Title
Non-perturbative models of intermittency in drift-wave turbulence: towards a probabilistic theory of anomalous transport

Journal
Nuclear Fusion, 43(9)

ISSN
0029-5515

Authors
Kim, Eun-jin
Diamond, PH
Malkov, M
et al.

Publication Date
2003-09-01

DOI
10.1088/0029-5515/43/9/321

Peer reviewed

Author's final version
Published as:
Nuclear Fusion **43** (9), 961, 2003
https://doi.org/10.1088/0029-5515/43/9/321

Nonperturbative Models of Intermittency in Drift Wave Turbulence: Towards a Probabilistic Theory of Anomalous Transport

Eun-jin Kim[1], P.H. Diamond[1], M. Malkov[1], T.S. Hahm[2], K. Itoh[3], S.-I. Itoh[4],
S. Champeaux[1], I. Gruzinov[1], O. Gurcan[1], C. Holland[1], M.N. Rosenbluth[1], A. Smolyakov[5]

[1] *Department of Physics, University of California San Diego, La Jolla, CA 92093-0319, USA*

[2] *Princeton University, Princeton Plasmas Physics Lab., PO Box 451, Princeton, NJ 08543, USA*

[3] *National Institute for Fusion Science, Toki, 509-5292, Japan*

[4] *Research Institute for Applied Mechanics, Kyushu University, Kasuga 816-8580, Japan*

[5] *University of Saskatchewan, Saskatoon, S7N5E2, Canada*

Abstract

Two examples of non-perturbative models of intermittency in drift wave turbulence are presented. One is a calculation of the probability distribution function (PDF) of ion heat flux due to structures in ion temperature gradient turbulence. The instanton calculus predicts the PDF to be a stretched exponential. The second is a derivation of a bi-variate Burgers equation for the evolution of the drift wave population density in the presence of radially extended streamer flows. The PDF of fluctuation intensity avalanches is determined. The relation of this to turbulence spreading, observed in simulations, is discussed.

PACS: 47.65.+a 05.45. 02.50.-r

1. INTRODUCTION

There is now a plethora of evidence from simulation and experiment that plasma turbulence is highly intermittent, and that turbulent transport has a fundamentally "bursty" character [1–3]. It is thus necessary to develop a probabilistic theory of plasma transport, focusing on calculating the probability distribution function (PDF) of flux, rather than anomalous transport coefficients. This follows from, say, the need to understand the frequency of large heat discharges from the plasma, which in turn affects the distribution of peak heat loads on the confinement vessel. Interestingly, intermittent transport often results from rare, large events which are accompanied by coherent structures such as zonal flows, streamers, blobs, and vortices. These structures are well known to play a crucial role in transport dynamics [4,5]. For instance, zonal flows (mainly poloidal flows that are radially localized) inhibit the radial transport by shearing eddies making up turbulence, while streamers (radially elongated and poloidally localized flows) enhance it. Therefore, two of the most fundamentally important questions in the prediction of transport are (i) the formation of coherent structures and (ii) the effects of these structures on transport. Of particular interest is the question of how 'non-locality' or fast transport phenomena are linked to structures and intermittency.

Most of the works dealing with the first issue (the formation of structures) has so far adopted a mean field theoretical view on the basis of quasi-linear closure and ray chaos (see TABLE 1) [4,5]. Although much insight has been gained in this approach, the formation of structure may be a strongly nonlinear phenomenon, whose description requires a non-perturbative method. Indeed, renormalized perturbation theory can easily make an exponentially large error in predicting PDFs in cases where structure formation is crucial. In particular, the formation of structure can be triggered by noise, in which case the PDF of the formation of structure itself is a quantity of ultimate interest. (For instance, an interesting issue is the prediction of the PDF of L→H transition [6].) On the other hand, these coherent structures, once formed, can cause intermittent and bursty transport. Especially, intermittent transport leads to non-Gaussian PDF of flux. The deviation from Gaussian statistics manifests the failure of random phase approximation, underscoring the need for a non-perturbative method. Note that rare events contributing to PDF tails can play a major role in transport when PDF tails are significantly enhanced over those of a Gaussian PDF.

In this paper, we discuss two examples of non-perturbative theoretical models of intermittency in drift wave turbulence. First, in Section 2, we study the effects of coherent structures in shaping the tail of the PDF of heat flux H due to curvature driven ion temperature gradient (ITG) modes using the instanton calculus. The essence of the instanton calculus is that it treats the calculation of the probability for a structure to emerge from the vacuum (laminar) state in the presence of forcing via a steepest descent approximation of the path integral which determines the transition probability. Thus, the instanton calculus recovers effects from *all* orders in perturbation theory (see TABLE 1), unlike quasi-Gaussian closures which retain effects to $O(\tilde{\phi}^4)$. The 'product' of this calculation is an expression for the tail of heat flux PDF, which is found to scale as $\exp[-cH^{3/2}]$. Here, c is a constant. In Section 3, we present a simple model of turbulence propagation and spreading and its impact on intermittency in the drift wave intensity. The key non-perturbative element here is the use of general symmetry principles, rather than quasi-linear iteration, to derive an equation for evolution about the self-organized fluctuation intensity profile. The principal 'products' of this analysis are a bi-variate Burgers-type equation which describes the evolution in x and k_θ of $\tilde{N}$, and an estimate of the PDF of spatial avalanches of fluctuation intensity and the resulting spreading. Section 4 consists of a discussion and concluding remarks.

2. NON-PERTURBATIVE COMPUTATION OF PDF FLUX

Coherent structures often accompany bursty and intermittent transport, leading to a non-Gaussian PDF of flux. In particular, this means that heat and particle loads may be concentrated in "large events", the frequency of which should be determined. This effect of coherent structure on the PDF of flux is investigated in the following. A non-perturbative method that is utilized in our analysis is called the instanton method. Before proceeding with the computation of PDF by using this method, we provide some explanation for the physical meaning of instantons.

In a classical dynamical system, instantons give the transition probability amplitude between two (stationary) states which have different nonlinear structures (e.g., see [7]). This may be understood intuitively as follows. Associated with each coherent structure is a (nonlinear) solution which takes certain value of an ideal (topological) invariant such as the number of vortices (or $\pm$ vacua in ϕ^4 model). In the presence of dissipation and an external noise, the ideal invariant is broken, and thus there is a finite probability that a system evolves from one state to another with different solutions. Instantons capture the probability of the transition between two nonlinear solutions (or structures). Since this transition occurs rapidly in time, instantons are temporally localized (as its name indicates) and can thus naturally be related to the burstiness of events (see Figure 1).

To exploit this idea in the prediction of PDF of flux in an analytically tractable manner, we take one structure to be the vacuum and the other to be a non-trivial entity. That is, we assume that a system is initially in a quiet state with no energy when an external random forcing is turned on. As the forcing injects energy into the system, there is a finite probability of the formation of coherent structures in the long time limit. Instantons capture the creation process of these structures. Once these structures are formed, they participate in transport, thereby contributing to the tails of the PDF of flux. As may be clear from this argument, the PDF tails of flux will then be determined once the transition probability amplitude to various structures is available. Unfortunately, the latter requires the knowledge of a complete set of coherent structures in a system, which is surely an unobtainable goal. Therefore, to utilize an instanton method, some insight is necessary as to what kind of structure is likely to be excited by a given forcing. One possible candidate for this nonlinear structure, which we are going to use, is an exact nonlinear solution of the dynamical equation in the absence of dissipation and forcing. Once their spatial form is fixed, instantons then give the probability of transition to different amplitude of this solution. For instance, an instanton for a dynamical variable u takes the form of $u(\mathbf{x},t) = F(t)u_0(\mathbf{x})$ with $F(t \to -\infty) = 0$. Here, $u_0(\mathbf{x})$ denotes the spatial form of a coherent structure and $F(t)$ is a temporally localized amplitude, representing its creation process (see Figure 1). The distribution of $F(t)$ determines the PDF of any flux which is a function of u in the long time limit (see Figure 2).

In the following, we utilize instantons to compute the PDF of the heat flux in a curvature driven ITG turbulence. For simplicity, we take a simple two-dimensional slab model where $k_{\|} \simeq 0$ is assumed besides ensuring the adiabaticity of electrons for drift waves (DW). In this model, the instability of DW originates from the bad curvature where $\nabla p \cdot \nabla B > 0$. By keeping FLR effect for electric potential ϕ to first order in $(\rho_i^2 k^2) \ll 1$, we employ the following governing equations for ϕ and pressure perturbation p [8]

$$\partial_t(1-\nabla_\perp^2)\phi - \left[\phi + \tau p, \nabla_\perp^2 \phi\right] + \tau\left[\partial_i \phi, \partial_i p\right]$$
$$+ v_* \left\{1 - 2\epsilon_n + \tau(1+\eta_i)\nabla^2\right\}\partial_y \phi - 2\epsilon_n v_* \tau \partial_y p = f\,, \tag{1}$$
$$\partial_t p + [\phi, p] + v_*(1+\eta_i)\partial_y \phi = 0\,. \tag{2}$$

Here, the notation is standard; x and y denote the local radial and poloidal directions, respectively; $\tau = T_{i0}/T_{e0}$, $\epsilon_n = L_n/R$, $\eta_i = L_n/L_T$, $L_n = -(\partial_x \ln n_0)^{-1}$, $R = -(\partial_x \ln B_0)^{-1}$, $L_T = -(\partial_x \ln T_{i0})^{-1}$, and $v_* = \rho_s/L_n$; square brackets denote Poisson brackets, i.e., $[A,B] = \partial_x A \partial_y B - \partial_y A \partial_x B$; f is an external random forcing for electric potential ϕ. Eqs. (1) and (2) are non-dimensionalized by measuring the length, velocity, ϕ, and p in units of ρ_s, c_s, and T_e/e, and p_{i0}. Note that the linear instability condition for Eqs. (1) and (2) is [8]

$$(1 - 2\epsilon_n - \tau(1+\eta_i)k^2)^2 < 8\tau(1+\eta_i)\epsilon_n(1+k^2)\,, \tag{3}$$

which can be satisfied only when $\epsilon_n \propto \nabla p \cdot \nabla B > 0$. The external forcing f introduces a random noise in the system and thus leads to the formation of a coherent structure (instanton). To simplify analysis, we assume the Gaussian statistics for the forcing with white noise in time as follows:

$$\langle f(\mathbf{x},t) f(\mathbf{x}',t')\rangle = \delta(t-t')\kappa(\mathbf{x}-\mathbf{x}')\,, \tag{4}$$

and $\langle f \rangle = 0$. Note that the use of Gaussian forcing here is for convenience to formulate the PDFs in terms of path integral as shall be clear below and that other statistics for the forcing should be explored [9]. We further assume κ in Eq. (4) is roughly parabolic for $|\mathbf{x}-\mathbf{x}'| < L$, with the form

$$\kappa(\mathbf{x}-\mathbf{x}') = \kappa_0 J_0(k_f|\mathbf{x}-\mathbf{x}'|)\,, \tag{5}$$

and vanish for $|\mathbf{x}-\mathbf{x}'| > L$. Thus, L may be considered the coherence length of the forcing. Here $L \lesssim \alpha_{01}/k_f$ with α_{01} being the first zero of J_0. This particular form of κ was chosen for computational convenience.

The Gaussian statistics of the forcing [10] allows us to formally express the probability for the heat flux pv_x at $\mathbf{x}=\mathbf{x}_0$ to take value H in terms of a path integral [10–15]:

$$P(H;\mathbf{x}_0) = \langle \delta(pv_x|_{\mathbf{x}_0} - H)\rangle = \int d\lambda e^{i\lambda H} I_\lambda\,, \tag{6}$$

where

$$I_\lambda = \int \mathcal{D}\phi\mathcal{D}\overline{\phi}\mathcal{D}p\mathcal{D}\overline{p}\, e^{-S_\lambda}\,.$$

In Eq. (6), the angular brackets denote the average over the random forcing f, and S_λ is the effective action given by

$$\begin{aligned} S_\lambda = -i\int d^2x dt \Big\{ \overline{\phi}\Big\{ (1-\nabla^2)\partial_t\phi - \Big[\phi + \tau p, \nabla_\perp^2\phi\Big] + \tau[\partial_i\phi, \partial_i p] \\ + v_*\left(1 - 2\epsilon_n + \tau(1+\eta_i)\nabla^2\right)\partial_y\phi - 2\epsilon_n v_*\tau\partial_y p\Big\} + \overline{p}\Big\{\partial_t p + [\phi, p] + v_p\partial_y\phi\Big\}\Big\} \\ + \frac{1}{2}\int d^2x d^2x' dt \overline{\phi}(\mathbf{x})\kappa(\mathbf{x}-\mathbf{x}')\overline{\phi}(\mathbf{x}') + i\lambda\int d^2x dt (pv_x)\delta(t)\delta(\mathbf{x}-\mathbf{x}_0)\,. \end{aligned} \tag{7}$$

Here, $v_p = v_*(1+\eta_i)$; $\overline{\phi}$ and $\overline{p}$ are conjugate variables, playing the role of Lagrange multipliers. Since we are interested in PDF tails, we can calculate the path integral in Eq. (6) by a saddle–point method for large H (and λ). Saddle–point solutions for ϕ, p which minimize the effective action with the initial condition $\phi(t \to -\infty) = p(t \to -\infty) = 0$, constitute instantons. Once instanton solutions are found, the effective action, and consequently the PDF tails, can be straightforwardly computed to leading order.

As discussed previously, the spatial form of instantons can be linked to coherent structures which are exact nonlinear solutions of a dynamical system. In the absence of external forcing and dissipation, the coupled equations (1) and (2) support such exact nonlinear solutions $p = \alpha\phi$ and $\phi(\mathbf{x},t) = \psi(\mathbf{x},t)$ with $\alpha = v_*/U$ [16,17]. Here, $\psi(\mathbf{x},t) = \psi(x, y-Ut)$ is a modon solution [18], which is a bipolar vortex soliton, propagating at speed U perpendicular to both the density gradient and toroidal magnetic field;

$$\psi_< = [c_1 J_1(kr) + \beta r/k^2] \cos\theta \,,$$
$$\psi_> = c_2 K_1(sr) \cos\theta \,. \tag{8}$$

Here $\psi_< = \psi(r<a)$ and $\psi_> = \psi(r>a)$; $r = \sqrt{x^2 + y'^2}$, $\tan\theta = y'/x$, $y' = y - Ut$, $\beta = U(k^2+s^2)$, $s^2 = [U - v_*(1-2\epsilon_n) + 2\epsilon_n v_* \tau\alpha]/(U + \tau(1+\eta_i)v_*)$; $c_1 = -s^2 Ua/k^2 J_1(ka)$, $c_2 = Ua/K_1(sa)$, and $J_1'(ka)/J_1(ka) = (1 + k^2/s^2)/ka - kK_1'(sa)/sK_1(sa)$; U is the velocity of a modon; a is the size of the core region; J_1 and K_1 are the first Bessel and the second modified Bessel functions.

This modon solution has two free parameters a and U. In order for a localized modon solution to exist ($s^2 > 0$), the following should be satisfied [17]

$$(1-2\epsilon_n)^2 < 8\epsilon_n \tau (1+\eta_i) \,, \tag{9}$$

and $U > v_*$ or $U < -\tau(1+\eta_i)v_*$. *Interestingly, Eq. (9) is the very condition for linear instability (see Eq. (3)) as* $\rho_i k \to 0$. *That is, on large scales* ($\gg \rho_i$)*, a vortex solution (modon) can exist only while linear waves are unstable.* Note that in the presence of magnetic shear, the size of a modon should be smaller than the characteristic scale of magnetic shear.

In the following analysis, we take the two parameters U and a for a modon to be fixed parameters, with U satisfying the aforementioned localization conditions. We then assume that an instanton, causing bursty transport, has the spatial form given by a modon with unknown time-dependent amplitude. That is,

$$p = \alpha\phi \,, \phi(\mathbf{x},t) = F(t)\psi(\mathbf{x},t) \,. \tag{10}$$

As may be clear from Eq. (8), the modon solution is symmetric in the poloidal direction (y) with no global heat flux. This is why the PDF of the *local* heat flux is under consideration. Note that the use of the special modon solution here is motivated mainly by convenience. In general, the instanton procedure could be implemented for any empirical eigenfunction ψ. The time variation of ϕ, i.e., $F(t)$ in Eq. (10), representing the excitation of a modon by an external forcing, can be associated with the degree of burstiness of an event. This temporal evolution $F(t)$ is to be computed by saddle-point approximation once the spatial integral in Eq. (7) is performed. To this end, we expand the conjugate variable $\overline{\phi}$ in terms of Bessel and Fourier series:

$$\overline{\phi}_< = \sum_{m,n} J_m \left(\frac{\alpha_{mn}}{a} r\right) \left[a_{mn}(t) \sin m\theta + b_{mn}(t) \cos m\theta\right] ,$$
$$\overline{\phi}_> = \sum_{m,n} K_m \left(q_{mn} r\right) \left[\overline{a}_{mn}(t) \sin m\theta + \overline{b}_{mn}(t) \cos m\theta\right] , \qquad (11)$$

where $\overline{\phi}_< = \overline{\phi}(r < a, \theta, t)$ and $\overline{\phi}_> = \overline{\phi}(r > a, \theta, t)$; $a_{mn}(t)$, $b_{mn}(t)$, $\overline{a}_{mn}(t)$, and $\overline{b}_{mn}(t)$ are unknown functions of time, which are to be determined by solving saddle-point equations.

By using Eqs. (5) and (10)–(11) in (7), we reduce S_λ to an integral with respect to time only:

$$S_\lambda = -i \int dt \sum_n [\dot{F}(\overline{A}_n \overline{b}_{1n} + A_n b_{1n}) + F(F-1) B_n a_{2n}]$$
$$+\kappa_0 \int dt \sum_{m,n} [(D_n b_{1n} + \overline{D}_n \overline{b}_{1n})(D_m b_{1m} + \overline{D}_m \overline{b}_{1m}) + E_m E_n a_{2m} a_{2n}] + i\lambda \int dt F^2 \xi_0 \delta(t) \, . \quad (12)$$

Here, $\xi_0 = p v_x(\mathbf{x}_0) = -\alpha\psi\partial_y\psi(\mathbf{x}_0)$ is local heat flux associated with the modon solution. $A_n/\pi = c_1(1+k^2+\alpha)\int_0^a dr r J_1(kr) J_1(z_{1n}) + (1+\alpha)\beta \int_0^a dr r^2 J_1(z_{1n})/k^2$, $B_n/\pi = -c_1 k\beta(1+\alpha\tau)\int_0^a dr r J_2(kr) J_2(z_{2n})/2$, $D_n/\pi = \int_0^a dr r J_1(k_f r) J_1(z_{1n})$, $E_n/\pi = \int_0^a dr r J_2(k_f r) J_2(z_{2n})$, $\overline{A}_n/\pi = c_2(1-s^2+\alpha)\int_a^L dr r K_1(q_{1n} r) K_1(sr)$, and $\overline{D}_n/\pi = \int_a^L dr r K_1(q_{1n} r) J_1(k_f r)$; $z_{1n} = \alpha_{1n} r/a$ and $z_{2n} = \alpha_{2n} r/a$; α_{in} is nth zero of J_i (i.e. $J_i(\alpha_{in}) = 0$); q_{1n} is a constant. It is interesting to note that the coefficients A_n, $\overline{A}_n$, and B_n involve the projection of the conjugate variable onto the modon, while D_n, $\overline{D}_n$, and E_n contain the projection of the forcing onto the conjugate variable. Thus, the (spatial) 'overlap' between the forcing and modon, represented by non-vanishing projection of the forcing onto modon, is necessary for the existence of a non-trivial solution for F. This projection is likely to be to be maximized when the characteristic scale of the forcing is comparable to that of the modon. Even though the instanton calculus, formulated on the basis of a known nonlinear solution as a coherent structure, cannot address the question of which structures are formed by a given forcing, these coefficients do contain the information on how efficiently a given coherent structure is excited by the forcing. *In more general terms, which coherent structure is likely to be generated is determined by the nature of the forcing, with different forcings giving rise to different manifestations of intermittency.*

By minimizing S_λ (7) with respect to independent variables $F(t)$, $a_{2n}(t)$, $b_{1n}(t)$, $\overline{a}_{2n}(t)$, and $\overline{b}_{1n}(t)$, we obtain the following saddle-point equations:

$$-iA_n\partial_t F + 2\kappa_0 \sum_m (D_m b_{1m} + \overline{D}_m \overline{b}_{1m}) D_n = 0\,, \tag{13}$$

$$-iB_n F(F-1) + 2\kappa_0 \sum_m E_n E_m a_{2m} = 0\,, \tag{14}$$

$$-i\overline{A}_n\partial_t F + 2\kappa_0 \sum_m (D_m b_{1m} + \overline{D}_m \overline{b}_{1m}) \overline{D}_n = 0\,, \tag{15}$$

$$\sum_n \Big[A_n \partial_t b_{1n} + \overline{A}_n \partial_t \overline{b}_{1n} - B_n(2F-1)a_{2n}\Big] = -2\lambda F(t)\delta(t)\xi_0\,. \tag{16}$$

Eq. (16) implies that b_{1n} and $\overline{b}_{1n}$ have a discontinuity at $t=0$, since the physical quantity $F(t)$ is a smoothly varying function of time. Furthermore, as conjugate variables propagate backwards in time in the presence of dissipation [12,13], $b_{1n}=0$, $\overline{b}_{1n}=0$, and $a_{2n}=0$ for $t \geq 0$. We thus integrate Eq. (16) for a small time interval $t \in [-\epsilon, 0]$ ($\epsilon \ll 1$) to obtain the relation at $t=-\epsilon$,

$$\sum_n \Big[A_n b_{1n} + \overline{A}_n \overline{b}_{1n}\Big] = 2\lambda F_0 \xi_0\,, \tag{17}$$

where $F_0 = F(t=0)$. Note that the discontinuities in b_{1n} and $\overline{b}_{1n}$ at $t=0$ are directly related to the non-vanishing value of F_0. For $t<0$, the coupled equations (13)–(16) yield an equation for F as

$$\partial_{tt}F - \gamma(F^2 - F)(2F-1) = 0\,, \tag{18}$$

where $\gamma = \sum_m B_m B_m / Q \sum_n E_n E_n$ and $Q = \sum_m A_m A_m / \sum_n D_n D_n = \sum_m \overline{A}_m \overline{A}_m / \sum_n \overline{D}_n \overline{D}_n$. The solution to Eq. (18), with the boundary conditions $F(t=0) = F_0$ and $F(t \to -\infty) = 0$, is easily found to be:

$$F(t) = \frac{1}{1 - \frac{F_0 - 1}{F_0} \exp\{-\sqrt{\gamma} t\}}, \tag{19}$$

while the value of F_0 is determined by Eqs. (13), (15), and (17) as

$$F_0 = 1 + \frac{i4\kappa_0 \lambda}{\sqrt{\gamma} Q} \xi_0 .$$

As can be seen from Eq. (19), the instanton is localized within a time interval proportional to $1/\sqrt{\gamma}$.

The instanton solution (19), with the help of Eqs. (13)–(16), then gives us the saddle–point action to leading order in λ, as $S_\lambda(0) \simeq -\frac{i}{3} h \lambda^3$ where $h = \xi_0^3 q^2$ and $q = |4\kappa_0 / \sqrt{\gamma} Q|$. Finally, the PDF tails for local heat flux H can easily be computed by performing the remaining λ integral in Eq. (6), with the result

$$P(H; \mathbf{x}_0) \sim \exp\left\{ -\frac{2}{3q} \left(\frac{H}{\xi_0} \right)^{3/2} \right\}, \tag{20}$$

for $H/\xi_0 > 0$. Eq. (20) is the probability of finding heat flux H, normalized by ξ_0, at $\mathbf{x} = \mathbf{x}_0$. This can be viewed as a transition probability amplitude from an initial vacuum state to final states with different values of H due to the presence of a modon in the long time limit. Interestingly, it is a stretched exponential, exhibiting non-Gaussian statistics and thus a non-trivial intermittency in the heat flux. The stretched exponential PDF tail implies that a coherent structure enhances heat transport over Gaussian prediction. This is consistent with the expectation that a coherent structure is efficient in transport owing to its coherent behavior. Note that in the absence of forcing (i.e., $\kappa_0 \to 0$), $P \to 0$, simply because the instanton cannot form without the forcing. Note also that Eq. (20) should be thought of as the PDF of local heat flux in the saturated state, as the physics of linear growth and the dynamics of structure formation are not addressed in its derivation.

It is interesting that the 3/2 exponent in Eq. (20), which follows from $S_\lambda \sim \lambda^3$, is due to the quadratic nonlinearity as shown by simple dimensional estimate in [19]. In our case, this can be seen by balancing various terms in S_λ in the limit as $\lambda \to \infty$ as $\lambda\phi^2 \sim \phi\overline{\phi} \sim T\overline{\phi}\phi^2 \sim T\overline{\phi}^2$, from which it follows: $\phi \sim \lambda$, $\overline{\phi} \sim \lambda^2$, $T \sim \lambda^{-1}$, and $S_\lambda \sim \lambda^3$. Here, T is the typical time scale of the instanton. Note that a similar stretched exponential PDF tail for local Reynolds stress was obtained in a simple drift wave turbulence [14,15]. Nevertheless the coefficient q in the exponent of the PDF in Eq. (20) does contain the information about coherent structures, which is model dependent. It is also possible that the exponent may be different in the case where there are multiple coherent structures in the system. Furthermore, if the PDF for cross correlation between ϕ and p can be found in ITG, it may lead to the PDF of heat flux which is different from the PDF of Reynolds stress in Hasagawa-Mima turbulence.

In summary, we have demonstrated, by using a non-perturbative method (instanton calculus), that a coherent structure can lead to intermittent heat flux, with enhanced PDF tail over Gaussian prediction. Although our result (stretched exponential PDF tail) does indicate the important role that a coherent structure plays in transport, it also reveals some of weaknesses of the instanton method. First, it naturally tends to predict an exponential form for PDF as it is computed by a steepest descent approximation to the functional integral. Second, the exponent 3/2 follows from the quadratic nonlinearity in the system. Third, to utilize the instanton method, we had to assume a priori the spatial form of a coherent structure to be an exact solution of the system with fixed parameter values. That is, the mechanism of formation of the structure itself was not addressed in this framework. Therefore, the extension of the instanton method to overcome these limitations and/or the exploration of other non-perturbative calculation of PDF of flux will be desirable.

3. A SIMPLE MODEL OF INTERMITTENT TURBULENCE PROPAGATION IN DRIFT WAVE TURBULENCE

As noted previously, recent theoretical, computational and experimental advances have revealed that transport is a bursty, intermittent process, comprised of a gas of avalanches.

In simple terms, avalanches may be thought of as intermittent secondary cells, which extend many 'average' correlation lengths of the underlying turbulence [20]. Radially extended, poloidally localized secondary cells are called streamers. Streamers are generated by modulational interaction processes, as are zonal flows, and regulated by shearing feedback on the basic instabilities (here 'shearing' refers to distortion by poloidally sheared radial flows which generates large k_θ) and by subscale instability [21–23]. This mechanism, together with the consequent evolution of the underlying drift wave population, has been described by a multi-component 'predator-prey' type system. The tendency of the system to form extended avalanche structures (albeit intermittent ones) has also been associated with deviations from gyro-Bohm scaling [24] and other "nonlocality" phenomena.

Recently, however, large scale gyrokinetic simulations have indicated that the turbulence may spread, or propagate, to regimes where the basic drift-ITG modes are linearly stable [25]. This phenomenon is hereafter referred to as *turbulence spreading* and can contribute to perceived 'intermittency' by:

a) introducing fast time scale variation in the fluctuation intensity profiles. Here, the 'fast' time scale is the nonlinear interaction or mixing rate.

b) introducing non-locality to the fluctuation energy balance. Here non-locality refers to the fact that the local fluctuation intensity in a linearly stable region must necessarily be determined by a balance between spatial flux of energy from the unstable region with local energy dissipation via coupling to small scales. In this regard it is worthwhile to note that virtually all existing models of turbulent transport tacitly presume a *local* balance of excitation (i.e., growth) and dissipation. Since streamers naturally produce radial flows which can advect the local drift wave population, it is only natural to explore the effect of the streamers on turbulence spreading.

In this section, we present a simple model for the intermittent propagation of *fluctuation intensity* via secondary radial flows. We emphasize that the "stuff" being propagated is fluctuation energy, not heat or particles. However, it is obvious that the latter must necessarily follow the former so that turbulence spreading certainly plays a role in avalanche dynamics.

Also, this model should be viewed as complementary to that presented in [25] which treats the spreading as a diffusion process.

The local fluctuation population density is $N(x, k_\theta, t)$, which satisfies the wave kinetic equation

$$\frac{\partial N}{\partial t} + v_g \frac{\partial N}{\partial y} + V \frac{\partial N}{\partial x} - \frac{\partial}{\partial y} (\omega + k_r V) \frac{\partial N}{\partial k_\theta} = C(N) \,. \tag{21}$$

Here, V is purely radial (corresponding to a streamer), so that only the k_θ-dependence of N need be considered, v_g is the poloidal direction group velocity (i.e., $v_g = v_*(1 - k_\theta^2 \rho_s^2)/(1 + k_\perp^2 \rho_s^2)^2$ for drift waves), and $C(N)$ represents a collision integral which accounts for linear processes as well as non-action-conserving wave interactions. In general: $C(N) = \gamma_k N - \Delta\omega_k N^2/N_0$. The effect of the streamer flow is manifested in the advection term $V \partial_x N$ and in the streamer shearing term $\partial_y(\omega + k_r V)\partial_{k_\theta} V$. Thus, the effect of the streamer flow is to drive both a spatial flux in x and a wave-number flux in k_θ. The spatial flux produces turbulent spreading by relaxing intensity gradients, while the wavenumber flux generates large k_θ by shearing. Note that the shearing-induced spreading in k_θ is the self-regulation mechanism for the streamer flow. Thus, the drift-wave population necessarily evolves via dual-fluxes in x and k_θ, due to intensity transport events and shearing events, respectively.

The obvious self-regulation of the system suggests the conjecture that the profiles of $N(x, t)$ and $N(k_\theta, t)$ hover near self-organized critical (SOC) profiles in x and k_θ. Thus, Eq. (21) implies that $\tilde{N} = N - N_{soc}$ evolves according to:

$$\frac{\partial \tilde{N}}{\partial t} + v_g \frac{\partial \tilde{N}}{\partial y} + \frac{\partial}{\partial x} \Gamma_x(\tilde{N}) + \frac{\partial}{\partial k_\theta} \Gamma_{k_\theta}(\tilde{N}) = \tilde{C}(N) \,. \tag{22}$$

Here, Γ_x is the flux in x, and Γ_{k_θ} is the flux in k_θ. By definition of the SOC state, $\Gamma_x(\tilde{N}) \to 0$ and $\Gamma_{k_\theta}(\tilde{N}) \to 0$ as $\tilde{N} \to 0$, as there should be no flux when the system adopts precisely the SOC configuration.

In the usual approach, Eq. (22) is now closed using a Fokker-Planck calculation, so that Γ_x and Γ_{k_θ} are represented as diffusion in position and wave-number space, respectively, i.e.

$$\Gamma_x = -D_x \frac{\partial \tilde{N}}{\partial x} \,, \tag{23}$$

$$D_x = \sum_{\mathbf{q}} |\mathbf{V_q}|^2 R(\mathbf{k},\mathbf{q}) \,, \tag{24}$$

$$\Gamma_{k_\theta} = -D_{k_\theta} \frac{\partial \tilde{N}}{\partial k_\theta} \,, \tag{25}$$

$$D_{k_\theta} = \sum_{\mathbf{q}} k_r^2 q_\theta^2 |\mathbf{V_q}|^2 R(\mathbf{k},\mathbf{q}) \,, \tag{26}$$

where $R(\mathbf{k},\mathbf{q})$ is a resonance function determining the effective correlation time [26]. However, noting that the streamer flow is driven by $\tilde{N}$, it is possible to use symmetry constraints to determine the forms of Γ_x and Γ_{k_θ} without recourse to perturbation theory. In particular, the (assumed) existence of a SOC profile suggests that local excesses of N (i.e., $\tilde{N} > 0$) should run downhill while local deficits of N (i.e., $\tilde{N} < 0$) should run uphill. In this 'dual-SOC' system, x and k_θ evolution evolve independently, so for each k'_θ, there is a SOC profile $N_{soc}(x, k'_\theta, t)$ and for each x', there is a SOC profile $N_{soc}(x', k_\theta, t)$. Thus, both Γ_x and Γ_{k_θ} must each satisfy the joint reflection symmetry constraint [27]. Thus, for each k_θ, $\tilde{N} \to -\tilde{N}$ and $x \to -x$ must leave Γ_x invariant, while for each x, $k_\theta \to -k_\theta$ must leave Γ_{k_θ} invariant. Thus, $\Gamma_x(\tilde{N})$ must have the form

$$\Gamma_x = \sum_m \left(\alpha_{x,2m} \tilde{N}^{2m} + \beta_{x,m} \left(\frac{\partial \tilde{N}}{\partial x} \right)^m + \cdots \right) . \tag{27}$$

Similarly, $\Gamma_{k_\theta}(\tilde{N})$ must have the form

$$\Gamma_{k_\theta} = \sum_m \left(\alpha_{k_\theta,2m} \tilde{N}^{2m} + \beta_{k_\theta,m} \left(\frac{\partial \tilde{N}}{\partial x} \right)^m + \cdots \right) . \tag{28}$$

In lowest order, then

$$\Gamma_x = \frac{\alpha_x}{2} \tilde{N}^2 - \beta_x \frac{\partial \tilde{N}}{\partial x} \,, \tag{29}$$

$$\Gamma_{k_\theta} = \frac{\alpha_{k_\theta}}{2} \tilde{N}^2 - \beta_{k_\theta} \frac{\partial \tilde{N}}{\partial k_\theta} \,, \tag{30}$$

where α's and β's are coefficients which follow from the underlying model. The determination of α, β is discussed in Appendix. $\tilde{N}$ then evolves according to

$$\frac{\partial \tilde{N}}{\partial t} + v_g \frac{\partial \tilde{N}}{\partial y} + \alpha_x \tilde{N} \frac{\partial \tilde{N}}{\partial x} + \alpha_{k_\theta} \tilde{N} \frac{\partial \tilde{N}}{\partial k_\theta} - \beta_x \frac{\partial^2 \tilde{N}}{\partial x^2} - \beta_{k_\theta} \frac{\partial^2 \tilde{N}}{\partial k_\theta^2} = -\gamma \tilde{N} + \tilde{f} \,. \tag{31}$$

Here, we have used the linearized form of $C(N)$ and have imposed $C(N) \rightarrow 0$ for $N \rightarrow N_{soc}$. $\tilde{f}$ represents noise. It is important to note that this model cannot actually predict N_{soc}, but only can describe the dynamics of deviations from it.

Eq. (31) is a bivariate noisy Burgers equation for the evolution of the drift wave population in x and k_θ. Its bivariate structure is due to the fact that turbulence spreading (due to streamer flows) is necessarily accompanied by wave-packet evolution due to straining by the same streamer flow. The physical significance of the various terms in Eq. (31) is clear. The $\beta_x \partial_x^2 \tilde{N}$ is spatial diffusion of turbulence intensity, while $\beta_{k_\theta} \partial_{k_\theta}^2 \tilde{N}$ is diffusion in k_θ space, much like the familiar induced diffusion process for the evolution of wave-packets in the presence of larger scale strain fields. The term $\alpha_x \tilde{N} \partial_x \tilde{N}$ refers to self-advection of drift waves by the streamer flow fields they produce. Similarly $\alpha_{k_\theta} \tilde{N} \partial_{k_\theta} \tilde{N}$ may be thought of as nonlinear self-refraction of drift waves by the streamer flow fields they drive. The absence of cross-terms in Eq. (31) is consistent with the vanishing of cross-terms in the corresponding quasi-linear calculation. Note that Eq. (31) may be viewed as a limiting case of a more general system of the form:

$$\frac{\partial \tilde{N}}{\partial t} + v_g \frac{\partial \tilde{N}}{\partial y} + \alpha_x V \frac{\partial \tilde{N}}{\partial x} + \alpha_{k_\theta} V' \frac{\partial \tilde{N}}{\partial k_\theta} - \beta_x \frac{\partial^2 \tilde{N}}{\partial x^2} - \beta_{k_\theta} \frac{\partial^2 \tilde{N}}{\partial k_\theta^2} = -\gamma \tilde{N} + \tilde{f}\,, \qquad (32)$$

$$\left(\frac{\partial}{\partial t} + i\omega + \frac{1}{\tau} \right) V \sim \sum_{k_\theta} \tilde{N}\,. \qquad (33)$$

This has the form of a generic prey-predator system, with the flow V as the predator and the population fluctuation $\tilde{N}$ as the prey. In the limit of quasi-stationary flow, $V \sim \tau \tilde{N}$, Eq. (31) can be recovered. We consider this limit, as it constitutes the simplest possible model.

A central question here is concerned with the nature of the self-organized criticality in x and k_θ. The basic idea of spectral shape as an SOC is discussed in [28]. $N(k_\theta, t)_{soc}$ can be viewed as the spectrum formed by shearing events (i.e., jumps in k_θ) in response to noise stimulus. Indeed, the familiar Kolmogorov spectrum of fluid turbulence is, in essence, a particularly simple form of a SOC, where the cellular automata rule (ala' sandpiles) is replaced by the requirement of constant dissipation at all scales. Here, it is also very important to note that the spectra shown as products of computer simulations are invariably heavily time-averaged, so that it is very likely that the *instantaneous* spectrum exhibits complex structure, consistent with experience from sandpiles.

The question of the validity of presuming SOC in configuration space (i.e. x) is considerably more subtle. Certainly, it is a generic property of drift-wave interaction that nonlinear couplings necessarily involve spatial transport of fluctuation energy, since the couplings involve radial derivatives of fluctuations. This may also be seen by noting that using standard closure theory, the nonlinear damping of fluctuation energy can be written as a sum of radial diffusion and radially local decorrelation, i.e.:

$$
\begin{aligned}
\frac{\mathcal{E}_{\mathbf{k}}}{\tau_{c,\mathbf{k}}} &= \sum_{\mathbf{k}'}(\mathbf{k}\cdot\mathbf{k}'\times\hat{z})^2\frac{c^2}{B_0^2}|\hat{\phi}(\mathbf{k}')|^2 R(\mathbf{k},\mathbf{k}')\mathcal{E}_{\mathbf{k}} \\
&\simeq -\frac{\partial}{\partial x}\sum_{\mathbf{k}'_\theta} k'^2_\theta\frac{c^2}{B_0^2}|\hat{\phi}(\mathbf{k}')|^2 R(\mathbf{k},\mathbf{k}')\frac{\partial}{\partial x}\mathcal{E}_{\mathbf{k}} + \sum_{\mathbf{k}'_\theta} k_\theta^2\frac{c^2}{B_0^2}\left|\frac{\partial\hat{\phi}(\mathbf{k}')}{\partial x}\right|^2 R(\mathbf{k},\mathbf{k}')\mathcal{E}_{\mathbf{k}} + \cdots \\
&= -\frac{\partial}{\partial x}D_{r,\mathbf{k}}(\mathcal{E})\frac{\partial\mathcal{E}_{\mathbf{k}}}{\partial x} + k_\theta^2 D_{\theta,\mathbf{k}}(\mathcal{E})\mathcal{E}_{\mathbf{k}} + \cdots .
\end{aligned}
\tag{34}
$$

Thus, nonlinear interaction must necessarily work to relax fluctuation intensity gradients, resulting in amplitude dependent transport (i.e., diffusion) of fluctuation energy. The competition between nonlinear diffusion ($D = D(\mathcal{E})$) and noisy excitation then is very similar to the competition between deposition and toppling which characterizes a sandpile SOC. It is less clear what sets the local "critical gradient" for relaxation. One likely possibility is when $\mathcal{E}$ and $\partial_x\mathcal{E}$ are sufficiently large enough so that turbulent spreading exceeds local damping. The physics underlying this is discussed below. It should also be noted that the simple scenario discussed here is limited to cases of modest fluctuation intensity gradients. Should $\partial_x\mathcal{E}$ become very large, the Reynolds stress drive of poloidal flows, itself proportional to $-\partial_x\langle\tilde{v}_r\tilde{v}_\theta\rangle \sim \partial_x\mathcal{E}$, will become large enough to overcome frictional damping, resulting in the generation of a shear layer which reduces $\mathcal{E}$ and the associated transport. This suggests that the fluctuation intensity flux is necessarily *bi-stable*, and so the simple model discussed here applies to only the first (modest $\partial_x\mathcal{E}$) branch.

As Eq. (31) is a noisy, bivariate Burgers equations, it necessarily implies that the drift wave population dynamics consists of shocks in x and k_θ. The physics of shocks in k-space is discussed in [28]. Shocks in k_θ represent *coherent* shearing phenomena, with $k_\theta \sim t$. These correspond to strong but brief shearing events with shearing time τ_s less than the shearing field auto-correlation time τ_{ac} (i.e., $\tau_s < \tau_{ac}$). Thus, shocks in k_θ correspond to bursts of straining where poloidally sheared radial flows ballistically produce large k_θ. This is a class of shearing process *not* captured by quasilinear closures of the wave kinetic equation.

The physics of shocks in x-space is both more familiar and more germane to the problem of understanding turbulent spreading. To focus on spatial dynamics, we integrate Eq. (31) over k_θ and y, yielding

$$\frac{\partial \tilde{N}}{\partial t} + \alpha_x \tilde{N} \frac{\partial \tilde{N}}{\partial x} - \beta_x \frac{\partial^2 \tilde{N}}{\partial x^2} = -\gamma \tilde{N} + \tilde{f}\,. \qquad (35)$$

Note this is just a noisy Burgers equation, with an additional linear damping term (which follows from $C(N)$ and the requirement that $C(N) \to 0$ for $N \to N_{soc}$)! Thus, a number of results follow directly from the enormous body of knowledge available concerning Burgers equation, the favorite laboratory animal of turbulence theorists. First, Eq. (35) suggests that turbulence spreading will be "fast" (i.e., will occur super-diffusively), with $x \sim t$, as in Burgers shocks. Second, fast turbulence spreading will be most virulent near marginality of the underlying drift waves. This follows from a familiar text-book example [29], which states that in the limit of small β_x and $\tilde{f} \to 0$, a finite time singularity (i.e., shock) in $\tilde{N}$ will form if $|\alpha_x \partial_x \tilde{N}| > \gamma$ – i.e., the initial (negative) slope must be sufficient for steepening to overcome dissipation. As dissipation here is linked to linear growth via the constraints on $C(N)$, this is equivalent to requiring that the underlying drift waves be 'near' marginality. Note also that $|\partial_x \tilde{N}| > \gamma/\alpha_x$ is the effective "toppling condition" for the onset of turbulence spreading, and thus defines the effective critical spatial gradient in $\tilde{N}$.

A third result concerns the PDF of $\tilde{N}$, denoted by $P(\tilde{N})$. $P(\tilde{N})$ is of obvious interest, as it gives the PDF of shocks, which may be thought of spreading events. The most fundamental aspect of Burgers dynamics is the asymmetry between shock regions with $\partial_x \tilde{N} < 0$ and ramp regions with $\partial_x \tilde{N} > 0$. Thus, the natural dependence of P is on $\partial_x \tilde{N}$, or equivalently, $\Delta \tilde{N}$; the local jump in $\tilde{N}$. $P(\Delta \tilde{N})$ is asymmetric, as the system tends to amplify negative slopes (i.e. form shock fronts) and to flatten positive slopes (i.e. smooth ramps). In the case of white noise forcing (and $\gamma \to 0$), application of the instanton calculus yields $P(\Delta\tilde{N}) \sim \exp[-c\Delta\tilde{N}^3/r]$ for $\Delta\tilde{N} > 0$ [12]. Here $\Delta\tilde{N} \sim \tilde{N}'r$. This is in good agreement with numerical calculations. For $\Delta\tilde{N} < 0$, the arguments of Chekhlov and Yakhot [30] imply that for white noise and $\Delta\tilde{N} \ll 0$, $P(\Delta\tilde{N}) \sim \Delta\tilde{N}^{-4}$, also in good agreement with simulations.

While these observations suggest that noisy Burgers turbulence may be good paradigm for the dynamics of turbulence spreading, the reader is cautioned that the true dynamics are governed by a *bivariate* Burgers equation. Thus, propagation in x is necessarily accompanied by shearing, generation of large k_θ and thus coupling to high k_θ damping. Hence, turbulence spreading and k-space energy transfer are *intertwined*, so that coupling to damped k_θ will act to limit the range of turbulence propagation in space! Obviously, then, the inclusion of the precise value of $\alpha_x/\alpha_{k_\theta}$ is critical to predicting the actual range of turbulent spreading in a concrete example.

4. CONCLUSIONS

In this paper we have presented two non-perturbative models of intermittency in drift wave turbulence. In the first example, the instanton calculus was used to analytically calculate the tail of the local PDF of ion heat flux due to structure in a curvature driven ITG turbulence. The result $P(H) \sim \exp[-cH^{3/2}]$, a non-Gaussian PDF, was obtained. Note that the instanton calculation can, in principle, be implemented for any 'empirical eigenfunction' which is a stationary solution of the underlying nonlinear equation. Thus, this methodology could be used to compute PDFs from numerical or experimental data, provided one had a good understanding of the underlying plasma model. In the second example, concepts from SOC-theory and symmetry constraints on the spatial and momentum flux were used to derive a bi-variate Burgers equation for the evolution of $\tilde{N}(x, k_\theta, t)$, the fluctuation in the drift wave density about the SOC state induced by radially extended streamer flows. The PDF of $\tilde{N}$ ($P(\tilde{N})$) associated with spatial avalanching was obtained. For $\tilde{N} < 0$, $P(\tilde{N}) \sim \tilde{N}^{-4}$, while $P(\tilde{N})$ decays faster than Gaussian for $\tilde{N} \gg 0$. The relation of spatial avalanching to turbulence spreading, which is observed in simulations, was discussed. Future work here should focus on quantitative comparisons of theory with simulation results for concrete cases, including specially designed numerical experiments.

While the two calculations considered here are quite different in assumptions, structure, and methodology, it is interesting to note that both end in the prediction of a PDF for fluctuation properties. The PDF is, indeed, the common element of the two lines presented here. Future work will focus on alternative approaches to calculating turbulence PDFs, and on critical comparisons of existing approaches with simulations and experiments.

Acknowledgments We thank G. Falkovich, and X. Garbet for stimulating discussions. This research was supported by U.S. DOE FG03-88ER 53275.

APPENDIX A: DETERMINATION OF α AND β COEFFICIENTS

Here, in interest of brevity, we discuss only the determination of α, β for zonal flow strain case – a similar method will be applied to the streamer case. The coefficients α, β enter the parameterization of the self-refraction of drift wave packets, via its nonlinear ($\propto N\partial N/\partial k_r$) and linear diffusive ($-\beta\partial^2 N/\partial k_r^2$) pieces, respectively. These coefficients can be determined in (at least!) two ways, namely via scaling analyses ala' Connor and Taylor [31] or by (substantially equivalent) physically motivated estimates. For example, the coefficient β accounts for the (minimal) linear induced diffusion in k_r due to random shearing. The induced diffusion coefficient [32] is:

$$D_{k_r} = \sum_{\mathbf{q}} k_\theta^2 |V_q'|^2 \tau_{c\mathbf{q}}.$$

Here q is the radial wave-vector of the zonal flow strain field. As β is to account for the *minimal* diffusion, one can take $k_\theta^2 \sim \overline{k_\theta^2}$ (a typical mean-square turbulence wave-vector),

$\tau_{c\mathbf{q}} \sim \gamma_k^{-1}$ (from the triad resonance) and $\sum_{\mathbf{q}} |V_q'|^2 \gtrsim (c\langle E_r\rangle'/B_0)^2$ (taking the mean field shear as having scale on which the electric field is smoothest). Note the last approximation is not appropriate in transport barrier regimes, where turbulence and zonal flows are extinguished. Thus, $\beta \sim \overline{k_\theta^2}\gamma_k^{-1}(c\langle E_r\rangle'/B_0)^2$ appears as a reasonable estimate. As to α, it is the coefficient for nonlinear refraction by self-driven zonal flows. This can be estimated by considering the balance of flow damping with generation by Reynolds stresses. Specifically,

$$\left(\frac{\partial}{\partial t} + \gamma_d\right) V_E = -\frac{\partial}{\partial r}\langle \tilde{v}_{Er}\tilde{v}_{E\theta}\rangle_q$$

where the brackets denote an average over a scale smaller than q^{-1}. At stationarity,

$$V_q' \simeq \overline{\frac{C(q,k)}{\gamma_d} N(\mathbf{k})}$$

where γ_d is the flow damping, $C(q,k)$ is a Kernel determined by modulational stability analysis [26] and $N(\mathbf{k})$ is the wave spectrum. The bar denotes an average over $\mathbf{k}$. Noting that the refractive nonlinearity in wave kinetics is $-k_\theta V_q' \partial N/\partial k_r$, *breaking* the average straightforwardly yields the estimate $\alpha \sim \frac{\overline{k_\theta C(q,k)}}{\gamma_d}$. Obviously, the regime where $\gamma_d \to 0$ presents special complications. These are discussed in [33].

REFERENCES

[1] Politzer, P.A., Phys. Rev. Lett. **84** (2000) 1192.

[2] Beyer, P., et al, Phys. Rev. Lett. **85** (2000) 4892.

[3] Drake, J.F., Guzdar, P.N., Hassam, A.B., *ibid.* **61** (1988) 2205.

[4] Holland, C., Diamond, P.H., Phys. Plasmas **9** (2002) 3857.

[5] Champeaux, C., et al., "Towards a continuum theory of avalanches", DPP-2001, RI1.5..

[6] Itoh, S.-I., Itoh, K., J. Phys. Soc. Japan **69** (2000) 408.

[7] Spineanu, F., Vlad, M., "On the nature of intermittency in weakly dissipative system", http://arXiv.org/abs/physics/0208037 (2002).

[8] Horton, W., Choi, D.I., Tang, W.M., Phys. Fluids **24** (1981) 1077.

[9] Medina, E., Hwa, T., Kardar, M, Yi-Cheng Zhang, Y.-C. Phys. Rev. A, **39** (1989) 3053.

[10] Zinn-Justin, J., "Field Theory and Critical Phenomena" (Clarendon: Oxford) (1989).

[11] Wyld, H.W., Ann. Phys. **14** (1961) 143.

[12] Gurarie, V., Migdal, A., Phys. Rev. E **54** (1996) 4908.

[13] Falkovich, G., Kolokolov, I., Lebedev, V., Migdal, A., Phys. Rev. E **54** (1996) 4896.

[14] Kim, E., Diamond, P.H., Phys. Plasmas **9** (2002) 71.

[15] Kim, E., Diamond, P.H., Phys. Rev. Lett. **88** (2002) 225002.

[16] Shukla, P.K., Phys. Scr. **36** (1987) 500.

[17] Hong, B.G., Romanelli, F., Ottaviani, M., Phys. Fluids B **3** (1991) 615.

[18] Larichev, V.D., Reznik, G.M., Dokl. Akad. Nauk SSSR **231** (1976) 1077.

[19] Balkovsky, E., Falkovich, G., Kolokolov, I., Lebedev, V., Phys. Rev. Lett. **78** (1997)

1452.

[20] Diamond, P.H., Hahm, T.S., Phys. Plasmas **2** (1995) 3640.

[21] Champeaux, S., Diamond, P.H., Phys. Lett. A **288** (2001) 214.

[22] Diamond, P.H., et al., Nucl. Fusion **41** (2001) 1067.

[23] Cowley, S.C., et al., Phys. Fluids B **3** (1991) 2767.

[24] Nevins, W.M., LLNL, private communication (2002).

[25] Lin, Z., Hahm, T.S., Ethier, S., et al., "Size scaling of turbulent transport in tokamak plasmas", (IAEA, Lyon, 2002) IAEA-CN-96/TH1/1.

[26] Diamond, P.H., et al, Fusion energy 1998 (Proc. 17th Int. Conf. break Yokohama, 1998) (Vienna: IAEA) CD-ROM file TH3/1. and http://www.iaea.org/programmes/ripc/phjysics/start.htm

[27] Hwa, T., Kardar, M., Phys. Rev. A **45** (1992) 7002.

[28] Diamond, P.H., Malkov, M., Phys. Scr. **T98** (2002) 63.

[29] Whitham, G.B., "Linear and Nonlinear Waves" (Wiley, New York) (1974), Chap. 2.

[30] Chekhlov, A., Yakhot, V., Phys. Rev. E **52** (1995) 5681.

[31] Connor, J., Taylor, J.B., Nuclear Fusion, **17** (1977) 1047.

[32] Sagdeev, R.Z., Galeev, A.A., "Nonlinear plasma theory" (Benjamin, New York) (1969).

[33] Kim, E., Diamond, P.H., Phys. Plasmas, **9** (2002) 4530.

TABLE 1

Contrast	Formation of structure (conventionally)	Effect of structure on PDF flux
Method	Mean field theory (quasi-linear closure)	Instantons (non-perturbative)
Assumption	Ray chaos, random coupling, etc	Spatial form of coherent structure

Figure Captions

Fig. 1. The time evolution of F, showing the temporal localization of instanton. The latter can be interpreted as a 'burst'.

Fig. 2. The uncertainty in an external forcing determines the PDF for F, which in turn determines the PDF of the flux.