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Author
Mandelstam, Stanley.

Publication Date
1969-09-02

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LAWRENCE RADIATION LABORATORY
UNIVERSITY of CALIFORNIA BERKELEY

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RELATIVISTIC QUARK MODEL BASED ON THE VENEZIANO REPRESENTATION.
II.   GENERAL TRAJECTORIES[*]

Stanley Mandelstam

Department of Physics and Lawrence Radiation Laboratory
University of California
Berkeley, California

September 2, 1969

ABSTRACT

The model previously proposed is extended to include multi-quark trajectories. Once any trajectories with more than a single quark and anti-quark are included, it is necessary to include trajectories where the number of quarks plus the number of anti-quarks, which we call the total quark number, is arbitrarily large. The necessary factorization properties of the multi-particle Veneziano amplitudes will hold provided the intercept of the leading trajectory is a polynomial function of the total quark number, and the degeneracy of the levels on all but the leading trajectory will increase with the order of the polynomial. It is possible to construct two different models depending on whether one allows non-planar duality diagrams. The model with non-planar diagrams resembles more closely the non-relativistic harmonic-oscillator quark model, and the non-planar duality diagrams must be associated with the non-planar Veneziano amplitudes discussed in a previous paper. One can introduce $SU(3)$ symmetry breaking by making the intercept depend on the number of strange and non-strange quarks separately, and one then obtains a modified Gell-Mann-Okubo mass formula.

I. INTRODUCTION

A relativistic quark model has been proposed and applied to meson trajectories by Mandelstam[1] and by Bardakci and Halpern.[2] In the present paper we wish to extend the model to other trajectories. We shall discuss the general properties of the multi-quark trajectores, as well as the symmetry properties of the three-quark states. The spin and unitary-spin degrees of freedom will only be mentioned in so far as they are connected with the symmetry properties. We hope to treat the more detailed spin properties of the baryon trajectories in a subsequent paper.

Within the framework of the model presented in I, it appeared that one need not introduce resonances consisting of more than two quarks. Once one requires the presence of three-quark states, however, it is necessary to introduce trajectories where the number of quarks and anti-quarks is arbitrarily large. We shall examine such trajectories in Sec. 2. For baryon anti-baryon scattering, it has already been pointed out by Rosner[3] that exotic resonances with two quarks and two anti-quarks must occur in the intermediate states, and one can apply similar reasoning to more complicated reactions. Following Delbourgo and Salam,[4] we shall refer to the number of quarks plus the number of anti-quarks as the total quark number, and resonances with an arbitrarily large total quark number must be present. Our model in its present form does not appear to require trajectories with a net quark number greater than three, though it can certainly accommodate such trajectories. Until we know how to extend our

model beyond the narrow-resonance approximation we cannot answer the
question whether trajectories of baryon number greater than one occur
in this approximation, or only in higher orders.

If our model is to be at all acceptable on experimental and
theoretical grounds, it is necessary that the mass of the lightest
particle with a given total quark number be an increasing function of
the quark number. As long as the resonances with a total quark number
of four or greater are sufficiently heavy, they will decay rapidly into
resonances with smaller total quark numbers, and they will not appear
experimentally as narrow resonances. We therefore have to inquire
whether the model allows different trajectories to have different
intercepts. The question to be investigated concerns the factorization
properties of the multi-particle Veneziano amplitude,[5,6] which lie at
the basis of the relativistic quark model. The original treatment of
such factorization properties assumed that all trajectories had the
same intercept.

We shall show in Sec. 2 that the residues at the poles of
the multi-particle Veneziano amplitude can still be expressed as a
finite sum of factored terms, provided that the intercept of the
leading trajectory is a polynomial function of the total quark
number. The degeneracy of the resonances will depend on the order of
the polynomial. If the intercept is a linear function of the total
quark number, the degeneracy of the spectrum of ordinary mesons,
i.e., of mesons with a total quark number of two, will be the same
as in a model without nucleons and exotic resonances. The degeneracy
of the spectrum of other resonances will be an increasing function

of the total quark number, as is to be expected on intuitive grounds.
If the intercept is a higher polynomial function of the total quark
number, the degeneracy of all ordinary or exotic resonances on non-
leading trajectories will be increased, and even the degeneracy of the
resonances on the first subsidiary trajectory will increase indefinitely
with the order of the polynomial.

In order that the spectrum of resonances be not too complicated,
we therefore have to postulate that the intercept of the leading
trajectory be a polynomial of low degree in the total quark number. We
might be tempted to assume that the intercept was a linear function
of the total quark number. The masses of highly exotic resonances
would then be proportional to the square root of the total quark
number, however, and those of sufficiently high charge and hypercharge
would be stable. It is not absolutely prohibited that such a model
might be the appropriate narrow-resonances approximation to nature,
since the range of validity of the model will probably not extend
to resonances of very high mass. A model without the infinite system
of stable exotic mesons is obviously to be preferred, and the simplest
such model is one where the intercept of the leading trajectory is a
quadratic function of the total quark number.

It is possible to construct two different models with all the
features mentioned above, depending upon whether one allows non-planar
as well as planar duality diagrams. The models with only planar
diagrams and with all diagrams will be discussed in Secs. 3 and 4
respectively. According to the general principles of the relativistic

quark model, each duality diagram is to be associated with the

topologically similar multi-particle Veneziano diagram. The non-

planar duality diagrams will therefore be associated with the non-

planar Veneziano amplitudes which we have discussed in a previous paper.[7]

Of the two models, that containing non-planar as well as planar

diagrams has the closer resemblance to the non-relativistic harmonic-

oscillator quark model. All baryon states on the leading trajectory of

such a model are symmetric in the three quarks, and we shall hereafter

refer to it as the symmetric quark model. The planar-diagram model,

on the other hand, posesses some non-symmetric states as well. The

presence of one such state, the $\ell = 0$ $\underline{70}$ has already been pointed

out by Mandula, Rebbi, Slansky, Weyers and Zweig[8]. While the symmetric

quark model is preferable in this respect, the planar-diagram model

is not in definite contradiction with experiment, since the $\ell = 0$ $\underline{70}$

is only weakly coupled and may simply appear as a contribution to the

continuum. We shall discuss such an interpretation in Sec. 3.

The planar-diagram model is simpler than the symmetric quark

model in regard to the spectrum of resonances on the leading trajectory.

As has been shown in Ref. 5 and 6, the leading trajectory in a planar

Veneziano amplitude has no "orbital" degeneracy, though there will

of course be a degeneracy associated with the spin and unitary-spin

degrees of freedom. The extra degeneracy associated with the exotic

resonances affects only the subsidiary trajectories, and no trajectory

is infinitely degenerate. In non-planar Veneziano amplitudes, the

resonances on the leading baryon trajectory are degenerate, and the

degeneracy increases with the angular momentum. The spectrum of

resonances on the leading baryon trajectory of our symmetric quark
model is identical to that of the non-relativistic quark model,
where the degeneracy also increases with the angular momentum.[9]

The meson-baryon coupling constants in the planar-diagram model
are two-thirds as large as in the symmetric quark model. A convenient
comparison of the $B\bar{B}M$ and MMM coupling constants may be made on the
basis of the coupling with the neutral vector mesons. We showed in I
that it was possible to adjust a single parameter in such a way that
the coupling constant of the vector mesons to the other mesons was
universal, and we can treat the $B\bar{B}M$ vertex in a similar manner. On
comparing the $B\bar{B}M$ and MMM coupling constants, we then find that the
symmetric quark model gives results in accord with vector meson
universality, while the couplings of the vector mesons to the baryons
in the planar-diagram model are too small by a factor $\frac{2}{3}$.

The models constructed in I and in Secs. 2,3 and 4 of the
present paper possess exact $SU(3)$ symmetry. A convenient method
of introducing symmetry breaking is to assume that the intercept of
the leading trajectory is a polynomial function, not only of the total
quark number, but of the number of strange and non-strange quarks
separately. The resulting formula for the mass splitting resembles
the Gell-Mann-Okubo formula without the term responsible for the $\Sigma\Lambda$
mass difference. If we assume that the intercept is a quadratic
function of the quark models, our formula differs in detail from the
Gell-Mann-Okubo formula. It contains an extra parameter, but we are
able to fit the baryon and meson multiplets with a single formula,

whereas the usual Gell-Mann-Okubo formula involves the masses of the
baryon multiplets and the squares of the masses of the pseudo-scalar
octet. It is also worth pointing out that our present derivation does
not depend on the smallness of the SU(3) symmetry breaking.

II.   EXOTIC RESONANCES

The general principles of the model are the same as those

applied to meson trajectories in I. For meson-baryon scattering we

shall have the processes represented by Fig. 1(a) and its crossed

diagram and by Fig. 1(b), possibly together with further processes

represented by non-planar diagrams which we shall discuss in Sec. 4 .

In momentum space the diagrams correspond to multi-particle Veneziano

amplitudes[10] with the external particles in the same cyclic order as

in the diagrams. From such multi-particle amplitudes one can factor out

meson-baryon amplitudes with external particles of any spin. The lines

in Fig. 1 also represent delta functions in spin and unitary spin. The

coupling therefore has the form originally proposed by Capps[11] and, in

their internal degrees of freedom, the diagrams are the duality

diagrams used by Harari,[12] by Rosner,[13] by Matsuoka, Minomiya and

Sawada,[14] and, in a rather different form, by Neville.[15]

The diagram for baryon anti-baryon scattering is shown in

Fig. 1(c); there will be a similar diagram with the s- and t-

channels interchanged. It will be noticed immediately that the

resonances in the t- channel are exotic mesons consisting of two

quarks and two anti-quarks. Rosner[3] was the first to draw attention

to such exotic mesons, and their presence is a necessary feature of

our model. By viewing Fig. 1(c) from the s- channel, we can see that

it must be present. The intermediate state is then an ordinary meson,

and the B$\overline{\text{B}}$M coupling is required for consistency with the meson-baryon

amplitude.

One can employ similar reasoning to prove the existence of exotic baryons consisting of four quarks and one anti-quark. A diagram for the process $B\bar{B}B \rightarrow B\bar{B}B$ is shown in Fig. 1(d). Such a diagram must be present in order to represent the sequence of exchanges shown in Fig. 2. The resonances in the t-channel of Fig. 1(d) are the exotic baryons under discussion. It is evident that exotic mesons and baryons with an arbitrarily large number of quarks and anti-quarks are present in the model. This type of reasoning leaves open the question of the existence of resonances with a net quark number greater than three.

We have pointed out that a model with an infinity of meson and baryon resonances would be completely unacceptable unless the masses of the resonances increase with the total quark number. We now wish to investigate the factorization properties of our amplitude when the intercepts of the trajectories depend on the total quark number, and to prove the results already quoted in the introduction.

We begin by restricting ourselves to planar amplitudes. Figure 3 represented a general diagram for such an amplitude, and we are interested in the factorization properties when the diagram is divided by the dashed line. As in Ref. 5, we write the Veneziano integrand in the form:

$$I = I_1 I_2 \prod_{i=1}^{j} \prod_{k=j+1}^{N} (1-w\, y_i y_k)^{-S_{ik} - S_{i+1,\,k-1} + S_{i,\,k-1} + S_{i+1,k}},$$

$$(2.1)$$

where

$$w = u_{ij} \tag{2.2a}$$

$$y_i = u_{ij} \cdots u_{1,\, j-1} \qquad\quad i \leq 2 \leq j-1 \tag{2.2b}$$

$$y_j = 1, \; y_1 = 0 \tag{2.2c}$$

$$y_k = u_{n,\, j+2} \cdots u_{N,\, k} \quad j+2 \leq k \leq N-1 \tag{2.2d}$$

$$y_{j+1} = 1, \; y_N = 0 \tag{2.2e}$$

$$S_{ik} = \alpha_{ik}(s_{ik}) \quad \text{if} \qquad k \geq i+1 \tag{2.2f}$$

$$S_{jj} = S_{j+1,\, j+1} = 0 \tag{2.2g}$$

$$S_{j+1,\, j} = -1 \quad . \tag{2.2h}$$

We have used the notation of Chan and Tsou[10] for the u's. The subscripts i k refer to the channel with particles i to k in clockwise order round the diagram. I_1 and I_2 are the Veneziano integrands associated with the two halves of the diagram.

In units for which the slope of the trajectory is unity, we may write:

$$S_{ik} = s_{ik} + b_{ik}, \;\; k \geq i+1 \tag{2.3}$$

where b_{ik} is the intercept of the leading trajectory in the channel ik. We supplement (2.3) with the further definitions:

$$b_{j,j} = b_{j+1,\, j+1} = -\mu^2 \tag{2.4a}$$

$$b_{j+1} = 1 \quad . \tag{2.4b}$$

The quanity μ^2 in (2.4a) is the mass of the quark, so that $b_{j,j}$ and $b_{j+1,\;j+1}$ are defined as the intercept which a trajectory would have it the mass of its lowest member were equal to the quark mass. We may then write (2.4a) as:

$$I = I_1 I_2 \prod_{i=1}^{j} \prod_{k=j+1}^{N} (1-w\, y_i y_k)^{-2p_i\, p_k + \epsilon_{ik}}, \qquad (2.5a)$$

where

$$\epsilon_{ik} = -b_{ik} -b_{i+1,\, k-1} + b_{i,\, k-1} + b_{i+1,k} . \qquad (2.5b)$$

If all the intercepts are the same, so that all the quantities b_{ik} are zero except $b_{j+1,j}$, it follows from (2.5b) that all ϵ's are zero except $\epsilon_{j,j+1}$. The expression (2.5a) thus has the factorization properties described in Refs. 5 and 6. If the ϵ's are not zero the factorization properties become more complicated and, unless restrictions are imposed on the b's, the residue at a pole cannot be written as a finite sum of factored terms at all.

We now assume that the intercept b_{ik} is a polynomial function of the total quark number and, to begin, we shall suppose that it depends linearly on the quark number. Most of the ϵ's are then equal to zero. Let us examine the channel ℓp, for instance (Fig. 3). If we denote the total quark number by ν, we may write

$$v_{\ell p} = 3 \quad v_{\ell+1,\, p-1} = 3, \quad v_{\ell,\, p-1} = 4, \quad v_{\ell+1,\, p} = 2 \; .$$

$$(2.6)$$

As long as b_{ik} depends linearly on v_{ik}, it follows from (2.5b) and (2.6) that $\epsilon_{\ell p} = 0$. We may treat the majority of the variables ϵ_{ik} in a similar way. The points i and k will be on the right and the left of the diagram respectively, and let us imagine a dotted line drawn just above these points. The quantity $v_{ik} - v_{i+1,\, k}$ will be equal to ± 1 according to whether the quark line drawn to the point i crosses this dotted line or not. Exactly the same rule serves to determine the value of $v_{i,k-1} - v_{i+1,\, k-1}$, so that ϵ_{ik}, given by (2.5b), is zero.

The only ϵ's for which this reasoning falls are the quantities such as ϵ_{mq} (Fig. 3), where the points m and q are joined by a quark line. In that case

$$v_{mq} = 0, \quad v_{m+1,\, q-1} = 0 \quad v_{m,\, q-1} = 1, \quad v_{m+1,\, q} = 1 \; ,$$

$$(2.7)$$

and ϵ_{mq} is not zero. Thus, in Fig. 3, only two of the ϵ's are non-zero, $\epsilon_{j,\, j+1}$ and ϵ_{mq}. These two ϵ's correspond to the two quark lines which cross the diagram from left to right. No ϵ is associated with the line 1N, since the factor $(1-wy_1 y_N)^{-2p_1 p_N + \epsilon_{1N}}$ is absent by virtue of (2.2c) and (2.2e). The quantity $\epsilon_{j,\, j+1}$ is non-zero even if the intercept of all trajectories is the same, but the other

non-zero ϵ's such as ϵ_{mq} are new features of our present model.
The number of non-zero ϵ's is equal to the number of quark lines
crossing the diagram besides the top line or, in other words, to the
total quark number of the channel minus one.

We observed in Ref. 5 that the term $\epsilon_{j,j+1}$ in the
exponent increased the degeneracy of the resonances, and any other
non-zero ϵ will have a similar effect. As long as the number of
non-zero ϵ's is finite and independent of the number of external
lines, we may easily repeat the reasoning of Ref. 5 and 6 to show that
the residue at each pole is equal to the sum of a finite number of
factorizable terms. The resonances on the leading trajectory will be
non-degenerate, but the degeneracy of all other resonances will
increase with the number of non-zero ϵ's. In ordinary-meson channels,
where the total quark number is two, the only non-zero ϵ is $\epsilon_{j,j+1}$,
so that the degeneracy of the resonances is exactly the same as in a
model where all leading trajectories have the same intercept. The
degeneracy of the resonances in other channels will be an increasing
function of the total quark number.

We turn next to the case where the intercept b_{ik} is a
quadratic function of the total quark number.

Thus

$$-b_{ik} = (\alpha \, v_{ik})^2 + \beta \, v_{ik} + \gamma \; . \qquad\qquad (2.8)$$

For later reference we may generalize our formula to systems where we
have several different quarks, e.g. strange and non-strange quarks in
a model with broken $SU(3)$. Equation (2.8) then becomes:

$$-b_{ik} = \left(\sum_r \alpha_r \nu_{r,ik} \right)^2 + \sum_r \beta_r \nu_{r,ik} + \gamma \qquad (2.9)$$

where $\nu_{r,ik}$ is the number of quarks of type r in the channel ik.[16]

For simplicity we shall limit our investigation to the spectrum of ordinary mesons. The only lines which pass from the left to the right of Fig. 3 are then the top and the bottom lines. The channels ik may be divided into four classes as shown in Fig. 4(a)-(d); the number of vertical quarks lines in arbitrary. In the first case (Fig. 4(a):

$$\epsilon_{ik} = \left(\sum_r \alpha_r \nu_{r,ik} \right)^2 + \sum_r \beta_r \nu_{r,ik} + \gamma + \left(\sum_r \alpha_r \nu_{r,ik} - \alpha_i - \alpha_k \right)^2$$

$$+ \sum_r \beta_r \nu_{r,ik} - \beta_i - \beta_k + \gamma - \left(\sum_r \alpha_r \nu_{r,ik} - \alpha_i \right)^2$$

$$- \sum_r \beta_r \nu_{r,ik} + \beta_i - \gamma - \left(\sum_r \alpha_r \nu_{r,ik} - \alpha_k \right)^2 - \sum_r \beta_r \nu_{r,ik} + \beta_k - \gamma,$$

i.e.,

$$\epsilon_{ik} = 2 \, \alpha_i \alpha_k \; . \qquad\qquad (2.10)$$

The parameters α_i, α_k, β_i, β_k represent α_r and B_r for the quarks in the external lines i and k. With the configuration shown in Fig. 4(b), one again obtains the result $\epsilon_{ik} = 2\,\alpha_i \alpha_k$. With Figs. 4(c) and 4(d), $\epsilon_{ik} = -2\,\alpha_i \alpha_k$. We may therefore write the general equation

$$\epsilon_{ik} = 2\,\epsilon_i \alpha_i\,\epsilon_k \alpha_k \quad \text{unless} \quad i = j \text{ and } k = j + 1 \quad , \qquad (2.11)$$

where ϵ_i is equal to $+1$ if the quark leading from the external line i goes towards the top of the diagram, and equal to -1 if the quark does towards the bottom of the diagram.

As in all previous examples, the value of ϵ_{ik} when $i = j$ and $k = j + 1$ will not be given by the equation valid for other values of i and k.

If we substitute (2.11) in (2.5a), we obtain the equation

$$I = I_1 I_2 \prod_{i=1}^{j} \prod_{k=j+1}^{N} (1 - w\, y_i y_k)^{-2p_i p_k + 2\epsilon_i \alpha_i \epsilon_k \alpha_k} (1-w)^{\epsilon'} \quad ,$$

$$(2.12a)$$

where

$$\epsilon' = \epsilon_{j,j+1} - 2\epsilon_j \alpha_j\,\epsilon_{j+1}\alpha_{j+1} \quad . \qquad (2.12b)$$

We may rewrite (2.12a) as the exponential of a logarithm so that it becomes:

$$I = I_1 I_2 \exp \left\{ 2 \sum_{r=1}^{\infty} \frac{w^r}{r} \left[\left(\sum_{i=1}^{j} p_i y_i^{\,r} \right) \left(\sum_{k=j+1}^{n} p_k y_k^{\,r} \right) \right. \right.$$

$$\left. \left. - \left(\sum_{i=1}^{j} \epsilon_i \alpha_i y_i^{\,r} \right) \left(\sum_{k=j+1}^{n} \epsilon_k \alpha_k y_k^{\,r} \right) - \epsilon' \right] \right\} \,. \qquad (2.13)$$

On expanding the exponential in (2.13), we notice that the coefficient

of w^r is indeed equal to the sum of a finite number of factored terms.

As the second term within the square bracket does not involve the scalar

product $p_i\, p_j$, it does not contribute to the leading trajectory,

which remains non-degenerate. The term in question does contribute

to the subsidiary trajectories, so that the degeneracy of the spectrum

is greater than in a model where all trajectories have the same intercept.

One may treat more complicated cases in the same way. If the

intercept is a polynomial function of the total quark number, the

degeneracy of a particular resonance will increase with the order of

the polynomial and with the total quark number of the resonance. All

resonances are finitely degenerate, but the degeneracy of even the first

subsidiary ordinary-meson trajectory increases indefinitely with the

order of the polynomial. One would therefore expect the intercept to

be a polynomial of low degree in the total quark number and, as we

explained in the introduction, the optimal choice is a quadratic

function if we demand that there be no infinite family of highly exotic

stable particles.

We may easily generalize our results to the two-quark channels of a non-planar amplitude. It has been shown in Ref. 7 that the factorization properties in such channels are the same as those of planar amplitudes. We had assumed that all trajectories had the same intercept, but we can easily prove a similar theorem in a model where the intercept depends upon the total quark number. All results obtained in this section for the factorization properties of two-quark-channel are therefore true in models with planar and non-planar diagrams.

If the intercept of the leading trajectory is a quadratic or higher polynomial in the total quark number, we cannot predict the masses of the exotic mesons. The masses of the ordinary mesons and baryons only give us two points with which to determine the coefficients of the polynomial. If the quadratic term were absent we would obtain a mass of 1.6 - 1.8 BeV for the exotic mesons with a total quark number of four. The quadratic term will increase this value, since it must be negative in order that the mass of the highly exotic resonances be an increasing function of the total quark number.

With a sufficiently large quadratic term, the exotic mesons will have a considerable Q-value for S-wave decay into ordinary mesons. They may therefore be too broad to appear experimentally as resonances. In our present model there is no coupling between one exotic meson and two ordinary mesons, but the linear-trajectory dynamical scheme is only to be regarded as a weak-coupling approximation to nature, and the coupling in question will occur when we improve on the approximation.

III. THE PLANAR-DIAGRAM MODEL

In this and the following section we are interested in the
symmetry properties of multi-quark states and, in particular, of
three-quark states. One may construct two models with different
symmetry properties. In the present section, we shall examine a
model with only planar duality diagrams.

The scattering of ordinary mesons and baryons will involve
the duality diagrams of Figs. 1(a) and (b), together with the same two
diagrams drawn upside down. If s and u are the meson-baryon channels,
t the meson-meson channel, the st and tu terms will correspond to
Fig. 1(a), the su term to Fig. 1(b). The even orbital-angular-
momentum states in the s-channel will be given by the sum of Figs. 1(a)
and (b), the odd orbital-angular-momentum states by their difference.

We observe that Fig. 1(b) may be obtained from Fig. 1(a)
by interchanging the top and bottom quarks on the right-hand side,
followed by twisting the entire right-hand side through 180°. The
last operation does not affect the meaning of the diagram in any way,
so that the two diagrams are related to one another by interchanging a
pair of quarks in the final state. It follows that the purely
anti-symmetrical state and the purely symmetrical state will be
absent from the even and odd orbital-angular-momentum trajectories
respectively. The even orbital-angular-momentum trajectories will
possess symmetrical multiplets and multiplets of mixed symmetry or,
in SU(6), the $\underset{\sim}{56}$ and $\underset{\sim}{70}$ representation.[17] Odd angular-
momentum trajectories will posses multiplets of mixed symmetry and

anti-symmetrical multiplets or in SU(6), the $\underset{\sim}{70}$ and $\underset{\sim}{20}$

representations. The $\underset{\sim}{20}$ will of course not contribute to meson-baryon

scattering. These results have been obtained by Mandula, Rebbi,

Slansky, Weyers and Zewig.[8]

A more detailed analysis shows that the amplitudes represented

by Figs. 1(a) and (b) involve the $\underset{\sim}{56}$ and the $\underset{\sim}{70}$ in the following

proportions:

$$\text{Fig. 1(a):} \qquad 15 \cdot \underset{\sim}{56} + 16 \cdot \underset{\sim}{70} \qquad\qquad (3.1\text{a})$$
$$\text{Fig. 1(b):} \qquad 15 \cdot \underset{\sim}{56} - 8 \cdot \underset{\sim}{70} \qquad\qquad (3.1\text{b})$$

We confirm that the amplitude for states of odd angular momenta, which

is obtained by subtracting (3.1b) from (3.1a), is a pure $\underset{\sim}{70}$. States

of even angular momenta involve the $\underset{\sim}{56}$ and the $\underset{\sim}{70}$ in the ratio 15/4.

The existence of the $\underset{\sim}{70}$ multiplet in trajectories of even

orbital angular momentum represents a difference between the planar-

diagram model and the non-relativistic symmetric quark model. The

spectrum of our present model is not restricted to states which are

symmetric in the spin, unitary-spin and orbital degrees of freedom of

the quarks. No $\underset{\sim}{70}$ multiplets with even angular momentum have been

observed in pion-nucleon phase shift analyses. Before we dismiss the

planar-diagram model out of hand, however, we should investigate possible

changes which may occur when we improve on the narrow-resonance

approximation.

The linear Regge trajectories will acquire a curvature as the

coupling is turned on. If the forces are attractive, the curves of

Re α against s will move upward from their positions in the Born

approximation, just as in potential theory. Since the weak-coupling

linear trajectories are inclined at 45° to the horizontal, this upward

movement will be accompanied by a movement to the left, and the Q-value

for a resonance with a given angular momentum will be decreased. The

most strongly bound particles or resonances will therefore lie in the

channels with the strongest attractive forces. We may go further and

make the interpretation that the resonances on a trajectory which is

not moved a substantial distance to the left by the coupling will be so

wide that they appear experimentally as part of the continuum. The

dynamical scheme will now have the property, expected in a bootstrap

model, that particles or narrow resonances exist only in those channels

where the attractive forces are sufficiently strong. We actually used

such an interpretation in I, where we assumed that the V' and Π' did

not correspond to particles or narrow resonances, and that no narrow

resonances exsisted on repulsive trajectories.

We have seen that the attractive forces in the $\underset{\sim}{70}$ - even - ℓ

channels are indeed much weaker than in the $\underset{\sim}{56}$ - even - ℓ channels;

the squares of the coupling constants are in the ratio 4/15. We may

therefore assume that the forces in the $\underset{\sim}{70}$ - even - ℓ channels are

not sufficient to produce narrow resonances. The S-wave states in

such channels would correspond to the 11, 13 and 31 states of the

pion-nucleon system. The latter two have not been observed at an

energy below 1.8 BeV. The prominent 11 Roper resonance is usually

assigned to the second trajectory, and we are not proposing to change

that assignment. If the resonances in question had approximately the

same energy as the $\ell=1$ resonances they could easily escape detection. Their coupling to the πN system would be about one-third as great as that of the $\ell=1$ resonances. On the other hand, if their partial width for decay into the $\bar\pi\pi N$ mode is estimated from that of the Roper resonance with the appropriate phase-space correction, it is found that their total width would be roughly comparable to that of the $\ell=1$ resonances.

A second difference between the planar-diagram model and the non-relativistic quark model lies in the nature of the spectrum of resonances with higher angular momentum. In our present model, all resonances on the leading trajectory are non-degenerate in their orbital degrees of freedom. The degeneracy of resonances on non-leading trajectories depends on the total quark number, but no trajectory is infinitely degenerate. In the non-relativistic harmonic-oscillator quark model the degeneracy of the resonances will again depend on the total quark number, but now the resonances on the leading trajectory will be degenerate if the total quark number is greater than two; the degeneracy will be roughly proportional to the $(\nu-2)$th power of the angular momentum.

From the duality diagrams of Fig. 1 one may immediately construct diagrams for the vertices. Thus, from Figs. 1(a) and 1(b), one can infer that the $B\bar{B}M$ vertex is given by the sum of Fig. 5(a) and 5(b). From Figs. 1(c) one infers that the vertex which couples an exotic meson to a baryon anti-baryon pair is given by the sum of Figs. 5(c) and 5(d). In general, by referring to Fig. 1(d) and

more complicated diagrams, one may conclude that any exotic baryon or meson is represented by a diagram such as Fig. 6, where the top two quarks have their arrows pointing in the same direction, following which all arrows alternate until the bottom two quarks again have their arrows pointing in the same direction. The vertex between any three particles can now be drawn as in Figs. 5(a) - (d), with no lines crossing. Each vertex will consist of the sum of two terms with different cyclic ordering of the three particles. Note that it is impossible to draw a vertex such as Fig. 5(e); at least one quark line must pass between any pair of particles.

For processes where the number of baryons plus anti-baryons exceeds four, half of the diagrams will occur with minus sign owing to the fermi statistics of the baryons. A particular diagram can be chosen and given a plus sign; any diagram obtained from the selected diagram by interchanging an odd number of baryon pairs or anti-baryon pairs is then given a minus sign.

One may construct more complicated models which possess the exotic mesons and baryons just discussed, together with exotic mesons and baryons where more than two adjacent quark lines have their arrows in the same direction. For instance, one can have a model where states of baryon number two exists in the narrow-resonance approximation. The method of constructing the diagrams is straightforward.

An important feature of the planar-diagram model is that it does not possess vector-meson universality. The ratio between the different MMV vertices and between the different $\overline{BB}V$ vertices is in

accord with vector-meson universality, provided one fixes certain

mixing parameters as has been explained in I. However, the ratio

between the $B\bar{B}V$ vertex and the MMV vertex is two thirds of that

predicted by vector-meson universality, since the meson in Figs. 5(a)

and (b) cannot interact with the middle quark of the baryon.

In summary, we may mention the following three features of

the planar-diagram model

i)   The baryon trajectories of even orbital angular momentum

possess a $\underset{\sim}{70}$ multiplet as well as a $\underset{\sim}{56}$.

ii)   The resonances on the leading trajectory have no orbital

degeneracy, as opposed to the states of the harmonic-oscillator quark

model.

iii) The model does not possess vector-meson universality, since

the meson can only interact with the two outer quarks of a multi-quark

particle.

Though point i) may be regarded as a drawback of the model, we have

seen that it does not necessarily imply that the model should be rejected.

With regard to point (ii), the degeneracy of the higher resonances on

the leading trajectory has thus far received no experimental support,

and we may therefore prefer our present model to the harmonic-oscillator

model on the grounds of simplicity of the spectrum   Point iii), like

point i), may be regarded as a drawback of the model, since vector-

meson universality is an appealing feature which may possibly be

helpful in constructing a representation of current algebra. However,

since our model in its present form cannot make exact quantitative

predictions of coupling constants, we should not regard the violation

of vector-meson universality as strong experimental evidence against

the model.

By including non-planar as well as planar diagrams, we can

construct a model which possess only symmetrical three-quark states on

the leading trajectory. It therefore agrees with the symmetric

harmonic-oscillator model in points i) and ii), and it does possess

vector-meson universality. We turn now to a discription of this model.

IV.   THE SYMMETRIC QUARK MODEL

If all the baryon states are to be symmetric in the three

quarks, it is necessary that a quark in an intermediate state of a

scattering process should be able to go into any quark in the initial

or final states. For meson-baryon scattering one would therefore expect

diagrams such as Figs. 7(a) and (b) as well as Figs. 1(a) and (b).

There are fifty-four diagrams in all, since each of the three quarks

of the initial and final nucleon may annihilate with the

anti-quark of the meson and, in addition, the three quarks which pass

from the initial to the final state may do so in six possible ways.

According to the principles of the relativistic quark model,

each duality diagram is associated with a topologically similar

multi-particle Veneziano diagram. Corresponding to Figs. 7(a) and (b)

we therefore have the diagrams of Fig. 8(a) and (b), where we have

drawn dashed lines across the meson and nucleon. These diagrams

represent the scattering of ten external quarks, and they are to be

interpreted in the sense of Ref. 7; we construct multi-particle

Veneziano amplitudes with resonances in those channels for which Figs.

8(a) and (b) prossess intermediate states. By factorizing the meson-

baryon amplitude from the ten-point Veneziano amplitude, we obtain the

amplitude for the scattering of mesons and baryons of arbitrary spin.

Of the fifty-four duality diagrams, the eighteen where the

quark of the incoming meson passes into the quark of the outgoing

meson (Figs. 1(a) and 7(b)) have intermediate states in the s and t

channels. They therefore correspond to an st term in meson-baryon scattering. The remaining thirty-six, such as Figs. 1(b) and 7(a), correspond to an su term in meson-baryon scattering.

In duality diagrams for processes where the total number of baryons and anti-baryons exceeds four, some of the diagrams will again occur with minus signs owing to the fermi statistics of the baryons or the para-fermi statistics of the quarks. Each quark will be given a three-valued degree of freedom in addition to its spin, unitary-spin and orbital degrees of freedom. The three quarks in a single baryon have different values for this degree of freedom. We insert an extra minus sign for every pair of crossed lines between two quarks with the same value of the new degree of freedom.

The vertex diagrams are obtained by combining the quarks in all possible ways. The meson-baryon vertex will consist of the sum of eighteen diagrams, since the quark and the anti-quark lines of the meson may pass into any of the three quark lines of the nucleon, and the remaining two nucleon lines may pass into one another in two possible ways. Vertices such as Fig. 5(e) are excluded. In order that the vertices combine consistenty in both the s and t channels to give the duality diagrams such as Figs. 1(a) and 7(b), it is necessary to multiply all $B\bar{B}M$ vertices and all meson-baryon amplitudes by a factor $\frac{1}{6}$.

Our present model does possess vector-meson universality, because any of the baryon quarks in the $B\bar{B}M$ vertex may pass into the meson quarks. The BBM coupling constant is $\frac{3}{2}$ as large as in the planar-diagram model, there being eighteen vertex diagrams and an overall factor $\frac{1}{6}$, as opposed to the two diagrams in the latter model.

We next examine the factorization properties of our amplitude, and we shall show that the spectrum of resonances on the leading trajectory is indeed the same as in the non-relativistic harmonic-oscillator quark model. In the analysis of non-planar diagrams given in Ref. 7, we had shown that the most direct formula had to be modified in order to obtain the amplitude with the simplest spectrum of intermediate states. We shall repeat some of the formulas here in order to exhibit the relationsip between planar and non-planar diagrams.

The general non-planar diagram has been represneted in Fig. 9(a). All the solid and dashed lines in Figs. 8(a) and 8(b) have here been represented by solid lines, and we are interested in the spectrum in the channel cut by the dashed line. The factorization properties of the amplitude can be obtained from an expansion of the Veneziano integrand in powers of w, the integration variable corresponding to the channel in question. If we are only interested in the leading trajectory, we may drop all but the highest power of the angular momentum for a given power of w. The expansion for the general unmodified non-planar amplitude has the form:

$$I_1 I_2 \sum_{n=0}^{\infty} (n!)^{-1} (2\, P_1 P_2 + 2\, P_5 P_4 + 2\, P_7 P_8 - 2\, P_1 P_4 - 2\, P_1 P_8 - 2\, P_5 P_2$$

$$- 2\, P_5 P_8 - 2\, P_7 P_2 - 2\, P_7 P_4)^n \, w^{n-\alpha-1},$$

$$(4.1)$$

where I_1 and I_2 are the Veneziano integrands for the two halves of the diagram, and α is the trajectory function for the channel of interest. The P's are defined by the equations:

$$P_\alpha = \sum_\beta p_{\alpha\beta} \, y_{\alpha\beta} \quad , \tag{4.2}$$

where $y_{\alpha\beta}$ is the product of the u's for all left-hand sub-channels which include the particles $\alpha\beta$ and 6 if $\alpha\beta$ is on the left of the diagram, or the product of the u's for all right-hand sub-channels which include the particles $\alpha\beta$ and 3 if $\alpha\beta$ is on the right of the diagram.

Let us compare this formula with the corresponding formula for a planar diagram. Figure 9(b) represents a general such diagram, the middle solid line corresponding to the middle quark line of Figs. 2(a) and (b). The expansion of the amplitude in powers of w has been given in Refs. 5 and 6. With neglect of lowest powers of angular momentum, it has the form

$$I_1 I_2 \sum_{n=0}^{\infty} (n!)^{-1} \left\{ 2 \left(\sum_\beta p_{1\beta} \, \tilde{y}_{1\beta} + p_6 \, \tilde{y}_6 + \sum_\beta p_{7\beta} \, \tilde{y}_{7\beta} \right) \right.$$

$$\left. \times \left(\sum_\beta p_{2\beta} \, \tilde{y}_{2\beta} + p_3 \, \tilde{y}_3 + \sum_\beta p_{8\beta} \, \tilde{y}_{8\beta} \right) \right\}^n w^{n-\alpha-1} \quad . \tag{4.3}$$

The variables $\tilde{y}$ have been defined in Ref. 5, as well as in Sec. 2 of the present paper. We may eliminate the momenta p_6 and p_3 by using the equations

$$\sum_{\beta} p_{1\beta} + p_6 + \sum_{\beta} p_{7\beta} = \sum_{\beta} p_{2\beta} + p_3 + \sum_{\beta} p_{8\beta} \approx 0 \quad,$$

to leading order in the angular momentum. On doing so and making use

of the relations between the u's, we may rewrite (4.3) as follows:

$$I_1 I_2 \sum_{n=0}^{\infty} (n!)^{-1} \left\{ 2 \left(-P_1 + P_7 \right)\left(-P_2 + P_8 \right) \right\}^n w^{n-\alpha-1}$$

$$= I_1 I_2 \sum_{n=0}^{\infty} (n!)^{-1} \left(2 P_1 P_2 + 2 P_7 P_8 - 2 P_1 P_8 - 2 P_7 P_2 \right)^n w^{n-\alpha-1} \quad,$$

$$(4.4)$$

where the P's are defined as in (4.2). The variables P_4 and P_5

are zero for the diagram under consideration, as there are no terms in

the summation (4.2) when r is equal to 4 or 5. We thus observe that

the same formula (4.1) holds for planar and non-planar diagrams.

In Ref. 7 we showed that one can obtain a simpler spectrum of

intermediate states by multiplying the Veneziano integrand by the function

$$\prod_{r \, > \, s} \left(1 - z_{rs} \right)^{-P_r P_s} \qquad (4.5)$$

The product is over pairs of momenta from different groups $11 \cdots 21$, $51 \cdots 41$ and $71 \cdots 81$ of Fig. 9(a); the variable z_{rs} is defined as the product of the u's for all channels which include one of the particles r and s and one of the particles 3 and 6. In the modified formula, (4.1) becomes replaced by the expression

$$I_1 I_2 \sum_{k=0}^{\infty} (n!)^{-1} (2 P_1 P_2 + 2 P_5 P_4 + 2 P_7 P_8 - P_1 P_4 - P_1 P_8 - P_5 P_2 - P_5 P_8$$

$$\times -P_7 P_2 - P_7 P_4)^n w^{n-\alpha-1}$$

$$= I_1 I_2 \sum_{n=0}^{\infty} (n!)^{-1} \left\{ \frac{1}{3} (2 P_1 - P_5 - P_7)(2 P_2 - P_4 - P_8) + \frac{1}{3} (-P_1 + 2 P_5 - P_7) \right.$$

$$\times (-P_2 + 2 P_4 - P_8) + \frac{1}{3} (-P_1 - P_5 + 2 P_7)(-P_2 - P_4 + 2 P_2) \left. \right\}$$

$$\times w^{n-\alpha-1} . \tag{4.6}$$

We may also modify the planar amplitudes by multiplying the Veneziano integrand by (4.5). The effect is again to replace (4.1) by (4.6). As in (4.4), at least two of the P's will be zero for a planar diagram. In a model with only planar diagrams we would not modify the formula in this way, since it results in a complication of the spectrum; the leading trajectory, which previously had no orbital degeneracy, now becomes infinitely degenerate. In our present model, however, we already have an infinitely degenerate leading trajectory due to the non-planar

diagrams, and we shall now prove that the effect of modifying the planar

amplitudes is to remove the non-symmetrical quark states.

In Fig. 9(a), the three solid lines cut by the dashed line

represent the transfer of three quarks from the left to the right

of the diagram. Let us denote the quarks on the left of the diagram

by the indices 1,5 and 7, those on the right by the indices 2,4 and 8.

The tensor $I_1 (2 P_1 - P_5 - P_7)^{n_1} (-P_1 + 2 P_5 - P_7)^{n_2} (-P_1 - P_5 + 2 P_7)^{n_3}$

corresponds to an initial state where the three quarks are in an

n_1 th, n_2 th and n_3 th level of the harmonic-oscillator spectrum.

Spurious states are eliminated by the relation

$(2 P_1 - P_5 - P_7) + (-P_1 + 2 P_5 - P_7) + (-P_1 - P_5 + 2 P_7) = 0$. The curly

brackets of (4.6) may then be represented symbolically by the

expression

$$\delta_{12} \, \delta_{54} \, \delta_{78} \, , \qquad\qquad (4.7)$$

the subscripts on the Kronecker δ referring to states of the

harmonic-oscillator spectrum. Since the lines going across the

diagram also represent quark lines in a duality diagram, we may

interpret the subscripts as referring to spin and isotopic-spin indices

as well as to states of the harmonic-oscillator specturm.

In addition to Fig. 9(a), there will be five further diagrams

where the groups 1, 5 and 7 are joined to the groups 2,4 and 8 in all

possible ways. When we take the sum of all six diagrams, (4.7) becomes

replaced by

$$\delta_{12}\,\delta_{54}\,\delta_{78} + \delta_{14}\,\delta_{58}\,\delta_{72} + \delta_{18}\,\delta_{52}\,\delta_{74} + \delta_{14}\,\delta_{52}\,\delta_{78} + \delta_{12}\,\delta_{58}\,\delta_{74}$$

$$+ \delta_{18}\,\delta_{54}\,\delta_{72}\,. \qquad (4.8)$$

It is evident from (4.8) that the only intermediate states are those which are symmetric in the space, spin and isotopic-spin degree of freedom of the quarks taken together.

The factor (4.5) will of course modify the meson-baryon amplitudes which have been factored out of the general amplitude. We discussed this point in Ref. 7 and, although we confined our attention in that paper to non-planar diagrams, the same reasoning applies to planar diagrams. The st and tu terms are unaffected by the modification, and they will be given by an ordinary beta function. The su term, on the other hand, is given by the formula:

$$\int_0^1 dx\; x^{-\alpha(s)-1}\,(1-x)^{-\alpha(u)-1}\,\left\{1-x(1-x)\right\}^{\frac{1}{2}(s+t\,-\,M^2-\mu^2)}\,.$$

$$(4.9)$$

Let us verify that the $l=0$ and $l=1$ states consist of a pure $\underset{\sim}{56}$ and a pure $\underset{\sim}{70}$ respectively, as implied by the symmetry of the quark states. The st and su terms will again involve the $\underset{\sim}{56}$ and $\underset{\sim}{70}$ in the proportions given by (3.1a) and (3.1b). In the present model there are twice as many su diagrams as st diagrams, so that the sum of the diagrams will give a pure $\underset{\sim}{56}$. For the $l=1$ resonance it is not

difficult to show that the last factor of (4.9) reduces the residue in
the su term by a factor $\frac{1}{2}$. The multiplet constitution is therefore
obtained simply by subtracting (3.1b) from (3.1a), and the
resonance is a pure $\underset{\sim}{70}$.

   We conclude with the remark that the duality diagrams which
we have drawn in Figs. 7 and 8 are the simplest possible non-planar
diagrams. In general, we could construct non-planar diagrams with
any number of crossed lines, and it is necessary that all diagrams be
present in a consistent bootstrap scheme. In Ref. 7 we confined our
attention to diagrams with only one pair of crossed lines, but it would
be surprising if the methods could not be extended to the general case.
We must assume that such an extension is possible if we are to complete
our scheme in the narrow-resonance approximation.

V. SU(3) SYMMETRY BREAKING

In the SU(3)- symmetric model which we have examined thus far, the mass of the lowest resonance on the leading trajectory depends only on the total quark number. An obvious method of introducing SU(3) symmetry breaking is to make the mass depend on the number of strange and non-strange quarks separately. The octet character of the mass formula then arises naturally out of the theory. One does not obtain a sufficiently general mass formula by this method, as the formula for the mass splitting of the nucleon octet contains only one term, the Σ and Λ having the same mass. It may be necessary to introduce another type of SU(3) symmetry breaking or, alternatively, the term responsible for the $\Sigma\Lambda$ mass difference may appear as a higher-order effect in the width of the resonances.

Let us assume that the intercept depends quadratically on the total quark number; we have shown in Sec. 2 that this is the simplest possible assumption if we are to avoid stable particles of high charge and strangeness. From Eq. (2.4), we find that the mass of the lowest resonance on the leading trajectory is given by the formula

$$\mu^2 = (\alpha_n \nu_n + \alpha_s \nu_s)^2 + \beta_n \nu_n + \beta_s \nu_s + \gamma \ , \qquad (5.1)$$

where ν_s and ν_n are the number of strange and non-strange quarks. We may rewrite (5.1) in the form

$$\mu^2 = \left\{\alpha_n \nu + (\alpha_s - \alpha_n)\nu_s\right\}^2 + \beta_n \nu + (\beta_s - \beta_n)\nu_s + \gamma, \qquad (5.2)$$

where ν is the total quark number. Hence

$$\mu^2 = A\nu_s^2 + B\nu_s + C \qquad\qquad (5.3a)$$

where

$$A = (\alpha_s - \alpha_n)^2 \qquad\qquad (5.3b)$$

$$B = 2\alpha_n\nu(\alpha_s - \alpha_n) + \beta_s - \beta_n \qquad\qquad (5.3c)$$

$$C = \alpha_n^2\nu^2 + \beta_n\nu + \gamma \quad . \qquad\qquad (5.3d)$$

We notice that the constant A is independent of the multiplet, while B is a function of the total quark number. If the symmetry-breaking parameters are small, A will be a small quantity of second order.

Equation (5.3a) provides us with a mass-splitting formula. We may compare it directly with experiment for the spin $\frac{3}{2}$ decuplet and for the vector meson nonet, where each particle contains a definite number of strange quarks. We shall not compare the formula with experiment for the nucleon octet, as the $\Sigma\Lambda$ term is absent. The mass splitting of the pseudo-scalar octet cannot be treated rigorously until have a model where the $\underset{\sim}{35}$ is not degenerate with the η'. In such a model the η would not contain a definite number of strange quarks. We shall make the assumption, justified by the success of the Gell-Mann-Okubo mass forumla, that one can simply put ν_s for the η equal to its average value of $\frac{4}{3}$.

We then find that the masses of the three multiplets are reasonably well represented by the following formulas:

Spin - $\frac{3}{2}$ decuplet : $\qquad$ $\mu^2 = 0.02 \, \nu_s^2 + 0.17 \, \nu_s + 1.56$

Vector-meson nonet : $\qquad$ $\mu^2 = 0.02 \, \nu_s^2 + 0.18 \, \nu_s + 0.59$

Pseudo-scalar octet: $\qquad$ $\mu^2 = 0.02 \, \nu_s^2 + 0.20 \, \nu_s + 0.02$.

The formula for the psuedo-scalar octet is the least accurate, since it would give a mass of 575 MeV for the η, as opposed to the experimental mass of 550 MeV. However, our present formula is not appreciably worse than the ordinary Gell-Mann-Okubo formula.

If the constant A in (5.3a) were zero, we would obtain a Gell-Mann-Okubo formula in the squares of the masses, without the $\Sigma-\Lambda$ term. If $A = B^2/4C$, we obtain a Gell-Mann-Okubo formula in the masses. By leaving A arbitrary we have one more parameter than the ordinary Gell-Mann-Okubo formula, but we can fit the three multiplets under consideration with a mass-splitting formula of the same type. The term $A\nu_s^2$ is of second order in the symmetry breaking, and in any case is represents a fairly small effect, but the fit with such a term is definitely better than that with a simple Gell-Mann-Okubo formula in the squares of the masses.

Another point worth mentioning is that our formula is not a perturbation formula in the symmetry breaking, at any rate for the spin - $\frac{3}{2}$ decuplet and the vector meson nonet. We nowhere assume that the SU(3) symmetry breaking terms are small.

VI. CONCLUDING REMARKS

At first sight our model appears to differ from the usual quark models by the presence of exotic mesons of arbitrarily high total quark number. The distinction between ordinary and exotic resonances is that the latter have a higher Q-value for decay across a given centrifugal barrier. It is to be expected of any reasonable model that poles should occur in all channels of the S-matrix. The channels would differ from one another according to the distance of the poles from the real axis. Our present model may be favored from the point of view of nucleon democracy, since it admits of no difference in principle between two and three-quark channels on the one hand, and multi-quark channels on the other.

Since the total quark number in our model can assume any positive integral value, it may be of interest to attempt to Reggeize the quark number in the manner proposed by Delbourgo and Salam.[4] We shall not investigate this point in the present paper, however.

We have left open the question of choosing between the planar-diagram model and the symmetric quark model. The two models do not differ drastically from one another in their experimental predictions and, as neither is meant to be an exact representation of nature, it may be difficult to make such a choice. The difference between the coupling constants provides one obvious possible method of distinguishing between the models. Using the fact that vector-meson universality is fairly well satisfied in nature, we might decide in favor of the symmetric quark model. We should bear in mind the

limitations in accuracy of our model, however and, in particular, the

model predicts a value for the ratio $g_{\pi\rho\omega} / g_{\pi\pi\rho}$ which is somewhat

smaller than the experimental value. If one compares the meson-baryon

coupling constants with the constant $g_{\pi\rho\omega}$ instead of with $g_{\pi\pi\rho}$,

one obtains roughly equally good results with the two models. The

"experimental" determination of the coupling constant $g_{\pi\rho\omega}$ is of

course subject to some uncertainty,

If the higher resonances on the leading baryon trajectory

turn out to be complex, we would have strong evidence in favor of

the symmetric quark model. It would be difficult to draw any conclusion

from a failure to resolve the resonances. The degenerate trajectories

may be so close together that the spacing between the resonances is

small compared to their width, in which case they would appear

experimentally as a single resonance. On the other hand, the degeneracy

may be broken to a considerable extent, so that the relationship between

the degenerate resonances is not evident. The resonances which are

increased in mass by the breaking of the degeneracy may not appear

experimentally as narrow resonances.

It is quite possible that the two models are equally good

representation of nature in the narrow-resonance approximation, and

that the differences between them are of the order of magnitude of

the effects due to finite widths of the resonances. Each model possesses

trajectories which do not appear in the other, and the symmetric quark

model has a greater degree of degeneracy. The extra trajectories, and

the breaking of the degeneracy, could appear as higher-order effects.

It may even be that the two models are different starting-points which
eventually lead to the same dynamical scheme. We cannot discuss such
questions until we are able to improve on the narrow-resonance
approximation, however.

ACKNOWLEDGMENTS

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16. One could also write down a more general formula in which the first term was an arbitrary quadratic function of the variables $v_{r,ik}$. The degeneracy of the spectrum would be greater than that corresponding to (2.9).

17. For simplicity we shall express our results in the framework of SU(6). If we adopt the interpretation given in I, the narrow resonances do form representations of SU(6), through the trajectories form representations of SU(6, 6) or SU(12).

FIGURE CAPTIONS

Fig. 1. Duality diagrams for various processes.

Fig. 2. A sequence of reactions contained in Fig. 1(d).

Fig. 3. A general n-point duality diagram.

Fig. 4. Various channels i k in Fig. 3.

Fig. 5. Vertex functions in the planar-diagram model.

Fig. 6. Quarks in an exotic resonance.

Fig. 7. Non-planar duality diagrams.

Fig. 8. Non-planar Veneziano amplitudes.

Fig. 9. Non-planar and planar multi-particle Veneziano amplitudes.

(a)

(b)

(c)

(d)
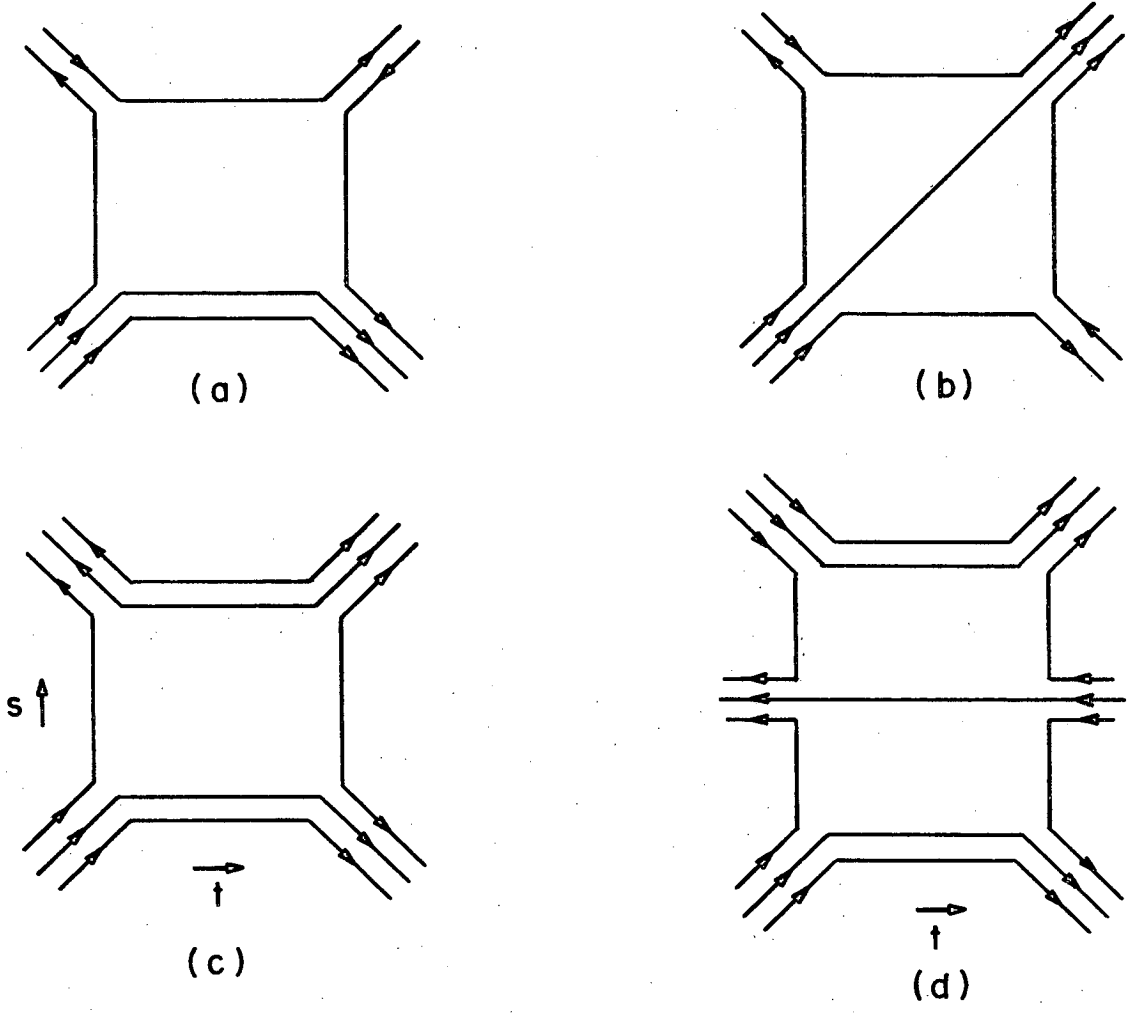
XBL699-3676

Fig. 1
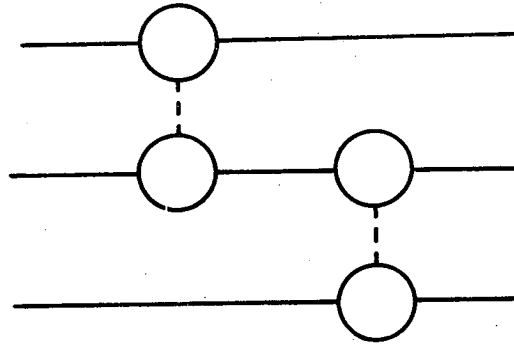
XBL699-3673

Fig. 2
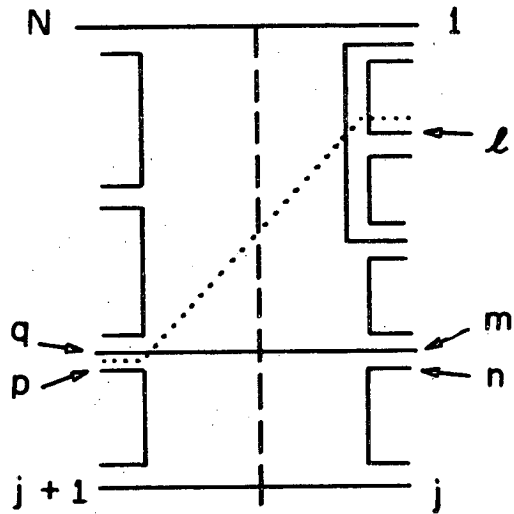
XBL699-3670

Fig. 3

XBL699-3669

Fig. 4

(a)   (b)

(c)   (d)

(e)
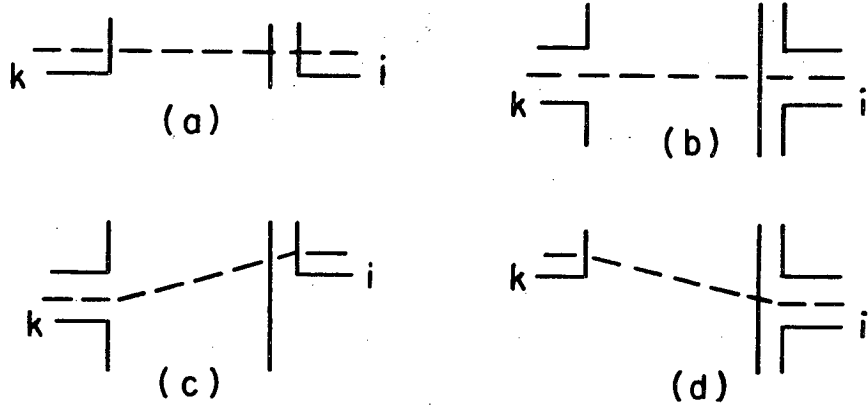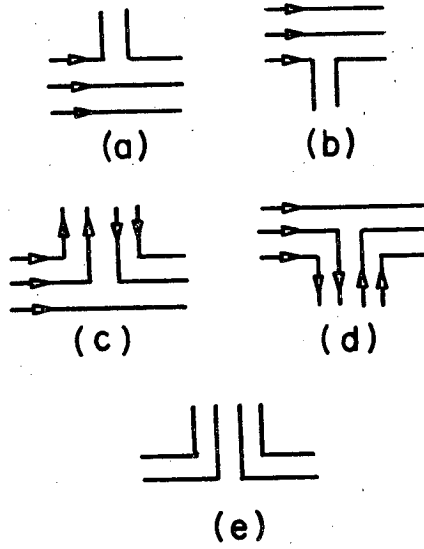
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Fig. 5

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Fig. 6

(a)

(b)

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Fig. 7

(a)

(b)

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Fig. 8

(a)

(b)

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Fig. 9