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COLOR ZITTERBEWEGUNG

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COLOR ZITTERBEWEGUNG

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DISCLAIMER

II. WHY THE WILSON LOOP INTEGRAL?

Halpern, Jevicki, and Senjanović have shown how the "classical quark particle" couples to quantum chromodynamics using a particle dynamics description of field theory.[3,4] I will amplify on this and the connection between Wilson loop integrals and Green's functions. For simplicity scalar quarks will be used.

Mesons consist of an SU(3)-color quark and antiquark. Gauge invariant vectors occurring in such a state are of the form $\phi_\alpha(x)\,\phi_\alpha^\dagger(x)\Omega$ or more generally

$$\phi_\alpha^\dagger(x)\left[P\exp\left(ig\int_x^y A\cdot d\ell\right)\right]_{\alpha\beta}\phi_\beta(y)\Omega\,, \qquad (2.1)$$

or some linear combination of the above folded into wave functions. Ω is the physical vacuum. P stands for path ordered product. Analyzing mesons is thus reduced to analyzing the objects of Eq. (2.1). The propagator of two of these is

$$\left\langle\Omega\middle|T\left\{\phi_\alpha(x')\left[P\exp\left(-ig\int_{x'}^{y'}A\cdot d\ell\right)\right]_{\alpha\beta}\phi_\beta^\dagger(y')\right.\right.$$
$$\left.\left.\times\,\phi_\gamma^\dagger(x)\left[P\exp\left(ig\int_x^y A\cdot d\ell\right)\right]_{\gamma\delta}\phi_\delta(y)\right\}\middle|\Omega\right\rangle. \qquad (2.2)$$

Using particle dynamics, neglecting quark-antiquark vacuum loops, and omitting the annihilation graph of Figure 1a, Eq. (2.2) is

$$\left\langle\Omega_G\middle|\int_0^\infty d\sigma\int_0^\infty d\tau\int_{y(0)=y}^{y(\sigma)=y'}\mathcal{D}y\int_{x(0)=x'}^{x(\tau)=x}\mathcal{D}x\right.$$
$$\times\exp\left\{i\int_0^\sigma\left(\frac{\dot y^2}{4}-m^2\right)+i\int_0^\tau\left(\frac{\dot x^2}{4}-m^2\right)\right\}$$
$$\left.\times\,\mathrm{Tr}\left[P\exp\left(ig\oint A\cdot d\ell\right)\right]\middle|\Omega_G\right\rangle. \qquad (2.3)$$

Ω_G is the vacuum for a pure Yang-Mills theory. The path in the path-ordered product is illustrated in Fig. 1b: it goes from y to y' along $y(\sigma)$, from y' to x', from x' to x along $x(\tau)$, and finally from x to y. The paths from y' to x' and x to y are the ones given in the wave function of Eq. (2.2).

The relevant factor for mesons is $\langle\Omega_G|\mathrm{Tr}\left[P\exp(ig\oint A\cdot d\ell)\right]|\Omega_G\rangle$. This provides a direct link between the meson propagator and Wilson loop integrals. For baryons one proceeds analogously. The relevant factor is

$$\left\langle\Omega_G\middle|\epsilon_{\alpha\beta\gamma}\,\epsilon_{\lambda\mu\nu}\left[P\exp\left(ig\int_{P_1}A\cdot d\ell\right)\right]_{\alpha\lambda}\right.$$
$$\times\left[P\exp\left(ig\int_{P_2}A\cdot d\ell\right)\right]_{\beta\mu}$$
$$\left.\times\left[P\exp\left(ig\int_{P_3}A\cdot d\ell\right)\right]_{\gamma\nu}\middle|\Omega_G\right\rangle. \qquad (2.4)$$

COLOR ZITTERBEWEGUNG*

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ABSTRACT

I discuss the proper procedure for a classical evaluation of a non-Abelian Wilson loop integral. The naive procedure is incorrect due to fluctuations in color, a phenomenon I call color zitterbewegung. A formalism for circumventing this complication is proposed. It is likely that the statistical mechanics of disorder theory is necessary in Wilson loop calculations.

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I. INTRODUCTION

The Wilson loop integral[1] is the principal test for quark confinement. More importantly, loop integrals enter directly in calculating quark Green's functions, as shown in Section II. A thorough knowledge of such integrals is therefore important. In an Abelian theory, one understands completely how to apply the test and how it leads to the charge-anticharge static potential. For non-Abelian theories, one assumes a straightforward generalization: replace $\exp[ie\oint A\cdot d\ell]$ by the trace of the analogous path ordered product. Its vacuum expectation value gives the quark-antiquark static potential. This paper will show that the evaluation procedure for non-Abelian theories differs vastly from the $U(1)$ case. This can be seen by examining a few situations. In fact, the naive procedure leads to bizarre results, as demonstrated in Section III. What is the proper procedure? I present an ansatz in Section IV, and, in doing so, uncover an effect which drastically contrasts non-Abelian gauge theories from their Abelian counterparts. I call it color zitterbewegung. An understanding of this phenomenon is vital for an understanding of color in strong interactions.

This paper is closely related to the problem of defining classical chromodynamics which has been examined by several authors.[2] The fact that these authors are forced into sophisticated approaches is probably due to color zitterbewegung and probably indicates a tacit awareness of the phenomenon. Actually, these authors may be providing different ways of dealing with this phenomenon. In this paper I would like to make this phenomenon manifest and suggest an alternative avenue of attack.

The p_i are made up of three parts as indicated in Fig. 2. For example, p_1 consists of an initial state path $p_1{}^{o}$, appearing in the initial baryon wavefunction; a piece $p_1(x_1(\tau_1))$ over which one integrates, and a final state path $p_1{}^{f}$. $p_1{}^{o}$ and $p_1{}^{f}$ occur in the Green's function to be evaluated.

The quark confinement criterion is

$$\left\langle \mathrm{Tr}\left[P \exp(ig \oint A\cdot d\ell)\right] \right\rangle \sim \exp[-C(\text{area})] \,, \tag{2.5}$$

where the loop encompasses a large area. Contrary to one's intuition, the evaluation of (2.5) is tricky. The naive evaluations of the next section lead to strange consequences.

III. EQUATION OF MOTION FOR THE GAUGE FIELDS IN THE PRESENCE OF A WILSON LOOP

The equations of motion are easily obtained for $A_\mu{}^{\gamma}$ in the presence of a Wilson loop. Parametrize the curve by the variable, τ, so that $x(\tau)$ traces out the path in space-time. The equation of motion is

$$\mathrm{Tr}\left[P \exp(ig \int_0^1 A\cdot\dot{x}\, d\tau)\right] D^\mu F^\gamma_{\mu\nu} = g \int_0^1 d\tau\, \dot{x}_\nu\, \delta^4(x - x(\tau))$$
$$\times \mathrm{Tr}\left\{ \left[P \exp(ig \int_0^\tau A\cdot\dot{x}\, d\tau)\right] \frac{\lambda^\gamma}{2} \times \left[P \exp(ig \int_\tau^1 A\cdot\dot{x}\, d\tau)\right]\right\} . \tag{3.1}$$

λ^α are the Gell-Mann matrices of SU(3). Equation (3.1) has an unusual property. $\left[P \exp(ig \int_0^1 A\cdot\dot{x}\, d\tau)\right]$ is an element of SU(3),

hence of the form $\exp(i\, \vec{\alpha}\cdot\frac{\vec{\lambda}}{2})$. When substituted into Eq. (3.1) one gets a complex current source! For SU(2) color, for example, the current is pure imaginary. What is happening? Such a result would be horrendous for unified models of weak and electromagnetic interactions since the derivation of Eq. (3.1) holds for the generators surviving spontaneous symmetry breaking. The photon would couple to a complex current. What has gone wrong? Consider the simple situation where $A(x)$ is restricted to be proportional to λ^3. Then

$$D^\mu F^\gamma_{\mu\nu} = \frac{\delta_{\gamma 3}}{2}\, g \int_0^1 d\tau\, \dot{x}_\nu\, \delta^4\big(x - x(\tau)\big)$$
$$\times \left[\exp(ig \int_0^1 A^3\cdot\dot{x}\, d\tau) - \exp(-ig \int_0^1 A^3\cdot\dot{x}\, d\tau)\right]$$
$$\times 1\Big/\left[1 + \exp(ig \int_0^1 A^3\cdot\dot{x}\, d\tau) + \exp(-ig \int_0^1 A^3\cdot\dot{x}\, d\tau)\right] . \tag{3.2}$$

From Eq. (3.2), it is clear one has averaged over the three charge states of λ^3. If instead of summing over these three terms, one restricts oneself to say the 11 term (i.e., $\mathrm{Tr}\, M = \sum_\alpha M_{\alpha\alpha} \to M_{11}$), then one obtains a respectable current. In this simple example, it is clear what one should do. Replace

$$\left\langle \mathrm{Tr}\left[P \exp(ig \int A\cdot d\ell)\right] \right\rangle \text{ by } \sum_{\alpha=1}^{3} \left\langle \left[P \exp(ig \int A\cdot d\ell)\right]_{\alpha\alpha} \right\rangle , \tag{3.3}$$

calculate three equations of motion, determine the appropriate classical solutions, and then add the contributions. Only in this manner can one obtain the classical (or semiclassical) approximation. One must calculate saddle points before taking traces. Of course, if one knew the functional integrals over A exactly, the distinction between the above two procedures would disappear, however, when doing perturbation theory or semiclassical methods one is expanding about a saddle point: the above distinction is essential. Does the solution for this simple case work in general? The answer is no and leads one to the concept of color zitterbewegung.

IV. COLOR ZITTERBEWEGUNG

The replacement [Eq. (3.3)] fails in non-trivial cases. There is a physical reason for this. Color may flicker (i.e. sudden change from one color to another) at some point along a curve because of colored gluons. These gluons can carry color from one point on the curve to another. In general, color will be changing rapidly so that Wilson loops present a very disordered physical process similar to the one dimensional Ising model. In this respect, non-Abelian gauge theories differ greatly from Abelian ones.

To illustrate how the replacement given by Eq. (3.3) fails consider Fig. 3. Let x_o be the starting and ending point of the path ordered product occurring in Eq. (3.3). Let y be some intermediate point and let σ be the corresponding value of τ, that is, $x(\sigma) = y$. Break up the path ordered product into two products:

$$\left[P \exp ig \oint_{x_o}^{x_o} A\cdot d\ell\right]_{\alpha\alpha} = \sum_{\beta} \left[P \exp ig \int_{x_o}^{y} A\cdot d\ell\right]_{\alpha\beta} \times \left[P \exp ig \int_{y}^{x_o} A\cdot d\ell\right]_{\beta\alpha} \qquad (\alpha \text{ not summed}). \tag{4.1}$$

Terms for which $\beta \neq \alpha$ can be considered as processes where color has changed from α to β between the points x_o and y. The saddle point for a particular α is

$$\left[P \exp ig \int_0^1 \dot{x}\cdot A\, d\tau\right]_{\alpha\alpha} D^{\mu} F^{\gamma}_{\mu\nu}$$

$$= \sum_{\beta,\delta,\eta} \Bigg\{ g \int_0^{\sigma} d\tau\, \dot{x}_{\nu}\, \delta^4(x - x(\tau)) \left[P \exp ig \int_0^{\tau} A\cdot\dot{x}\, d\tau\right]_{\alpha\delta} \left(\frac{\lambda^{\gamma}}{2}\right)_{\delta\eta} \times \left[P \exp ig \int_{\tau}^{\sigma} A\cdot\dot{x}\, d\tau\right]_{\eta\beta} \left[P \exp ig \int_{\sigma}^{1} A\cdot\dot{x}\, d\tau\right]_{\beta\alpha}$$

$$+ g \int_{\sigma}^{1} d\tau\, \dot{x}_{\nu}\, \delta^4(x - x(\tau)) \left[P \exp ig \int_0^{\sigma} A\cdot\dot{x}\, d\tau\right]_{\alpha\beta} \times \left[P \exp ig \int_{\sigma}^{\tau} A\cdot\dot{x}\, d\tau\right]_{\beta\delta} \left(\frac{\lambda^{\gamma}}{2}\right)_{\delta\eta} \left[P \exp ig \int_{\tau}^{1} A\cdot\dot{x}\, d\tau\right]_{\eta\alpha} \Bigg\} \qquad (\alpha \text{ not summed}). \tag{4.2}$$

Equation (4.2), like Eq. (3.2), can be interpreted as an averaging over colors, β, at the point y. Like Eq. (3.2) it can lead to complex currents. The solution to this problem is the same: pull the summation over β outside, evaluate each of the three saddle points, and then add their contributions.

In general, color zitterbewegung may occur anywhere along the curve. For a lattice approximating segmented curve with N products, the proper procedure is to calculate the saddle point for a given configuration of color and sum over all possibilities. To explicitly write this becomes cumbersome and I leave it as an exercise. There would be 3^N saddle point contributions for SU(3). Contrast this with a U(1) theory where only $1^N = 1$ contribution occurs. There will be an enhancement compared to the Abelian case. Perturbation theory about the trivial vacuum $A = 0$ would be suppressed compared to non-zero A since, writing $\exp(ig\, A\cdot\Delta x) \approx 1 + ig\, A\cdot\Delta x$, there would be only a single zero'th order term (in g), N first order terms, etc., compared to 3^N terms for a random A.

A lattice approximation is unsatisfactory from a calculational point of view because N must go to infinity and summations become awkward to do. There is, however, a formalism which neatly takes the limit $N \to \infty$ and performs the summations automatically. It uses anticommuting variables. One may write[4,5]

$$\mathrm{Tr}\left[P \exp ig \int_0^1 A\cdot\dot{x}\, d\tau\right] = \sum_\alpha \iint \mathcal{D}\eta^\dagger \mathcal{D}\eta\ \eta_\alpha^\dagger(0)$$

$$\times \exp\left\{\int_0^1 \eta(\tau^-)\dot{\eta}^\dagger(\tau)d\tau + ig\int_0^1 \eta(\tau^-)\, A\cdot\dot{x}\ \eta^\dagger(\tau)dt\right\} \eta_\alpha(1)\ . \tag{4.3}$$

Equation (4.3) is a functional integral over anticommuting variables. At each τ there are three η_α's and three $\eta_\alpha^\dagger$'s $(\alpha = 1, 2, 3)$. Summation in (4.3) is implied (that is, $\eta(\tau^-)\, A\, \eta^\dagger(\tau) = \sum_{\alpha\beta} \eta_\alpha(\tau^-)\, A_{\alpha\beta}\, \eta_\beta^\dagger(\tau)$). τ^- means $\tau-\epsilon$: the anticommuting variables are not multiplied at the same τ point.

Take the vacuum expectation value of (4.3) and extract the anticommuting functional integral:

$$\left\langle \mathrm{Tr}\left[P \exp(ig \int A\cdot d\ell)\right]\right\rangle = \sum_\alpha \iint \mathcal{D}\eta^\dagger \mathcal{D}\eta\ \ \eta_\alpha^\dagger(0)\, \eta_\alpha(1)$$

$$\times \exp\left(\int_0^1 \eta(\tau^-)\, \dot{\eta}^\dagger(\tau)d\tau\right)$$

$$\times \left\langle \exp\left(ig \int_0^1 \eta(\tau^-)\, A\cdot\dot{x}\ \eta^\dagger(\tau)\, d\tau\right)\right\rangle\ . \tag{4.4}$$

The equation of motion for A becomes

$$D^\mu F_{\mu\nu}^\gamma = g\int d\tau\, \dot{x}_\nu\, \eta(\tau^-)\, \frac{\lambda^\gamma}{2}\, \eta^\dagger(\tau)\, \delta^4(x - x(\tau))\ . \tag{4.5}$$

The proper procedure is to first solve Eq. (4.7) treating the anticommuting variables as c-number sources. This will give a saddle point contribution to $\langle \exp(ig \int_0^1 \eta(\tau^-)\, A\cdot\dot{x}\ \eta^\dagger(\tau)d\tau)\rangle$ yielding an effection action for the η's. The integral over anticommuting variables must then be solved. One must proceed in the prescribed order. It is not possible to evaluate the η integral first. This procedure is the way to obtain the correct quark-antiquark potential.

Many problems in statistical mechanics can be expressed in terms of anticommuting functional integrals.[6] After obtaining the effective anticommuting variable action, it may be possible to relate (4.4) to a one dimensional statistical mechanics problem. One expects long-ranged interactions since gauge potentials "connect" distant τ points. One also expects a disorder phenomenon corresponding to zitterbewegung in color. Equation (4.4) is thus probably related to a dimer or Ising model with long-ranged interactions.

V. CONCLUSION

Color zitterbewegung is a new phenomenon in non-Abelian gauge theories. Dynamically it will result in color bremsstrahlng. In QED a charged particle must accelerate to emit radiation. In QCD a charged particle need not accelerate since a sudden change in color produces a sudden change in the current. This leads to the emission of soft gluons. The dynamical behavior of color bremsstrahlung will thus determine the long-ranged zitterbewegung force. It is possible that such a force can lead to confinement. The next step will be to obtain numerical estimates of this force.

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FOOTNOTES AND REFERENCES

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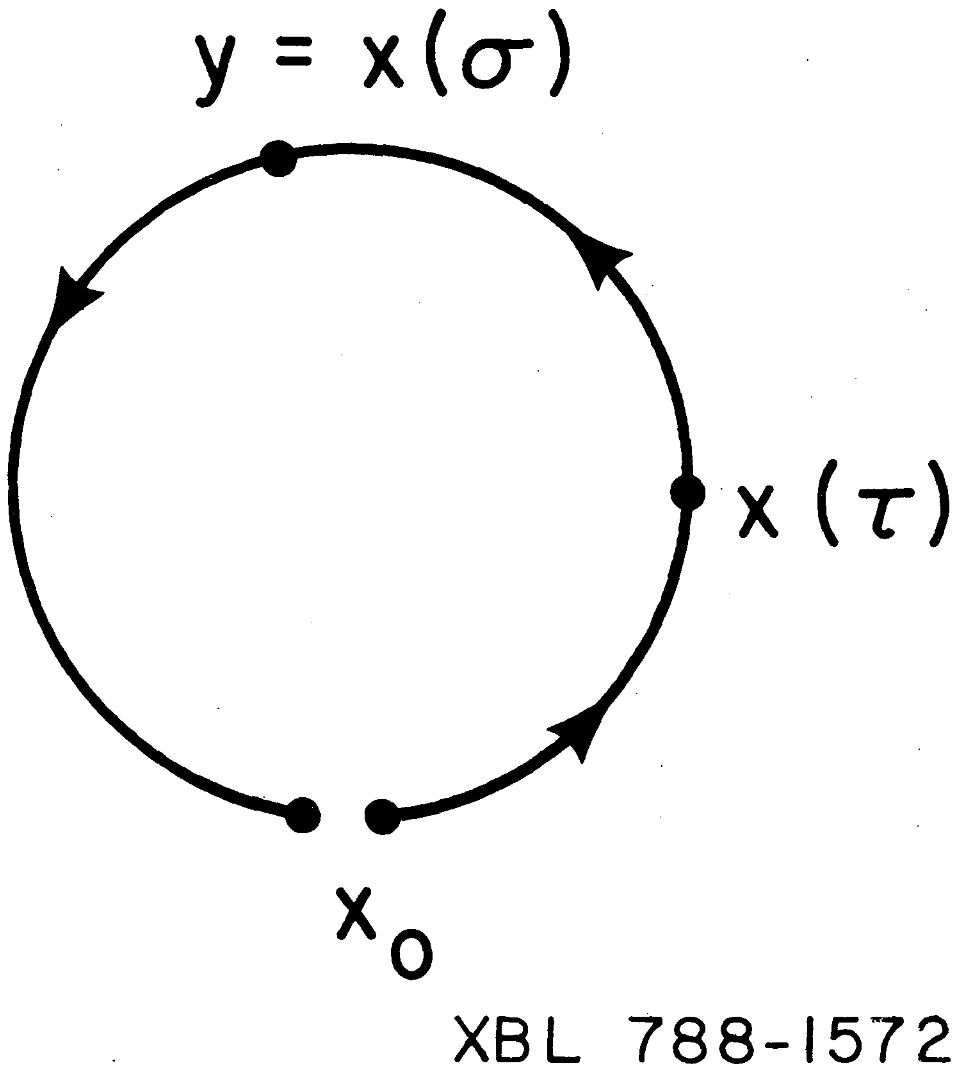

Fig. 3

FIGURE CAPTIONS

Figure 1a. This is a quark-antiquark annihilation graph. The light line represents the quark trajectories. The solid lines are the path ordered products occurring in the initial and final meson wavefunctions.

Figure 1b. The initial quark begins at y, travels along the light line, and is destroyed at y'. The antiquark is created at x and proceeds to x' where it is annihilated. This diagram represents the meson-meson propagator.

Figure 2. This is the baryon-antibaryon propagator. Three quarks emerge and proceed along three paths $p_i(\tau)$ $(i = 1,2,3)$ until they are destroyed. p_i^o and p_i^f are the path ordered products occurring in the initial and final baryon wavefunctions.

Figure 3. This is a path ordered product beginning at x_o and ending at x_o. The trace is not to be taken. The product can be written as two products, one from x_o to y and one from y to x_o. In Eq. (4.2) two terms result: one when $x(\tau)$ comes before y (as illustrated here) and one when $x(\tau)$ comes after y.

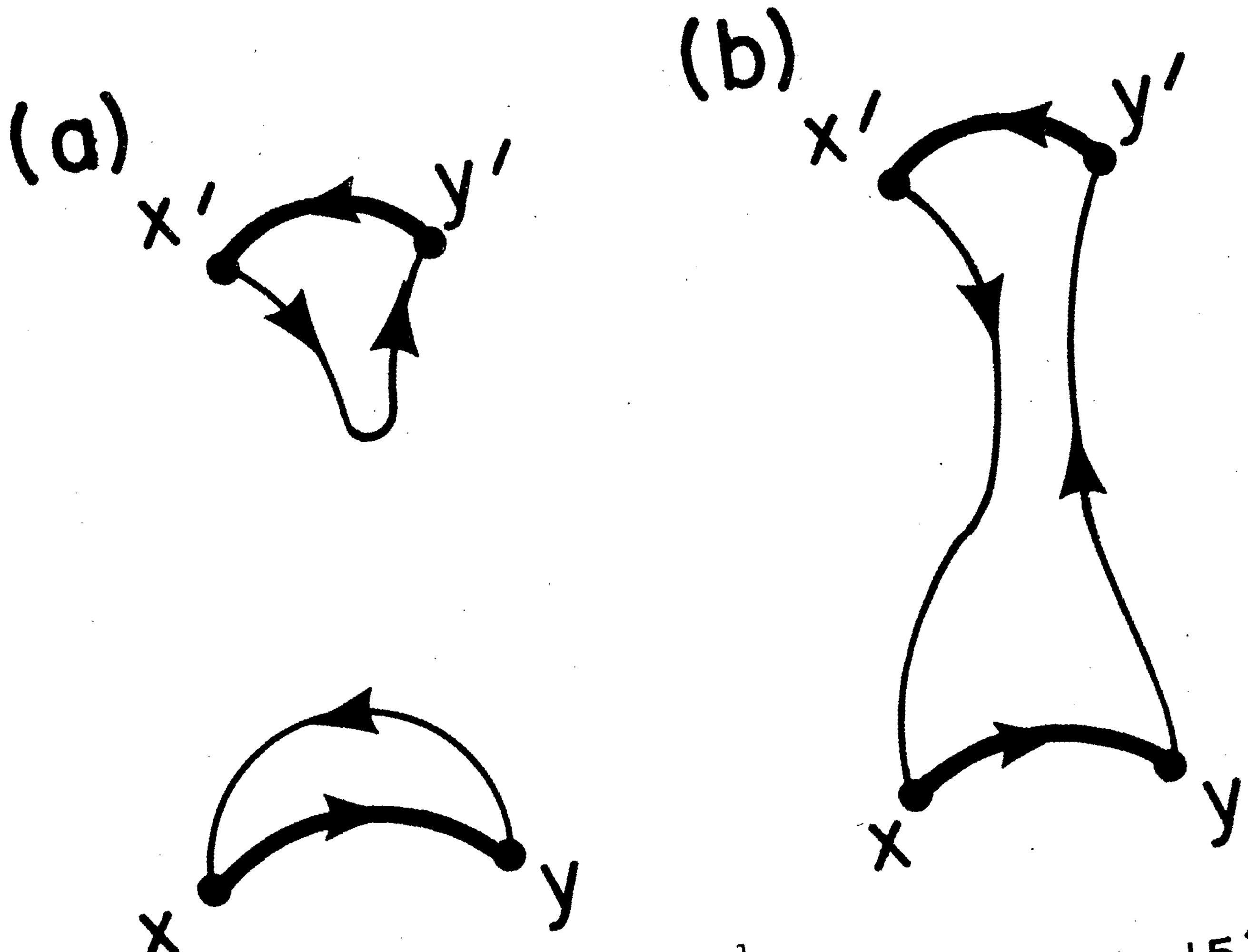

Fig. 1

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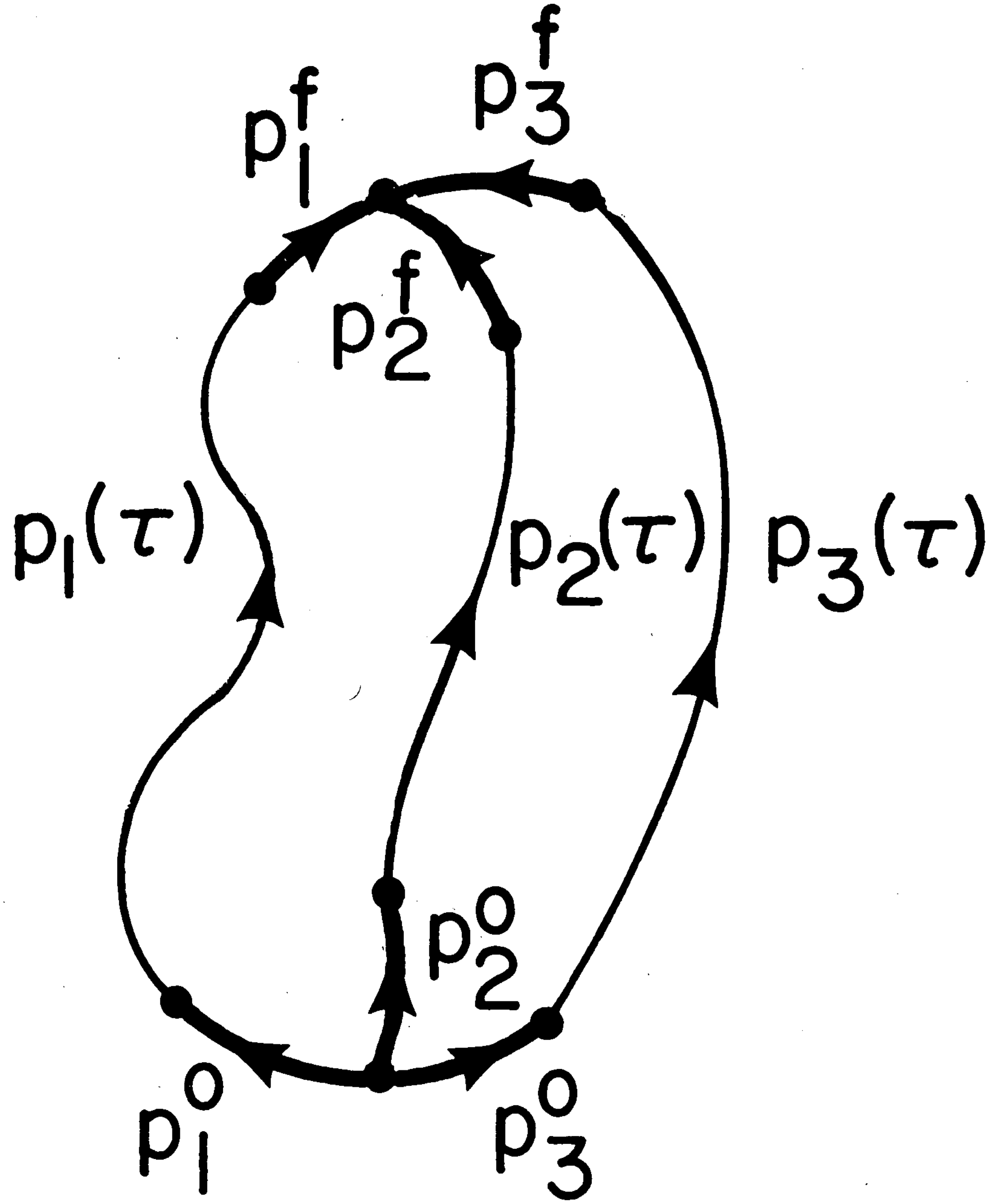

Fig. 2

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