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MODES OF ACCELERATION OF IONS IN A 3-DEE CYCLOTRON

M. Jakobson, M. Heusinkveld, and L. Ruby

February 29, 1952

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MODES OF ACCELERATION OF IONS IN A 3-DEE CYCLOTRON

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February 29, 1952

This analysis shows that a symmetrical three-dee cyclotron accelerating ions below relativistic velocities has the property that ions of differing e/m ratios can be accelerated without changing the frequency of the electrical power supplied to the dees or the value of the magnetic field. The energy gain per revolution of the ions which can be accelerated in the various modes are found, considering three identical dees of arbitrary angular widths. For these calculations it has been assumed that a step-function voltage change occurs at each edge of the dees.

Ordinarily the three dees are driven with voltages of equal amplitudes. The phase relations of these voltages may be of three types: (1) The voltages may all be in phase; (2) the phases may be 120 degrees from each other, with the phase of the B dee following that of the A dee, and the phase of the C dee following that of the B dee, designated the ABC sequence; (3) The phases may be 120 degrees from each other but in the reverse sequence, namely ACB. A phase relation of type (2) will be considered first in analyzing the possibilities for ion acceleration. The voltages on the dees can be expressed:

$$V_A = V_2 \, e^{i\omega_0 t}$$

$$V_B = V_2 \, e^{i(\omega_0 t - 2/3\,\pi)}$$

$$V_C = V_2 \, e^{i(\omega_0 t - 4/3\,\pi)}$$

Here V_2 is the amplitude of the voltages with respect to ground, ω_o is the angular frequency of the electrical power supplied to the cyclotron, and t is time.

As the ion moves along its circular path in the cyclotron, it will complete a revolution in time $t = 2\pi \frac{m}{eH} = \frac{2\pi}{\Omega}$, where Ω is its angular frequency. If it passes through the center of the A dee at time $t = t_o$, it will receive impulses at the successive gaps at the times

$$t_1 = t_o + \frac{\theta}{2\Omega}$$

$$t_2 = t_o + \frac{2\pi}{3\Omega} - \frac{\theta}{2\Omega}$$

$$t_3 = t_o + \frac{2\pi}{3\Omega} + \frac{\theta}{2\Omega}$$

$$t_4 = t_o + \frac{4\pi}{3\Omega} - \frac{\theta}{2\Omega}$$

$$t_5 = t_o + \frac{4\pi}{3\Omega} + \frac{\theta}{2\Omega}$$

$$t_6 = t_o + \frac{6\pi}{3\Omega} - \frac{\theta}{2\Omega}$$

where θ is the width of each dee expressed in radians.

The energy that this ion receives in going around the cyclotron once is found as the sum of the energy contributions at the six dee edges. In order that the voltage on the A dee be zero at t = 0, the imaginary part of the voltage expressions is taken. Putting in the values of time as given above, the energy gain per revolution is

$$\Delta W = eV_2 \text{ I.P.} \Big[\exp\left\{ i\omega_o\ (t+\theta/2\Omega)\right\} - \exp\left\{ i\left[\omega_o(t_o+2\pi/3\Omega - \theta/2\Omega) - 2\pi/3\right]\right\}$$
$$+ \exp\left\{ i\left[\omega_o(t_o+2\pi/3\Omega + \theta/2\Omega) - 2\pi/3\right]\right\} - \exp\left\{ i\left[\omega_o(t_o+4\pi/3\Omega - \theta/2\Omega) - 4\pi/3\right]\right\}$$
$$+ \exp\left\{ i\left[\omega_o(t_o+4\pi/3\Omega + \theta/2\Omega) - 4\pi/3\right]\right\} - \exp\left\{ i\omega_o(t_o+6\pi/3\Omega - \theta/2\Omega)\right\}\Big].$$

The quantity e is the total charge of the ion. Calling the fixed phase angle $\omega_o t_o = \emptyset$ and simplifying the expression,

$$\Delta W = eV_2 \text{ I.P.}\Big[\exp i\emptyset \left[\exp(i\omega_o\theta/2\Omega) - \exp(-i\omega_o\theta/2\Omega)\right] \Big\{ 1 + \exp\left[(2\pi i/3)(\omega_o/\Omega - 1)\right] + \exp\left[(4\pi i/3)(\omega_o/\Omega - 1)\right]\Big\}\Big].$$

In this expression the term $\exp i\emptyset$ gives the simple sinusoidal variation of energy gain with original phase angle, the quantity

$$\left[\exp(i\omega_o\theta/2\Omega) - \exp(-i\omega_o\theta/2\Omega)\right]$$

gives the effect of the dee width on the energy gain, and the quantity in braces gives the effect of the three-dee three phase periodicity without reference to the particular geometry of the dees.

Calling $\left(\frac{\omega_o}{\Omega} - 1\right)\frac{2\pi}{3} = f$ and simplifying further,

$$\Delta W = 2\ eV_2 \sin\left(\frac{\omega_o}{\Omega}\right)\left(\frac{\theta}{2}\right) \text{ Re } \left[e^{i\emptyset} \sum_{r=o}^{2} e^{irf}\right].$$

This equation is easily extended to the case where the ion makes s revolutions, where s is an integer, in which the gain per revolution is

$$\Delta W = 2\ eV_2 \sin\left(\frac{\omega_o}{\Omega}\right)\left(\frac{\theta}{2}\right) \text{ Re } \left[e^{i\emptyset} \frac{1}{s} \sum_{r=o}^{3s-1} e^{irf}\right].$$

If after several revolutions the ion reaches the center of the first dee at an rf phase identical to that at which the energy summation was begun, the series can be terminated to give a unique value for average energy gain per revolution. If no such repeat interval exists, the series does not terminate and the energy gain per revolution approaches zero:

$$\lim_{s \to \infty} \frac{1}{s} \sum_{r=o}^{3s-1} e^{irf} = 0$$

If the ion does reach the center of the first dee at an rf phase identical to that at which the summation was begun, then $3sf=2\pi m$ and $e^{i3sf}=1$, where s and m are integers and s gives the number of revolutions of the ion.

If $e^{if} \neq 1$, the summation can be expressed

$$\sum_{r=o}^{3s-1} e^{irf} = \frac{e^{i3sf} - 1}{e^{if} - 1}.$$

Since $e^{i3sf} = 1$, the summation is seen to equal zero.

Taking the remaining possibility that $e^{if} = 1$, the series terminates at s = 1, which has the physical meaning that the ion makes one revolution in an integral number of rf cycles. In this case the summation is

$$\sum_{r=o}^{2} e^{irf} = 3$$

and the quantity f is further restricted to the values $f = 2\pi n$. The energy gain per revolution becomes

$$\Delta W = 6\ eV_2\ \sin\left(\frac{\omega_o}{\Omega}\right)\left(\frac{\theta}{2}\right)\cos\phi.$$

Since $f = \left(\frac{\omega_o}{\Omega} - 1\right)\frac{2\pi}{3}$, the acceptable ratios $\frac{\omega_o}{\Omega}$ are found to be $\frac{\omega_o}{\Omega} = 3n + 1$ for this mode in which the ion travels in the same direction as the rf phase rotation.

For a given magnetic field, positive ions will travel in one direction around the cyclotron and the negative ions will travel in the opposite direction, such that a given phase sequence ABC will be in the direction of rotation of the positive ions but opposite to that of the negative ions. Consequently the foregoing analysis is valid for positive ions when the rf phase sequence is ABC, and for negative ions when the phase sequence is ACB.

The geometric factor of the dees, $\sin\left(\frac{\omega_o}{\Omega}\right)\left(\frac{\theta}{2}\right)$, may further limit the number of ions which may be accelerated, i.e. it is an amplitude modulating factor for energy gain which may be zero for a given dee angular width for some acceleration mode which would otherwise be acceptable.

For the case in which the ion travels in a direction opposite to the phase sequence the foregoing analysis still holds, but with the redefinition of the quantity f:

$$f = \left(\frac{\omega_o}{\Omega} + 1\right) \frac{2\pi}{3}.$$

Using the requirement that $f = 2\pi n$, the permissible values of $\frac{\omega_o}{\Omega}$ are

$$\frac{\omega_o}{\Omega} = 3n + 2,$$

where n may take integral positive values including zero. This is valid for positive ions when the phase sequence is ACB or for negative ions when the sequence is ABC, using the convention established earlier that a position ion passes through the A, B, and C dees in consecutive order. Designating by V_3 the amplitude of the voltages in this sequence, the energy gain per revolution is given by the expression

$$\Delta W = 6\ eV_3 \sin\left(\frac{\omega_o}{\Omega}\right)\left(\frac{\theta}{2}\right) \cos \phi.$$

For the case in which the three dees are excited in phase, the quantity f becomes $f = \left(\frac{\omega_o}{\Omega}\right)\left(\frac{2\pi}{3}\right)$, and the permissible modes of acceleration are $\frac{\omega_o}{\Omega} = 3n$. In this case the sign of the charge is immaterial. The energy gain per revolution is again $\Delta W = 6\ eV_1 \sin\left(\frac{\omega_o}{\Omega}\right)\left(\frac{\theta}{2}\right) \cos \phi$.

The factor $\cos \phi$ in the equations for energy gain gives the variation of energy gain with the position of the ion with respect to the phase of the dee voltages, maximum gain always occurring when the ion is at the center of any dee when the instantaneous voltage on that dee is zero, and either becoming negative or becoming positive as required by the particular mode considered. The gain drops to zero as the rf phase varies $\pm 90^\circ$, corresponding to $\pm \frac{\Omega}{\omega_o} \times 90$ degrees of angular displacement of the ion from the center of the dee when the instantaneous voltage is zero. Other considerations, however, may limit this angle to values less than this. The number of ion bunches present per ion cycle is $\frac{\omega_o}{\Omega}$.

Table I gives the results of this analysis in tabular form, with numerical values being computed for the maximum energy gain per revolution for the cases of 60 degree, 90 degree, and 120 degree dees. For dees of other angular widths or dees in which the fringing fields are included, the same $\frac{eH}{m}$ ratios will give the ions eligible for acceleration, but with other values of energy gain per revolution, including zero in particular cases, as indicated by the θ degree column. In this table the variable n has been so defined that resulting negative values of $\frac{\omega_o}{\Omega}$ indicate negative ions and positive values of $\frac{\omega_o}{\Omega}$ indicate positive ions. In these results the magnitude of the charge of the ion is to be included in the factor e in computing energy gain per revolution.

Table II indicates the ions of H_2, D_2, and He which can be accelerated in the three modes of a 3 dee cyclotron with 60° dees and electrical power of frequency $\omega_0 = \left(\frac{eH}{m}\right)_{proton}$.

These calculations have been made for conditions of exact balance of amplitudes of the voltages and exact phase conditions. In practice these conditions are not ordinarily met - slight unbalances in amplitudes or deviations in phase may occur. From the theory of symmetrical components, however, any arbitrary voltage distribution among the three dees both as to amplitude and as to phase may be expressed as the sum of the three symmetrical components as analyzed earlier - the ABC mode, the ACB mode, and the mode in which the three dees are excited in phase. This can be expressed mathematically

$$V_A = V_1 \, e^{i(\omega_0 t + \alpha)} + V_2 \, e^{i(\omega_0 t + \beta)} + V_3 \, e^{i(\omega_0 t + \gamma)}$$

$$V_B = V_1 \, e^{i(\omega_0 t + \alpha)} + V_2 \, e^{i(\omega_0 t + \beta - 2\pi/3)} + V_3 \, e^{i(\omega_0 t + \gamma + 2\pi/3)}$$

$$V_C = V_1 \, e^{i(\omega_0 t + \alpha)} + V_2 \, e^{i(\omega_0 t + \beta - 4\pi/3)} + V_3 \, e^{i(\omega_0 t + \gamma + 4\pi/3)}.$$

Here V_A, V_B, and V_C are the actual voltages on the dees, including both amplitude and phase, V_1 is the amplitude of the "in-phase" component, V_2 is the amplitude of the ABC component, and V_3 is the amplitude of the ACB symmetrical component. The phase factors α, β, and γ are constant phase angles arising from the resolution of the dee voltages into the symmetrical components.

If the dee voltages are nearly balanced in one of the phase sequences, then the amplitudes of the other two symmetrical components will be small. Since to a first approximation the effects of these voltages on accelerating ions are linear, each symmetrical component V_1, V_2, or V_3 can be considered separately, and the effect of the unbalanced dee voltages can be found by adding the separate effects of each component. It is observed from the formulas for permissible modes of acceleration that no ion can be accelerated by more than one of the three components, for a given magnetic field strength and electrical frequency.

These results are significant in that under identical operating conditions several different types of ions may be accelerated, either intentionally or as contamination products. Only ions with integral multiples of the quantity ω_0/Ω are eligible for acceleration. For example, if the cyclotron is driven in phase sequence ABC to accelerate H^+ in its lowest mode ($\omega_0 = \Omega$), then it will also accelerate $_4He^+$ and $_{14}N^{++}$ at the same time. In addition, if the voltages on the dees are unbalanced such that the ACB and A = B = C symmetrical components are also present, then many other ions can be accelerated simultaneously. An ion with a non-integral ratio of ω_0/Ω, however, such as He^{3++} cannot be accelerated under these conditions of operation, but can be accelerated only if the magnetic field or the electrical frequency of the cyclotron is changed. Since submultiples of ω_0/Ω are excluded, if the cyclotron is adjusted to accelerate $\dot{H}H^+$ or D^+ in the lowest mode, then H^+ cannot be accelerated.

In practice the voltage will not change in a step-function manner at the dee edges, but will change gradually over a distance comparable to the vertical aperture of the dees. When this fringing distance is comparable to the distance that an ion travels in an rf cycle, which will be the case for modes of operation where ω_0/Ω is large, the ion will gain very little energy in crossing any of the dee edges, and there will be no significant acceleration of these ions relative to those of lower ω_0/Ω ratios.

In the foregoing analysis it has been assumed that the magnetic field is constant as a function of radius, resulting in a constant value of the ion frequency Ω. In order to focus the beam, however, the magnetic field ordinarily is decreased with increasing radius, resulting in a lowering of the ion frequency as the orbit of the ion expands. For an ion which is in resonance at the center of the cyclotron the phase angle ϕ will no longer be constant but will vary as the ion moves outward. The permissible limits of variation of ϕ are $\pm$ 90 degrees; outside of these limits the ion will lose energy to the electric field. Since the total phase shift ϕ with respect to the electrical frequency is the sum of the increments per ion revolution, a certain threshold dee voltage is required to limit the number of revolutions necessary to obtain the desired energy and thereby to maintain the phase angle ϕ within the limits given.

At exact resonance an ion makes a revolution in time $T_0 = 2\pi/\Omega_0$ while for an off-resonance ion it is $T = 2\pi/\Omega$. The change in the phase angle per ion revolution is

$$\frac{\Delta\phi}{\Delta n} = \omega_0(T - T_0) = 2\pi\omega_0\ (1/\Omega - 1/\Omega_0).$$

With respect to the magnetic field,

$$\frac{\Delta\phi}{\Delta n} = \frac{2\pi\omega_o}{\Omega_o}\left(\frac{H_o - H_r}{H_r}\right).$$

The gain in energy per revolution is

$$\frac{\Delta W}{\Delta n} = 6\ eV_o \cos\phi \sin\left(\frac{\omega_o}{\Omega}\right)\left(\frac{\theta}{2}\right).$$

Combining, and replacing differences by differentials,

$$\cos\phi\ d\phi = \frac{2\pi}{6\ eV_o \sin\left(\frac{\omega_o}{\Omega}\right)\left(\frac{\theta}{2}\right)}\left(\frac{\omega_o}{\Omega_o}\right)\left(\frac{H_o - H_r}{H_r}\right) dW.$$

Integrating,

$$\sin\phi_2 - \sin\phi_1 = \frac{\pi}{3\ eV_o}\ \frac{\omega_o}{\Omega_o}\int_0^W \frac{1}{\sin\left(\frac{\omega_o}{\Omega}\right)\left(\frac{\theta}{2}\right)}\left(\frac{H_o - H_r}{H_r}\right) dW.$$

Since $\sin\left(\frac{\omega_o}{\Omega}\right)\left(\frac{\theta}{2}\right)$ is nearly constant, it is taken outside the integral.

Letting $\sin\phi_2 - \sin\phi_1 = 2$, the threshold voltage V_o is obtained:

$$V_o = \frac{\pi}{6e}\frac{\omega_o}{\Omega_o}\frac{1}{\sin\left(\frac{\omega_o}{\Omega_o}\right)\left(\frac{\theta}{2}\right)}\int_0^W\left(\frac{H_o - H_r}{H_r}\right) dW.$$

This formula is of value in comparing threshold voltages for various modes of acceleration for a certain ion when the electrical frequency may be adjusted but the magnetic field is to be held practically fixed. In this case a given energy corresponds to a given radius so that the value of the integral will not change and a direct comparison of the threshold voltage may be made. This formula shows that the threshold voltage varies directly as the order of the mode, with a modulating factor $1/\sin\left(\frac{\omega}{\Omega_0}\right)\left(\frac{\theta}{2}\right)$.

For example, in accelerating H_1^+ in the cyclotron with three θ-degree dees, the ratios of the threshold voltage in the lowest ABC mode, where $\omega_0/\Omega_0 = 1$, to that in the lowest ACB mode, where $\omega_0/\Omega_0 = 2$, is

$$\frac{V_{ABC}}{V_{ACB}} = \frac{\left[\frac{\omega_0}{\Omega_0}\ \frac{1}{\sin\left(\frac{\omega_0}{\Omega_0}\right)\left(\frac{\theta}{2}\right)}\right]_{ABC}}{\left[\frac{\omega_0}{\Omega_0}\ \frac{1}{\sin\left(\frac{\omega_0}{\Omega_0}\ \frac{\theta}{2}\right)}\right]_{ACB}}$$

$$= \frac{\sin\theta}{2\sin\frac{\theta}{2}} = \cos\frac{\theta}{2}.$$

This result shows that the threshold voltage for the mode in which ω_0/Ω_0 is 2 is always greater than the mode in which ω_0/Ω_0 is 1.

Similarly the ratio of the threshold of the lowest ABC mode, where $\omega_0/\Omega_0 = 1$, to that of the lowest A = B = C mode, where $\omega_0/\Omega_0 = 3$, is

$$\frac{V_{ABC}}{V_{A=B=C}} = \frac{\sin\frac{3\theta}{2}}{3\sin\frac{\theta}{2}} = 1 - \frac{4}{3}\sin^2\frac{\theta}{2}$$

Again the threshold voltage is the mode in which $\omega_0/\Omega_0 = 3$ is always higher than that in which $\omega_0/\Omega_0 = 1$. Table III lists the relative threshold voltages for the three modes with dees of various angular widths.

If the electrical frequency is to be held fixed but the magnetic field is to be varied to accelerate this ion in various modes, the value of the integral in the expression for the threshold voltage will change because of the change in radius required for a given energy. The kinetic energy relation $W = 1/2\ m\Omega^2\ r^2 + 1/2\ m\dot{r}^2$ can be used to relate the quantities in the integral. This procedure will also be valid for different ions.

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TABLE I

3 DEES - 120° SEPARATION

Angular Width of Dees	60° Dees			90° Dees			120° Dees			θ Dees		
Mode of Acceleration	ABC	ACB	A=B=C	ABC	ACB	A=B=C	ABC	ACB	A=B=C	ABC	ACB	A=B=C
Ions Accelerated $\frac{\omega_0}{\Omega}$ =	3n+1	3n+2	3n except 6n	3n+1	3n+2	3n	3n+1	3n+2	None	3n+1	3n+2	3n
					except 4n					except $\frac{\omega_0}{\Omega} = \frac{2n\pi}{\theta}$		
Energy gain per turn												
n even integer (pos or neg)	$3eV_0$	$5.2eV_0$	$6eV_0$	$4.2eV_0$	$6eV_0$	$4.2eV_0$	$5.2eV_0$	$5.2eV_0$		$6eV_0 \left\lvert \sin(3n+1)\frac{\theta}{2} \right\rvert$	$6eV_0 \left\lvert \sin(3n+2)\frac{\theta}{2} \right\rvert$	$6eV_0 \left\lvert \sin 3n \frac{\theta}{2} \right\rvert$
n odd integer (pos or neg)	$5.2eV_0$	$3eV_0$	"	$6eV_0$	$4.2eV_0$	"	"	"				

ω_0 = rf angular frequency

Ω = ion frequency = $\frac{eH}{m}$

n = integer

Number of bunches in each case is $\frac{\omega_0}{\Omega}$ per ion cycle

(Assumes V_0 (voltage maximum on the dees) the same, and a step function change at the edge of the dees)

e = total charge of ion

θ = dee angular width

TABLE II

IONS THAT CAN BE ACCELERATED IN A THREE DEE CYCLOTRON
WITH 60° DEES
WITH $\omega_o = \left(\frac{eH}{m}\right)_{Proton}$.

Mode	Ion	Gain Per Turn	Energy at Radius R	Number of Turns (Min)
ABC	H^+	$3\ eV_o$	W	$\frac{W}{3\ eV_o}$
	D_2^+	$5.2\ eV_o$	$\frac{W}{4}$	$\frac{W}{20.8\ eV_o}$
	He^+	"	"	"
	D^-	"	$\frac{W}{2}$	$\frac{W}{10.4\ eV_o}$
ACB	H_2^+	"	"	"
	D^+	"	"	"
	He^{++}	"	W	$\frac{W}{5.2\ eV_o}$
	H^-	$3\ eV_o$	W	$\frac{W}{3\ eV_o}$
A=B=C	None			

TABLE III

RELATIVE THRESHOLD VOLTAGE

Dees	ABC Mode with $\frac{\omega_o}{\Omega} = 1$	ACB Mode with $\frac{\omega_o}{\Omega} = 2$	A=B=C Mode with $\frac{\omega_o}{\Omega} = 3$
60°	1.00	1.15	1.50
90°	0.707	1.00	2.15
120°	0.577	1.15	∞
θ°	$\frac{1}{2\sin\frac{\theta}{2}}$	$\frac{1}{\sin\theta}$	$\frac{3}{2\sin\frac{3\theta}{2}}$

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