248	-186649	181248	310648	-374348	105949	-374348
2.30	-136849	926447	-155449	926447	290248	-317648	925448	-317648
2.35	-124449	193847	-128349	193847	270548	-268948	808248	-268948
2.40	-113049	-409047	-104849	-409047	251548	-226948	705448	-226948
2.45	-102549	-901147	-845148	-901147	233448	-190948	615148	-190948
2.50	-929148	-129948	-669348	-129948	216148	-159848	535848	-159848
2.55	-840648	-161648	-517348	-161648	199848	-133148	466148	-133148
2.60	-759548	-186548	-386448	-186548	184448	-110248	404748	-110248
2.65	-685048	-205648	-273948	-205648	169948	-904647	350848	-904647
2.70	-616848	-219748	-177448	-219748	156248	-735447	303348	-735447
2.75	-554248	-229548	-952147	-229548	143548	-590447	261648	-590447
2.80	-497048	-235848	-254347	-235848	131648	-466547	224948	-466547
2.85	-444748	-239148	334147	-239148	120448	-360847	192648	-360847
2.90	-396948	-239848	826647	-239848	110148	-270947	164348	-270947
2.95	-353348	-238448	123548	-238448	100548	-194747	139448	-194747
3.00	-313548	-235248	157048	-235248	915947	-130547	117748	-130547
3.10	-244448	-224948	205348	-224948	757147	-315046	820147	-315046
3.20	-187448	-210748	234048	-210748	621847	363746	549147	-363746
3.30	-140848	-194348	247848	-194348	507347	810746	345147	810746
3.40	-102948	-176848	250748	-176848	410847	108647	193547	108647
3.50	-723647	-159048	245548	-159048	330047	123747	826046	123747
3.60	-480247	-141448	234948	-141448	262747	129847	324245	129847
3.70	-288347	-124648	220548	-124648	207147	129547	-518646	129547
3.80	-139447	-108848	203848	-108848	161347	124947	-884546	124947
3.90	-258046	-942247	185948	-942247	123947	117547	-111147	117547
4.00	588246	-808547	167648	-808547	935646	108447	-123347	108447
4.10	119947	-687647	149548	-687647	691346	984846	-127847	984846
4.20	162147	-579547	132148	-579547	496246	882846	-126947	882846
4.30	189147	-483747	115748	-483747	342046	782446	-122347	782446
4.40	204347	-399647	100348	-399647	221446	686246	-115147	686246
4.50	210447	-326247	862847	-326247	128446	596246	-106447	596246
4.60	209447	-262847	735147	-262847	578345	513346	-968946	513346
4.70	203447	-208547	620447	-208547	535644	438246	-871046	438246
4.80	193647	-162447	518347	-162447	-326245	370846	-774246	370846
4.90	181347	-123547	428347	-123547	-590845	311046	-681146	311046
5.00	167547	-910346	349547	-910346	-764945	258546	-593546	258546
5.10	152847	-642346	281347	-642346	-868945	212846	-512546	212846
5.20	137947	-423446	222647	-423446	-919345	173346	-438646	173346
5.30	123247	-247046	172647	-247046	-929545	139546	-372046	139546
5.40	109047	-107046	130447	-107046	-910445	110846	-312746	110846
5.50	955346	209844	951146	209844	-870445	866645	-260446	866645

TABLE 11

βL	A_3	B_3	F_3	G_3	A_4	B_4	F_4	G_4
5.60	829546	850745	659346	850745	-816645	664645	-214646	664645
5.70	713446	146246	420946	146246	-754145	497545	-174946	497545
5.80	607646	189446	228746	189446	-687045	360545	-140846	360545
5.90	512146	217846	764045	217846	-618645	249445	-111746	249445
6.00	426846	234446	-419745	234446	-550945	160345	-871645	160345
6.10	351246	241446	-131646	241446	-485845	900044	-665845	900044
6.20	285046	241146	-197346	241146	-424345	353844	-495045	353844
6.30	227446	235246	-243046	235246	-367145	-617443	-354845	-617443
6.40	177846	225146	-272446	225146	-314845	-369444	-240945	-369444
6.50	135546	212046	-288646	212046	-267345	-588844	-149545	-588844
6.60	997345	197046	-294346	197046	-224845	-737144	-774044	-737144
6.70	698645	180946	-291946	180946	-187145	-828544	-214244	-828544
6.80	452045	164246	-283246	164246	-154045	-875144	-210544	-875144
6.90	251145	147646	-270046	147646	-125145	-887344	52344	-887344
7.00	90184	131346	-253546	131346	-100245	-873344	74454	-873344
7.10	-36354	115746	-235046	115746	-789144	-840344	89154	-840344
7.20	-133445	101046	-215346	101046	-608444	-793844	97914	-793844
7.30	-205645	87254	-195146	87254	-456844	-738444	102045	-738444
7.40	-256845	74644	-175046	74644	-330844	-677844	102545	-677844
7.50	-290745	63164	-155446	63164	-227244	-614944	100345	-614944
7.60	-310545	52824	-136746	52824	-143244	-551744	96024	-551744
7.70	-318845	43594	-119146	43594	-760643	-490144	90414	-490144
7.80	-318145	35444	-102746	35444	-232943	-431144	83894	-431144
7.90	-310445	28304	-87654	28304	17304	-375644	76854	-375644
8.00	-297445	22124	-73974	22124	47674	-324144	69594	-324144
8.10	-280745	16804	-61674	16804	69554	-27704	62354	-27704
8.20	-261445	12294	-50714	12294	84474	-234344	55314	-234344
8.30	-240545	84924	-41044	84924	93754	-196144	48594	-196144
8.40	-218945	53434	-32584	53434	98514	-162144	42274	-162144
8.50	-197245	27674	-25254	27674	99734	-132344	36434	-132344
8.60	-175945	69304	-18974	64304	98234	-106344	31084	-106344
8.70	-155345	-94284	-13654	-94284	94704	-83864	26244	-83864
8.80	-135945	-22014	-91844	-22014	89704	-64694	21914	-64694
8.90	-117645	-31374	-54904	-31374	83724	-48474	18064	-48474
9.00	-100845	-38014	-24784	-38014	77124	-34884	14694	-34884
9.10	-85414	-42384	-64554	-42384	70224	-23644	11754	-23644
9.20	-71494	-44884	18274	-44884	63254	-14464	9217	-14464
9.30	-59024	-45864	32714	-45864	56394	-7073	7053	-7073
9.40	-47964	-45644	43324	-45644	49784	-1234	5225	-1234
9.50	-38244	-44474	50704	-44474	43534	3280	3697	3280
9.60	-29784	-42594	55404	-42594	37694	6673	2435	6673
9.70	-22494	-40194	57894	-50194	32314	9124	1406	9124
9.80	-16284	-37444	58604	-37444	27414	10804	5817	10804
9.90	-11054	-34474	57894	-34474	22994	11834	-6691	11834
10.00	-6698	-31404	56094	-31404	19054	12354	-5652	12354

TABLE 12

$A = e^{x} \sin x$ $\qquad$ $C = e^{-x} \sin x$

$B = e^{x} \cos x$ $\qquad$ $D = e^{-x} \cos x$

X	A	B	C	D
0.05	525449	104951	475449	950050
0.10	110350	109951	903349	900350
0.15	173650	114851	128650	851050
0.20	242650	119751	162650	802450
0.25	317650	124451	192650	754550
0.30	398950	128951	218950	707750
0.35	486550	133351	241650	661950
0.40	580950	137451	261050	617450
0.45	682150	141251	277350	574150
0.50	790450	144651	290750	532250
0.55	905950	147751	301550	491850
0.60	102851	150351	309850	452950
0.65	115951	152451	315950	415550
0.70	129751	154051	319950	379850
0.75	144351	154851	321950	345650
0.80	159651	155051	322350	313050
0.85	175751	154451	321150	282050
0.90	192651	152851	318450	252750
0.95	210351	150451	314550	224950
1.00	228751	146851	309550	198750
1.05	247851	142151	303550	174150
0.10	267751	136251	296650	150950
1.15	288251	129051	289050	129350
1.20	309451	120351	280750	109150
1.25	331251	110051	271850	903449
1.30	353551	981550	262650	729049
1.35	376351	844850	252950	567749
1.40	399651	689250	243050	419149
1.45	423251	513750	232850	282649
1.50	447051	317050	222550	157849
1.55	471051	979749	212250	441348
1.60	495051	144650	-201850	589548
1.65	519051	411950	-191450	151949
1.70	542851	705250	-181150	235348
1.75	566251	102551	-170950	309749
1.80	589151	137451	-160950	375549
1.85	611351	175251	-151150	433349
1.90	632651	216151	-141550	483549
1.95	652951	260151	-132150	526649
2.00	671851	307451	-123050	563149

TABLE 12

X	A	B	C	D
2.05	689251	358151	-114250	593549
2.10	704951	412251	-105750	618249
2.15	718451	469851	-974849	637549
2.20	729651	531151	-895849	652049
2.25	738251	595951	-820049	662049
2.30	743751	664551	-747649	668049
2.35	746051	736851	-678549	670149
2.40	744551	812851	-612749	668949
2.45	739051	892551	-550349	664649
2.50	729051	975951	-491249	657649
2.55	714251	106352	-435449	648149
2.60	694051	115352	-382849	636449
2.65	668151	124752	-333449	622849
2.70	635951	134552	-287249	607549
2.75	597051	144552	-243949	590849
2.80	550851	154952	-203749	572949
2.85	496951	165552	-166249	554049
2.90	434851	176452	-131649	534249
2.95	363851	187552	-996648	513849
3.00	283451	198852	-702548	492849
3.10	923050	221752	-187348	450149
3.20	143251	-244952	-237948	-406949
3.30	427651	-267752	-581848	-364249
3.40	765751	-289652	-852848	-322649
3.50	116152	-310152	-105949	-282749
3.60	161952	-328152	-120949	-245049
3.70	214352	-343052	-130949	-209649
3.80	273552	-353552	-136849	-176949
3.90	339752	-358652	-139249	-146949
4.00	413252	-356852	-138649	-119749
4.10	493752	-346852	-135649	-952648
4.20	581252	-326952	-130649	-735148
4.30	675252	-295352	-124349	-543848
4.40	775052	-250352	-116849	-377348
4.50	879952	-189752	-108549	-234148
4.60	988552	-111552	-998848	-112748
4.70	109953	-136251	-909448	-112647
4.80	121053	-106352	819848	-720047
4.90	131953	-250452	731548	-138848
5.00	142353	-420952	646148	-191148
5.10	151853	-619952	564448	-230448
5.20	160153	-849252	487348	-258448
5.30	166753	-111053	415448	-276748
5.40	171053	-140553	349048	-286648
5.50	172653	-173453	288348	-289648

TABLE 12

X	A	B	C	D
5.60	170753	-209753	233448	-286748
5.70	164553	-249453	184248	-279248
5.80	153453	-292453	140648	-268048
5.90	136453	-338553	102448	-254048
6.00	112753	-387353	692647	-238048
6.10	812152	-438353	408547	-220548
6.20	409452	-491053	168647	-202248
6.30	915651	544453	308746	183648
6.40	701452	597753	193647	165048
6.50	143053	649553	323447	146848
6.60	229053	698553	423847	129248
6.70	328953	742853	498347	112548
6.80	443653	780553	550347	968347
6.90	573953	809453	582947	822047
7.00	720453	826753	599047	687447
7.10	883453	829653	601447	564847
7.20	106354	814853	592547	454147
7.30	125854	778753	574547	355347
7.40	147054	717453	549347	268047
7.50	169554	626753	518747	191747
7.60	193454	502053	484347	125747
7.70	218254	338753	447447	694546
7.80	243754	131653	409147	221046
7.90	269454	124053	-370347	170546
8.00	294954	433753	-331847	488046
8.10	319554	802353	-294347	739246
8.20	342554	123454	-258347	931546
8.30	363054	173554	-224247	107247
8.40	380054	230954	-192147	116747
8.50	392454	295854	-162447	122447
8.60	398854	368654	-135247	124947
8.70	397954	449454	-110447	124747
8.80	388054	538054	-881646	122247
8.90	367354	634554	-683346	118047
9.00	333954	738254	508546	112447
9.10	285754	848754	-356346	105847
9.20	220554	964854	-225246	984946
9.30	136154	108555	-113746	907146
9.40	299453	120855	-204945	826946
9.50	100354	-133255	-562545	-746446
9.60	257354	-145355	-118046	-666946
9.70	443454	-157055	-166546	-589746
9.80	660854	-167755	-203246	-515946
9.90	911854	-177255	-229546	-446146
10.00	119855	-184855	-246946	-380946