Title
Comments on â��Supraluminous Waves and the Power Spectrum of an Isotropic, Homogeneous Plasmaâ��

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model is that the charge is not conserved instantaneously, but only on the average over a cycle.[10,11] It should be pointed out that with this collision model, the collision frequency ν appears only in the time-varying part, but not in the time-independent part of the Boltzmann equation. Consequently, Eq. (1) cannot be derived directly from this approach. However, there is no difficulty in deriving the ac conductivity from the time-varying parts of Boltzmann and Maxwell equations. Therefore, for the system considered, the time-independent current density, J_0, and electric field intensity, E_0, are not necessarily governed by Eq. (1). The consistency of analysis must not be judged solely on the basis of Eq. (1) (Ohm's law).

If Scharer's objection is that of the usage of the particular collision model because it would not lead to the simple Ohm's law, then I can well understand. However, the usage of Ohm's law has its limitations[8] too. Next, I would like to show that there is no difficulty or inconsistency in satisfying the time-independent part of the Maxwell's field equations by taking $J_{0z} = 0$ and $E_{0z} = \text{const}$; static field equations are

$$\nabla \times \mathbf{E}_0 = 0, \quad \text{and} \quad \nabla \times \mathbf{H}_0 = \mathbf{J}_0. \tag{2}$$

The first equation is obviously satisfied. The second equation yields

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = J_{0z}. \tag{3}$$

Since a one-dimensional analysis is being considered, i.e., $\partial/\partial x = \partial/\partial y = 0$, the left-hand side of Eq. (3) vanishes so that J_{0z} must also be zero.

Finally, I agree that if the electrostatic field E_{0z} was not sufficiently weak, then a shifted Maxwellian should be used for the time-independent electron distribution function. However, this form of distribution function would only approximately satisfy the time-independent part of the Boltzman equation. The dispersion equation then would have to be modified accordingly. Furthermore, for appropriate conditions when electron streaming occurs in plasma, wave–plasma interaction would be expected.[12]

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Comments on "Supraluminous Waves and the Power Spectrum of an Isotropic, Homogeneous Plasma"

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Calculations of the power spectrum of fluctuations in a plasma have previously been based on the time-asymptotic solution of a test-particle problem. Certain poles corresponding to waves with phase velocities $\omega/k \geq c$ have been omitted in these calculations. In a recent paper, these poles have been considered by Lerche who finds that they lead to terms that persist in time. When included, they lead to significantly different results for the power spectrum. It is shown that these persistent terms are caused by the impulsive acceleration of the test particle that is inherent in the mathematical model. and are, therefore, physically unacceptable.

We shall begin our considerations with the solution of the test-particle problem as given by Lerche[1]

$$\mathbf{E}(\mathbf{x}, t) = \frac{1}{(2\pi)^4} \int d\mathbf{k} \int_{i\sigma-\infty}^{i\sigma+\infty} d\omega \cdot \exp\,[i(\mathbf{k}\cdot\mathbf{x} - \omega t)]\mathbf{E}(\mathbf{k}, \omega), \tag{1}$$

$$\mathbf{E}(\mathbf{k}, \omega) = \mathbf{E}_L(\mathbf{k}, \omega) + E_T(\mathbf{k}, \omega),$$

$$D_L(\mathbf{k}, \omega)\mathbf{E}_L(\mathbf{k}, \omega) = \frac{4\pi q\mathbf{k} \exp\,(-i\mathbf{k}\cdot\mathbf{x}_0)}{k^2(\omega - \mathbf{k}\cdot\mathbf{v}_0)},$$

$$D_T(\mathbf{k}, \omega)\mathbf{E}_T(\mathbf{k}, \omega) = -\frac{4\pi q\omega}{c^2k^4} \frac{[(\mathbf{k}\times\mathbf{v}_0)\times\mathbf{k}] \exp\,(-i\mathbf{k}\cdot\mathbf{x}_0)}{(\omega - \mathbf{k}\cdot\mathbf{v}_0)}.$$

Following Lerche we can approximate $D_L(\mathbf{k}, \omega)$ and $D_T(\mathbf{k}, \omega)$ as follows:

$$D_L(\mathbf{k}, \omega) \cong 1 - \frac{\Omega^2}{\omega^2}, \qquad D_T(\mathbf{k}, \omega) \cong 1 - \frac{\Omega_*^2}{c^2k^2} - \frac{\omega^2}{c^2k^2}. \tag{2}$$

We shall further approximate these functions by assuming $\Omega = \Omega_* = 0$, i.e., we shall neglect the plasma density completely and still obtain supraluminous waves in their most transparent form.

According to the "adiabatic" approximation[2] only the pole $\omega = \mathbf{k}\cdot\mathbf{v}_0$ is retained. This leads to the time-asymptotic solution

$$\mathbf{E}(\mathbf{x},\,\infty) = \int \frac{d\mathbf{k}}{(2\pi)^2} \exp\,[i\mathbf{k}\cdot(\mathbf{x} - \mathbf{x}_0 - \mathbf{v}_0 t)] \left(\frac{-\,4\pi i\mathbf{k}q}{k^2 - [(\mathbf{k}\cdot\mathbf{v}_0)^2/c^2]} + \frac{4\pi i q}{c^2}\,\frac{(\mathbf{k}\cdot\mathbf{v}_0)\mathbf{v}_0}{k^2 - [(\mathbf{k}\cdot\mathbf{v}_0)^2/c^2]}\right). \tag{3}$$

This integration is simple and the result is

$$\mathbf{E}(\mathbf{x},\,\infty) = \frac{q(\mathbf{x} - \mathbf{x}_0 - \mathbf{v}_0 t)}{|\mathbf{x} - \mathbf{x}_0 - \mathbf{v}_0 t|^3}\,\frac{[1 - (v_0/c)^2]}{(1 - (v_0/c)^2 + (v_0/c)^2\{[v_0\cdot(\mathbf{x} - \mathbf{x}_0 - \mathbf{v}_0 t)]^2/v_0^2\,|\mathbf{x} - \mathbf{x}_0 - \mathbf{v}_0 t|^2\})^{3/2}}. \tag{4}$$

We now return to a consideration of the poles we have omitted at $\omega = \pm\,ck$ that arise from $D_T(\mathbf{k},\,\omega)$. If we do not neglect Ω_*, the poles will be at $\omega = \pm(\Omega_*^2 + c^2k^2)^{1/2}$ in which case $\omega/k > c$. It is simpler to neglect Ω^*, and it will be apparent that the physical result of including the pole is in any case similar.

Rather than directly carry out the ω and $\mathbf{k}$ integrations of Eq. (1), we shall solve the test particle problem in a different way which is less tedious if one's objective is to display $\mathbf{E}(\mathbf{x},\,t)$. We employ the potentials

$$\Phi(\mathbf{x},\,t) = \int d\mathbf{x}' \int dt'\,\frac{\rho(\mathbf{x}',\,t')}{|\mathbf{x} - \mathbf{x}'|}\cdot\delta\left(t' - t + \frac{|\mathbf{x} - \mathbf{x}'|}{c}\right), \tag{5}$$

$$\mathbf{A}(\mathbf{x},\,t) = \frac{1}{c}\int d\mathbf{x}' \int dt'\,\frac{\mathbf{j}(\mathbf{x}',\,t')}{|\mathbf{x} - \mathbf{x}'|}\cdot\delta\left(t' - t + \frac{|\mathbf{x} - \mathbf{x}'|}{c}\right). \tag{6}$$

Following Lerche we assume

$$\rho(\mathbf{x}',\,t') = q\;\delta(\mathbf{x}' - \mathbf{x}_0 - \mathbf{v}_0 t'), \qquad t' > 0,$$
$$\mathbf{j}(\mathbf{x}',\,t') = q\mathbf{v}_0\;\delta(\mathbf{x}' - \mathbf{x}_0 - \mathbf{v}_0 t'), \qquad t' > 0, \tag{7}$$

and

$$\rho(\mathbf{x}',\,t') = 0,\ \mathbf{j}(\mathbf{x}',\,t') = 0 \quad \text{for} \quad t' < 0.$$

By Fourier transforming Eqs. (5) and (6) it follows that

$$\begin{aligned}\mathbf{E}(\mathbf{x},\,t) &= \frac{1}{(2\pi)^4}\int d\mathbf{k}\int_{i\sigma-\infty}^{i\sigma+\infty} d\omega\,\exp\,[i(\mathbf{k}\cdot\mathbf{x} - \mathbf{k}\cdot\mathbf{x}_0 - \omega t)]\left(\frac{4\pi q\mathbf{k}}{k^2(\omega - \mathbf{k}\cdot\mathbf{v}_0)} - \frac{4\pi q\omega}{c^2k^2}\,\frac{(\mathbf{k}\times\mathbf{v}_0)\times\mathbf{k}}{(\omega - \mathbf{k}\cdot\mathbf{v}_0)[k^2 - (\omega^2/c^2)]}\right)\\ &= -\frac{\partial}{\partial\mathbf{x}}\,\Phi(\mathbf{x},\,t) - \frac{1}{c}\,\frac{\partial}{\partial t}\,\mathbf{A}(\mathbf{x},\,t) - \frac{1}{(2\pi)^4}\int d\mathbf{k}\int_{i\sigma-\infty}^{i\sigma+\infty} d\omega\,\frac{4\pi q\omega\mathbf{k}}{(k^2c^2 - \omega^2)k^2}\exp\,[i(\mathbf{k}\cdot\mathbf{x} - \mathbf{k}\cdot\mathbf{x}_0 - \omega t)].\end{aligned} \tag{8}$$

Note that we have reserved the symbol $\mathbf{E}(\mathbf{x},\,t)$ for the quantity actually calculated by Lerche.

The last term in Eq. (8) can easily be integrated, The result is

$$\begin{aligned}&\frac{1}{(2\pi)^4}\int d\mathbf{k}\int_{i\sigma-\infty}^{i\sigma+\infty} d\omega\,\frac{4\pi q\omega\mathbf{k}}{(k^2c^2 - \omega^2)k^2}\cdot\exp\,\{i[\mathbf{k}\cdot(\mathbf{x} - \mathbf{x}_0) - \omega t]\}\\ &= -4\pi q\,\frac{\partial}{\partial\mathbf{x}}\int_{-\infty}^{t} dt'\,\frac{1}{|\mathbf{x} - \mathbf{x}_0|}\cdot\left[\delta\left(t' - \frac{|\mathbf{x} - \mathbf{x}_0|}{c}\right) - \delta(t')\right]\\ &= -\frac{q(\mathbf{x} - \mathbf{x}_0)}{|\mathbf{x} - \mathbf{x}_0|^3} \quad \text{for} \quad \frac{|\mathbf{x} - \mathbf{x}_0|}{c} > t > 0\\ &= 0 \quad \text{for} \quad t < 0 \quad \text{or} \quad t > \frac{|\mathbf{x} - \mathbf{x}_0|}{c}.\end{aligned} \tag{9}$$

The calculation of Φ or $\mathbf{A}$ proceeds as follows:

$$\Phi(\mathbf{x},\,t) = q\int_{\tau=-t}^{\infty} d\tau\,\frac{\delta[f(\tau)]}{|\mathbf{x} - \mathbf{x}_0 - \mathbf{v}_0 t - \mathbf{v}_0\tau|}, \tag{10}$$

$$f(\tau) = \tau + |\mathbf{x} - \mathbf{x}_0 - \mathbf{v}_0 t - \mathbf{v}_0\tau|/c,$$

$$\delta[f(\tau)] = \frac{1}{f'(\tau_0)}\,\delta(\tau - \tau_0),$$

where

$$\tau_0 = -\frac{|\Delta\mathbf{x}|\,\gamma_0^2}{c}\left\{\frac{\mathbf{v}_0\cdot\Delta\mathbf{x}}{|\Delta\mathbf{x}|\,c} + \left[1 - \beta_0^2 + \beta_0^2\,\frac{(\mathbf{v}_0\cdot\Delta\mathbf{x})^2}{|\Delta\mathbf{x}|^2\,v_0^2}\right]^{1/2}\right\},$$

$$f'(\tau_0) = \frac{1}{\gamma_0^2}\left\{1 - \left[1 + \frac{|\Delta\mathbf{x}|\,c}{\mathbf{v}_0\cdot\Delta\mathbf{x}}\cdot\left(1 - \beta_0^2 + \beta_0^2\,\frac{(\mathbf{v}_0\cdot\Delta\mathbf{x})^2}{|\Delta\mathbf{x}|^2\,v_0^2}\right)^{-1/2}\right]\right\}.$$

The abbreviations $\beta_0 = v_0/c$, $\gamma_0 = (1 - \beta_0^2)^{-1/2}$ and $\Delta\mathbf{x} = \mathbf{x} - \mathbf{x}_0 - \mathbf{v}_0 t$ have been employed

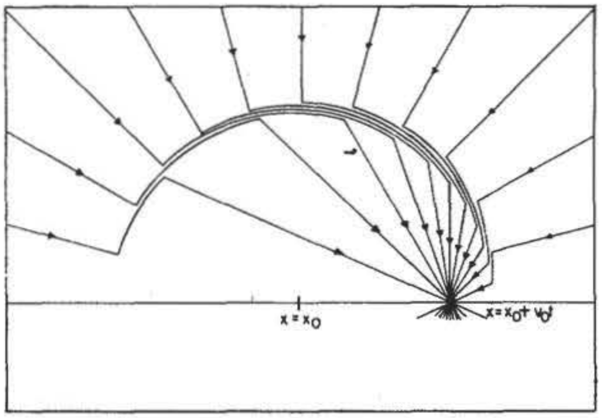

FIG. 1. Electric field due to an impulsively accelerated test charge.

$$\Phi(\mathbf{x}, t) = \frac{q}{|\Delta \mathbf{x}|}\left(1 - \beta_0^2 + \beta_0^2 \frac{(\mathbf{v}_0 \cdot \Delta \mathbf{x})^2}{|\Delta \mathbf{x}|^2 v_0^2}\right)^{-1/2}, \qquad (11)$$

provided that $|\tau_0| < t$ and $t > 0$. $\Phi = 0$ when either condition is not satisfied. From the expression for $|\tau_0|$ the condition $|\tau_0| < t$ can be reduced to

$$|\mathbf{x} - \mathbf{x}_0| < ct.$$

Since $\mathbf{A}(\mathbf{x}, t) = (\mathbf{v}_0/c)\, \Phi\, (\mathbf{x}, t)$,

$$-\frac{\partial}{\partial \mathbf{x}} \Phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} = -\frac{\partial}{\partial \mathbf{x}} \Phi - \frac{\mathbf{v}_0}{c}\left(\mathbf{v}_0 \frac{\partial}{\partial \mathbf{x}}\right) \Phi.$$

After carrying out the indicated algebra the result for Eq. (8) is that $\mathbf{E}(\mathbf{x}, t)$ is given exactly by Eq. (4), provided that $|\mathbf{x} - \mathbf{x}_0| < ct$ and $t > 0$. When $|x - x_0| > ct$ and $t > 0$,

$$E(\mathbf{x}, t) = q(\mathbf{x} - \mathbf{x}_0)/|\mathbf{x} - \mathbf{x}_0|^3 .$$

If $t < 0$, $\mathbf{E}(\mathbf{x}, t) = 0$. These results are illustrated in Fig. 1. It is clear that this is the electric field for an impulsively accelerated charge. An electromagnetic wave centered at $\mathbf{x} = \mathbf{x}_0$ is radiated. In the limit where $t \to \infty$ the electromagnetic wave will have moved an infinite distance from $\mathbf{x}_0$ and Eq. (4) is indeed the asymptotic solution. The difference between the result of Eq. (4) and the result illustrated in Fig. 1 is entirely due to the poles $\omega = \pm kc$. If a great many such test charges are superposed to simulate an infinite homogeneous plamsa as Lerche has done, it is clear that at any finite location there will be supraluminous contributions to the power spectrum that do not decay. If we do not neglect Ω, Ω^*, and consider plasma effects, the same kind of result is to be expected with some minor modifications due to the fact that electromagnetic waves have a different dispersion relation in a plasma. Thus it seems clear that supraluminous waves are due to impulsive acceleration inherent in the mathematical model of a moving test charge. The results obtained by Lerche are essentially correct, mathematically, but are physically unacceptable.

[1] I. Lerche, Phys. Fluids 11, 413 (1968) [Eqs. (9), (15), and (16)]. Note that Eq. (16) contains an error in that $\mathbf{E}_T(\mathbf{k}, \omega)$ should be in the direction $(\mathbf{k} \times \mathbf{v}_0) \times \mathbf{k}$ rather than $(\mathbf{k} \times \mathbf{v}_0)$. Also $\omega + \mathbf{k} \cdot \mathbf{v}_0$ should be replaced by $\omega - \mathbf{k} \cdot \mathbf{v}_0$.

[2] N. Rostoker, Nucl. Fusion 1, 101 (1960).

Reply to Comments by Norman Rostoker

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(Received 21 April 1969)

The errors in Lerche's paper[1] noted by Rostoker in his first footnote are indeed correct. But they are not the major issue in the question of the existence, or otherwise, of supraluminous waves.[2]

The basic problem is how one sets up the test particle approach to waves in a plasma. There appear to be several schools of thought on this problem. First there is the technique, exemplified by Klimontovich,[3] in which, for a *non*steady (in time) process, *all* effects are "switched on" instantaneously at time $t = 0$. For a *steady* process, *all* initial effects are carefully omitted and all the relevant integrals are run from time $t = -\infty$ to $t = T$ by *fiat*. This dichotomy of the steady-state versus nonsteady-state plasma raises many questions concerned with how one goes from $t = 0$ to $t = -\infty$ as a process "approaches" a steady-state level. It also raises the problem of what comprises a steady state and, starting from $t = 0$, how one knows that a steady state is being approached.

Second, there is a method suggested by Thompson,[4] in which all processes are switched on at time $t = 0$ but some collisional damping is introduced. Then if one introduced the damping everywhere, *all* waves damp and there is no persistent Cerenkov wake at $\omega = \mathbf{k} \cdot \mathbf{v}_0$, nor any persistent supraluminous waves at $\omega \cong \pm\omega_p$ since all effects die out as time progresses. It has been proposed that damping be included only in the supraluminous waves and not in the Cerenkov wake term, thus eliminating (on a long-time basis) the supraluminous waves and leaving only the Cerenkov wake. However, it can equally well be argued that damping can be introduced preferentially into the Cerenkov wake term and not into the supraluminous terms. This, of course, gives exactly the opposite long time behavior. On physical grounds there is no reason for