Structural Behavior of Mass Concrete Beams

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STRUCTURAL BEHAVIOR OF MASS CONCRETE BEAMS

A Report of an Investigation

by

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to

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INTRODUCTION

In accordance with Contract No. DA-45-164-CIVENG-66-275 between the Walla Walla District of the U.S. Army Engineers and the Regents of the University of California, dated 31 January 1966, the University of California undertook studies aimed at determination of the thermal stresses near the base of a large concrete gravity dam, including the time-dependent effects of creep and variation of elastic properties of the material, and taking into account the rate of construction and the program of artificial cooling.

The complete program of investigation has been organized into two phases. Phase I of the program includes evaluation of tensile creep properties of concrete including simulation of beam tests, reported in Reference I, by finite element technique to determine whether the tensile creep effect is adequately modeled by the computer program.

In Reference 1, tests on 6 concrete beams had been reported. The beams were 24 inches by 24 inches by 8 feet long and were tested under conventional 'third point' loading test for modulus of rupture. A special feature of the tests was that strain gages were embedded in the beams to record directly the strains both in compression and in tension. The instruments were located at midspan, placed 2 inches below the top and above the bottom faces of the beams, and were corrected for temperature, shrinkage, and other loads incident upon the beams prior to the bending tests.

The strain data obtained from the tests showed that, in general, strains near the face in tension were somewhat greater than the corresponding strains near the compression face. The difference increased significantly as the beam approached rupture.

Assuming linear variation in stress over the beam cross-section, and extrapolating measured strains to outer fibers, the laboratory reported different values of the elastic modulus for tensile and compressive loadings.

It is known from theory of elasticity that the stress distribution in an elastic beam is, in general, nonlinear. The assumption of linearity is often accepted as fairly valid in engineering design for large span-depth ratios, but is inapplicable to relatively small span-depth ratios, since shear and thickness deformation are neglected in elementary beam theory.

To determine the influence of the actual force boundary conditions as they existed for the beam tests, an analysis was undertaken using the finite element method. The theory of this method is now well known and no attempt will be made to explain it in this preliminary report. As this analysis was intended to be 'typical' in nature, only one beam and one loading condition were selected for analysis. It was considered that other analyses would only be repetitive and would not influence the conclusions.

Another analysis was carried out using assumptions similar to those made by the Waterways Experiment Station (WES) in its reports, Reference 1. This study assumed linear variation of strain over the cross-section of the beam. Different elastic moduli for concrete in tension and in compression were assumed and stresses computed from strains. Expressions for the location of the neutral axis, for the elastic moduli, and for the bending moment, were derived in terms of fiber strains and measured strains.

ANALYSIS OF THE BEAM AS AN ELASTIC MEMBER

The case selected for analysis was the beam designated as No. 1 in Report 3, December 1964 of Reference 1. Stresses and displacements corresponding to a live load $P_t = 32,245$ lbs. and including effects of the dead load of the beam were computed.

The beam was treated as a case of two-dimensional stress. For a beam width of 24 inches, this assumption appears to be justified. The analysis, therefore, considered a one-inch thick beam carrying proportionate loads. Geometry of the beam in the plane of the loads was taken as the same as in the laboratory tests. It was assumed that the elastic modulus is constant in the range of stresses the beam was subjected to. A value of 6.38×10^6 psi corresponding to Beam 1 of Report 3, Reference 1 was adopted. Poisson's ratio was taken as 0.2. Linear elastic behavior was assumed for the study.

For finite element idealization, the beam of 24 inches by 96 inches was divided into 576 square elements of 2" x 2" size. Taking advantage of symmetry of geometry as well as of loading, about the midspan, the actual analysis covered 288 elements. Total number of nodal points was 325.

In the finite element analysis, all loads and forces are treated as acting on nodal points. The computer program yields displacements and stresses at centers of the square elements. The dead load was simulated by imposing an upward acceleration on the system. The live loads in this case were actually located at nodal points.

The displacement boundary conditions for the analysis were:

a. Horizontal displacement at all points of the cross-section at midspan is zero.

b. Vertical displacement of the support is zero.

Results of the analysis are plotted in the drawings appended to this report. Fig. 1 shows the loading arrangement. Figs. 2 and 3 show, respectively, the deformation of sections which were vertical before loading and the distribution of horizontal stress on the cross-sections. Sections selected are located at midspan and at 6", 18", 24", and 30" from the midspan for displacements and at 1", 7", 19", 25" and 31" from midspan for stresses. Sections under the loads and at the support have not been considered as, necessarily, the localized influence of concentrated loading will predominate at these sections and this is of no interest to the present investigation. Figure 4 shows a comparison of strains calculated using the theory of elasticity and those calculated on the basis of linear distribution. It is clear that the linear distribution theory gives, near the extreme fibers, compressive strains which are too large and tensile strains which are too small as compared to the elasticity solution which is close to the lab observations.

PROPERTIES OF A BEAM SECTION HAVING DIFFERENT ELASTIC MODULI IN TENSION AND COMPRESSION

Following the broad assumptions made by the WES laborabory in Reports 1 and 3, Reference 1, a beam section was examined allowing for the elastic modulus in tension being different from that in compression. It was assumed that strain was linear over the cross-section, and that no axial forces existed.

Linearity of strain yields:

$$k = \frac{e_c}{e_c + e_t} = \frac{1}{1 + (\frac{e_t}{e_c})}$$

where,

kd = depth of neutral axis below the compression face,

d = total depth of the section,

e_c, e_t = the maximum compressive and tensile strains respectively.

Axial force equilibrium gives:

$$k = \frac{E_t e_t}{E_t e_t + E_c e_c} = \frac{1}{1 + \frac{E_c}{E_t} \cdot \frac{e_c}{e_t}}$$

where,

E_c and E_t are the elastic modulus in compression and tension respectively.

From the two values of k we obtain:

$$\frac{E_c}{E_t} = \frac{e_t^2}{e_c^2}$$

From the moment equilibrium, using the relations established above, the following expressions result:

$$E_c = \frac{3(e_c + e_t)}{e_c^2} \quad \frac{M}{bd^2}$$

$$E_t = \frac{3(e_c + e_t)}{e_t^2} \quad \frac{M}{bd^2}$$

where,

M = the bending moment at the cross-section,

b = width of the beam.

If strains are measured at a distance d' inside each face, then, if the measured strains are e_c' , e'_t , linearity of strain implies

$$e_c = \frac{(d-d')e_c + d'e_t'}{d - 2d'}$$

and $$e_t = \frac{d'e_c' + (d-d')e_t'}{d - 2d'}$$

These expressions have been used to analyse the test data for Beams 1, 2 and 3 of Report 3, Reference 1 for the load range $2P_L = 24$ kips to $2P_L = 56$ kips. Results are tabulated in Table I which also shows the approximate values obtained by the WES assuming linear variation in stress over the entire cross-section.

DISCUSSION

The finite element analysis of the beam shows that the stress distribution over the various cross-sections of the elastic beam is, as expected, non-linear. For sections near the middle of the beam span, the departure from linearity is relatively small except near the extreme fibers. In this zone, in general, the tensile stress on extreme fibers is greater than the compressive stress on corresponding locations. The difference is nearly 10% for the midspan section for the load considered in the analysis, i.e. $P_t = 32{,}245$ lbs. On sections away from the midspan, there is significant departure from linearity over most of the cross-section.

In the beams tested at the WES laboratory, strainmeters were located at the midspan section and placed 2" inside from the extreme fibers. Strainmeter records showed that the tensile strains were greater than the corresponding compressive strains in most cases. In the light of the finite element analysis reported herein, the difference in strains is logically explained by and is consistent with the theory of elasticity.

The analysis based on the assumption of linear strain and stress distribution, and different elastic moduli in compression and tension is only of academic value. Compared with the WES laboratory values, it is seen that the difference in the maximum compressive and tensile strains is somewhat less and the difference in elastic moduli greater than that reported. Also, the maximum compressive stress is, in general, larger than the maximum tensile stress. The difference in the elastic moduli - about 70% for a strain difference of 30% - is rather large. However, it does not appear to be rational to give importance to results based on several arbitrary assumptions all of which are open to serious objection.

PRELIMINARY CONCLUSIONS

The studies reported herein show that in tests of the type reported in Reference 1 elementary beam theory does not apply since the span-depth ratios of the test beams are relatively small. An elastic finite element analysis of the test beam indicated that the tensile strains are somewhat larger than the corresponding maximum compressive strains. Therefore, it can be concluded that part of the difference in compressive and tensile strains in the beams is due to the geometry of the test specimen, and not due to a difference of material properties in tension and compression. As the specimen appraoches rupture, the difference would obviously become more pronounced because of the influence of cracking.

The test data reported by the WES laboratory in Reference 1 show precisely the above discrepancy in compressive and tensile strains. The difference in strains observed is not unduly large (this tends to narrow down substantially if strains are computed taking first load application as the 'zero') as to cast doubts on the elastic modulus being constant. It is believed that the information available at this stage does not warrant a conclusion that concrete possesses different elastic moduli in tension and compression ranges of stress.

It is proposed, therefore, to carry on all further work on this project covered by this contract, on the assumption of a uniform modulus of elasticity for concrete.

REFERENCES

1. Concrete Laboratory Studies, Dworshak (Bruce's Eddy) Dam, Idaho; Miscellaneous Paper No. 6-613, Report 1 and Report 3; U.S. Army Engineer Division Laboratory, North Pacific; U.S. Army W.E.S., Vicksburg, Mississippi.

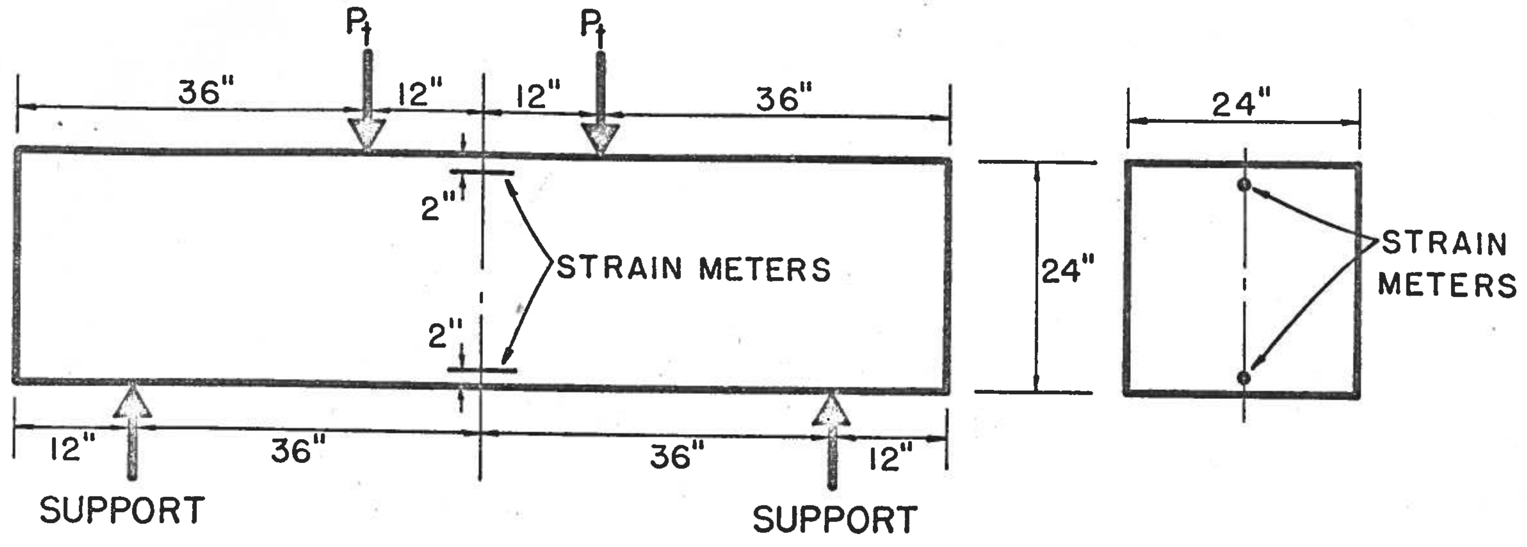
TABLE I

Quantity	Beam 1		Beam 2		Beam 3	
	WES Lab. Value	Per Present Analysis	WES Lab. Value	Per Present Analysis	WES Lab. Value	Per Present Analysis
Neutral Axis	---	0.435	---	0.465	---	0.458
Max. Tens. Strain	36.0	35.2	38.4	37.9	39.6	39.0
Max. Comp. Strain	26.4	27.0	32.4	32.9	32.4	33.0
Elastic Modulus (Tensile)	4.61×10^6	4.18×10^6	4.28×10^6	4.11×10^6	4.08×10^6	3.95×10^6
Elastic Modulus (Compressive)	6.38×10^6	7.10×10^6	5.06×10^6	5.45×10^6	4.94×10^6	5.50×10^6
Max. Tens. Stress	167	147	167	156	167	154
Max. Comp. Stress	167	192	167	179	167	182

Notes:

i. The beams are No. 1, 2, and 3 of Report 3, December 1964, Reference 1.

ii. Calculations are for load variation from $2P_L = 24$ kips to $2P_L = 56$ kips.



NOTES:

THE BEAM SELECTED FOR ANALYSIS IS NO. 1 OF REPORT 3, REFERENCE 1.
FOR ANALYSIS, $P_t = 32,245$ LBS, $W = 154.22$ LBS/CFT.

DISPLACEMENT BOUNDARY CONDITIONS WERE:

1.) NO HORIZONTAL DISPLACEMENT ON VERTICAL SECTION AT MIDSPAN.
2.) NO VERTICAL DISPLACEMENT AT SUPPORTS.

FIG. 1 LOADING ARRANGEMENT

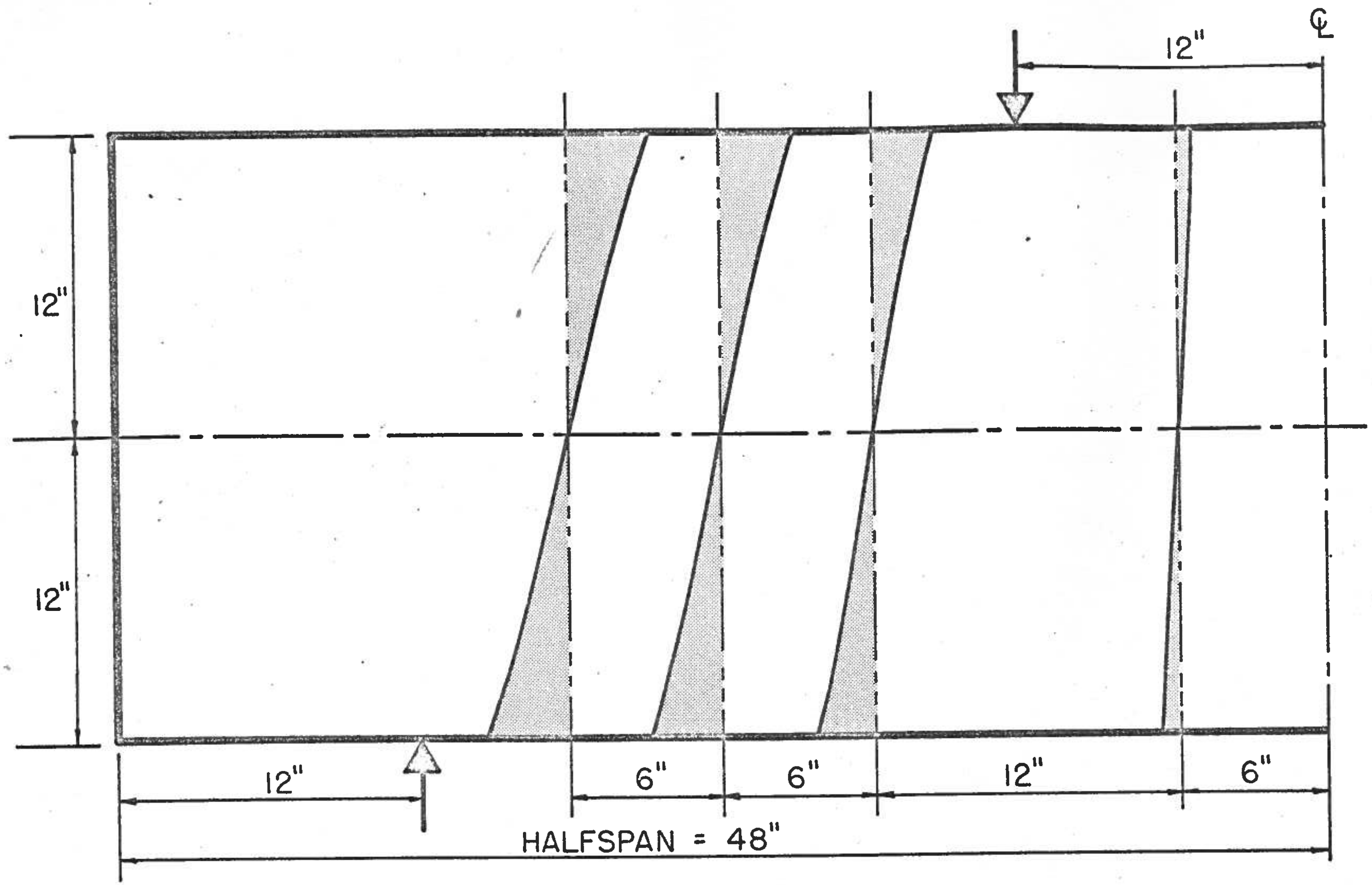


FIG. 2 HORIZONTAL DISPLACEMENT OF VERTICAL SECTIONS

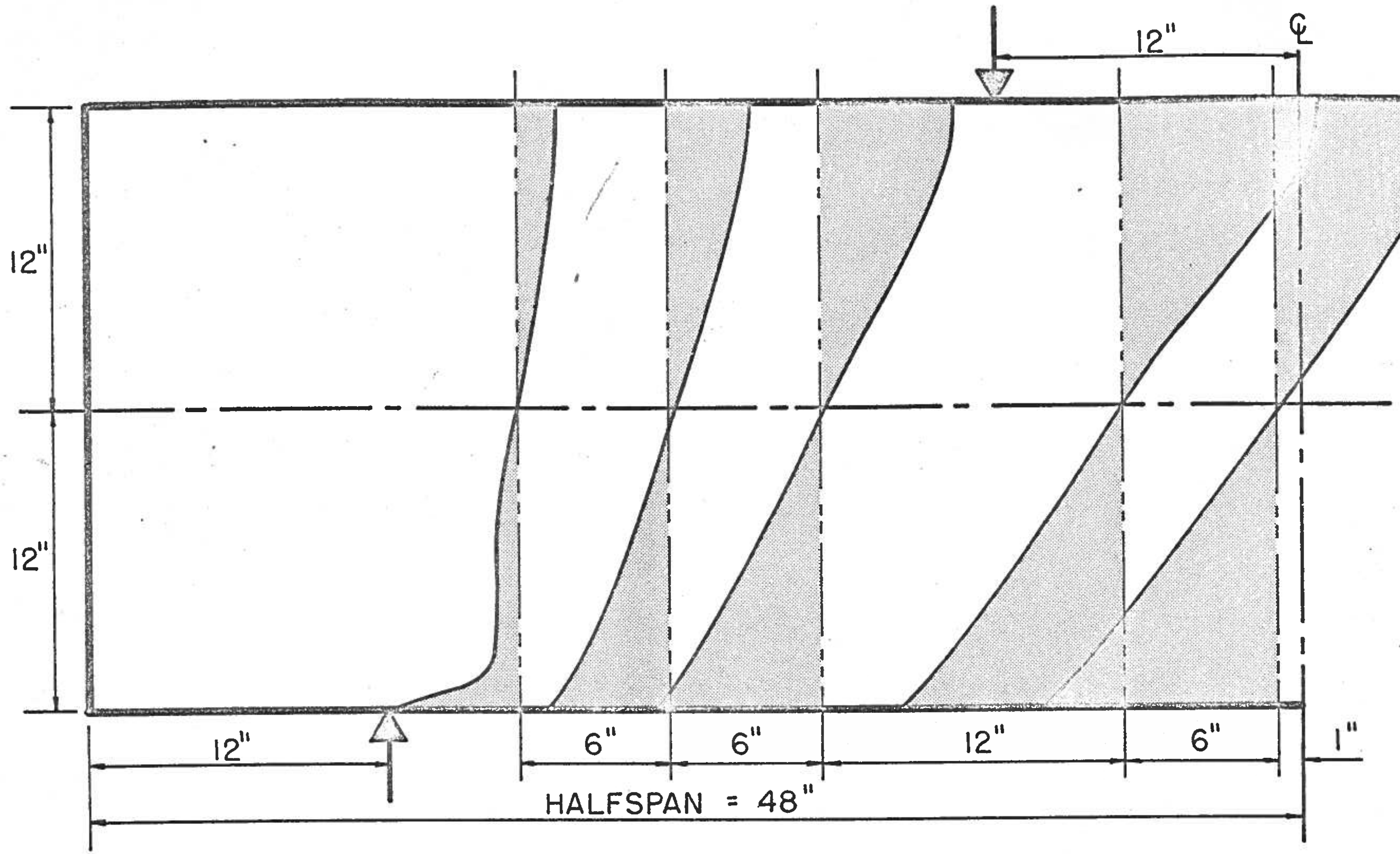
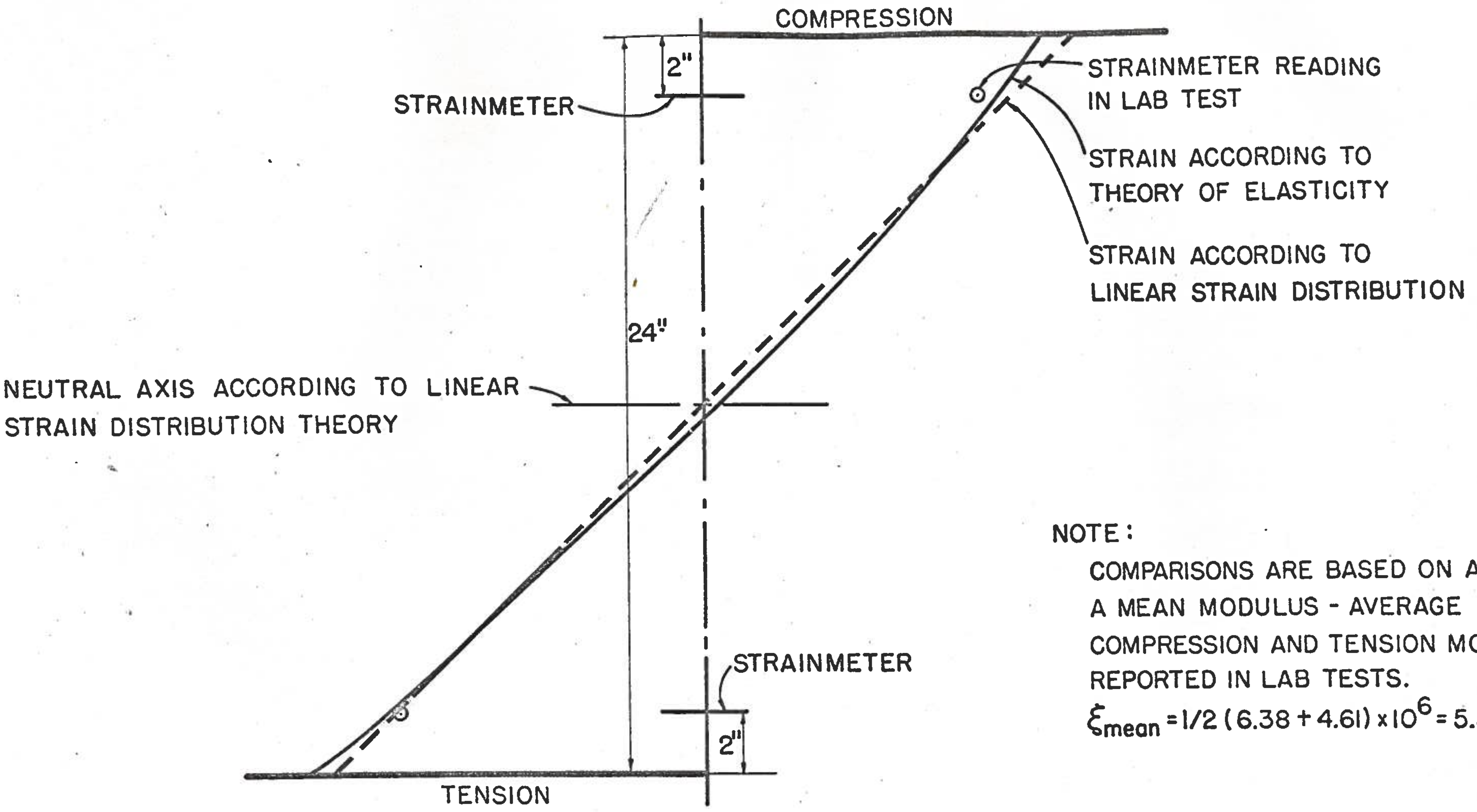


FIG. 3 HORIZONTAL STRESSES ON VERTICAL SECTIONS



NOTE:

COMPARISONS ARE BASED ON ASSUMING A MEAN MODULUS - AVERAGE OF THE COMPRESSION AND TENSION MODULI REPORTED IN LAB TESTS.

$\xi_{mean} = 1/2\,(6.38 + 4.61) \times 10^6 = 5.5 \times 10^6$ psi

FIG. 4 STRAIN DISTRIBUTION AT MIDSPAN ($P_t = 28{,}245$ lbs)