Lawrence Berkeley National Laboratory
Recent Work

Title
PION-NUCLEUS CHARGE EXCHANGE REACTIONS IN THE ISOBAR-DOORWAY MODEL

Permalink
https://escholarship.org/uc/item/7rd5k5t9

Authors
Nagarajan, M.A.
Wang, W.L.

Publication Date
1974-06-01

eScholarship.org Powered by the California Digital Library
University of California

Submitted to Physical Review C

LBL-2934
Preprint

PION-NUCLEUS CHARGE EXCHANGE REACTIONS IN THE ISOBAR-DOORWAY MODEL

LAWRENCE
RADIATION LABORATORY

JUL 25 1974

LIBRARY AND
DOCUMENTS SECTION

M. A. Nagarajan and W. L. Wang

June 1974

Prepared for the U. S. Atomic Energy Commission
under Contract W-7405-ENG-48

TWO-WEEK LOAN COPY

This is a Library Circulating Copy which may be borrowed for two weeks. For a personal retention copy, call Tech. Info. Division, Ext. 5545

LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA

LBL-2934

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

NUCLEAR REACTIONS: Pion-Nucleus Charge Exchange;

(3,3) Resonance; isobar-doorway model

PION-NUCLEUS CHARGE EXCHANGE REACTIONS IN THE ISOBAR-DOORWAY MODEL*

M. A. Nagarajan and W. L. Wang

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

June 1974

ABSTRACT

The isobar-doorway model, previously applied to the study of pion-nucleus elastic scatterings, is generalized to the treatment of single and double charge exchange reactions. The charge exchange amplitudes are calculated within the framework of distorted wave Born approximation, where the resonant distortion of the elastic scattering wave functions in the initial and final channels as well as the resonant part of the charge exchange interaction are explicitly taken into account. In the isobar-doorway model, it is shown that the transition amplitude depends upon quantities related to the elastic scattering and to a charge exchange amplitude which contains nonresonant initial and final pion wave functions. The strong energy dependence in various components of single and double charge exchange amplitudes is clearly displayed in the model; this energy dependence due to the (3,3) isobar in the reaction mechanism (including initial and final state interaction) may also be separated out from the nuclear structure information as contained in the nuclear form factors.

I. INTRODUCTION

During recent years, pion-nucleus scattering has received a great deal of attention. A general review of the subject can be found in the article by Koltun.[1] In this paper, we shall concentrate only on the charge exchange reactions, the single and double charge exchange reactions being referred to as SCX and DCX, respectively. We shall be mainly interested in these reactions in the vicinity of the (3,3) resonance. In this energy region, we expect the interaction mechanism to considerably simplify due to the dominance of the $\Delta(1231)$ resonance.

There have been several investigations of the mechanism of the charge exchange reaction. A general review may be found in Becker and Batusov.[2] The simplest calculations of the charge exchange process have been in terms of the production and decay of the Δ particle. In SCX, the pion absorbs a nucleon of the target to form a Δ which decays by emitting a pion of different charge and forming the analogue nucleus, while, in DCX, two nucleons are converted into Δ particles which decay into the "double analogue" nucleus. The work of Parsons, Trefil and Drell,[3] Becker and Marč,[4] Becker and Schmidt,[5] and Barshay and Brown[6] fall under the above category. The effect of the $\Delta(1231)$ resonance in the initial and final channels have however not been considered in these impulse approximation calculations.

The initial and final state interaction effects have been considered by various authors. Kaufman and Hower[7] used a simple absorption reduction factor. Lucci and Picchi,[8] and Bjørnenak, et. al.[9] have used the Glauber approximation. Recently, Kaufmann, Jackson, and Gibbs[10] have calculated the SCX and DCX amplitudes using a multiple scattering theory. There are also optical model calculations by Koren,[11] and by Kerman and Logan.[12] There

also exist DWBA calculations due to Charlton, Eisenberg, and Jones,[13] and Rost and Edwards.[14] Recently, Miller and Spencer have done a coupled channel calculation for these reactions.[15] The initial and final distorted wave functions utilized in the above calculations are usually calculated from a first order approximation to the optical potential. These predict the asymptotic form of the elastic scattering wave functions, but a calculation of the SCX and DCX amplitudes require also the wave functions at small distances. The study of various improvements on these theories has received much attention.

In this work, we shall discuss the application of the isobar-doorway model[16] to the charge exchange reactions. It is appropriate to recall the main ideas of the model. One separates the pion-nucleon interaction into a resonant and a nonresonant part. One assumes that the resonant part of the interaction creates an isobar compound state, i.e. a nucleon hole and a Δ particle in the target. One further assumes that the scattering and reaction phenomena are largely determined by the detailed properties of the isobar compound system. In an elastic scattering process, one assumes that the isobar is formed in the elastic channel and then decays into both elastic and inelastic channels. The doorway state picture emerges if one further assumes that the coupling between the elastic and inelastic channels is solely through the isobar compound state.

In this paper, we do not discuss coupled channel effects. We do believe that the coupled channel effects should be taken into account. We shall discuss it in a later publication. We restrict our present formulation within the framework of the DWBA. We shall consider an extension of the isobar-doorway model, wherein we shall replace the elastic scattering by the elastic and charge exchange channels. In the extended isobar-doorway model, we shall show that the transition amplitude contains strong energy dependence due to the resonance effects in the initial and final channel wave functions as well as in the charge exchange interaction.

II. ISOBAR-DOORWAY FORMALISM

We shall consider both the single charge exchange and double charge exchange reactions in the isobar-doorway model. For simplicity, we give detailed formulation for the SCX reactions, since the extension to the DCX reactions is straightforward and the results will be given only at the end of this section.

For SCX reaction, we consider the following process:

$$\pi_1 + A_1 \longrightarrow \pi_2 + A_2 \tag{1}$$

where 1 and 2 denote the initial and final charge states of the pions. The incident channel has a π_1 (π^+ or π^-) interacting with the target nucleus A_1; the outgoing channel has a π_2 (generally π^o for SCX reactions) with the residual nucleus A_2. The Hamiltonian of the system may be written as

$$H = H_b + K_\pi + V \tag{2}$$

where H_b (r_1, r_2 ... r_A) is the baryon Hamiltonian with baryon coordinates r_1, ... r_A. The baryon Hamiltonian also describes the motion of the excited state of the nucleon (i.e., isobar). We allow at most one isobar in the system. In Eq. (2), the pion kinetic energy operator is K_π and the π-nucleus interaction is V. This π-nucleus interaction V describes the elastic scattering as well as the charge exchange process, along with all other reactions. We may separate this interaction into two parts:

$$V = V_o + V_R \tag{3}$$

where the non-resonant interaction V_o contains operators of the following form

$$V_o = f_{\pi N}\, a_\pi^+ a_N^+ a_N a_\pi \tag{4}$$

with $f_{\pi N}$ as the strength of the interaction (a_π^+ and a_N^+ are creation operators of a pion and a nucleon, respectively). The Coulomb interaction is included in V_o. The resonant interaction V_R has the following form

$$V_R = G_{\pi N\Delta}\, a_\Delta a_N^+ a_\pi^+ + \text{h. c} \tag{5}$$

where $G_{\pi N\Delta}$ depends on the coupling strength and the quantum numbers of π, N and Δ. For details of the resonant interaction, we refer to the nonrelativistic form of Kerman and Kisslinger[17] and a field theory calculation of Δ-exchange.[18] However, in the following discussion, we do not need the explicit form for the interaction. The nonresonant interaction contains the π-nucleon s-wave and $T = \frac{1}{2}$ p-wave interactions. The resonant interaction is the $T = \frac{3}{2}$ p-wave interaction.

We may now define the various isobar-nucleus states in terms of the eigenstates of the baryon Hamiltonian H_b: (1) The initial target state $|\Phi_1\rangle$ and the final residual nuclear state $|\Phi_2\rangle$ may be defined as

$$H_b(r_1 \ldots r_A)\,|\Phi_i(r_1 \ldots r_A)\rangle = E_i\,|\Phi_i(r_1 \ldots r_A)\rangle \tag{6}$$

where i = 1 or 2. The coordinates $(r_1 \ldots r_A)$ refer to those of the nucleons of the nucleus; (2) The isobar-doorway states $|\Phi_{n\alpha}\rangle$ may be defined as

$$H_b(r_1 \ldots r_A)\,|\Phi_{n\alpha}(r_1 \ldots r_{A-1}, \Delta_\alpha)> = (E_n + \varepsilon_\alpha)\,|\Phi_{n\alpha}(r_1 \ldots r_{A-1}, \Delta_\alpha)> \tag{7}$$

where $(r_1 \ldots r_{A-1})$ refers to the coordinates of the (A-1) nucleons with energy E_n, and Δ_α denotes the coordinate of the isobar with a single-particle energy ε_α in the nucleus. The isobar in the state $|\Phi_{n\alpha}(r_1 \ldots r_A, \Delta_\alpha)>$ is assumed to be a stable particle (the narrow resonance approximation); the width of the resonance is restored latter by coupling the doorway states $|\Phi_{n\alpha}>$ to the inelastic and the open channels under our consideration.

To formulate our problem, it is useful to use the projection-operator techniques of Feshbach.[19] We define the elastic pion scattering states as the P-space (the π-continuum space). We have two charge states in the P-space, so we separate the P-space into two parts: P_1 and P_2. We define the P_1 and P_2 operators as

$$P_1 = |\Phi_1(r_1 \ldots r_A)><\Phi_1(r_1 \ldots r_A)| \tag{8}$$

and

$$P_2 = |\Phi_2(r_1 \ldots r_A)><\Phi_2(r_1 \ldots r_A)| \tag{9}$$

which project onto the P_1- and P_2-spaces, or the initial and the final nuclear states, respectively. We next define the Q-space operator, Q, which projects onto the isobar-doorway states $|\Phi_{n\alpha}>$ as

$$Q = \sum_{n,\alpha} |\Phi_{n\alpha}(r_1 \ldots r_{A-1}, \Delta_\alpha)><\Phi_{n\alpha}(r_1 \ldots r_{A-1}, \Delta_\alpha)| \quad . \tag{10}$$

We finally define the q-space operator, q, which projects onto the rest of the Hilbert space (the compound inelastic states) defined by the system Hamiltonian, i.e.;

$$q = 1 - P - Q \quad . \tag{11}$$

It is easy to see that the operators P, Q, and q satisfy the projection operator and the orthogonality conditions.

The π-nucleus interaction V_o and V_R may now be shown to satisfy the following conditions: (1) The nonresonant interaction V_o does not connect P and Q spaces:

$$Q\, V_o\, P_i = 0 \qquad i = 1,2 \quad , \tag{12}$$

and

$$P_i\, V_o\, P_j \neq 0 \qquad i,j = 1,2 \quad , \tag{13}$$

and similarly (2) the resonant part V_R satisfies

$$P_i\, V_R\, Q \neq 0 \qquad i = 1,2 \quad , \tag{14}$$

and

$$P_i\, V_R\, P_j = Q\, V_R\, Q = 0 \qquad i,j = 1,2 \quad . \tag{15}$$

We have now properly specified our projection operators. We may proceed to solve the scattering problem.

The complete wave function of the system $|\Psi\rangle$ is described by the Schrödinger equation

$$(E - H)|\Psi\rangle = 0 \quad , \tag{16}$$

where E is the energy of the system. This equation may be rewritten as the following coupled equations, by using the projection-operator techniques;[19]

$$(E - H_{11})P_1|\Psi> = H_{12}P_2|\Psi> + H_{1Q}Q|\Psi> \tag{17a}$$

$$(E - H_{22})P_2|\Psi> = H_{21}P_1|\Psi> + H_{2Q}Q|\Psi> \tag{17b}$$

$$(E - H_{QQ})Q|\Psi> = H_{Q1}P_1|\Psi> + H_{Q2}P_2|\Psi> + H_{Qq}q|\Psi> \tag{17c}$$

$$(E - H_{qq})q|\Psi> = H_{qQ}Q|\Psi> \quad , \tag{17d}$$

where we have used the usual notations: $H_{ij} = P_i\ H\ P_j$, $H_{iQ} = P_i\ H\ Q$, and $H_{QQ} = Q\ H\ Q$, etc. In Eq. (17), we have already made the doorway-state hypothesis: there is no coupling between P- and q-spaces, or

$$P_i\ V_o\ q = P_i\ V_R\ q = 0 \quad , \quad i = 1,2 \quad . \tag{18}$$

We have assumed that the elastic scattering states in both the incident and final channels are not <u>directly</u> coupled to the compound inelastic states. It is clear that the assumption on the reaction mechanisms in the preceeding discussion is identical to the one used in the elastic scattering.[16]

To simplify Eq. (17), let us introduce an effective Q-space Hamiltonian by eliminating the q-space, i.e.,

$$\tilde{H}_{QQ} = H_{QQ} + H_{Qq}(E - H_{qq})^{-1}\ H_{qQ} \quad . \tag{19}$$

We obtain an effective Hamiltonian for the P-space as

$$\mathcal{H} = H + H(E - \tilde{H}_{QQ})^{-1}\ H \quad , \tag{20}$$

and write the coupled-channel equations for the charge exchange reactions as

$$(E - \mathcal{H}_{11})P_1|\Psi\rangle = \mathcal{H}_{12}P_2|\Psi\rangle \tag{21a}$$

$$(E - \mathcal{H}_{22})P_2|\Psi\rangle = \mathcal{H}_{21}P_1|\Psi\rangle \quad , \tag{21b}$$

where $\mathcal{H}_{ij} = P_i \mathcal{H} P_j$. Equation (21) may be solved for the desired T-matrix. These coupled equations are, however, rather complicated. In order to exhibit the important dynamical effects of the isobar resonance, we choose to introduce a distorted wave Born approximation (DWBA) for the T-matrix. The DWBA SCX amplitude is defined as

$$T = \langle\chi_2^{(-)}|\mathcal{H}_{21}|\chi_1^{(+)}\rangle \quad , \tag{22}$$

where the distorted wave functions $|\chi_i^{(\pm)}\rangle$ are the homogeneous solutions of Eq. (21), i.e., we have

$$(E - \mathcal{H}_{ii})|\chi_i^{(\pm)}\rangle = 0 \qquad i = 1,2 \quad . \tag{23}$$

Equations (22) and (23) are the basic starting point of our discussions.

Formally our results so far are rather similar to the (p,n) reactions. However, for the case of pion-nucleus charge exchange reactions, both the wave functions $|\chi^{(\pm)}\rangle$ and the interaction $\mathcal{H}_{21}$ have strong energy dependence due to the (3,3) resonance. The fact that the π-nucleus interaction is strong and resonating indicates important initial- and final-state interactions. To show these effects explicitly, we have to study the behavior of the continuum wave functions (the distorted waves). This may be done conveniently by using the isobar-doorway model, where the resonant and the nonresonant parts of the pion wave functions are treated separately.

We write Eq. (23) explicitly as

$$(E - H_{ii} + H_{iQ}(E - \tilde{H}_{QQ})^{-1} H_{Qi})|\chi_i^{(\pm)}> = 0 \quad . \tag{24}$$

The distorted wave $|\chi_i^{(\pm)}>$ may be formally obtained in terms of the nonresonant and resonant parts as

$$|\chi_i^{(\pm)}> = |\phi_i^{(\pm)}> + |\phi_i^{R(\pm)}> \quad , \tag{25}$$

where the nonresonant wave function $|\phi_i^{(\pm)}>$ is the solution of the homogeneous equation,

$$(E - H_{ii})|\phi_i^{(\pm)}> = 0 \quad , \tag{26}$$

and the resonant wave function $|\phi_i^{R(\pm)}>$ is related to the nonresonant part by

$$|\phi_i^{R(\pm)}> = \frac{1}{E - H_{ii} \pm i\eta} P_i V_R Q \frac{1}{E - \tilde{H}_{QQ} + H_{Qi}(E - H_{ii} \pm i\eta)^{-1} H_{iQ}} Q V_R P_i |\phi_i^{(\pm)}> \quad . \tag{27}$$

In order to simplify Eq. (27), let us introduce the isolated doorway approximation[19] and represent the inverse operators by the eigenstates of the operator H_{QQ}, as defined in Eq. (10). We may write Eq. (27) as

$$|\phi_i^{R(\pm)}(\vec{k})> = \int \frac{d\vec{k}'}{E - E_{k'} \pm i\eta} |\phi_i^{(\pm)}(\vec{k}')> T_i^{(\pm)}(\vec{k}', \vec{k}; E) \quad , \tag{28}$$

where we have denoted k as a continuum energy variable in $|\phi_i^{(\pm)}(\vec{k})>$, the resonant T-matrix $T_i^{(\pm)}$ is defined as

$$T_i^{(\pm)}(\vec{k}',\vec{k};\ E) = \sum_{n,\alpha} \frac{\langle \phi_i^{(\pm)}(\vec{k}')|V_R|\Phi_{n\alpha}\rangle\langle\Phi_{n\alpha}|V_R|\phi_i^{(\pm)}(\vec{k})\rangle}{E - (E_n + \varepsilon_\alpha + \Delta_{n\alpha i}^{\uparrow} + \Delta_{n\alpha}^{\downarrow}) + \frac{i}{2}(\Gamma_{n\alpha i}^{\uparrow} + \Gamma_{n\alpha}^{\downarrow})} \quad , \tag{29}$$

where the continuum width $\Gamma_{n\alpha i}^{\uparrow}$ and shift $\Delta_{n\alpha i}^{\uparrow}$ to the ith channel are defined as

$$\Delta_{n\alpha i}^{\uparrow} - \frac{i\Gamma_{n\alpha i}^{\uparrow}}{2} = \langle\Phi_{n\alpha}|V_R(E - H_{ii} + i\eta)^{-1} V_R|\Phi_{n\alpha}\rangle \quad . \tag{30}$$

The compound width $\Gamma_{n\alpha}^{\downarrow}$ and shift $\Delta_{n\alpha}^{\downarrow}$ are defined as

$$\Delta_{n\alpha}^{\downarrow} - \frac{i\Gamma_{n\alpha}^{\downarrow}}{2} = \sum_q \frac{|\langle\phi_q|V_R|\Phi_{n\alpha}\rangle|^2}{E - \varepsilon_q + i\Gamma_{q/2}} \quad , \tag{31}$$

where we have used an energy averaging procedure by assigning a width parameter Γ_q to each q-state .[19] In Eq. (31), ε_q and $|\phi_q\rangle$ are the eigenenergies and the eigenstates of the q-space:

$$(\varepsilon_q - H_{qq})|\phi_q\rangle = 0 \quad , \tag{32}$$

which may contain inelastic continuum states. The resonant T-matrix $T_i^{(\pm)}$ as defined in Eq. (29) is very close to the T-matrix for the elastic scattering in channel i.[16] This fact may enable us to use the results from the elastic scattering calculation to estimate the matrix elements $T_i^{(\pm)}$. We shall later return to such approximation. The quantity $T_i^{(\pm)}(\vec{k}',\vec{K};\ E)$ will become one of the most important factors in our model for charge exchange reactions.

Substituting Eq. (25) into the DWBA T-matrix, we obtain the results in terms of a nonresonant SCX T_{SCX}^{NR} and a resonant SCX amplitude T_{SCX}^{R} as

$$T_{SCX} = T^{NR}_{SCX} + T^{R}_{SCX} \quad , \tag{33}$$

where the nonresonant SCX amplitude is given by

$$T^{NR}_{SCX} = \langle\phi_2^{(-)}|V_o^{21}|\phi_1^{(+)}\rangle + \langle\phi_2^{R(-)}|V_o^{21}|\phi_1^{(+)}\rangle$$
$$+ \langle\phi_2^{(-)}|V_o^{21}|\phi_1^{R(+)}\rangle + \langle\phi_2^{R(-)}|V_o^{21}|\phi_1^{R(+)}\rangle \quad , \tag{34}$$

where $V_o^{21} = P_2\, V_o\, P_1$ is the charge-exchange part of the π-nucleus interaction V_o, and the resonant SCX amplitude T^R_{SCX} is

$$T^{R}_{SCX} = \langle\phi_2^{(-)}|H_{2Q}(E - \tilde{H}_{QQ})^{-1} H_{Q1}|\phi_1^{(+)}\rangle$$
$$+ \langle\phi_2^{(-)}|H_{2Q}(E - \tilde{H}_{QQ})^{-1} H_{Q1}|\phi_1^{R(+)}\rangle$$
$$+ \langle\phi_2^{R(-)}|H_{2Q}(E - \tilde{H}_{QQ})^{-1} H_{Q1}|\phi_1^{(+)}\rangle$$
$$+ \langle\phi_2^{R(-)}|H_{2Q}(E - \tilde{H}_{QQ})^{-1} H_{Q1}|\phi_1^{R(+)}\rangle \tag{35}$$

All these terms are represented in Figs. 1 and 2, where, for simplicity, we have denoted the resonant charge exchange operator as

$$X_{ij} = H_{iQ}(E - \tilde{H}_{QQ})^{-1} H_{Qj} \tag{36}$$

Equations (34) and (35) give a complete DWBA description of the single charge exchange reactions. The four terms in T^{NR}_{SCX} or T^{R}_{SCX} are related. To show this property, we now take the Born amplitudes, i.e. the first terms in Eqs. (34) and (35), which are close to the amplitudes in the plane-wave Born approximation (PWBA). We define the nonresonant Born amplitude as

$$U_1^{NR}(\vec{k}',\vec{k}) = \langle\phi_2^{(-)}(\vec{k}')\,|V_o^{21}|\,\phi_1^{(+)}(\vec{k})\rangle \tag{37}$$

then we may show that

$$U_2^{NR}(\vec{k}',\vec{k}) = \langle\phi_2^{R(-)}(\vec{k}')\,|V_o^{21}|\,\phi_1^{(+)}(\vec{k})\rangle = \int \frac{d\vec{q}}{E - E_q + i\eta}\, T_2^{(-)}(\vec{q},\vec{k}';\ E)\ U_1^{NR}(\vec{q},\vec{k}) \tag{38}$$

$$U_3^{NR}(\vec{k}',\vec{k}) = \langle\phi_2^{(-)}(\vec{k}')\,|V_o^{21}|\,\phi_1^{R(+)}(\vec{k})\rangle = \int \frac{d\vec{q}}{E - E_q + i\eta}\, T_1^{(+)}(\vec{q},\vec{k};\ E)\ U_1^{NR}(\vec{k}',\vec{q}) \tag{39}$$

$$U_4^{NR}(\vec{k}',\vec{k}) = \langle\phi_2^{R(-)}(\vec{k}')\,|V_o^{21}|\,\phi_1^{R(+)}(\vec{k})\rangle = \iint \frac{d\vec{q}\ d\vec{q}'}{(E - E_q + i\eta)(E - E_{q'} + i\eta)}\, T_2^{(-)}(\vec{q},\vec{k}';E)$$

$$T_1^{(+)}(\vec{q}',\vec{k};\ E)\ U_1^{NR}(\vec{q},\vec{q}') \tag{40}$$

where $T_i^{(-)}$ are also defined by Eq. (29). The nonresonant SCX amplitude is then

$$T_{SCX}^{NR} = \sum_{i=1}^{4} U_i^{NR}(\vec{k}',\vec{k}) \tag{41}$$

It is clear from Eqs. (37) through (40) that once the Born amplitude $U_1^{NR}(\vec{k}',\vec{k})$ is known on and off the energy shell, we may determine the complete non-resonant amplitude in terms of the T-matrix $T_i^{(\pm)}(\vec{k}',\vec{k};\ E)$ defined by Eq. (29). We may follow exactly the same argument for the resonant exchange interaction. Let us define the Born amplitude:

$$U_1^{R}(\vec{k}',\vec{k};\ E) = \langle\phi_2^{(-)}(\vec{k}')\,|H_{2Q}(E - \tilde{H}_{QQ})^{-1}\ H_{Q1}\,|\phi_1^{(+)}(\vec{k})\rangle \tag{42}$$

where we have the explicit energy dependence. Then the other three terms in Eq. (35) are related to $U_1^R(\vec{k}',\vec{k};\ E)$ by exactly the same relations as given by Eq. (38) through (40) with V_o^{21} changed to X_{ij}, and $U_1^{NR}(\vec{q},\vec{q}')$ changed to $U_1^R(\vec{q},\vec{q}';\ E)$; the connection is also made by the use of the T-matrix $T_i^{(\pm)}$. The complete resonant SCX amplitude is then given as

$$T_{SCX}^R = \sum_{i=1}^{4} U_i^R(\vec{k}',\vec{k};\ E) \quad . \tag{43}$$

We have now shown that the complete DWBA amplitude for single charge exchange reactions is determined by $U_1^{NR}(\vec{k}',\vec{k})$, $U_1^R(\vec{k}',\vec{k};\ E)$, and $T_i^{(\pm)}(\vec{k}',\vec{k};\ E)$. The usefulness of the method may be viewed as follows. The energy dependence due to the resonance is completely explicit in these basic quantities. The basic Born amplitudes may be evaluated with minimum uncertainty in the pion wave functions since they are expressed only in terms of the nonresonant parts of the wave functions, which may be reliably evaluated from a first-order optical model calculation. Near the resonance where the nonresonant distortion may be neglected, we may be able to use plane waves for $|\phi_i^{\pm}>$.

Before we make approximations on these basic quantities, we would first like to extend the same formalism to DCX reactions in a second-order distorted wave approximation. For simplicity, we shall discuss only the resonant charge exchange interaction. We first extend Eqs. (23) through (29) to include the doubly-charge-exchanged channel, hereafter denoted as i = 3 channel in these equations. The resonant DCX reactions proceed through two successive interactions of X_{ij}. The simplest term with nonresonant internal and external pion wave functions is shown in Fig. 3. We shall call this simplest term the nonresonant Born term and denote it as $W_1(\vec{k}',\vec{k};\ E)$; it may be related to $U_1^R(\vec{k}',\vec{k};\ E)$ of Eq. (41) by

$$W_1(\vec{k}',\vec{k};\ E) = \int \frac{d\vec{q}}{E - E_q + i\eta}\, U_1^R(\vec{k}',\vec{q};\ E)\ U_1^R(\vec{q},\vec{k};\ E) \tag{44}$$

There are three other terms with nonresonant external pion wave functions. We may write these terms as (see Fig. 4)

$$W_2(\vec{k}',\vec{k};\ E) = \int \frac{d\vec{q}}{E - E_q + i\eta}\, U_1^R(\vec{k}',\vec{q};\ E)\ U_2^R(\vec{q},\vec{k};\ E) \tag{45}$$

$$W_3(\vec{k}',\vec{k};\ E) = \int \frac{d\vec{q}}{E - E_q + i\eta}\, U_2^R(\vec{k}',\vec{q};\ E)\ U_1^R(\vec{q},\vec{k};\ E) \tag{46}$$

$$W_4(\vec{k}',\vec{k};\ E) = \int \frac{d\vec{q}}{E - E_q + i\eta}\, U_3^R(\vec{k}',\vec{q};\ E)\ U_2^R(\vec{q},\vec{k};\ E) \tag{47}$$

It is readily shown that the Born term with nonresonant external pion wave functions is given as

$$W_1^B(\vec{k}',\vec{k};\ E) = \langle \phi_3^{(-)}(\vec{k}') | X_{32} | \frac{1}{E - \mathcal{H}_{22} + i\eta} | X_{21} | \phi_1^{(+)}(\vec{k}) \rangle$$

$$= \sum_{n=1}^{4} W_n(\vec{k}',\vec{k};\ E) \tag{48}$$

In Eqs. (45) through (47), we have also used $U_2^R(\vec{k}',\vec{k};\ E)$ and $U_3^R(\vec{k}',\vec{k};\ E)$, as defined in Eqs. (38) and (39), for simplification in notations. The diagrams corresponding to W_n (n = 2,3,4) are shown in Fig. 4. It is important to note that the Born term $W_1^B(\vec{k}',\vec{k};\ E)$ is also completely determined by the basic quanitites in the theory: $U_1^R(\vec{k}',\vec{k};\ E)$ and the T-matrix $T_i^{(\pm)}(\vec{k}',\vec{k};\ E)$. We do not need any extra parameters.

After we have obtained the Born term, it is straightforward to generalize our consideration to the cases with resonant external pion states. The results are given similar to Eqs. (38), (39) and (40), with the following replacements

$$U_i^{NR}(\vec{k}',\vec{k}) \longrightarrow W_i^B(\vec{k}',\vec{k};\ E),\ i = 1,2,3,4\ ,$$

and $$T_2^{(\pm)}(\vec{k}',\vec{k};\ E) \longrightarrow T_3^{(\pm)}(\vec{k}',\vec{k};\ E) \qquad [\text{with } i = 3 \text{ in Eq. (29)}]$$

where we have defined

$$W_2^B(\vec{k}',\vec{k};\ E) = \langle \phi_3^{R(-)}(\vec{k}') \left| X_{32}(E - \mathcal{H}_{22} + i\eta)^{-1} X_{21} \right| \phi_1^{(+)}(\vec{k}) \rangle \tag{49}$$

$$W_3^B(\vec{k}',\vec{k};\ E) = \langle \phi_3^{(-)}(\vec{k}') \left| X_{32}(E - \mathcal{H}_{22} + i\eta)^{-1} X_{21} \right| \phi_1^{R(+)}(\vec{k}) \rangle \tag{50}$$

and

$$W_4^B(\vec{k}',\vec{k};\ E) = \langle \phi_3^{R(-)}(\vec{k}') \left| X_{32}(E - \mathcal{H}_{22} + i\eta)^{-1} X_{21} \right| \phi_1^{R(+)}(\vec{k}) \rangle \tag{51}$$

The DCX amplitude in second-order distorted wave approximation is therefore given as

$$T_{DCX}^R = \sum_{i=1}^{4} W_i^B(\vec{k}',\vec{k};\ E) \tag{52}$$

We have now completed our formulation of the SCX and DCX reactions in the isobar-doorway model. Both SCX and DCX amplitudes are shown to depend on two basic quantities: $T_i^{(\pm)}(\vec{k}',\vec{k};\ E)$ of Eq. (29) and $U_1^R(\vec{k}',\vec{k};\ E)$ of Eq. (42), if we neglect the small nonresonant contributions from $U_1^{NR}(\vec{k}',\vec{k};\ E)$ (which

also appears in the nonresonant DCX amplitudes which have not been discussed here). These two quantities have clear dynamical meanings in the theory. In the next section, we shall use a phenomenological model for these quanitites, where they are related to the basic π-nucleon interactions.

III. A SIMPLE MODEL

In this section we shall show a model calculation of our basic quantities $T_i^{(\pm)}(\vec{k}',\vec{k};\ E)$ and $U_1^R(\vec{k}',\vec{k};\ E)$ with the same approximations used in Ref. 16. We first rewrite Eqs. (29) and (42) as

$$T_i^{(\pm)}(\vec{k}',\vec{k};\ E) = \sum_{n\alpha} \frac{\langle \phi_i^{(\pm)}(\vec{k}')|V_R|\Phi_{n\alpha}\rangle\langle\Phi_{n\alpha}|V_R|\phi_i^{(\pm)}(\vec{k})\rangle}{E - \tilde{E}_{n\alpha i} + \frac{i}{2}\tilde{\Gamma}_{n\alpha i}} \tag{53}$$

and

$$U_1^R(\vec{k}',\vec{k};\ E) = \sum_{n\alpha} \frac{\langle \phi_2^{(-)}(\vec{k}')|V_R|\Phi_{n\alpha}\rangle\langle\Phi_{n\alpha}|V_R|\phi_1^{(+)}(\vec{k})\rangle}{E - E_{n\alpha} + \frac{i}{2}\Gamma_{n\alpha}^{\downarrow}} \tag{54}$$

where we have introduced the following definitions:

(1) the resonance energy

$$\tilde{E}_{n\alpha i} = E_n + \varepsilon_\alpha + \Delta_{n\alpha i}^{\uparrow} + \Delta_{n\alpha}^{\downarrow} \tag{55}$$

(2) the modified isobar resonance energy

$$E_{n\alpha} = E_n + \varepsilon_\alpha + \Delta_{n\alpha}^{\downarrow} \tag{56}$$

(3) the quasi-total width

$$\tilde{\Gamma}_{n\alpha i} = \Gamma_{n\alpha i}^{\uparrow} + \Gamma_{n\alpha}^{\downarrow} \tag{57}$$

In the static model for the interaction V_R,[17] we may write (ignoring a phase factor)

$$\langle\phi_j^{(o)}(\vec{k}')|V_R|\phi_{n\alpha}\rangle\langle\phi_{n\alpha}|V_R|\phi_i^{(o)}(\vec{k})\rangle$$

$$\simeq \sqrt{\Gamma_{\pi N}^{\alpha}(k')\ \Gamma_{\pi N}^{\alpha}(k)}\ \ \eta_{ji}\ \frac{\hat{K}'\cdot\hat{K}}{2\pi}\ F_j^n(\vec{k}')\ F_i^n(\vec{k}) \qquad (58)$$

where $|\phi_i^{(o)}(\vec{k})\rangle$ is related to $|\phi_i^{(\pm)}(\vec{k})\rangle$ by a phase, K's refer to unit vectors of the pion momenta in the π-nucleon c.m. system. For the static model, we may use $\vec{K} = \vec{k}$. To simplify our discussions, we assume plane wave solutions for $|\phi_i^{(\pm)}(\vec{k})\rangle$ so that we may drop the designation of $(\pm)$ in our notations.[20] The factor η_{ji} depends on the isospin states of the transition $i \to j$. We have also defined the width

$$\Gamma_{\pi N}^{\alpha}(k) = 2\pi|\langle\vec{k}|V_R|\Delta_\alpha\rangle|^2 \qquad (59)$$

where $|\vec{k}\rangle$ is a state of the pion nuclear system, and $|\Delta_\alpha\rangle$ is the single particle state of Δ in the nucleus. The form factors are defined

$$F_i^n(\vec{k}) = \sum_{j=1}^{A}\langle\Phi_n(\vec{r}_1\ \ldots\ \vec{r}_{j-1}\ \vec{r}_{j+1}\ \ldots\ \vec{r}_A)|e^{i\vec{k}\cdot\vec{r}_j}|\Phi_i(\vec{r}_1\ \ldots\ \vec{r}_j\ \ldots\ \vec{r}_A)\rangle$$

$$= \int d\vec{r}\ e^{i\vec{k}\cdot\vec{r}}\ \rho_{n,i}(\vec{r}) \qquad (60)$$

$\rho_{ni}(\vec{r})$ is a product of a single particle wave function and a spectroscopic factor which depends upon the parentage of the state of the A nucleon system in terms of the state Φ_n of the (A-1) nucleon system. If we assume that the scattering is not far off-shell, i.e., $|\vec{k}| \approx |\vec{k}'|$, we may approximate

$$\Gamma_{\pi N}^{\alpha}(k') \approx \Gamma_{\pi N}^{\alpha}(k) \quad . \qquad (61)$$

since $\Gamma^{\alpha}_{\pi N}(k)$ is not as critically energy dependent as the energy denominator in Eqs. (53) and (54). We may rewrite Eqs. (53) and (54) as

$$T_i(\vec{k}',\vec{k};\ E) = \frac{\eta_{ii}}{2\pi} \sum_{n\alpha} \frac{\Gamma^{\alpha}_{\pi N}(k)\ \hat{K}' \cdot \hat{K}\ F^n_i(\vec{k}')\ F^n_i(\vec{k})}{E - \tilde{E}_{n\alpha i} + \frac{i}{2}\ \tilde{\Gamma}_{n\alpha i}} , \tag{62}$$

and

$$U^R_1(\vec{k}',\vec{k};\ E) = \frac{\eta_{21}}{2\pi} \sum_{n\alpha} \frac{\Gamma^{\alpha}_{\pi N}(k)\ \hat{K}' \cdot \hat{K}\ F^n_2(\vec{k}')\ F^n_1(\vec{k})}{E - E_{n\alpha} + \frac{i}{2}\ \Gamma^{\downarrow}_{n\alpha}} . \tag{63}$$

We now introduce the closure property of the isobar-doorway states and sum over $n\alpha$. We obtain[16]

$$T_i(\vec{k}',\vec{k};\ E) = \frac{\eta_{ii}}{2\pi}\ \frac{\Gamma_{\pi N}(E)\ \hat{K}' \cdot \hat{K}\ F_{ii}(\vec{k}',\vec{k})}{E - M(1236) - \delta E + \frac{i}{2}\ (\Gamma^i_{el} + \Gamma_{in})} \tag{64}$$

and

$$U^R_1(\vec{k}',\vec{k};\ E) = \frac{\eta_{21}}{2\pi}\ \frac{\Gamma_{\pi N}(E)\ \hat{K}' \cdot \hat{K}\ F_{21}(\vec{k}',\vec{k})}{E - M(1236) - \delta E^{\downarrow} + \frac{i}{2}\ \Gamma_{in}} \tag{65}$$

where we have approximated the average width $\langle \Gamma^{\alpha}_{\pi N} \rangle$ by the free Δ (1231) decay width $\Gamma_{\pi N}(E)$, and the average resonance energy $\langle \tilde{E}_{n\alpha i} \rangle$ by

$$\langle \tilde{E}_{n\alpha i} \rangle \approx M\ (1231) + \delta E \tag{66}$$

where δE is the total energy shift of the isobar energy in the nuclear medium.[16] We have also denoted the averaged quasi-total width as

$$\langle \tilde{\Gamma}_{n\alpha i} \rangle = \Gamma^i_{el} + \Gamma_{in} \tag{67}$$

where

$$\Gamma^{i}_{el} = \langle \Gamma^{\uparrow}_{n\alpha i} \rangle \qquad \text{and} \qquad \Gamma_{in} = \langle \Gamma^{\downarrow}_{n\alpha} \rangle \ . \tag{68}$$

The form factors are defined as

$$F_{ij}(\vec{k}',\vec{k}) = \sum_{n} F^{n}_{i}(\vec{k}') F^{n}_{j}(\vec{k}) \ . \tag{69}$$

We note that the difference between the total width Γ_{tot} and the quasi-total width is the decay width to charge-exchange channel. Since the charge-exchange width is very small compared to the total width, we may take

$$\Gamma^{i}_{el} + \Gamma_{in} \approx \Gamma_{tot} \tag{70}$$

In Eq. (65) we also have an average energy shift

$$\delta E^{\downarrow} = \langle \Delta^{\downarrow}_{n\alpha} \rangle \tag{71}$$

Since the energy shifts are probably small, the two energy shifts δE and $\delta E^{\downarrow}$ may be taken to be equal.

We have now reduced these basic quantities to their simplest forms. It is interesting to make the connection of charge exchange reactions to the elastic scattering in the isobar-doorway model. Compared to the results we have in Ref. 16, the $T_i(\vec{k}',\vec{k};\ E)$ is nothing but the off-shell extension of the <u>elastic</u> T-matrix in the isobar-doorway model (with Eq. (70)). We also find that $U^{R}_{1}(\vec{k}',\vec{k};\ E)$ is closely related to the modified π-nucleon t-matrix,[16]

$$\alpha(E)\ \tilde{t}_{33}(\vec{k}',\vec{k};\ E) \equiv \frac{\Gamma_{\pi N}(E)/2}{E - M(1236) - \delta E + i\Gamma_{in}/2}\ \hat{K}' \cdot \hat{K} \ , \tag{72}$$

by the off-diagonal form factor $F_{21}(\vec{k}',\vec{k})$ as

$$U_1^R(\vec{k}',\vec{k};\ E) = \frac{\eta_{21}}{2\pi}\,\alpha(E)\ \tilde{t}_{33}(\vec{k}',\vec{k};\ E)\ F_{21}(\vec{k}',\vec{k}) \quad . \tag{73}$$

In analogy to $U_1^R(\vec{k}',\vec{k};\ E)$, we may write

$$T_i(\vec{k}',\vec{k};\ E) = \frac{\eta_{ii}}{2\pi}\,\tilde{T}_{33}(\vec{k}',\vec{k};\ E)\ F_{ii}(\vec{k}',\vec{k}) \quad , \tag{74}$$

where

$$\tilde{T}_{33}(\vec{k}',\vec{k};\ E) = \frac{\Gamma_{\pi N}(E)/2}{E - M(1236) - \delta E + \frac{i}{2}\,(\Gamma_{el}^{i} + \Gamma_{in})}\ \hat{K}' \cdot \hat{K} \quad . \tag{75}$$

As we have shown in Ref. 16, the total width (i.e., $\approx \Gamma_{el}^{i} + \Gamma_{in}$ may be determined by a fit to the total cross section as a function of energy. By the same procedure, we may also estimate the inelastic width Γ_{in} by the ratio of the total reaction cross section $\sigma_r(E)$ to the total elastic cross section $\sigma_{el}(E)$. Therefore, all the necessary parameters in the model may be checked with the measurements of the total cross sections alone. The angular distributions of the SCX and DCX reaction cross section and of the elastic scattering may then be predicted within the same approximations.

The model is therefore shown to be capable of making direct and simple connections between the SCX and DCX reactions and the elastic scattering. Furthermore, the model is particularly useful in its separation of the resonant factors from the nonresonant parts. As examples, we shall explicitly show this property in the DWBA amplitudes. We first consider the nonresonant terms.

Since the treatment of the completely non-resonant interaction $U_1^{NR}(\vec{k}',\vec{k})$ of Eq. (37) is not in the model, we shall assume that it is known

from an appropriate optical-model calculation.[21] This term should have smooth energy dependence. The resonant terms of Eqs. (38) and (39) may be combined to the following form

$$U_2^{NR}(\vec{k}',\vec{k}) + U_3^{NR}(\vec{k}',\vec{k}) = \frac{1}{2\pi}|\tilde{T}_{33}(E)|[a(\vec{k}',\vec{k}) + b(\vec{k}',\vec{k})] \tag{76}$$

where we have defined

$$\tilde{T}_{33}(\vec{k}',\vec{k};\ E) \equiv |\tilde{T}_{33}(E)|\hat{K}' \cdot \hat{K} \tag{77}$$

$$a(\vec{k}',\vec{k}) = \eta_{22}\int d\vec{q}(E - E_q + i\eta)^{-1}(\hat{K}' \cdot \hat{q})\ F_{22}(\vec{k}',\vec{q})\ U_1^{NR}(\vec{q},\vec{k}) \tag{78}$$

$$b(\vec{k}',\vec{k}) = \eta_{11}\int d\vec{q}(E - E_q + i\eta)^{-1}(\hat{q} \cdot \hat{K})\ U_1^{NR}(\vec{k}',\vec{q})\ F_{11}(\vec{q}',\vec{k}) \tag{79}$$

We note that the energy dependence of Eq. (76) appears mainly in the factor $|\tilde{T}_{33}|$, since $a(\vec{k}',\vec{k})$ and $b(\vec{k}',\vec{k})$ are smooth functions of energy. The most resonating term in the non-resonant charge exchange reaction is then

$$U_4^{NR}(\vec{k}',\vec{k}) = \frac{\eta_{11}\eta_{22}}{4\pi^2}|\tilde{T}_{33}(E)|^2\ C(\vec{k}',\vec{k}) \tag{80}$$

where the smoothly energy-dependent factor $C(\vec{k}',\vec{k})$ is given as

$$C(\vec{k}',\vec{k}) = \iint d\vec{q}\ d\vec{q}'(E - E_q + i\eta)^{-1}(E - E_q' + i\eta)^{-1}(\hat{k}' \cdot \hat{q}')(\hat{k} \cdot \hat{q})$$

$$F_{22}(\vec{k}',\vec{q}')\ U_1^{NR}(\vec{q}',\vec{q})\ F_{11}(\vec{q},\vec{k}) \tag{81}$$

We next consider the resonant terms. The basic term is given in Eq. (73) with main energy dependence in $|\tilde{t}_{33}(E)|$ defined by

$$\alpha(E)\tilde{t}_{33}(\vec{K}',\vec{k};\ E) \equiv |\tilde{t}_{33}(E)|\ \hat{K}' \cdot \hat{K} \tag{82}$$

We may write

$$U_2^R(\vec{k}',\vec{k};\ E) + U_3^R(\vec{k}',\vec{k};\ E) = \frac{\eta_{21}}{4\pi^2}|\tilde{t}_{33}(E)||\tilde{T}_{33}(E)|[d(\vec{k}',\vec{k}) + e(\vec{k}',\vec{k})] \tag{83}$$

where

$$d(\vec{k}',\vec{k}) = \eta_{22}\int \frac{d\vec{q}(\hat{k}' \cdot \hat{q})(\hat{q} \cdot \hat{k})}{E - E_q + i\eta} F_{22}(\vec{k}',\vec{q})\ F_{21}(\vec{q},\vec{k}) \tag{84}$$

and

$$e(\vec{k}',\vec{k}) = \eta_{11}\int \frac{d\vec{q}(\hat{k}' \cdot \hat{q})(\hat{q} \cdot \hat{k})}{E - E_q + i\eta} F_{21}(\vec{k}',\vec{q})\ F_{11}(\vec{q},\vec{k}) \tag{85}$$

Finally we have

$$U_4^R(\vec{k}',\vec{k};\ E) = \frac{\eta_{22}\eta_{21}\eta_{11}}{8\pi^3}|\tilde{t}_{33}(E)||\tilde{T}_{33}(E)|^2\ f(\vec{k}',\vec{k}) \tag{86}$$

where

$$f(\vec{k}',\vec{k}) = \iint \frac{d\vec{q}\ d\vec{q}'(\hat{k}' \cdot \hat{q}')(\hat{q}' \cdot \hat{q})(\hat{q} \cdot \hat{k})}{(E - E_q + i\eta)(E - E_{q'} + i\eta)} F_{22}(\vec{k}',\vec{q}')\ F_{21}(\vec{q}',\vec{q})\ F_{11}(\vec{q} \cdot \vec{k}) \tag{87}$$

As noted before, the factors $d(\vec{k}',\vec{k})$, $e(\vec{k}',\vec{k})$ and $f(\vec{k}',\vec{k})$ have relatively smooth energy dependence, so that the strong energy dependence in all the

resonant terms is completely separated out in our final expressions. Furthermore, the energy dependence is in turn directly related to the energy dependence in the elastic scattering. We finally note that the main nuclear structure effects are contained in the form factors which are separated out from the main effects of the resonance energy dependence.

The same discussion may be extended to the DCX reactions, where the main energy dependence will also be contained in $|\tilde{t}_{33}(E)|$ and $|\tilde{T}_{33}(E)|$ factors. The extension is straightforward and will not be discussed here.

IV. CONCLUDING REMARKS

We have extended the isobar-doorway model for pion-nucleus scattering to the charge exchange reactions. We have shown that energy dependence of the SCX and DCX reaction amplitude in DWBA may be conveniently separated from the parts which contain nuclear structure information. The energy dependence depends on the pion optical potential, or the self-energy effects in the nuclear medium; it is therefore shown to be closely related to the elastic scattering amplitude.

Within the model, the SCX and DCX reactions may be treated on the same footing as the elastic scattering with common factors depending only on the energy and therefore may be consistently described by a simple parametrization from the energy dependence of the total cross sections. These factors may eventually be evaluated by a more detailed interaction model, such as a microscopic theory where the motion of the isobar is explicitly taken into account. The model we present here is general enough to allow variations in the detailed assumptions of the pion-nucleon interaction in the medium.

Finally, we would like to point out that the model has the distinctive feature of displaying explicitly the roles of the (3,3) resonance in the reaction dynamics, including the initial and final state interactions. The effect of the nonresonant background interactions is also properly retained in the formalism. The interplay of the resonant and the nonresonant interactions can be clearly exhibited in the model, as we have shown in our discussions.

FOOTNOTES AND REFERENCES

*Work performed under the auspices of the U. S. Atomic Energy Commission.

1. D. S. Koltun, Advances in Nucl. Phys. 3, 71 (1969), edited by Baranger and Vogt, Plenum Press (1969).
2. F. Becker and Yu. A. Batusov, Revista del Nuovo Cimento 1, 309 (1971).
3. R. G. Parsons, J. S. Trefil, and S. D. Drell, Phys. Rev. 138, B847 (1965).
4. F. Becker and Z. Marć, Nuovo Cimento 41B, 174 (1966).
5. F. Becker and C. Schmidt, Nucl. Phys. 18B, 607 (1970).
6. S. Barshay and G. E. Brown, Phys. Letters 16, 165 (1965).
7. S. B. Kaufman and C. O. Hower, Phys. Rev. 140, B1272 (1965).
8. M. A. Locci and P. Picchi, Nuovo Cimento 57A, 803 (1968).
9. K. Bjørnenak, J. Finjord, P. Ostland, and A. Reitan, Nucl. Phys. B20, 327 (1970).
10. W. B. Kaufmann, J. C. Jackson, and W. R. Gibbs, Phys. Rev. C 9, 1340 (1974).
11. M. Koren, Ph.D. Thesis, Massachusetts Institute of Technology (1969).
12. A. K. Kerman and R. K. Logan, in Nuclear Spectroscopy with Direct Reactions, ANL-6878, ed. F. E. Throw (1964), Vol. II, p. 236.
13. L. A. Charlton, J. M. Eisenberg, and W. B. Jones, Nucl. Phys. A171, 625 (1971).
14. E. Rost and G. W. Edwards, Phys. Letters 37B, 247 (1971).
15. G. A. Miller and J. E. Spencer, private communications. We thank Dr. G. Miller for discussions about their work.
16. L. S. Kisslinger and W. L. Wang, Phys. Rev. Letters 30, 1071 (1973); details to be published.
17. A. K. Kerman and L. S. Kisslinger, Phys. Rev. 180, 1483 (1969).

18. See, for example, M. G. Olson, L. Turner, and E. T. Osypowski, Phys. Rev. D 7, 3444 (1973). See also Ref. 6.

19. H. Feshbach, A. K. Kerman, and R. H. Lemmer, Ann. Phys. (New York) 41, 230 (1967).

20. We note that, if we keep only the resonant π-nucleon interactions, $\phi_i^{(\pm)}(\vec{k})$ will be plane waves, so this is a good approximation near the resonance energy.

21. An appropriate choice of the nonresonant optical potential for $|\phi^{(\pm)}>$ was discussed in Ref. 16. The nonresonant charge exchange interaction is contained in H_{21}. By definition, this nonresonant charge exchange interaction contains no effects of the isobar resonance and is rather weak, so a reliable estimate of $U_1^{NR}(\vec{k}',\vec{k})$ may be obtained in the usual multiple scattering formalism or in a first-order optical potential calculation where only the nonresonant π-nucleon interactions are considered.

FIGURE CAPTIONS

Fig. 1. Nonresonant charge exchange reactions. Diagram (a) is the (NR-NR-NR) term, where both initial and final pion states are nonresonant; diagrams (b) and (c) are the (NR-NR-R) and (R-NR-NR) terms where one of the pion state is resonating; diagram (d) is the (R-NR-R) term with both pion states resonating. Here we identify each term by (initial pion state - interaction — final pion state). The nonresonant pion states are shown as the dotted lines (-----) and the resonating pion states by wavy lines (∿∿∿∿).

Fig. 2. Resonant charge exchange through an isobar-doorway state. The order of the diagrams follows Fig. 1. (a) the (NR-R-NR) term; (b) the (NR-R-R) term; (c) the (R-R-NR) term, and (d) the (R-R-R) term.

Fig. 3. Double charge exchange (DCX) interaction through two successive isobar formations. In this diagram, all the pion states are nonresonant. For DCX interaction diagrams (Figs. 3 and 4), we denote the diagram by <initial pion state — intermediate pion state — final pion state>; Fig. 3 is therefore <NR-NR-NR>.

Fig. 4. Double-resonance DCX diagrams with nonresonant external pion states. Diagrams (a) and (b) are <NR-(NR,R)-NR> and <NR-(R,NR)-NR>, respectively, where the intermediate states contain one resonant component of the pion wave function, their values being given by Eqs. (45) and (46). Diagram (c) is the <NR-R-NR> term given by Eq. (47).

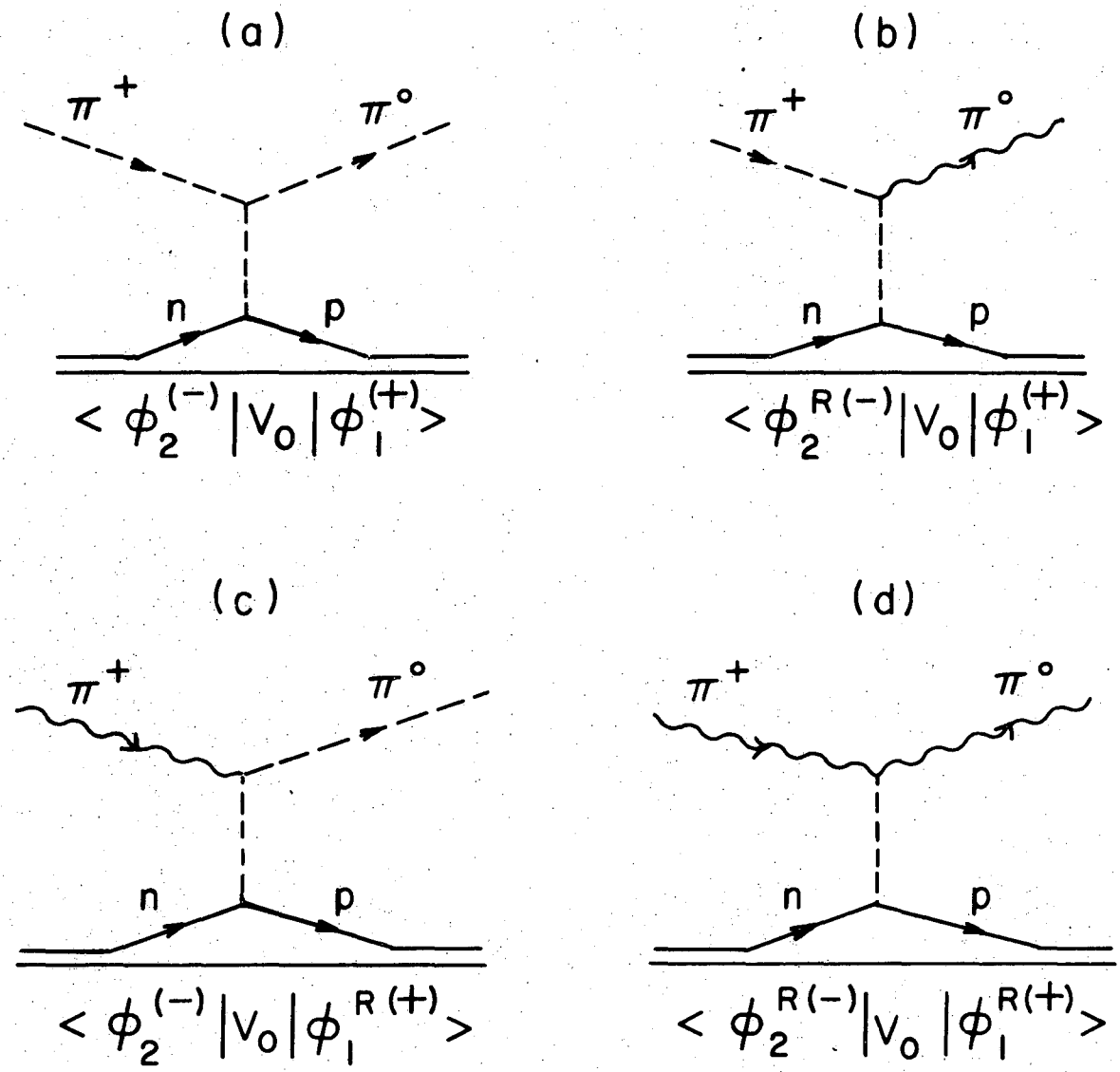
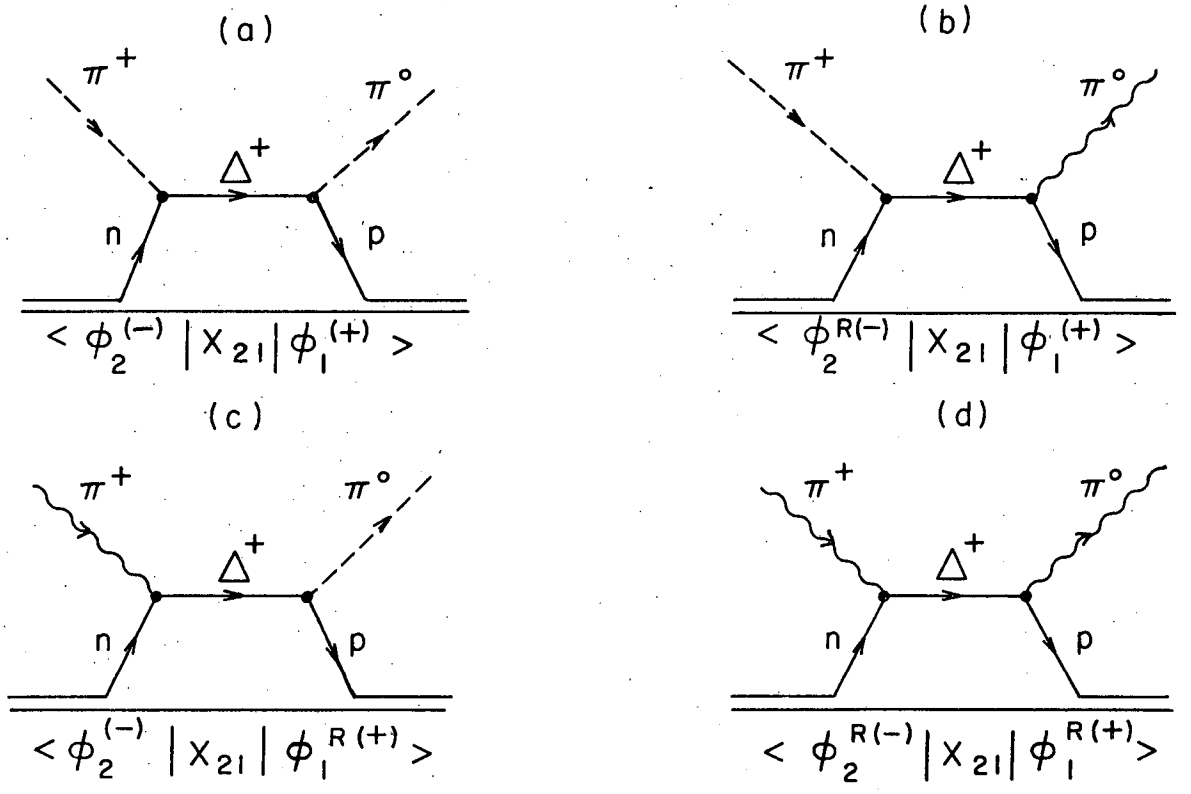
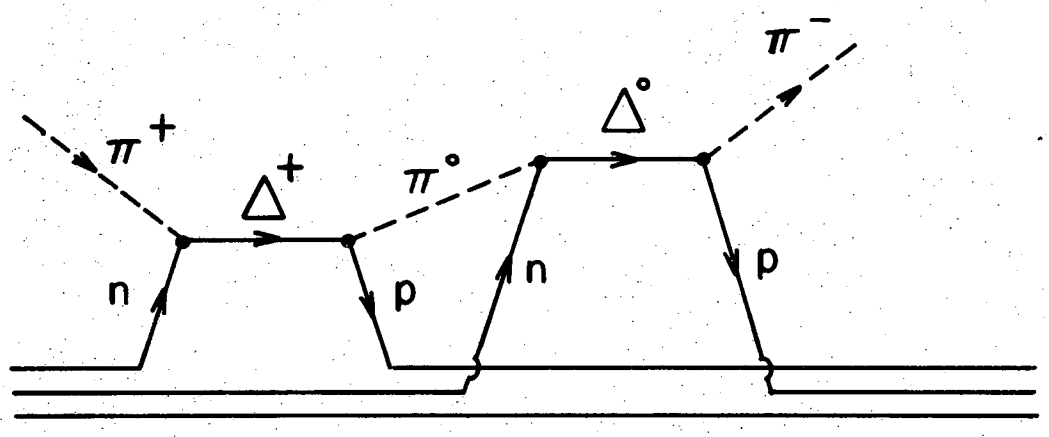

(a)
π+
π°
n
p
< φ2(−) | V0 | φ1(+) >
(b)
π+
π°
n
p
< φ2R(−) | V0 | φ1(+) >
(c)
π+
π°
n
p
< φ2(−) | V0 | φ1R(+) >
(d)
π+
π°
n
p
< φ2R(−) | V0 | φ1R(+) >
XBL746-3414

Fig. 1



XBL746-3415

Fig. 2

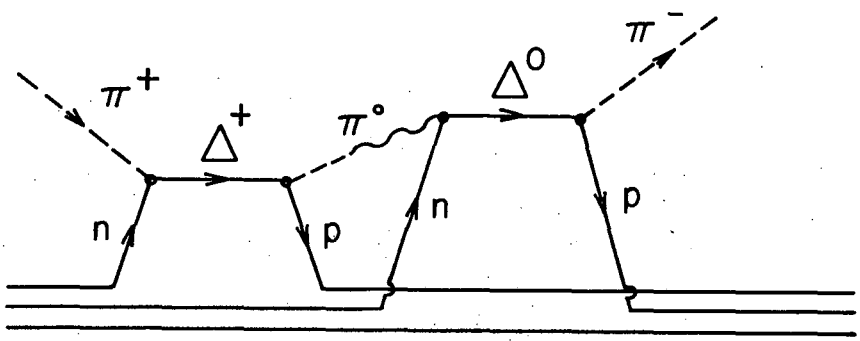


W_1 term

XBL746-3413

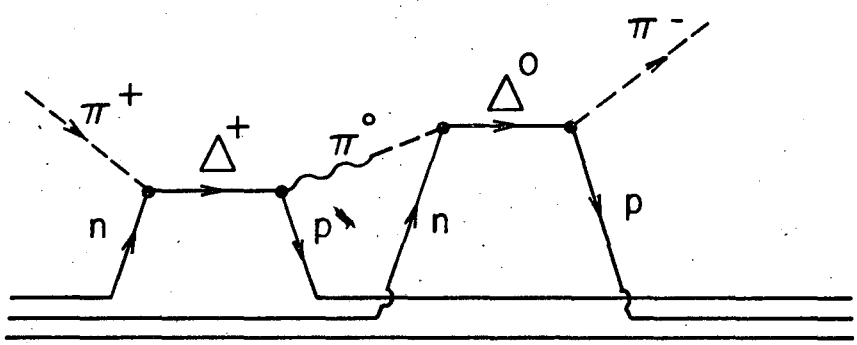
Fig. 3

(a)



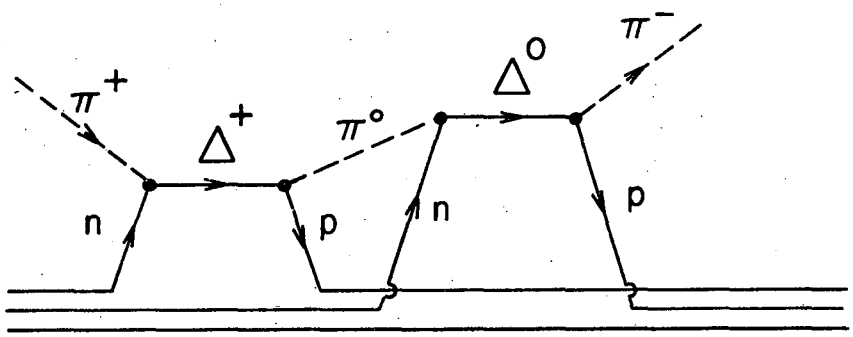
W_2 term

(b)



W_3 term

(c)



W_4 term

XBL746-3416

Fig. 4

LEGAL NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

TECHNICAL INFORMATION DIVISION
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720