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The analysis of interferences between nuclear and Coulomb scattering in pp and pd collisions has become an important means for obtaining information regarding the real parts of the nucleon-nucleon (NN') elastic scattering amplitudes.[1,2] Several experimental and theoretical studies of small-angle high-energy scattering by deuterons have been recently carried out.[1-6] The theoretical analyses are based upon the Glauber high-energy approximation[4] and have treated both single and double interactions. Such investigations are also of current interest because of questions recently raised[7] concerning the applicability of this approximation to high-energy scattering by deuterons, and the suggestion[7] that double interaction effects may vanish much more rapidly at high energies than is inferred from this approximation. Recent calculations[5] have indicated, however, that even for incident momenta up to 18 BeV/c this approximation yields results for deuteron total cross sections which agree fairly well with measurements. In this note we compare the theory with high-energy pd differential cross section measurements of elastic scattering $(d\sigma/d\Omega)_{e\ell}$ and of the sum of inelastic (i.e., $p+d \rightarrow p+n+p$) plus elastic scattering $(d\sigma/d\Omega)_{sc}$. Since we shall consider small-angle collisions, we shall explicitly include the single and double scattering effects due to the Coulomb interaction.

By means of the closure approximation, $(d\sigma/d\Omega)_{sc}$ may be approximated by[5] the expectation value of the squared modulus of an operator $F(\underset{\sim}{q},\underset{\sim}{s})$ taken with respect to the deuteron ground state, $(d\sigma/d\Omega)_{sc} = \langle |F(\underset{\sim}{q},\underset{\sim}{s})|^2\rangle$, where $\hbar\underset{\sim}{q}$ is the momentum transferred by the incident particle and $\underset{\sim}{s}$ is the projection of the internal coordinate $\underset{\sim}{r}$ of the deuteron on the plane perpendicular to the direction of the incident beam. The differential cross section for elastic scattering is given by $(d\sigma/d\Omega)_{e\ell} = |\langle F(\underset{\sim}{q},\underset{\sim}{s})\rangle|^2$. In the Glauber approximation $F(\underset{\sim}{q},\underset{\sim}{s})$ takes the form

$$F(\underset{\sim}{q},\underset{\sim}{s}) = (ik/2\pi) \int e^{i\underset{\sim}{q}\cdot\underset{\sim}{b}}\, \Gamma_{tot}\, (\underset{\sim}{b},\underset{\sim}{s})\, d^{(2)}\, \underset{\sim}{b} \tag{1}$$

where the integration is over the plane of impact parameters $\underset{\sim}{b}$ perpendicular to the direction of the incident beam. The function $\Gamma_{tot}$ is given by

$$\Gamma_{tot}\, (\underset{\sim}{b},\underset{\sim}{s}) = 1 - \exp[i\chi_n(\underset{\sim}{b}-\underset{\sim}{s}/2) + i\chi_p(\underset{\sim}{b}+\underset{\sim}{s}/2)] \tag{2}$$

where $\chi_n$ and $\chi_p$ represent phase shifts produced by the neutron and proton in their instantaneous positions.

To exhibit the effects of the Coulomb interaction we write

$$\chi_p(\underset{\sim}{b}) = \chi_c(\underset{\sim}{b}) + \chi_{ps}(\underset{\sim}{b}) \tag{3}$$

where $\chi_c(\underset{\sim}{b})$ is due to the Coulomb interaction alone and $\chi_{ps}(\underset{\sim}{b})$ is attributed to the pp strong interaction. Putting

$$\Gamma_j(\underset{\sim}{b}) = 1 - \exp[i\chi_j(\underset{\sim}{b})] \quad , \qquad j = c,\ ps,\ n, \tag{4}$$

we have the identity

$$\Gamma_{tot}(\underset{\sim}{b},\underset{\sim}{s}) = \Gamma_c(\underset{\sim}{b}+\underset{\sim}{s}/2) + \exp[i\chi_c(\underset{\sim}{b}+\underset{\sim}{s}/2)][\Gamma_{ps}(\underset{\sim}{b}+\underset{\sim}{s}/2)$$

$$+ \Gamma_n(\underset{\sim}{b}-\underset{\sim}{s}/2) - \Gamma_{ps}(\underset{\sim}{b}+\underset{\sim}{s}/2)\, \Gamma_n(\underset{\sim}{b}-\underset{\sim}{s}/2)] . \qquad (5)$$

This identity is to be used in Eq. (1). For high-energy pp scattering $\chi_c(0)$ is typically small[8] and $\chi_c(\underset{\sim}{b})$ varies slowly over a range of values for b which does not greatly exceed the range a of the nucleon-incident-particle strong interaction plus the average n-p separation $r_d$ in the deuteron. In addition, for $b \gg a+r_d$, both $\Gamma_n$ and $\Gamma_{ps}$ are negligibly small and therefore do not contribute to the integral (1). Thus for calculating differential cross sections it is a good approximation to replace the last three terms in Eq. (5) by $\exp(i\chi_1)\Gamma_{ps} + \exp(i\chi_2)\Gamma_n + \exp(i\chi_3)\Gamma_{ps}\Gamma_n$, where the constants $\chi_1$, $\chi_2$, and $\chi_3$ are appropriate "average" values of $\chi_c$. For example, $\chi_1$ is given by

$$\chi_1 = \int \chi_c \Gamma_{ps} d^{(2)}\underset{\sim}{b} / \int \Gamma_{ps} d^{(2)}\underset{\sim}{b} \quad .$$

We may express $\Gamma_n$, $\Gamma_{ps}$, and $\Gamma_c$ as Fourier transforms of the pn and pp strong-interaction elastic scattering amplitudes $f_n$ and $f_{ps}$, and the Coulomb scattering amplitude for the proton $f_c$, respectively, and obtain for $(d\sigma/d\Omega)_{sc}$ and $(d\sigma/d\Omega)_{e\ell}$ the following relations:

$$(d\sigma/d\Omega)_{sc} = \langle |\exp(-i\underset{\sim}{q}\cdot\underset{\sim}{s}/2)[f_c(\underset{\sim}{q}) + \exp(i\chi_1)\, f_{ps}(\underset{\sim}{q})]$$

$$+ \exp(i\underset{\sim}{q}\cdot\underset{\sim}{s}/2+i\chi_2)\, f_n(\underset{\sim}{q}) + (i/2\pi k)\exp(i\chi_3)$$

$$\times \int \exp(i\underset{\sim}{q}'\cdot\underset{\sim}{s})\, f_n(\underset{\sim}{q}/2+\underset{\sim}{q}')\, f_{ps}(\underset{\sim}{q}/2-\underset{\sim}{q}')d^{(2)}\underset{\sim}{q}'|^2\rangle \quad (6)$$

$$(d\sigma/d\Omega)_{e\ell} = |S(\underset{\sim}{q}/2)[f_c(\underset{\sim}{q}) + \exp(i\chi_1)f_{ps}(\underset{\sim}{q})] + S(-\underset{\sim}{q}/2)\exp(i\chi_2)\, f_n(\underset{\sim}{q}) + (i/2\pi k)\exp(i\chi_3) \times \int S(\underset{\sim}{q}')\, f_n(\underset{\sim}{q}/2+\underset{\sim}{q}')\, f_{ps}(\underset{\sim}{q}/2-\underset{\sim}{q}')d^{(2)}\underset{\sim}{q}'|^2 \tag{7}$$

where $S(\underset{\sim}{q})$ is the form factor for the deuteron ground state, $S(\underset{\sim}{q}) = \langle e^{i\underset{\sim}{q}\cdot\underset{\sim}{r}}\rangle$.

We have calculated these cross sections using the form $f_j(\underset{\sim}{q}) = (i+\alpha)(k\sigma_j/4\pi)\exp(-Aq^2/2)$, where $j = n$, ps, and using $S(\underset{\sim}{q})$ derived from the representation of the deuteron wave function given by Moravcsik.[9] No complete set of measurements for $\sigma_n$, $\sigma_{ps}$, $\alpha$, A, and $(d\sigma/d\Omega)_{e\ell}$ is yet available at a given nucleon momentum. In calculations, therefore, it may be necessary to utilize as input data measurements made at slightly different momenta. For each of the eight incident momenta considered, the values used for $\alpha$, $\sigma_{ps}$, and A were taken from pp measurements and the value used for $\sigma_n$ was taken directly from np measurements or indirectly from pd and pp measurements. More specifically, we have used the values given in Refs. 1, 2, and 3 which were obtained from or employed in their analyses of their pp and pd measurements. This determined 22 of the 32 values used, including all four in the calculations of $(d\sigma/d\Omega)_{sc}$ at 19.3 BeV/c. Of the other 10 values, 7 were taken from measurements[2,3,10,11] at momenta within ±0.2 BeV/c of the desired momenta, and the remaining 3 from measurements[11,12] at momenta within ±1 BeV/c.

In Fig. 1 we show the calculations for $(d\sigma/d\Omega)_{sc}$ at 19.3 BeV/c as a function of -t, the negative of the squared four-momentum transfer, together with the data.[1] The measurements present clear evidence for the influence of double scattering and indicate a rather large effect. The calculated scattering cross section, secured by integrating $(d\sigma/d\Omega)_{sc}$ without the Coulomb interaction, is 20.6 mb when double scattering is neglected and 18.0 mb when it is not neglected. The measured value[1] is 18.0±0.6 mb.

In Fig. 2 we compare calculations for $(d\sigma/d|t|)_{e\ell}$ at several momenta with measured values[3] and with fits[2] to more recent measurements. In Table I we compare data for integrated pd elastic scattering cross sections $\sigma_{e\ell}$ with the calculated values for which double scattering is treated and those (in parentheses) for which double scattering is neglected. The data in this table are from Ref. 2 except as noted.

Our calculations contain no adjustable parameters. We have used the high-energy approximation together with NN' data as input and have made predictions which we see are in good agreement with pd measurements.

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## FOOTNOTES AND REFERENCES

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Fig. 1. Differential cross sections for pd elastic plus inelastic scattering as a function of the squared four-momentum transfer $t$, at an incident proton momentum of 19.3 BeV/c.

Fig. 2. Differential cross sections for pd elastic scattering between 2.78 and 10.9 BeV/c. Measurements[3] (●,■) and fits[2] to measurements (-·-·-) are compared with calculations which neglect (------) and which include (———) double scattering. The points (■) for the momentum 4.85 BeV/c are from measurements[3] at 4.67 BeV/c.

Table I. Elastic pd scattering cross sections $\sigma_{e\ell}$. Values in parentheses are calculated with double scattering neglected.

| Momentum (BeV/c) | $\sigma_{e\ell}$-exp. (mb) | $\sigma_{e\ell}$-calc. (mb) | $\sigma_{e\ell}$-calc. (mb) |
|---|---|---|---|
| 2.78 | 9.5 ± 0.7 | 10.1 | (12.3) |
| 4.85 | 9.9 ± 0.8 | 11.1 | (13.4) |
| 6.87 | 9.5 ± 0.7 | 9.5 | (11.3) |
| 7.08[a] | 8.41± 0.73 | 8.23 | ( 9.89) |
| 8.89 | 9.5 ± 0.7 | 9.4 | (11.1) |
| 9.09[b] | 9.74± 1.10 | 8.46 | (10.16) |
| 10.90 | 9.3 ± 0.7 | 9.3 | (11.1) |

[a]For the angular interval $1.5^{\circ} < \theta_{c.m.} < 7.5^{\circ}$ (Ref. 3)

[b]For the angular interval $1.3^{\circ} < \theta_{c.m.} < 10^{\circ}$ (Ref. 3)

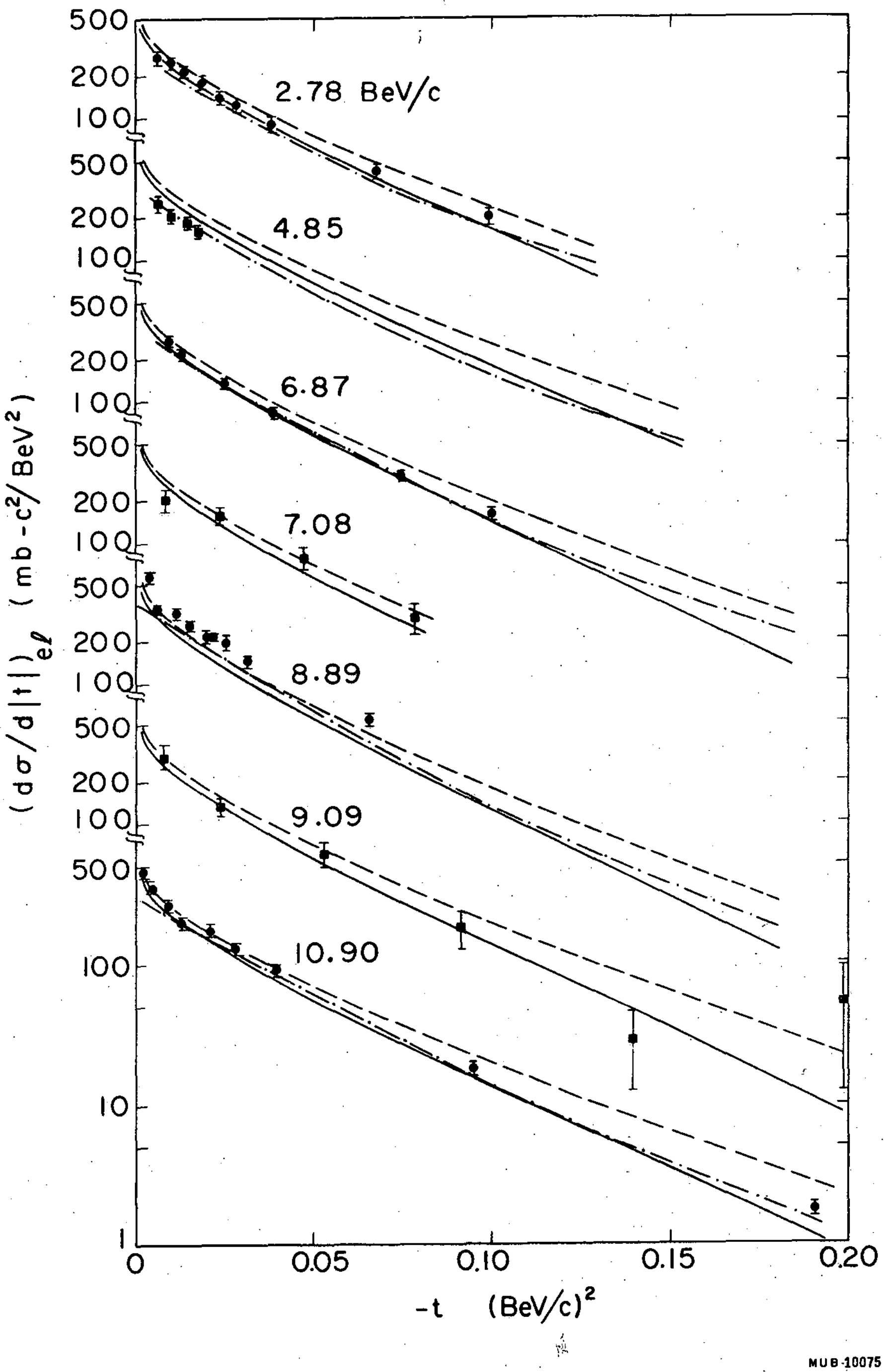
2.78 BeV/c
4.85
6.87
7.08
8.89
9.09
10.90
$(d\sigma/d|t|)_{el}$ (mb-c²/BeV²)
-t (BeV/c)²
0
0.05
0.10
0.15
0.20
500
200
100
10
1
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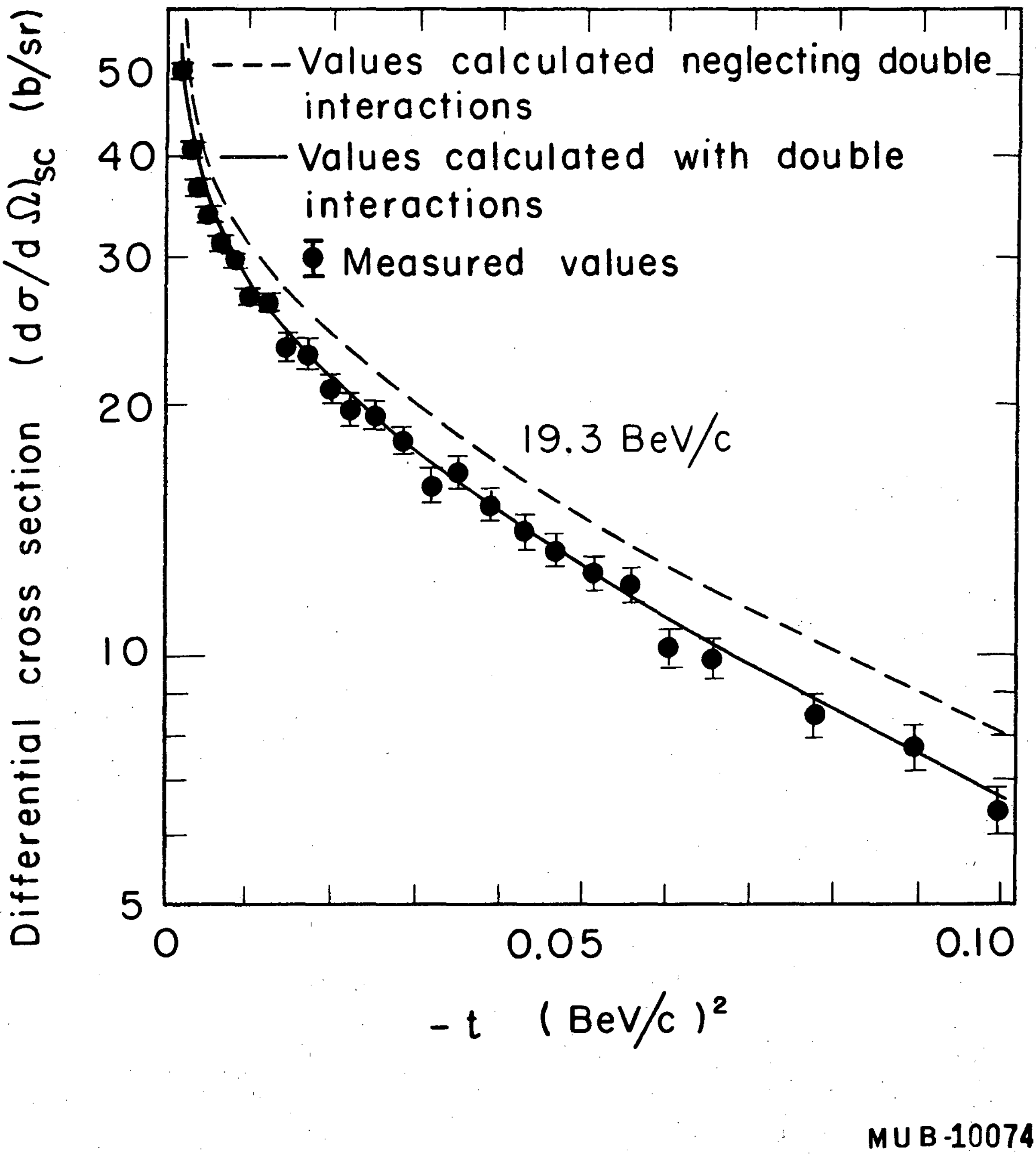
Fig. 1



Values calculated neglecting double interactions
Values calculated with double interactions
Measured values
19.3 BeV/c
Differential cross section $(d\sigma/d\Omega)_{sc}$ (b/sr)
50
40
30
20
10
5
0
0.05
0.10
$-t$ $(\mathrm{BeV}/c)^2$
MUB-10074


Fig. 2