Title
Oscillations in Population Sizes - From ecology to history

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Peter Turchin's interesting paper "Dynamical Feedbacks between Population Growth and Sociopolitical Instability in Agrarian States" studies preindustrial population cycles. The author considers three case studies (England 1450-1800, pre-1000 C.E. China and Rome), and finds that there is a correlation between population growth and decay and oscillations in the level of warfare in all three cases. This is consistent with the author's enlightening demographic-structural theory, which in my opinion is an excellent explanation of the observed cycles. Indeed, as population grows and approaches the carrying capacity of land, its negative influence on real wages and general well-being is felt more and more strongly. Peasants leave their land and migrate to towns, inflation grows, civil upheavals increase in frequency and the state collapses, thus leading to a drastic decrease in the population size. Once the population is significantly decreased, the internal warfare subsides and the cycle starts over.

That the sociopolitical instability level and population sizes oscillate with a phase shift is demonstrated very clearly in the paper. The author uses an ingenious method to convert (necessarily qualitative) historical accounts on wars and rebellions into a smoothly changing quantitative measure of instability. This is plotted against the population size. The resulting graphs can be analyzed by a number of methods, but even a naked eye can detect a delay in the war frequency with respect to population waves.

What I believe is interesting to discuss further is the author's proposal that the population cycles are driven by an exogenous variable, an external "factor Z", which according to P. Turchin is the sociopolitical instability. Indeed, from theoretical ecology we know that a classical and very well studied system that models oscillations is the Lotka-Volterra predator-prey model, together with a myriad of its extensions and modifications. This class of models includes the variable of interest (say, the population numbers of the prey) and another, algebraically independent but dynamically coupled variable, the prey. The foxes feed on the rabbits thus driving their population down. Once there is a shortage of rabbits, foxes starve and their population decreases. Rabbits can multiply in the less dangerous environment, thus providing more food for the foxes, and the cycle begins again (I apologize to the sophisticated reader for this naive description of the Lotka-Volterra dynamics). It is clear that in the absence of foxes, the population of rabbits would not exhibit oscillatory behavior, so the predator is a necessary component to drive the system through the cycles.

Based on the spirit of Lotka-Volterra type systems, P.Turchin suggests that internal warfare takes the role of the foxes and serves as a necessary second component in fueling the cyclic behavior. Intuitively speaking however I find it difficult to understand how warfare, which is nothing but an unfortunate fact of human behavior, an "extended phenotype" of *homo sapiens*, can become an independent entity, take a life of its own and exert a prey-like pressure on the population.

I would like to offer an alternative mathematical interpretation of the demographic-structural theory which does not require internal warfare to be an exogenous variable. This interpretation also comes from ecology, and is based on a time-lag logistic differential equation,

$$\dot{x}(t) = rx(t)\left[1 - \frac{x(t-\tau)}{K}\right], \qquad (1)$$

where $x(t)$ is the population size, r is the linear growth rate of the population, K is the carrying capacity and τ is the time-delay. This equation was first introduced by Hutchinson [1], solved by Cunningham [2], and reviewed in the article by Wangersky [3], where many modifications of this equation are also cited. The two relevant factors to note about this equation is that (1) its solutions exhibit oscillatory behavior in a certain parameter regime and (2) it does not require an exogenous variable to drive the oscillations.

Let us discuss the meaning of various terms in this equation and reason whether this is a suitable starting point for modeling historical dynamics. The term $rx(t)$ on the right hand side describes the rise of the population to the level determined by the carrying capacity of land. As the population size increases, the growth is delayed by the nonlinear term proportional to $x(t-\tau)$, which is the population size τ years ago. Returning to the demographic-structural theory of P. Turchin, we note that the warfare and the resulting state collapse negatively regulate the population size as it reaches K. Warfare can be very trivially translated into the death rate: the more intense the sociopolitical instability, the more people die. Therefore, "internal warfare" can be simply related to the negative term in equation (1), which is a linear function of $x(t-\tau)$.

Why is the warfare defined by the population size τ years ago? In other words, where does the time-delay come from? Perhaps this is the inertia of military conflicts. A conflict will not subside immediately after the population size is reduced to a level below the carrying capacity. Even though there is already enough land for everybody to feed on, the war does not stop. It continues until it is resolved, because the reason for the war is not likely to be the issues of land and how to distribute it among the parties involved. Reasons for civil upheavals are largely political, and even though at their roots they are based on the scarcity of resources, they are too removed from them to be immediately resolved once the population has decreased below the critical level. Another factor worth mentioning is vengeance or retaliation/punishment which play an important role in military conflicts in many cultures. The point is that the level of warfare at time t (the present) does not reflect the demographic situation of today, but is rather determined by the population level (and its needs) in the past, which is expressed by $x(t - \tau)$.

We can conclude that equations of type (1) capture the inertial nature of military conflict and sociopolitical instability and have a potential to be a starting point for modeling population cycles.

It is interesting that the biological interpretation of equation (1) in application to animal populations is quite different. There, the time-delay is usually the reflection of the gestation time-lag (the time-lapse between fertilization and birth). Depending on the relative length of this delay, it can cause significant oscillations in the population levels of a single species with a constant food supply and no predators (see the famous example of the Australian sheep blowfly analyzed by R.May [4]).

Returning to the material presented in the paper of P. Turchin, we can easily interpret the graphs of figures 4, 6 and 7 in the framework of this theory. Indeed, the dashed lines (sociopolitical instability) is equivalent to the death rate of the population and is a linear function of $x(t - \tau)$. When plotted against $x(t)$ it will be shifted with respect to it by the amount τ. The cyclic behavior presented in figure 4(b) is nothing but the *phase-portrait* (a graph of x vs its velocity) of a one-component oscillatory system. Finally, mathematical model (1) is consistent with the author's statement in the Discussion section that "instability affects population via its growth rate".

To conclude, I would like to offer an alternative mathematical interpretation of the author's demographic-structural theory which can attempt to explain the interesting data presented in the paper. This model, while exhibiting oscillatory behavior, does not require an exogenous factor to drive the oscillations. According to demographic-structural theory, sociopolitical instability plays a key role in the wave-like dynamics of population. In terms of equation (1), sociopolitical instability, through its inertial nature, acts as the time-delayed death term. This is consistent with the author's interpretation of the warfare acting upon the rate of change of the population. At the same time, the frequency of war does not have to be interpreted as an exogenous prey-like identity. Instead, we could use its inertial nature, such as political rather than directly land-related reasons for warfare and the tendency for retribution in military conflicts. These result in a time-delay in the mathematical system for the population level, and drive its oscillatory dynamics. The rest of the details and factors can be added on to make the equation more realistic.

References

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