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Synopsis

Stress-resultants along meridians for several selected axisymmetrical shells of revolution loaded symmetrically along free edges or with own weight are reported. The shells considered include the ones with zero, positive as well as negative Gaussian curvatures. The solutions are based on bending theory of thin shells. The numerical results were obtained by applying finite element solution using digital computer. Report of solutions for shells of negative Gaussian curvature appears to be novel.

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Introduction

Because of the great importance of the problem, stress perturbation in axisymmetrical shells of revolution caused by constraints or the application of edge loads received much attention from the investigators. This subject is discussed in all classical texts[1,2,3,4]. For shells of revolution of zero and positive Gaussian curvature, the available treatment of this topic is rather complete, although some numerical difficulties may be encountered in the solution of practical problems. For shells of negative Gaussian curvature, however, one finds only solutions based on the assumption of membrane action.

If membrane behavior is assumed for a shell with negative Gaussian curvature, the governing differential equations are of hyperbolic type[3,4,]. This would indicate direct propogation of edge stresses to the interior,[3] and raises the question of the basic stability of such structures[4]. The real behavior of structures having negative Gaussian curvature, however, is quite different. Nozzles of jet engines, chimneys for blast furnaces, water towers, and roof domes have been built as one-sheet hyperboloids and have performed satisfactorily. Approximation of such shells by a membrane is unrealistic and a more accurate theory must be used in the solution of such problems.

Formal mathematical difficulties prevent the solution of the bending problem for shells of negative Gaussian curvature. However, a sufficiently accurate method[5,6] for most practical purposes based on finite elements is capable of yielding the needed information. In this method any axisymmetrical shell of arbitrary meridian is approximated by a sequence of short conical or cylindrical ring elements. The stiffness or flexibility matrices of the elements are developed from known complete solutions of conical and cylindrical shells. The geometrical shape of the shell defines the force-transformation and displacement-transformation matrices. Upon application of a set of loads at the joints of the element assemblage, the displacement of the joints, and then the displacements and internal forces at the edges of the elements can be determined using the displacement method of structural analysis. After the basic edge forces in the conical or cylindrical elements are found, other internal (hoop) forces and displacements can be obtained from known shell solutions. By subdividing a shell into numerous conical elements, solutions with high degree of accuracy can be achieved.

Shells Analyzed

For the purposes of comparison four shells having the geometry shown in Fig. 1, and a cylinder shown in Fig. 2, were analyzed for various loadings. Shell A having a height of 100.3695 in., ends of 70 in. diameter, and meridians of 75 in. radius is a shell of positive Gaussian curvature. Shells B, C, and D are one-sheet hyperboloids of negative Gaussian curvature. Shell B has similar dimensions to shell A except that the center for the meridians curvature is external to the shell itself. Shell C's meridian is composed of two circular arcs. In shell D, which is similar to shell C, a flat cone is added at the top.

All shells were assumed fixed at the bottom and free at the top. With the exception of shell B, the wall thickness was taken as 0.5 in., unit weight was assumed to be 0.05 lbs./in.^{2}. For shell B the thickness was variously taken as 0.5 in., 1.0 in., 2.0 in., and 4.0 in. The elastic modulus was assumed as $E = 10^7$ psi, and Poisson's ratio $\nu = 0.2$. The number of finite elements used in each case is noted by the figures.

Altogether four different types of loadings were considered. Three edge loadings included a ring moment, a ring horizontal force, and a ring vertical load. Shell's own weight provided the fourth type of loading.

In Fig. 2 are plotted the meridianal moments caused by the application of a ring moment of 1,000 in.-lb./in. to five different shells. Both the classical and the finite element solution yielded virtually identical results for the cylindrical shell. The characteristic variation in the meridianal moment for this case is readily recognized. For shell A an analogous variation of moment is observed. Significantly, similar pattern of damped moment can be noted for shells B, C, and D of negative Gaussian curvature. In shell B nothing unusual by comparison with a shell of positive Gaussian curvature occurs as its thickness is varied. In shell D an extended region of almost constant meridianal moment near the free end is not very different from what one finds in flat plates subjected to a constant edge moment.

The variation of the meridianal moments due to horizontal ring force of 1,000 lb./in. for shells A, B, and C is shown in Fig. 3. It is interesting to note that in shells A and B the induced meridianal moment damps out in about the same manner.

A series of diagrams in Fig. 4 shows the variation in the stress-resultants caused by the vertical ring force of 1,000 lb./in. for shells A and B. Again it is noteworthy that with the exception of the in-plane meridianal stress-resultants N_S, which behave as would be expected, the corresponding cases are very similar. A small discrepancy in the values of Q_S occurs depending on whether the i-th or the j-th end of the conical element is considered; in the plot Q_S based on the i-th end of the cones is shown. In Fig. 5 curves analogous to the above are shown for shell C.

In Fig. 6 the meridianal moments caused by shell's own weight are plotted. As far as the finite element solution is concerned, the complexity of the structure and of the loading may be further increased provided axial symmetry is preserved. This example illustrates the comprehensive character of the method employed.

Conclusions

Based on the study of the above solutions for several special cases, some important general conclusions can be reached. The most important one is that regardless of the sign of the shell's Gaussian curvature, edge disturbances damp out. This dampening appears to be approximately the same for the same Gaussian curvature regardless of sign. One may surmise that this phenomenon will also occur for unsymmetrical loadings. Finally, membrane solutions for shells of negative Gaussian curvature can be particularly misleading.

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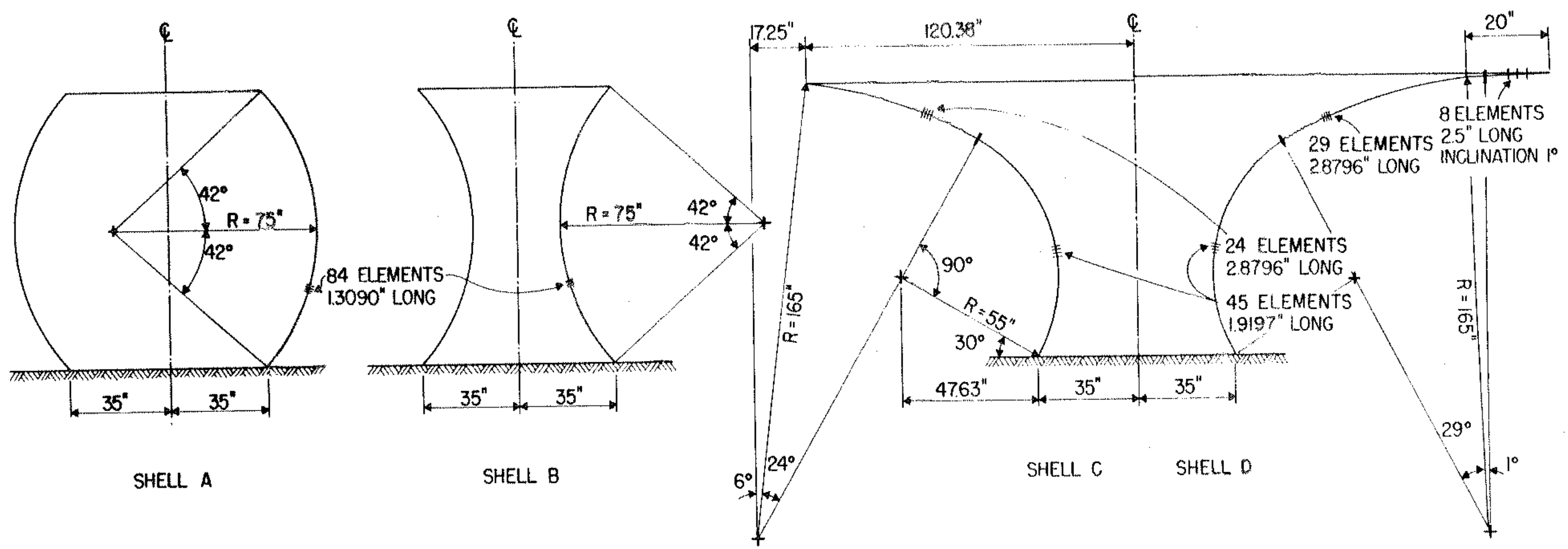

FIG. I GEOMETRICAL PROPERTIES OF SHELLS

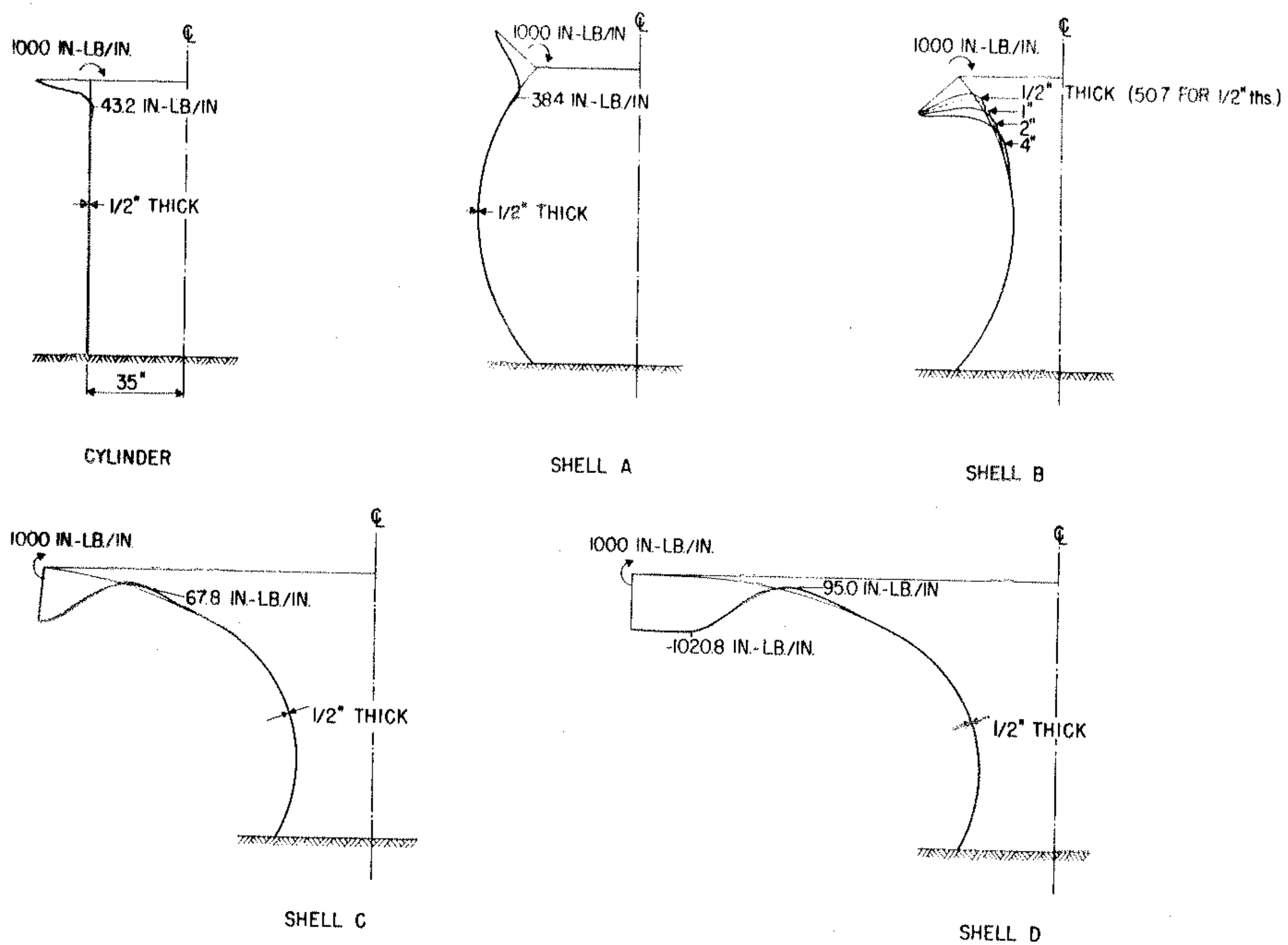

FIG. 2 MERIDIANAL MOMENTS M_S DUE TO EDGE MOMENT OF 1000 IN.-LB./IN.

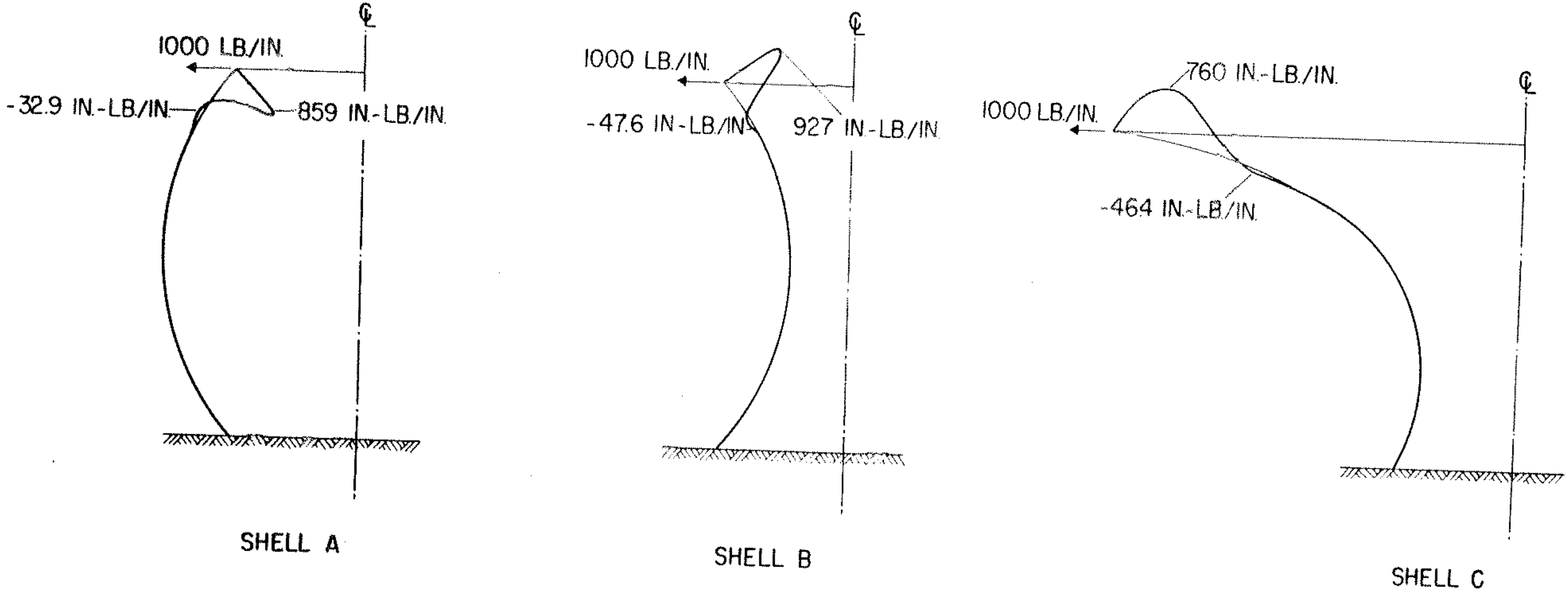

FIG. 3 MERIDIANAL MOMENTS DUE TO HORIZONTAL EDGE FORCE OF 1000 LB./IN.

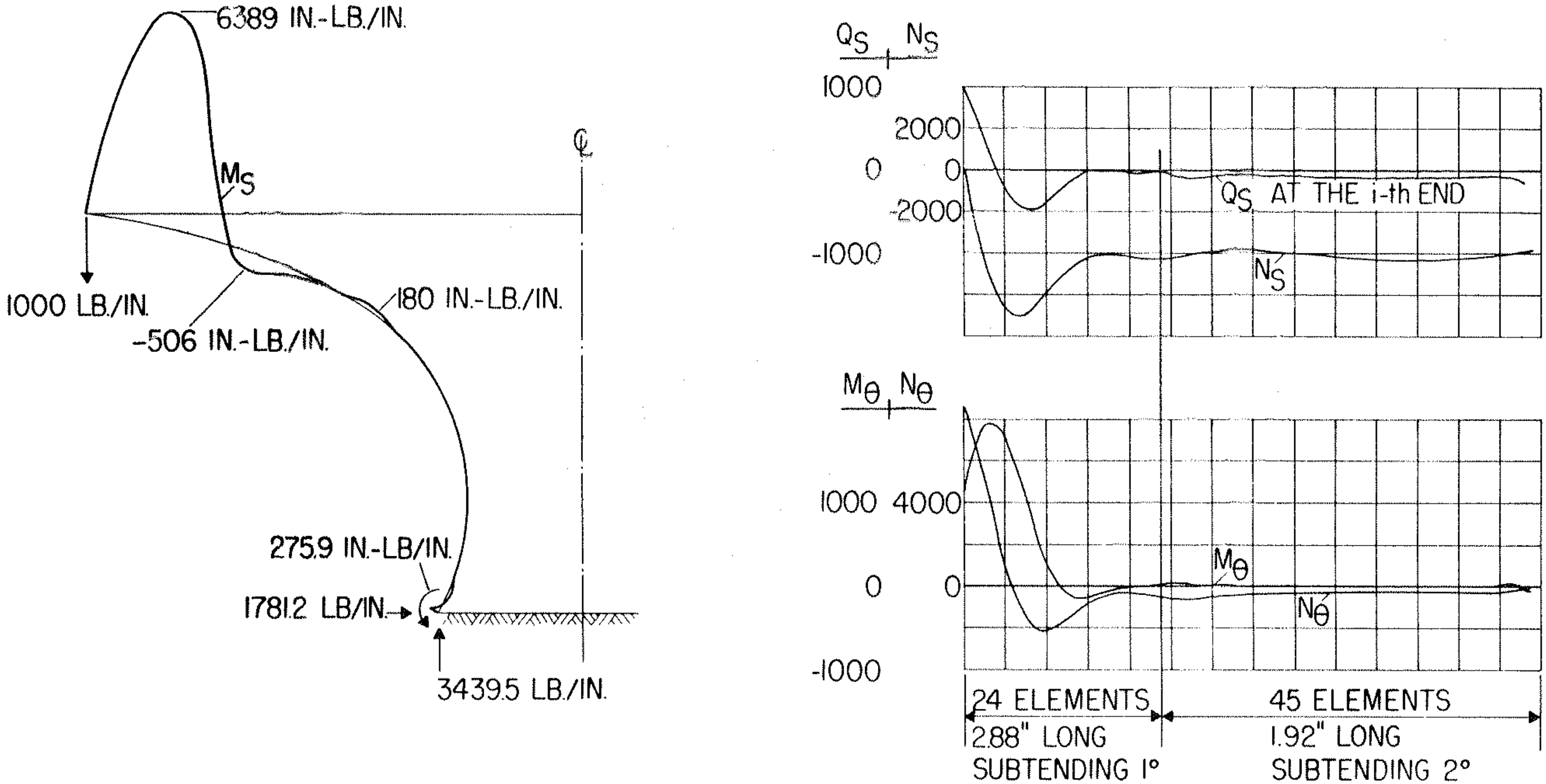

FIG. 5 STRESS-RESULTANTS IN SHELL C
CAUSED BY VERTICAL FORCE OF 1,000 LB./IN.

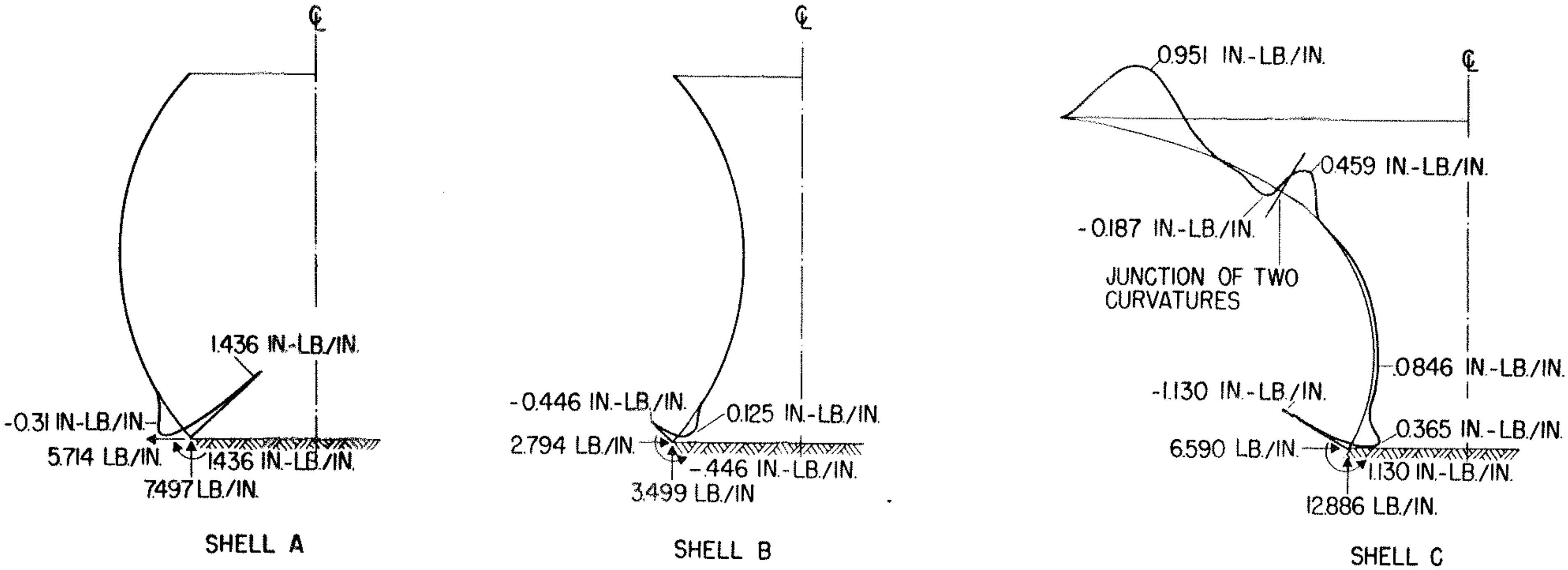

FIG. 6 MERIDIANAL MOMENTS DUE TO OWN WEIGHT OF 0.05 LB./IN.^{2}