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MULTIPLE SCATTERING OF HEAVY IONS

Berkeley, California

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ABSTRACT

The multiple scattering distributions of 164-MeV O^{16} ions from Au, Ni, and Al foils and of 400-MeV A^{40} ions from Al and Zapon foils have been measured. These measurements are of interest because they involve large values (2.9 to 30.8) of the Born parameter a (which = $Z_1 Z_2/137\beta$). The 1/e widths found in this experiment are in complete agreement with those calculated from Molière's Theory and verify its accuracy to within 2% for large values of a. However, the accuracy is not sufficient to verify the detailed shape of the calculated distributions. The results of earlier experiments involving values of $a > 1$ have also been compared with Molière's theory. These results, with a few exceptions, agree to within 5 or 10% with calculations based on the theory, and are scattered about the theoretical values in such a way that no systematic differences can be inferred. Molière's theory agrees well with experiment for large values of a because it relies on a classical calculation of scattering to account for deviations from the Born approximation. The theory of Nigam, Sundaresan, and Wu, which uses a second Born approximation, overestimates the widths of the distributions by as much as 60%.

INTRODUCTION

The problem of calculating a multiple-scattering distribution involves two steps: first one must be able to calculate the single scattering distribution; second, one must then be able to calculate how this single scattering affects the propagation of a beam of particles through matter. The second problem has apparently been satisfactorily solved by the Wentzel-Molière method.[1] The first step--which contains all the physics of the problem--has been subject to much controversy.[2,3] In regions where the Born approximation is strictly applicable, there is general agreement as to what the single scattering distribution should be. The Born approximation is applicable when the "Born parameter," α (which = $Z_1 Z_2/137\beta$) is much less than one. In regions where α is much greater than one, there are large differences between the predictions of the Molière theory[1,4] and those of the later theory of Nigam, Sundaresan, and Wu (NSW).[2] The experimental results previously available are not adequate to decide clearly between these two theories. Furthermore, no one had surveyed these results with the purpose of examining the validity of these theories for large values of α.

Most of the experimental determinations of multiple scattering have been done with electrons or positrons.[5-13] Since an electron of appreciable range is relativistic, the value of α involved is $Z_2/137$, and thus never exceeds about 0.6. This value of α is not large enough to make a clear distinction between the two theories. Several measurements have been made with α = 1.1 to 2.5 of the multiple scattering of protons, π mesons, and μ mesons in nuclear emulsion.[14-17] Two early experiments[18,19] on the scattering of alpha particles from metal foils involve large values of α (up to 22.7), but these give contradictory results. We have compared the results of references 14-19 with Molière's theory and have found no

systematic discrepancies. These measurements do not, however, decisively favor Molière's theory over the NSW theory. In the experiment presented here we measure the multiple scattering of O^{16} and A^{40} ions in thin foils, with values of a from 2.9 to 30.8. The experimental 1/e widths are in excellent agreement with Molière's theory and verify its accuracy to within 2% for large values of a. The NSW theory, on the other hand, overestimates the width of the distributions by as much as 60%.

Molière's theory is inapplicable if the nucleus cannot be considered a point charge without nuclear forces. If this condition is not met one must use a nuclear form factor to describe the distribution of charge in the nucleus, and in addition, for strongly interacting particles, one must consider the effects of the nuclear force (reference 15, page 285). For strongly interacting particles, if the mean free path for nuclear collisions is much greater than the thickness of the scattering foil, then the nucleus can be considered a point charge. In this experiment it can always be so considered; for instance, for 164-MeV O^{16} ions penetrating a 1.84-mg/cm^2 Ni foil, the probability of a nuclear collision is $\approx 6\times10^{-6}$.

EXPERIMENT

The experimental arrangement is shown schematically in Fig. 1. We measure the distribution of X coordinates of the particle tracks on the emulsion plate. If the effects of finite slit size and imperfect collimation can be neglected, then the distribution of X coordinates is the projected scattering distribution in the small-angle approximation. It has been shown by Barkas (reference 15, page 248) that the projected and space distributions are equivalent in the sense that one may be derived from the other provided only that the scattering has cylindrical symmetry. This is true even when the small angle approximation is not assumed.

The beam from the Berkeley heavy-ion accelerator was collimated by the two slits labeled No. 1 and No. 2 in Fig. 1. The scattering foil was placed immediately behind slit No. 2. The entire beam is recorded by a 4×4-in. nuclear emulsion plate with its surface perpendicular to the beam, as shown in Fig. 1. Slit No. 2 was about 0.5 mm wide. We found that if too narrow collimation was attempted, the fraction of particles scattering from the edges of the slit was undesirably large. The distance of the emulsion plate from the scattering foil was adjusted to give a distribution of convenient size at the emulsions. Distances varying from 90 to 336 cm were used. A stripper foil, located at the exit of the accelerator, reduced the beam to a known distribution of charge states. The stripped O^{16} beam is 98% fully charged, with 2% charge-7 ions, and a negligible number with lower charges. The stripped argon beam consists of 52% charge-18 ions, 39% charge-17 ions, and 9% charge-16 ions.[20]

The energy of the beam was measured by observing track ranges in 1×3-in. glass-backed emulsions placed so that the beam entered the emulsion with a dip angle of 10 deg. No foreign or energy-degraded ions were found in the O^{16} ion beam. The A^{40} beam, however, was found to contain a large amount of contamination. This required that the scattering distributions for the A^{40} beam be measured with emulsions placed at a small angle to the beam so that background tracks could be eliminated by range and track-width comparisons. (These discriminatory procedures could not be applied effectively if the beam entered the emulsion plate normally.)

We measured distributions obtained with no scattering foil so that we could evaluate the effects of imperfect collimation and slit scattering; in all cases these were found to be negligible. A list of the foils and beam ions used, together with parameters of interest, which are defined in the results section, appears in Table I.

EMULSION SCANNING

The vertical tracks were counted visually under 1000X magnification. The projected scattering distribution was measured by counting the number of tracks in narrow strips, 10 to 100μ wide, extending beyond the limits of the distribution in the slit direction. The width and spacing of the scanned areas were adjusted to achieve the desired statistics.

A somewhat different procedure was used in scanning the argon plates. In these cases, the emulsion plate, inclined at 10 deg with respect to the beam, did not record the entire distribution of tracks. Therefore, a profile of the beam perpendicular to the slit was taken. This gives the projected distribution correctly, provided that the distance of the particle tracks from the slit image remains small compared with the slit height. For this reason the argon distributions were not extended to large angles.

RESULTS AND DISCUSSION

Figures 2 through 6 show the experimental projected scattering distributions. These distributions are normalized so that $\int_{-\infty}^{\infty} f(\theta)\, d\theta = 2$. The errors shown are due entirely to counting statistics; no other appreciable errors are believed to be present. The solid curves are calculated according to Molière's theory of multiple scattering. [Discussions of Molière's theory have been given by H. Bethe, Phys. Rev. 89, 1256 (1953), and by W. T. Scott, reference 4!] It should be pointed out that this theory contains no free parameters. Table I summarizes the results of this experiment and compares them with two multiple-scattering theories, that of Molière, and that of Nigam, Sundaresan, and Wu.[2] These are discussed in a later section.

The full width at 1/e times the peak value of the distribution is used for comparing the experimental values with theory. We determined the best experimental values of the 1/e widths by least-squares fitting a Gaussian to the central peak of the experimental distribution. Experimental points were included in this fit if they had values of $f(\theta)$ greater than 10% of the maximum value. This avoided including the non-Gaussian tail of the distribution in the analysis. The errors given in the widths are external errors, derived from the errors in the coefficients of the least-squares fit.

COMPARISON WITH THEORY

In order to calculate multiple scattering distributions, one needs to know cross sections for very small angles, where the effects of electronic screening are important. Because a multiple scattering distribution is the result of a large number of single collisions, it does not reflect the detailed shape of the single-scattering distribution involved. For this reason, it is possible to describe the effects of electronic screening--so far as multiple scattering is concerned--by a single parameter χ_a called the screening angle. This was shown explicitly by Molière.[1,4] The parameter χ_a is easily calculated in the Born approximation for simple potentials approximating a Fermi-Thomas atom. Nigam, Sundaresan, and Wu used the Dalitz formula (2) to calculate χ_a up to the second Born approximation. They obtain the result,

$$\frac{\chi_a^2}{\chi_0^2} = 1 + 4\alpha\chi_0\left[(1-\beta^2)\ln\chi_0 + 0.2310 + 1.448\beta^2\right], \qquad (1)$$

where $\chi_0^2 = \frac{\lambda}{r_0}$, and r_0 is the Fermi-Thomas radius, $r_0 = 0.889\, a_0 z^{-1/3}$.

Since the small-angle approximation[3] requires that $a\chi_0 << 1$, χ_a never differs very much from χ_0 according to the NSW calculation.

Molière's approach is to calculate χ_a in the limit of small a (Born approximation) and in the limit of large a (classical approximation), and to interpolate quadratically for intermediate values.[22] He obtains

$$\chi_a^2/\chi_0^2 = 1.13 + 3.76\, a^2. \qquad (2)$$

Thus, according to Molière's result, χ_a differs greatly from χ_0 for large values of a. It is noted that Molière's single-scattering formula [Eq. (8.4) of reference 4] does not enter into the determination of Eq. (2)--except insofar as it may verify the interpolation scheme used to obtain (2). Nigam, Sundaresan, and Wu are thus incorrect in trying to trace the discrepancy between (1) and (2) to errors in Molière's single-scattering formula. Furthermore, these authors apparently failed to observe that the so-called "correction term" in Eq. (2) (the second term) actually represents a correct classical calculation for χ_a and that it is not correct to make assumptions about its validity for $a >> 1$ on the basis of a disagreement between this and their Born-approximation result. It should be pointed out that the NSW calculations differ from Molière's in ways additional to the discrepancies between the χ_a's. The NSW theory contains new terms arising from relativistic effects and from spin-dependent terms. These are, however, all too small in our work to affect the results. Furthermore, NSW use a free parameter μ in the form of the scattering potential,

$$V(r) = (Z_1 Z_2 e^2/r)\exp(-\mu r/r_0). \qquad (3)$$

NSW introduce μ to account for the fact that the form of (3) does not exactly fit the Fermi-Thomas distribution. The quantity $\chi_\mu = \mu\chi_0$ then replaces χ_0 in the theory. Molière, on the other hand, uses a more complicated expression

than (3), and fits the parameters to the Fermi-Thomas potential, thus introducing no free parameters. The parameter μ has been calculated[3] to be 1.12, but NSW found that in order to fit experimental distributions of multiply scattered electrons, they needed to use a value of $\mu = 1.8$. We have, therefore, calculated the 1/e widths for $\mu = 1.2$ and 1.8.

In Table I we compare the 1/e widths calculated according to Molière's theory with those obtained in this experiment. The agreement is excellent. Values of the Born parameter a are also given in Table I. It is seen that the second (classical) term of Eq. (2) is dominant in all cases, and the validity of this term is well demonstrated. We also list in Table I the 1/e widths according to the NSW theory to show that the Born approximation greatly overestimates the widths of these distributions. The theoretical curves for the NSW theory were calculated as outlined by Scott,[3] and his tabulated values of the NSW "D" functions were used. We included all terms of the expansion series for which the D functions were tabulated. The calculations for the Molière theory were performed as outlined by Molière,[1] using his tabulated values of the "f" functions.

COMPARISON WITH MOLIÈRE'S THEORY OF OTHER EXPERIMENTS THAT INVOLVE LARGE VALUES OF a

As mentioned earlier, the only other measurements of multiple scattering through foils that involve large values of a known to us are those of Geiger[18] and of Mayer.[19] These experiments were discussed by Williams[23] to show that a classical calculation of the multiple scattering distributions was appropriate to these cases. Geiger measured the most probable value of the spatial scattering angle for alpha particles, and Mayer measured the average projected angle of scattering. Both authors reduced their results to a foil thickness equivalent in stopping to 1 cm of air.

We calculated the foil thicknesses using the range-energy tables of Bichsel.[24] Tables II and III show the comparison between these experimental results and calculations according to Molière's theory. The results of Geiger are consistently higher than the calculations of Molière's theory by 5 to 10%. The results of Mayer are rather scattered about the theoretical values.

Four measurements[14-17] of the multiple scattering of protons and π and μ mesons in nuclear emulsion involve values of $a > 1$. These cases do not involve well-defined values of a, but we may give approximate values by assigning to Z the value for silver, which comprises about one half of the emulsion by weight. These measurements are taken along the particle's trajectory, and thus do not involve a definite value for β. We take an average value of β^2 over the range interval measured to arrive at a value of a for the purpose of judging how badly the Born approximation criteria are violated. Thus

$$a \approx \frac{47}{137 \langle \beta^2 \rangle} .$$

It should be emphasized that the calculational procedure for obtaining the theoretical results listed in Table IV are perfectly well defined even though we cannot give a definite value to a. Table IV summarizes the results of these emulsion measurements. The theoretical values for references 14 and 17 were calculated by using the relation of Voyvadic and Pickup,[12] which is derived from the Williams' theory with the employment of Molière's "γ" factor, rather than for Molière's theory itself.[25] This should make no difference in these comparisons, since Voyvadic and Pickup state that the results of this procedure agree to within 1% with results calculated entirely by Molière's theory. The agreement with theory is generally good. Two facts should be kept in mind in evaluating these results. First, the parameters measured

in an emulsion or cloud chamber experiment are not simply connected to the multiple scattering distribution, and a great amount of interpretation, involving various approximations, must be made before the results can be compared with multiple scattering theory. Second, errors other than statistical errors are generally present, and are not easily detected. These errors in general cause one to overestimate the scattering.

Finally, we summarize in Fig. 7 the results of this experiment and all other experiments known to us which involve values of $a > 1$. Against a we plot the percent disagreement of the experimental results with those calculated according to Molière's theory:

$$\Delta \text{ (percent)} = \frac{V \text{ (exptl)} - V \text{ (Molière)}}{V \text{ (Molière)}} \times 100,$$

where V stands for the quantity measured in the experiment (1/e width, "scattering constant," etc.).

ACKNOWLEDGMENTS

I would like to thank Dr. Barkas for his many helpful suggestions and criticisms, and Mr. Ridgway Banks who did most of the emulsion scanning.

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Table I. Summary of measurements done in this experiment and comparison with the results of Molière and of Nigam, Sundaresan, and Wu. (Angles are in degrees.)

Projectile	O^{16}	O^{16}	O^{16}	A^{40}	A^{40}
Target	Al	Ni	Au	Al	Zapon
Z, Projectile	8	8	8	18	18
Z, Target	13	28	79	13	-
Mass, projectile	16.0	16.0	16.0	39.94	39.94
Mass, target nucleus	26.98	58.71	197.0	26.98	-
Thickness (mg/cm^2)	1.74	1.83	3.12	0.320	0.100
a	5.07	10.9	30.8	11.4	2.9[a]
χ_a Molière	2.53×10^{-3}	6.99×10^{-3}	2.79×10^{-2}	3.53×10^{-3}	
χ_a, NSW, μ=1.12	2.86×10^{-4}	3.70×10^{-4}	5.20×10^{-4}	1.79×10^{-4}	
χ_a, NSW, μ=1.8	4.61×10^{-4}	5.94×10^{-4}	8.33×10^{-4}	2.87×10^{-4}	
$W_{1/e}$, μ=1.12	0.416	0.579	1.335	0.142	
$W_{1/e}$, μ=1.8	0.346	0.560	1.198	0.136	
$W_{1/e}$, Molière	0.339	0.446	0.751	0.108	0.052
$W_{1/e}$, exptl	0.339±0.006	0.445±0.009	0.739±0.017	0.109±0.005	.051±.001

[a] a is not well defined for a mixture of elements; we have assigned the value for carbon to this case. This is for comparison only; the calculational procedure does not involve this arbitrariness.

Table II. Comparison of Geiger's results for the most probable scattering angle with the results of Molière's theory. The alpha-particle energy was 6.26 MeV.

Element	Foil thickness (mg/cm^2)	a	Scattering angle (deg)	
			Observed	Molière
Au	0.386	19.9	2.1	1.96
Sn	0.277	12.6	1.5	1.43
Ag	0.267	11.8	1.5	1.40
Cu	0.206	7.3	1.1	1.05
Al	0.146	3.3	0.6	0.67

Table III. Comparison of Mayer's results for the average projected scattering angle with the results of Molière's theory. The alpha particle energy was 4.84 MeV.

Element	Foil thickness (mg/cm^2)	a	Scattering angle (deg)	
			Observed	Molière
Au	0.386	22.7	1.72	2.40
Pt	0.377	22.4	1.85	2.35
Ag	0.267	13.5	2.00	1.69
Cu	0.206	8.3	1.36	1.25
Al	0.146	3.7	0.93	0.80

Table IV. Comparison of multiple scattering measurements made in nuclear emulsion with Molière's theory.

Ref.	Particle	Energy (MeV)	$\langle \beta^2 \rangle$	a	Molière	Experimental
14	proton and μ Mesons	5–50	0.068	1.32	$K_c^{(a)} = 25.6$	26.1 ±0.7
14	protons	9–35	0.020	2.45	$K_c = 25.9$	27.5 ±0.5
15	μ Mesons	0–4.5	0.040	1.72	$K_0^{(b)} = 0.146$	0.149 ±0.008[c]
16	protons	0–55	0.062	1.37	$K_0 = 0.117$	0.105 ±0.005
17	π Mesons	3.1–7.2	0.076	1.24	$K = 26.3$	25.1 ±0.42[d]
17	μ Mesons	2.8–6.1	0.087	1.16	$K = 26.3$	25.8 ±0.42[d]

(a) For a definition of the scattering factor K_c and K, see reference 15, pages 294 - 296.

(b) For a definition of the scattering factor K_0, see reference 15, page 326. The calculations of K_0 from Molière's theory are taken from Fig. 24 of reference 16. The μ-meson results reported in reference 15 were obtained without a cutoff angle. The measurements in reference 16 were done with a cutoff angle of 4°.

(c) This value differs from the one given in reference 15. The original calculations used R instead of R_p in Eq. (8.11.7) of reference 15. This correction was suggested by Dr. Barkas.

(d) We have averaged the values for plus and minus particles, since no difference could be detected between them.

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22. The classical nature of the second term in Eq. (2) becomes clear if one examines article 9 of Molière paper (reference 4) and earlier sections referred to there. The following simple argument shows that the difference between χ_a for the Born and classical cases is properly represented by Eq. (2): χ_a is roughly equal to that angle through which a particle scatters if it passes an atom with impact parameter equal to the screening radius. The scattering angle for a pure Coulomb field in Born approximation is equal to $\lambda\!\!\!^{-}/b$, where b is the impact parameter. In the classical case, the scattering angle is $Z_1Z_2/(b \cdot 1/2\ mV^2)$. Thus the value of χ_a should differ in the Born and classical approximations by the factor $(2Z_1Z_2/bmV^2) \cdot bmV/\hbar = 2a$. A quadratic interpolation then gives $\chi_a^2/\chi_0^2 = 1 + 4a^2$, which is approximately equal to Molière's expression for χ_a.

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25. There is an error in the presentation of this relation Eq. (17), in reference 12. The factor $(1/\beta^2 + 0.30)^{-1}$ should multiply the argument of the logarithm. This same error occurs in reference 15.

FIGURE CAPTIONS

1. Schematic diagram of the experimental arrangement.
2. Multiple scattering distribution for 164-MeV O^{16} ions scattered from a Ni foil 1.827 mg/cm^2 thick. The solid curve is calculated according to Molière's theory.
3. Multiple scattering distribution for 164-MeV O^{16} ions scattered from an Au foil 3.12 mg/cm^2 thick. The solid curve is calculated according to Molière's theory.
4. Multiple scattering distribution for 164-MeV O^{16} ions scattered from an Al foil 1.74 mg/cm^2 thick. The solid curve is calculated according to Molière's theory.
5. Multiple scattering distribution for 418-MeV A^{40} ions scattered from an Al foil 0.32 mg/cm^2 thick. The solid curve is calculated according to Molière's theory.
6. Multiple scattering distribution of 418-MeV A^{40} scattered from a Zapon (plastic) foil 0.1 mg/cm^2 thick. The solid curve is calculated according to Molière's theory.
7. Comparison of multiple scattering measurements involving values of $a > 1$ with Molière's theory. See text for definition of the ordinate. △, data from Geiger (reference 18); ▲, data from Mayer (reference 19); ■, data from Gottstein (reference 14); □, data from Hughes and Sinclair (reference 10); ▽, data from Simon (reference 16); ▼, data from Barkas (reference 15); ●, data from this experiment.

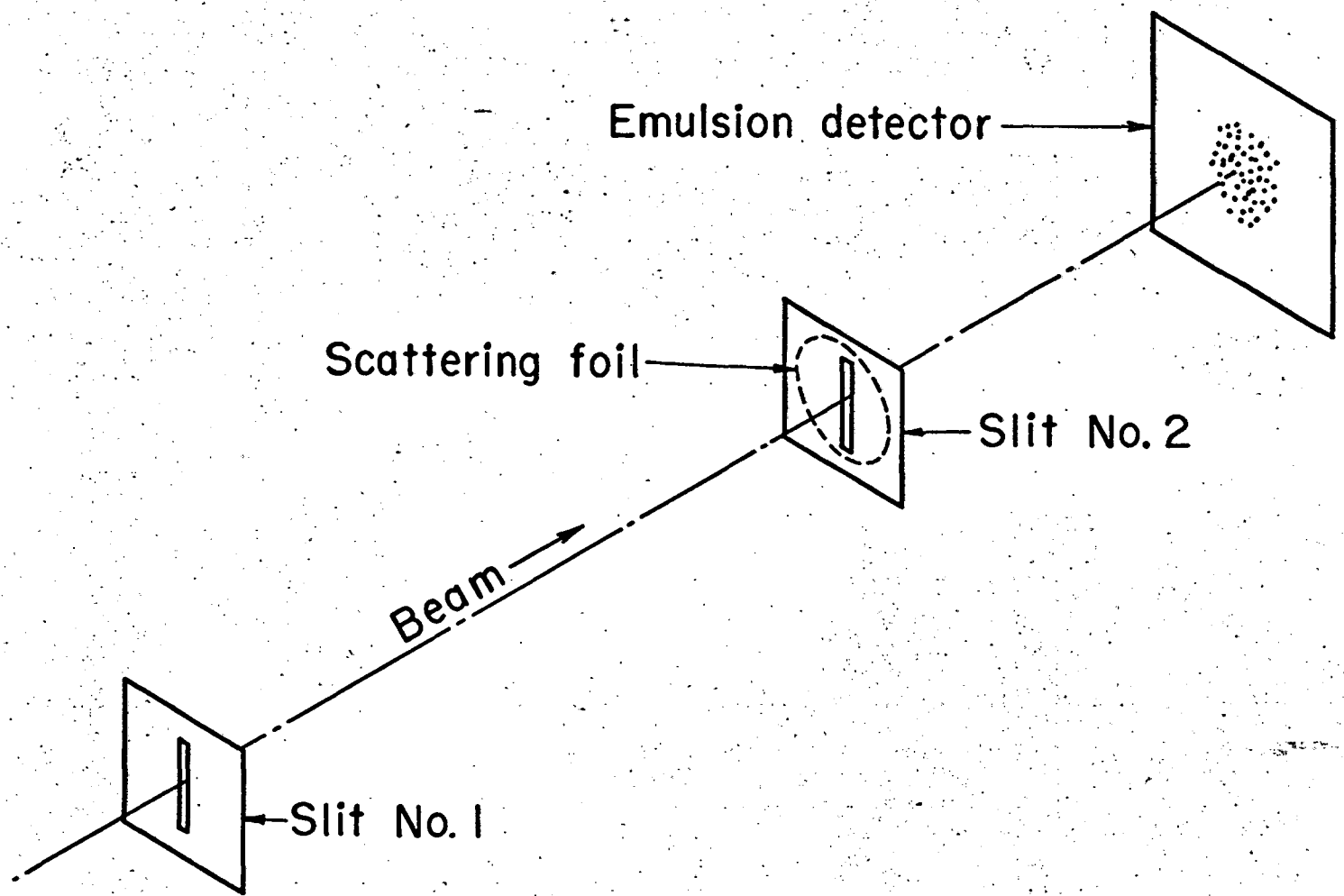


Fig. 1

MU-33054

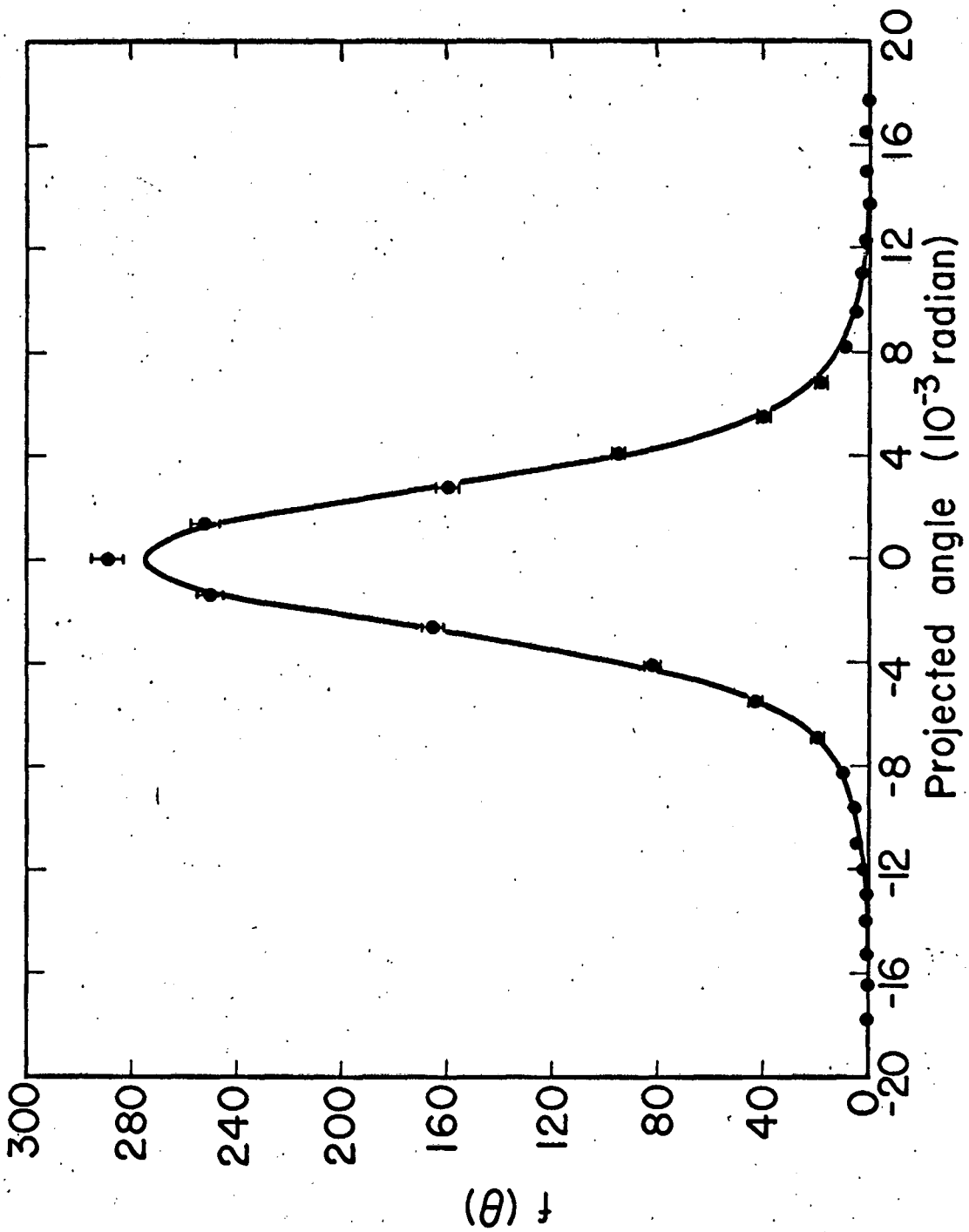

f (θ)
300
280
240
200
160
120
80
40
0
-20
-16
-12
-8
-4
0
4
8
12
16
20
Projected angle (10⁻³ radian)
MU-33030

Fig. 2

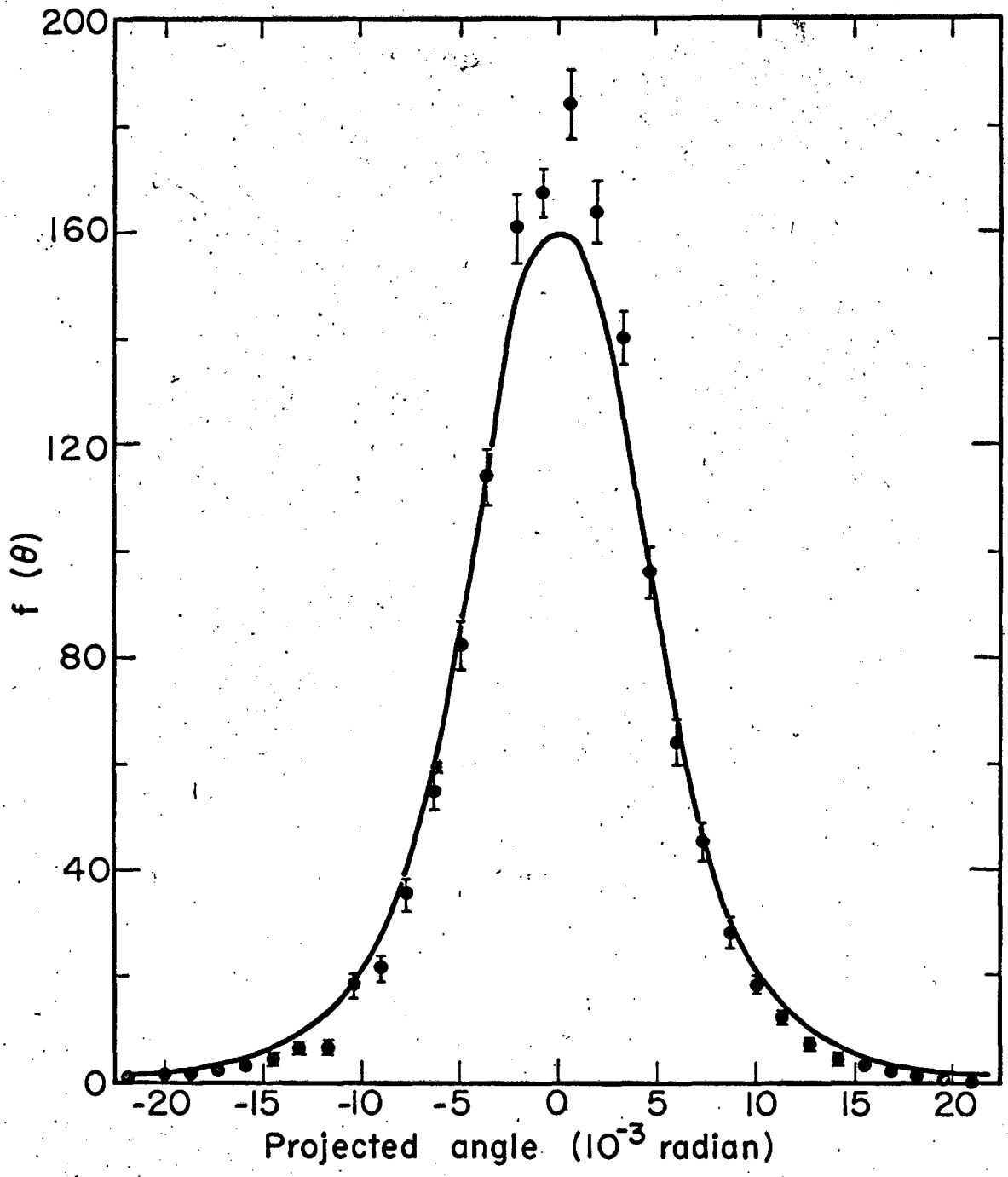


Fig. 3

MU-33052

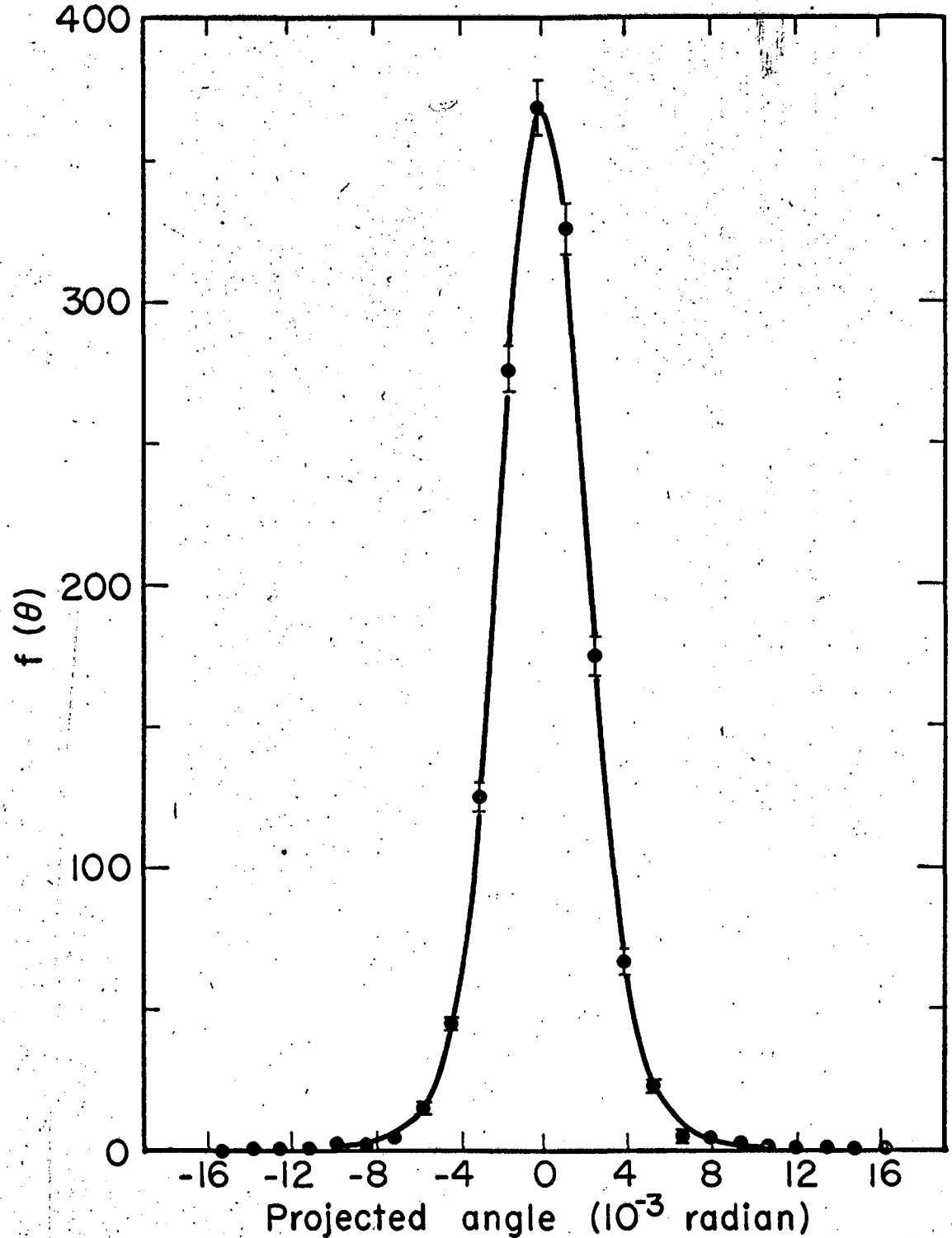
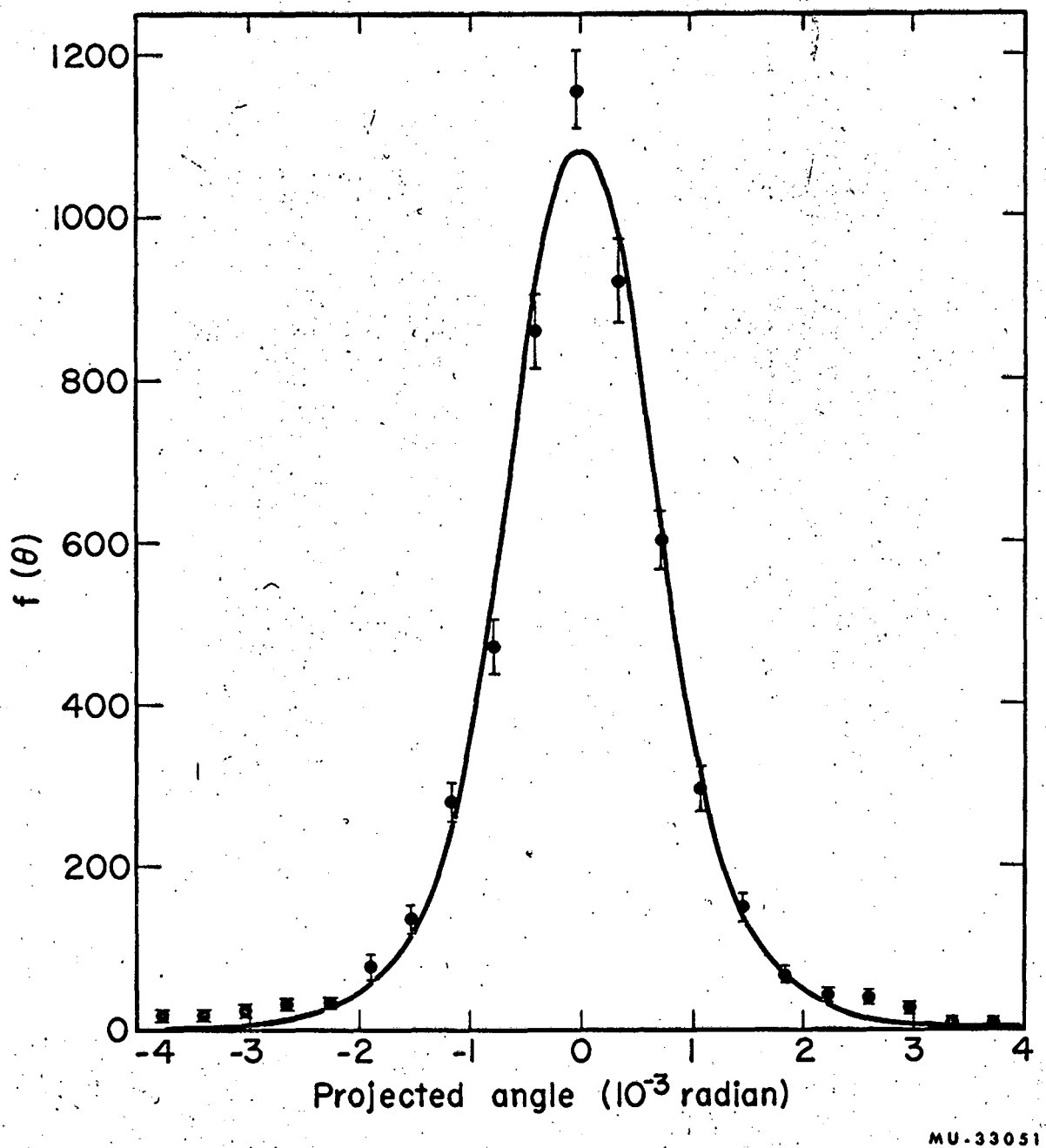


Fig. 4

MU-33053


1200
1000
800
600
400
200
0
f (θ)
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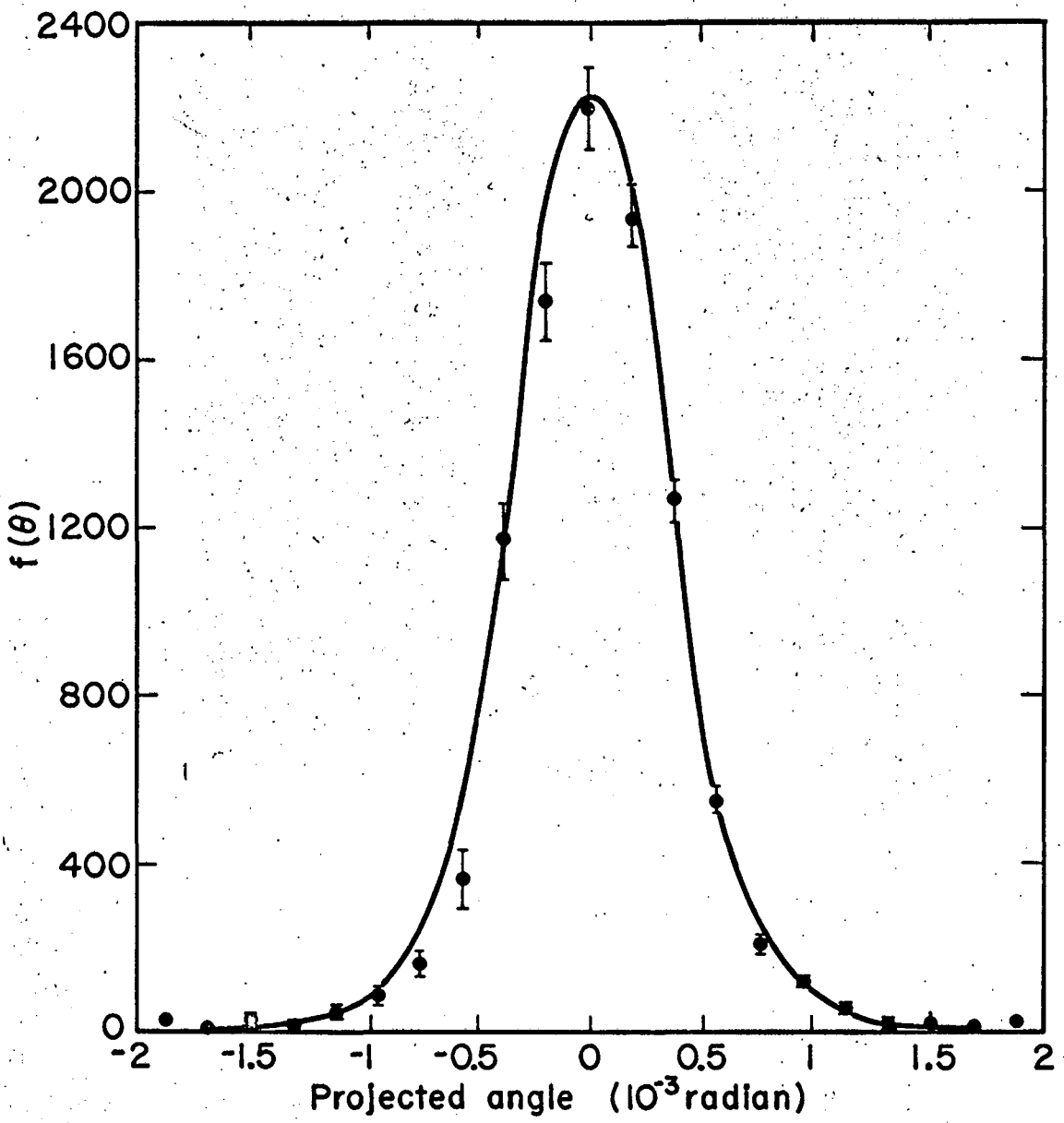
-3
-2
-1
0
1
2
3
4
Projected angle (10⁻³ radian)
MU-33051

Fig. 5



MU-33049

Fig. 6

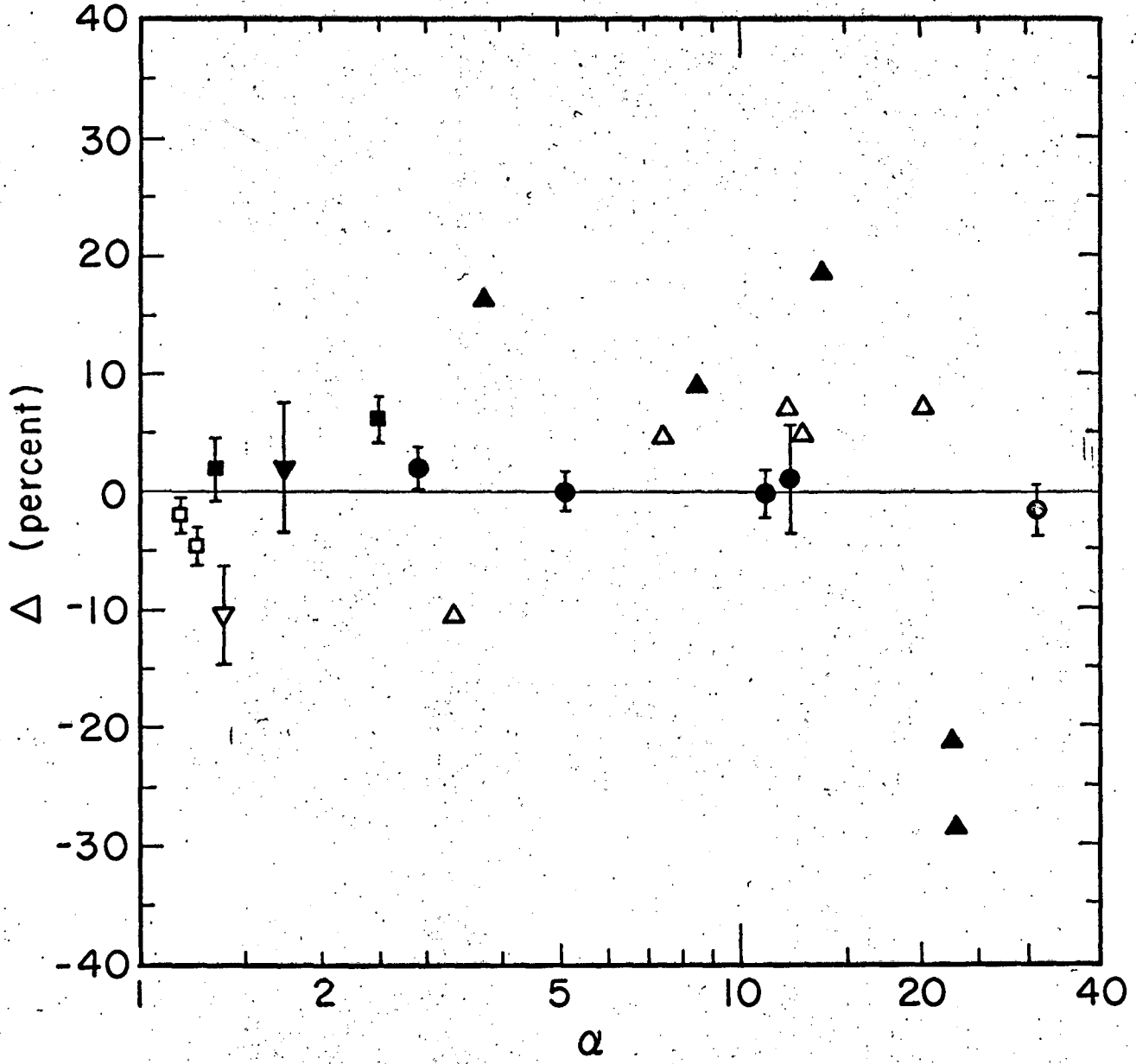


Fig. 7

MUB-3031