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**Authors**
Boudoukh, Jacob
Richardson, Matthew
Stanton, Richard
et al.

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*Jacob Boudoukh*[a], *Matthew Richardson*[b], *Richard Stanton*[c] *and Robert F. Whitelaw*[b]

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[1][a]IDC, NYU and NBER; [b]NYU and NBER; [c]U.C. Berkeley. Contact: Prof. Robert Whitelaw, NYU, Stern School of Business, 44 W. 4th St., New York, NY 10012, (212) 998-0338, rwhitela@stern.nyu.edu. We thank the Fisher Center for Real Estate and Urban Economics for financial support.

# Valuing Mutual Fund Companies


## ABSTRACT

Combining insights from the contingent claims and the asset-backed securities literatures, we study the economics of value creation in the asset management business. In particular, we provide a theoretical model and a closed form formula for the value of fund fees in the presence of the well known flow-performance relation, giving rise to interesting nonlinearities and volatility-related effects. The theoretical model sheds light on the role of fees, asset growth, asset and benchmark volatility, and the intensity of the flow-performance relation. To better understand the role of changing fund characteristics such as age and size on the fund value and fund risk, we estimate the empirical relation between returns and flows conditional on these characteristics for various asset classes. We study these effects using Monte Carlo simulations for various economically meaningful parameter values for specific asset classes. Measuring value as a fraction of assets under management, we find that both value and risk, systematic and idiosyncratic, decline in size and age. In addition, value is a complex, non-monotonic function of the fee charged on the fund.

# 1 Introduction

At the end of 2002, the mutual fund industry managed over $11 trillion dollars of assets worldwide according to data from the Investment Company Institute. Given the size of this industry, it is surprising that no literature has emerged which values mutual fund revenue streams. The literature to date has focused on documenting the performance of mutual funds, while a more recent literature links fund flows to this performance.[1] While there is considerable debate on the topic of whether mutual funds outperform relevant benchmarks, there is overwhelming evidence that fund flows depend on the ex post performance of mutual funds.

This paper develops a valuation methodology for the fee flow generated by these mutual funds ("mutual fund valuation" henceforth). Specifically, we employ a contingent claims approach to mutual fund valuation by recognizing that mutual funds have the essential characteristics of asset-backed securities. The key insight underlying our valuation approach is that in an efficient market the present value of a claim on the future value of an asset is simply a function of the current value of that asset. The result is that the value can be written in terms of three elements: (1) the current value of assets under management, (2) the volatility of assets under management (necessary for valuing option-like features induced by the link between fund flows and fund performance), and (3) a set of parameters that describe the fund (including the fees, the age, the size of assets, and parameters describing the flows into and out of the fund). Of particular interest, for the reasonable case where there is a positive link between flows and performance, the functional relation between the mutual fund value and the underlying net asset value of the fund is nonlinear.

We are not the first to recognize that valuation in the asset management industry can be done using the contingent claims approach. Goetzmann, Ingersoll and Ross (2003) value hedge fund management businesses under incentive fees and high-water mark provisions. While their paper focuses on features that are specific to the hedge fund industry, one could infer mutual fund values that are similar to some special cases of the model provided below by assuming no incentive fees and no high-water marks. In addition, Boudoukh, McAllister, Richardson and Whitelaw (2000) value the revenue streams from deferred load (so-called B and C shares) mutual funds. Some of the insights from these papers carry through here. However, neither paper addresses a critical and complicating feature of mutual fund valuation, namely the evidence that the flow of capital into the fund (and thus the growth rate of the assets) depends on changes in the fund's value and other characteristics

[1] See, for example, Elton, Gruber and Blake (1996), Gruber (1996), Carhart (1997), Chevalier and Ellison (1997) and Sirri and Tufano (1998), among others.

of the fund.[2] This issue is key from a valuation perspective and highly relevant for the mutual fund industry, and is the focus of our paper.

Methodologically, the closest analogy to our paper can be found in the mortgage-backed securities literature, and in particular empirical mortgage valuation models such as Schwartz and Torous (1989). That paper develops an approach for the valuation of a mortgage-backed security by incorporating an empirical model of mortgage prepayments into a simulation framework. Here, we develop a methodology for the valuation of mutual funds by building into our analysis an empirical model of fund flows. There are a number of similarities between the two approaches and the issues that arise. In the mortgage-backed case, prepayments depend on the underlying factor, i.e., interest rates, as well as particular characteristics of the mortgages, such as age, burnout, and type.[3] In the mutual fund case, fund flows also depend on the underlying factor, i.e., the change in the fund's net asset value (e.g., Chevalier and Ellison (1997) and Sirri and Tufano (1998)), as well as fund characteristics such as fund size (e.g., Sirri and Tufano (1998)), age (e.g., Chevalier and Ellison (1997)), and fund type (e.g., Bergstresser and Poterba (2002)), among other variables. Moreover, the techniques for implementing the valuation frameworks are based on the same underlying methodology, i.e., Monte Carlo simulation (e.g., Boyle (1977)).

This paper provides several contributions to the current finance literature. First, we develop a methodology for understanding the value and risk of mutual fund revenue streams. In particular, we provide a closed-form solution for a model that incorporates fund flows as a function of relative performance. Though the underlying fund flow model is kept simple for analytical purposes, it identifies some of the salient determinants of fund value. Mutual fund valuation depends both on the underlying asset value and its volatility, similar to asset-backed securities. Other determinants include asset growth and, in particular, the sensitivity of asset growth to relative performance. Interestingly, return volatility matters only to the extent flows correlate with net asset values. We also show that fees play an important role. On the one hand, higher fees increase current revenue, on the other hand, fees reduce growth. The combined effect is ambiguous, depending on the parameters of the system.

Second, using the existing empirical literature as an important guide, we build a more realistic empirical fund flow model and embed that model into a valuation framework for mutual funds. Though the empirical results provide some additional findings relative to

[2] Goetzmann, Ingersoll and Ross (2003) do discuss this issue for hedge funds, but do not build this into the valuation framework per se. Moreover, they argue that the incentive fee structure limits the amount of assets going into the fund.

[3] Schwartz and Torous (1989) assume mortgage termination is determined entirely by interest rate movements. More recent work includes additional factors such as house prices. See, for example, Kau, Keenan, Muller and Epperson (1992), and Downing, Stanton and Wallace (2003).

the current literature, the most important contribution lies in being able to link identified fund flow determinants to mutual fund values. Most significantly, our approach accounts for important state- and time-dependencies, such as how changes in funds' size and age affect value. In particular, across all asset classes, we establish the importance of the effects of age and size on asset flow properties such as growth, the sensitivity to performance, and volatility.

Last, armed with economically meaningful parameters, we provide detailed comparative static analyses of fund value and risk characteristics for funds of different asset classes for various fund sizes, ages, flow volatilities and fees. Values throughout are measured in terms of the present value of fees per dollar of assets under management. We show how age, size and the fee level affect fund value. In particular, older and/or larger funds are much less valuable, but are also less risky, when compared to young and/or small funds. This lower level of risk reflects both the volatility of the mutual fund value and its sensitivity to changes in the fund's net asset value and benchmark return. In addition, we show that mutual fund values are a complex, non-monotonic function of the fee charged on the fund.

The paper is organized as follows. Section 2 presents a closed-form solution (and corresponding analysis) for valuing mutual funds when fund flows depend on changes in the underlying fund's net asset value (NAV). In Section 3, we present an empirical fund flow model for different asset classes. In Section 4, we describe the valuation approach under these more general assumptions about fund flow specifications. The valuation methodology is then implemented and analyzed for its implications for mutual fund values and risk measurement. Section 5 makes some concluding remarks and suggests directions for future research.

# 2 A Closed-form Solution to Mutual Fund Valuation

We develop our theoretical model in continuous time since this makes it easier to derive closed form pricing results in a contingent claims framework. Note that throughout this paper we assume that mutual funds have zero alphas, that is, fund managers do not possess superior skill at choosing undervalued assets. We make this assumption in order to avoid the debate about the presence of excess performance or lack thereof in the mutual fund industry. (See, for example, Ippolito (1992), Elton, Gruber and Blake (1996), and Carhart (1997) for differing evidence, and Berk and Green (2002) for a recent theoretical paper addressing this issue.) As with the usual Black-Scholes framework, we assume that the net asset value of the fund, $S_t$, follows a lognormal diffusion process. Ceteris paribus, the NAV of the fund will decline because of fees paid to the manager of the fund company as well as cash distributions (i.e., dividends and capital gains) to investors. With respect to fees, we assume that the fee is paid continuously at (annualized) rate $c$. While in practice fees are generally taken out at discrete points, the fee as a constant proportion of assets is typical across all mutual funds. In this paper, we abstract from the issue of distributions.[4] The focus of this section is to characterize in a reasonable setting the relation between the fund's value and the underlying assets under management when flows depend on the fund's performance.

Under the risk-neutral measure, we can therefore write the NAV dynamics as:

$$dS_t = (r - c)S_t\, dt + \sigma S_t\, dZ_S, \tag{1}$$

where $r$ is the risk-free rate, $c$ is the management fee, $\sigma$ is the volatility and $Z_s$ is the Brownian motion associated with the underlying assets. In equation (1), mutual funds earn expected returns less than the underlying assets due to the fees paid to the mutual fund company. However, as long as the mutual fund cannot be shorted, there is no possibility of arbitrage.[5]

Let the number of shares in the fund be $N_t$, so that the total value of assets under management in the fund is

$$V_t = N_t S_t.$$

The goal of our paper is to better understand the interaction between $N$ and $S$ in light of the existing empirical evidence and the effect of this interaction on fund valuation. We treat the flow of monies into and out of the fund as a lognormal diffusion process governed by, among other factors, a constant mean $\nu$ and an independent volatility shock $\sigma_N$. As our empirical analysis shows there is an important fraction of flow variations which is unexplained. As long as this variation is not correlated with the underlying NAV of the fund and can be diversified away within a broader set of asset holdings, this noise will have little impact on the fund's value and priced risk (though not its idiosyncratic risk).

However, the assumption that net flows depend on the change in the underlying NAV is clearly the most important piece of evidence and we incorporate this stylized fact below. In particular, we assume that the flow of monies into and out of the fund depends on the contemporaneous return of the NAV of the fund relative to some benchmark. We specify this dependence as a linear relation. Assume that the benchmark is given by index, $I_t$, which

[4] Note that most distributions are reinvested in the funds, so that the effect on the underlying assets under management is small. That is, the NAV decline is offset by the increase in the number of shares of the fund.

[5] There does exist the important issue of why individuals invest in mutual funds in the first place under these circumstances. One possibility is that investors face lower transactions costs, either explicitly through trading constraints or implicitly through time constraints. Alternatively, mutual fund managers may possess superior investment skills, i.e., positive alpha.

follows a lognormal diffusion process. Under the risk-neutral measure, the dynamics of the index can be written as:

$$
\begin{aligned}
dI_t &= rI_t\,dt + \sigma_I I_t\,dZ_I, && (2)\\
dZ_S\,dZ_I &= \rho\,dt, && (3)
\end{aligned}
$$

where $Z_I$ is the index's Brownian motion, $\sigma_I$ its volatility and $\rho$ the correlation between the fund's NAV uncertainty and the index uncertainty.

Assuming that the funds flow in or out at a rate governed by a constant $\nu$, and that the net flows of the fund now depend linearly on its ex-post relative performance, we can write the net flows as

$$
\begin{aligned}
dN_t/N_t &= \nu\,dt + \theta\left([dS_t/S_t - r] - \gamma[dI_t/I_t - r]\right) + \sigma_N\,dZ_N, && (4)\\
&= (\nu - \theta c)\,dt + \theta\left(\sigma\,dZ_S - \gamma\sigma_I\,dZ_I\right) + \sigma_N\,dZ_N. && (5)\\
dZ_N\,dZ_S &= dZ_N\,dZ_I = 0. && (6)
\end{aligned}
$$

Several observations are in order. First, the parameter $\gamma$ describes the rule investors use to judge the fund's performance versus the benchmark index. For example, $\gamma = 1$ compares the return on the fund's NAV to the index's return without adjusting for relative risk. A number of existing studies, such as Sirri and Tufano (1998), Chevalier and Ellison (1997) and Bergstresser and Poterba (2002), use this rule. Alternatively, letting $\gamma$ equal the fund's beta, i.e. $\frac{\mathrm{cov}(dS_t/S_t, dI_t/I_t)}{\mathrm{var}(dI_t/I_t)}$, we are assuming fund investors impose a one-factor model for describing excess performance. Gruber (1996), Del Guercio and Tkac (2002), and most performance measurement studies use this (and more elaborate) rules. Lastly, $\gamma = 0$ is equivalent to investors looking at absolute rather than relative performance.

Second, by linking flows to net (and not gross) returns on the fund, there is a tendency for the net flows to decrease due to the negative effect of fees on NAVs. Of course, in practice, these fees may be related to differing alphas across funds.[6] As described earlier in this section, we are assuming that the managers of the funds do not have the ability to generate excess performance. In addition, there is considerable evidence linking both $\nu$ and $\theta$ to the size of the fund, the type of fund, the fee structure, the fund's age, etc. These features will be built into our empirical model of fund flows in the next section.

Third, one of the main stylized facts from the empirical literature is that there is a lag between the realization of fund performance and its effect on fund flows, one year being a typical lag chosen in this literature (e.g., Sirri and Tufano (1998)). In the continuous-time

[6] However, the evidence in the literature tends to support the opposite conclusion (e.g., Gruber (1996)).

framework above (though not in later sections), we abstract from this feature and assume the effect is instantaneous. This is not an important distinction for valuation. Since the fund's value depends on the present value of all future fees, and these fees depend on the assets and net flows, whether the flow occurs this period or next period is a second-order effect.

Finally, putting aside the idiosyncratic risk, $Z_N$, and index risk, $Z_I$, the stochastic shocks to both NAV and net flows are drawn from the same source. This common shock induces a nonlinear relation between NAV and the value of the fund. This aspect of valuation is new to the literature on mutual funds. Define $s_t \equiv \log S_t$, $i_t \equiv \log I_t$, and $n_t \equiv \log N_t$. By Ito's Lemma,

$$
\begin{aligned}
ds_t &= \left(r - c - \frac{1}{2}\sigma^2\right) dt + \sigma\, dZ_S, && (7)\\
di_t &= \left(r - \frac{1}{2}\sigma_I^2\right) dt + \sigma_I\, dZ_I, && (8)\\
dn_t &= \left(\nu - \theta c - \frac{1}{2}\theta^2\left[\sigma^2 + \gamma^2\sigma_I^2 - 2\rho\sigma\gamma\sigma_I\right] - \frac{1}{2}\sigma_N^2\right) dt \\
&\qquad + \theta\left(\sigma\, dZ_S - \gamma\sigma_I\, dZ_I\right) + \sigma_N\, dZ_N. && (9)
\end{aligned}
$$

Integrating these equations, we obtain

$$
\begin{aligned}
s_\tau &= s_t + \left(r - c - \frac{1}{2}\sigma^2\right)(\tau - t) + \sigma\int_t^\tau dZ_S. && (10)\\
i_\tau &= i_t + \left(r - \frac{1}{2}\sigma_I^2\right)(\tau - t) + \sigma_I\int_t^\tau dZ_I. && (11)\\
n_\tau &= n_t + \left(\nu - \theta c - \frac{1}{2}\theta^2\left[\sigma^2 + \gamma^2\sigma_I^2 - 2\rho\sigma\gamma\sigma_I\right] - \frac{1}{2}\sigma_N^2\right)(\tau - t) \\
&\qquad + \theta\sigma\int_t^\tau dZ_S + \theta\gamma\sigma_I\int_t^\tau dZ_I + \sigma_N\int_t^\tau dZ_N. && (12)
\end{aligned}
$$

From equations (10) – (12), $s_\tau$, $i_\tau$ and $n_\tau$ are normally distributed for all $\tau$.

Write $M_{t,T}$ for the present value of all fees between $t$ and some terminal horizon, $T$ (which might be infinite). Then

$$M_{t,T} = c\, E_t \int_t^T e^{-r(\tau - t)} V_\tau\, d\tau, \tag{13}$$

or, switching the integral and the expectation,

$$M_{t,T} = c \int_t^T E_t\left[e^{-r(\tau - t)} V_\tau\right] d\tau. \tag{14}$$

Since both $n_t$ and $s_t$ are normal, so is their sum, and

$$\begin{aligned} E_t\left[e^{-r(\tau-t)}V_\tau\right] &= E_t\left[e^{n_\tau+s_\tau-r(\tau-t)}\right], \\ &= \exp\left[E_t(n_\tau+s_\tau)+1/2\,\mathrm{var}_t(n_\tau+s_\tau)-r(\tau-t)\right]. \end{aligned} \quad (15)$$

From equations (10) and (12), we have

$$\begin{aligned} E_t(s_\tau+n_\tau) &= E_t(s_\tau)+E_t(n_\tau), \\ &= s_t+n_t+ \\ &\quad \Big(r+\nu-c(1+\theta)-\frac{1}{2}\sigma^2 \\ &\qquad -\frac{1}{2}\theta^2\left[\sigma^2+\gamma^2\sigma_I^2-2\rho\sigma\gamma\sigma_I\right]-\frac{1}{2}\sigma_N^2\Big)(\tau-t). \quad (16) \\ \mathrm{var}_t(s_\tau+n_\tau) &= \mathrm{var}_t(s_\tau)+\mathrm{var}_t(n_\tau)+2\,\mathrm{cov}_t(n_\tau,s_\tau), \\ &= \sigma^2(\tau-t)+\left\{\theta^2\left[\sigma^2+\gamma^2\sigma_I^2-2\rho\sigma\gamma\sigma_I\right]+\sigma_N^2\right\}(\tau-t) \\ &\qquad +2\left(\theta\sigma^2-\theta\rho\sigma\gamma\sigma_I\right)(\tau-t). \quad (17) \end{aligned}$$

Substituting into equation (15) and simplifying, we obtain

$$E_t\left[e^{-r(\tau-t)}V_\tau\right]=V_t\,e^{-[c(1+\theta)-\nu-\theta\sigma(\sigma-\rho\gamma\sigma_I)](\tau-t)}. \quad (18)$$

Finally, performing the integration in equation (14), we obtain[7]

$$\begin{aligned} M_{t,T} &= cV_t\int_t^T e^{-[c(1+\theta)-\nu-\theta\sigma(\sigma-\rho\gamma\sigma_I)](\tau-t)}\,d\tau, \\ &= \frac{c\left[1-e^{-[c(1+\theta)-\nu-\theta\sigma(\sigma-\rho\gamma\sigma_I)](T-t)}\right)}{c(1+\theta)-\nu-\theta\sigma(\sigma-\rho\gamma\sigma_I)}V_t. \end{aligned} \quad (19)$$

Letting the terminal horizon go to infinity, we have

$$M_t\equiv\lim_{T\to\infty}M_{t,T}=\frac{c}{c(1+\theta)-\nu-\theta\sigma(\sigma-\rho\gamma\sigma_I)}V_t \quad \text{for } c(1+\theta)>\nu+\theta\sigma(\sigma-\rho\gamma\sigma_I). \quad (20)$$

Equations (19) and (20) provide the intuition for understanding the relation between mutual fund values and the value underlying assets under management. Unlike hedge funds, mutual funds tend not to disappear. This is partly due to the lack of a high watermark, but also to the fact that poorly performing funds are generally merged into existing funds so the assets do not disappear (Gruber (1996)). Therefore, since a long horizon setting may be

[7] As long as $c(1+\theta)\neq\nu+\theta\sigma(\sigma-\rho\gamma\sigma_I)$. Otherwise, $M_{t,T}=c\,V_t(T-t)$.

appropriate for analyzing the present value of fees, we first focus our intuition on the infinite horizon case given by equation (20) and discuss the finite horizon case in a later subsection. The finite horizon case is relevant, however, to the extent that we present evidence in the empirical section that the age of the fund has important implications for the parameters governing equation (20). These implications are equivalent to shortening the life of the fund.

As a first pass at the intuition, consider the case in which fund flows are unrelated to fund performance, i.e., $\theta = 0$. This is the typical environment that has been studied in the theoretical literature to date and therefore serves as a useful benchmark. Substituting $\theta = 0$ into equation (20), we get

$$M_t = \frac{c}{c-\nu} V_t \quad \text{for } c > \nu. \tag{21}$$

Equation (21) is essentially the Gordon growth model in which the initial cash flow is the fee earned on the assets under management, i.e., $cV_t$. Define the other parameters of the Gordon growth model as $r^*$ for the discount rate and $g$ for the growth rate. In the present case, $g = r^* - c + \nu$. Since the assets are expected to grow at their expected return, adjusted for fees and exogenous growth, the expected return cancels. This is one of the central ideas of the paper, and for that matter any contingent claim analysis; that is, the present value of the future asset price is just today's price. The present value of the fees then depends on the remaining two components of the growth rate. The fee, $c$, leads to a lower growth rate as the NAVs decline in $c$, while $\nu$ represents general growth in net flows not related to the NAV.

Perhaps not surprisingly, this valuation formula suggests that the mutual fund business produces substantial revenue streams in a present value sense.[8] In particular, if we assume no growth in fund flows (i.e., $\nu = 0$), then the fund's value equals the current assets under management, $V_t$. In other words, in present value terms, the management company extracts all the value of the assets. This is analogous to the house eventually owning the pot in a poker game by taking a small cut of each hand played. According to equation (21) the present value can even exceed current assets under management. It may, in fact, reach infinity, for growth rates in flows that are larger than the management fee.

The more interesting case, and the primary contribution of the paper, is to consider $\theta \neq 0$. As described in the introduction and above, this is the more empirically relevant case. The valuation formula in equation (20) now has two additional terms affecting the growth rate of the cash flows. The first term is $c\theta$. Since $\theta$ measures the sensitivity of the

[8]This equation is similar in spirit to the valuation of a hedge fund contract in Goetzmann, Ingersoll and Ross (2003) if one assumes no high water mark and no incentive fee. In their paper, they focus on hedge funds and treat the number of shares as fixed. They do assume, however, a constant withdrawal rate which is analogous to a negative value of $\nu$ in our case.

net flows to the fund's NAV return relative to the benchmark, and the fees cause the NAVs to drift downward through time, $c\theta$ hurts the cash flow growth. The second term is $\theta\Sigma$, where we define $\Sigma \equiv \sigma(\sigma - \rho\gamma\sigma_I)$. $\Sigma$ has a particularly interesting economic interpretation. It represents the covariance between the fund's NAV return and the return relative to the benchmark, which determines net flows, namely $\Sigma = \text{cov}\,(dS_t/S_t, dS_t/S_t - \gamma dI_t/I_t)$. That is, $\theta\Sigma$ is relevant for pricing to the extent that the return driving net flows is also correlated with the return on NAV. This is the convexity effect that is common to option pricing and other asset-backed security pricing, such as mortgage-backed securities.

## 2.1 Comparative Statics

The analysis above explains the primary components of the valuation of mutual fund revenue streams. How do these components affect the valuation and under what conditions these components are important? Below, we provide some comparative statics analysis to help answer this question.

### 2.1.1 $\Sigma$

As long as the sensitivity of fund flow to performance is positive, i.e., $\theta > 0$, then the value of the fund will be increasing in the covariance term, $\Sigma$. Note that $\Sigma$ is increasing in $\sigma$ but decreasing in $\rho$, $\sigma_I$ and $\gamma$. Hence, the value of the fund is increasing in $\sigma$ but decreasing in $\rho$, $\sigma_I$ and $\gamma$.

This result relating value to the fund's $\sigma$ has also been documented by Goetzmann, Ingersoll and Ross (2003) but for completely different reasons. Their result derives from the incentive fee in which hedge funds take a proportion of the return above the high-water mark. Here, even with no incentive fee, this result carries through because fund flow depends on the fund's performance. Thus, many of the issues surrounding the incentives of fund managers discussed by Grinblatt and Titman (1989), Brown, Harlow and Starks (1996), Chevalier and Ellison (1997), Carpenter (2000), Das and Sundaram (2002) and Goetzmann, Ingersoll and Ross (2003) are relevant in our framework.

Of some interest, Brown, Harlow and Starks (1996) and Chevalier and Ellison (1997) analyze the incentives of mutual fund managers to increase the flows into the fund. Under the assumption that flows go more quickly into the very best performing funds (yielding an option-like payoff), or under the assumption that investors look at year-end returns so that poor performers at mid-year have different incentives, they argue and show empirically that managers may take more volatile positions.[9] In contrast, the result in equation (20)

[9]The empirical evidence in support of these claims is not universally accepted (e.g., Busse (2001)).

shows that neither the relative ranking nor nonlinearity in the flow-performance relation are important per se for the managerial incentive to take greater risk. This incentive arises quite naturally from the nonlinear payoff structure. This is true even if the flow-performance relation is linear (as in equation (4)). The empirical results of Chevalier and Ellison (1997), as well some of the previously cited literature, are, however, important. They find that $\theta$ varies across fund characteristics as well as across fund performance. The effect of volatility on the mutual fund's value depends very much on the sign and magnitude of $\theta$. A primary focus of our empirical analysis in Section 3 is, therefore, on characterizing the fund's $\theta$ as a function of the characteristics and performance of the fund.

In contrast, as long as $\theta > 0$, the value of the fund will be decreasing in the volatility of the index and in its correlation with the fund's NAV. Intuitively, the index serves as a counterweight to the fund's own return volatility. Take the extreme case where the fund and index are perfectly correlated. Then the "volatility" effect depends on the relative volatility of the fund versus the index, adjusted for $\gamma$. Thus, the incentives of managers described above relates to choosing assets which are both volatile and less correlated with the index subject to the constraints of a contract between investors and the fund (vis a vis the benchmark). Of some interest, this result does not depend on the "mutual fund tournament" explanation of Brown, Harlow and Starks (1996). Their effect will just further accentuate our result here.

One interesting example is that of index funds. To the extent index funds are perfectly correlated with the benchmark, and are risk-adjusted so that $\gamma$ equals the fund's $\beta$ in a one-factor model, the value of the fund will be unrelated to $\Sigma$. Moreover, if this fund were holding some liquid assets, such as money-market accounts, and it was being judged as an index fund, then its value would actually decrease in $\Sigma$. In this example the concavity induced by the fund investor's risk adjustment on the benchmark outweighs the convexity effect induced by the fund's own volatility. The example points out the relative importance of (i) the variance- covariance matrix between the fund and benchmark, and (ii) transparency between the fund and investors on what the appropriate benchmark should be.

### 2.1.2 $c$

Consider how the fund's value changes relative to the fees it charges. In the infinite horizon case, we find a somewhat surprising result. If $\nu + \theta\Sigma > 0$, then the mutual fund value actually falls as the manager increases the fee. The intuition is straightforward. Higher fees tend to decrease the net asset value of the fund over time. Since $\nu + \theta\Sigma > 0$ implies a positive growth rate in flows, the higher fee counteracts this positive growth. In a present value sense, it would be better to let the flows grow quickly and reap the present value of all future fees than take higher fees upfront. Note, however, that the present value of fees may actually fall if $\Sigma < 0$. In this latter case, the concavity in the payoff induced by the benchmark volatility overwhelms the convexity of the fund's uncertainty. Thus, it is possible that it would be better to take the higher fees upfront as future flows are expected to grow at a much slower rate.

#### 2.1.3 $\nu$

The growth rate in net flows, $\nu$, is positively related to the mutual fund's value. A higher growth rate in net flows leads to a permanent increase in fund flows which in turn increases the present value of the fees. To the extent that $\nu$ depends in practice on the fund's characteristics, the valuation result here serves to illustrate the importance of these characteristics on fund value.

#### 2.1.4 $\theta$

An increase in $\theta$ affects the mutual fund's value in the same direction as the sign of $\Sigma - c$. There are two relevant components. The effect of the first component,$\Sigma$, depends on whether the covariation between the fund's NAV return and its benchmark adjusted NAV return is positive or not. Assuming that the value is positive, then the intuition is that the greater the sensitivity the greater the convexity of the payoff between the mutual fund's value and the underlying NAV. In contrast, there is no ambiguity in the second component's effect, i.e., the fee, $c$. It is always negative since a higher value of $\theta$ means greater sensitivity to changes in NAV, which are lower due to the fees paid to the mutual fund. Because the literature, and corresponding empirical analysis in Section 3, show that $\theta$ varies with the fees charged, the fund's size, the fund's age and the type and magnitude of the fund's performance, the value of the mutual fund will vary across each of these characteristics. To the extent some of these characteristics are at the fund's discretion, the methodology and results here could be used to find the optimal structure of the fund.

### 2.2 The Nonlinearity Between the Fund's Valuation and Its NAV

$\theta$ plays a key role in understanding the relation between changes in the value of the fund, $M$, and changes in its NAV, $S$. To see this, rewrite equation (9) as

$$\begin{aligned} dn_t &= \left[\nu - \frac{1}{2}\theta(\theta-1)\sigma^2 - \frac{1}{2}\theta\gamma(\theta\gamma+1)\sigma_I^2 + \theta^2\rho\sigma\gamma\sigma_I + \theta r(\gamma-1) - \frac{1}{2}\sigma_N^2\right] dt + \\ &\quad \theta\, ds_t - \theta\gamma\, di_t + \sigma_N\, dZ_N. \end{aligned} \tag{22}$$

Integrating this and taking exponentials, we can write the number of shares of the fund at some future date, $N_{t+\tau}$, in terms of the size at time $t$ and the growth in net asset value. Defining the return on the fund's NAV and the index as, respectively, $R^S_{t,t+\tau} \equiv \left[\frac{S_{t+\tau}}{S_t}\right]$, and $R^I_{t,t+\tau} \equiv \left[\frac{I_{t+\tau}}{I_t}\right]$, we obtain

$$\frac{N_{t+\tau}}{N_t} = e^{\left[\nu-\frac{1}{2}\theta(\theta-1)\sigma^2-\frac{1}{2}\theta\gamma(\theta\gamma+1)\sigma_I^2+\theta^2\rho\sigma\gamma\sigma_I+\theta r(\gamma-1)-\frac{1}{2}\sigma_N^2\right]\tau+\sigma_N\left(\tilde{Z}_{N,t+\tau}-\tilde{Z}_{N,t}\right)}\left(\frac{{R^S_{t,t+\tau}}^{\theta}}{{R^I_{t,t+\tau}}^{\gamma\theta}}\right), \tag{23}$$

where $\tilde{Z}_{N,t}$ represents the sum of all the idiosyncratic shocks to net flows. Because the net flows, $dN_t$, depend on both $dS_t$, $dI_t$ and $dZ_N$, note that the number of shares in the future depends on both the underlying change in net asset value, the change in the index and the idiosyncratic shock.

From equations (20) and (23), we obtain

$$\frac{M_{t+\tau}}{M_t} = e^{\left[\nu-\frac{1}{2}\theta(\theta-1)\sigma^2-\frac{1}{2}\theta\gamma(\theta\gamma+1)\sigma_I^2+\theta^2\rho\sigma\gamma\sigma_I+\theta r(\gamma-1)-\frac{1}{2}\sigma_N^2\right]\tau+\sigma_N\left(\tilde{Z}_{N,t+\tau}-\tilde{Z}_{N,t}\right)}\left(\frac{{R^S_{t,t+\tau}}^{1+\theta}}{{R^I_{t,t+\tau}}^{\gamma\theta}}\right). \tag{24}$$

This is the capital gains component of the appreciation in $M_t$, but there is also a dividend. From equation (20), this is paid at an annualized fractional rate

$$cV_t/M_t = c(1+\theta) - \nu - \theta\sigma(\sigma - \rho\gamma\sigma_I).$$

The total return on $M_t$, assuming all dividends are reinvested, is therefore given by

$$\begin{aligned} R^M_{t,t+\tau} &= e^{[c(1+\theta)-\nu-\theta\sigma(\sigma-\rho\gamma\sigma_I)]\tau}\left(\frac{M_{t+\tau}}{M_t}\right), \\ &= e^{\left[(1+\theta)(c-\frac{1}{2}\theta\sigma^2+\theta\rho\sigma\gamma\sigma_I)-\frac{1}{2}\theta\gamma(\theta\gamma+1)\sigma_I^2+\theta r(\gamma-1)-\frac{1}{2}\sigma_N^2\right]\tau+\sigma_N\left(\tilde{Z}_{N,t+\tau}-\tilde{Z}_{N,t}\right)}\left(\frac{{R^S_{t,t+\tau}}^{1+\theta}}{{R^I_{t,t+\tau}}^{\gamma\theta}}\right) \end{aligned} \tag{25}$$

Define $\tilde{K} \equiv e^{\left[(1+\theta)(c-\frac{1}{2}\theta\sigma^2+\theta\rho\sigma\gamma\sigma_I)-\frac{1}{2}\theta\gamma(\theta\gamma+1)\sigma_I^2+\theta r(\gamma-1)-\frac{1}{2}\sigma_N^2\right]\tau+\sigma_N\left(\tilde{Z}_{N,t+\tau}-\tilde{Z}_{N,t}\right)}$. Then the sensitivity of the total return on $M_t$ to the fund's NAV return can be written as:

$$\frac{\partial R^M_{t,t+\tau}}{\partial R^S_{t,t+\tau}} = (1+\theta)\tilde{K}\left(\frac{{R^S_{t,t+\tau}}^{\theta}}{{R^I_{t,t+\tau}}^{\gamma\theta}}\right). \tag{26}$$

Using this measure, the risk of the fund is not constant as long as the net flows are sensitive to fund performance, i.e., $\theta$ is nonzero. Thus, returns on the fund's revenue stream are nonlinear in the fund's NAV. Interestingly, the risk depends upon the excess performance of the fund relative to the benchmark.

In order to better understand the nonlinearity we may want to factor out the idiosyncratic noise. We can write the expected return on $M_t$ between date $t$ and date $t+\tau$ as a nonlinear (convex) function of the relative return earned by an investor in the fund:

$$E_t\left[R^M_{t,t+\tau} \mid R^S_{t,t+\tau}, R^I_{t,t+\tau}\right] = e^{\left[(1+\theta)(c-\frac{1}{2}\theta\sigma^2+\theta\rho\sigma\gamma\sigma_I)-\frac{1}{2}\theta\gamma(\theta\gamma+1)\sigma_I^2+\theta r(\gamma-1)\right]\tau}\left(\frac{R^S_{t,t+\tau}{}^{1+\theta}}{R^I_{t,t+\tau}{}^{\gamma\theta}}\right). \quad (27)$$

Figure 1A graphs the return on the fund's value as a function of the return on the NAV of the fund and the excess performance of the fund for different values of $\theta$. Specifically, we assume that $\tau = 1$, $\sigma = 0.20$, $\sigma_I = .15$, $\rho = 0.75$, $\gamma = 1$ and $\nu = 0$. These parameters are chosen to coincide roughly with those governing the equity asset class, evidence of which is presented in the next section. The graph makes clear a few salient points. First, as $\theta$ increases, the return on the fund's value increases as a function of the excess return on the benchmark, $\frac{R_s}{R_I}$. Second, this increase occurs at an increasing rate. Third, in order to see the convexity more clearly, Figure 1B graphs cut-throughs of the relation between the return on the fund's revenue stream and $R_S$ (holding $R_I$ constant). At $\theta = 0$, the relation is linear by construction as net flows are insensitive to fund performance. As $\theta$ increases, however, the convexity is very transparent for large moves in the NAV return. This result implies that if a mutual fund business needs to hedge its risk, it must do so either dynamically or using volatility-based instruments (like options) to capture the convexity. The intuition is very similar to delta and gamma hedging with respect to options and the underlying stock.

## 2.3 The Finite Horizon Case

In the discussion so far, we have focused on the infinite horizon case for mutual fund valuation. In the next section the empirical evidence will show that even in an infinite horizon the parameters describing growth and flow-performance sensitivity vary over the fund's life to effectively shorten the fund's horizon. Comparing equations (19) and (20), it is immediate that

$$M_{t,T} = M_t\left(1 - e^{-[c(1+\theta)-\nu-\theta\sigma(\sigma-\rho\gamma\sigma_I)](T-t)}\right). \quad (28)$$

That is, the period $t$ finite horizon value of a fund that will die at $t+T$ is a fraction of its infinite value. It is easy to verify and intuitive that the fraction lost due to the fund being finite horizon is the $T$ period hence infinite horizon value $M_{t+T}$. The fraction lost due to the finiteness of the fund's life equals $e^{-[c(1+\theta)-\nu-\theta\sigma(\sigma-\rho\gamma\sigma_I)](T-t)}$ and reflects the capital gain (see equation (25)) which is lost once the fund dies.

To garner some intuition we derive the fund's half-life – the time it takes for the fund to accumulate half of its infinite value. Defining the half life horizon to be $T^* - t$ we get

$$T^* - t = \frac{\ln(2)}{c(1+\theta) - \nu - \theta\sigma(\sigma - \rho\gamma\sigma_I)}. \tag{29}$$

Sensibly, the half-life falls in the fee, $c$, as raising fees increases the rate at which managers' take their cut. Recall, however, that as fees rise there is an effect on fund value, so the managers, by raising fees, may expedite taking their cut of a smaller pie. The half life is unambiguously rising in growth. Everything else the same the fund rises in value and the same fees take longer to gather half of the fund's value in a present value sense. Similar insight holds for the effect of $\Sigma$ as long as $\theta$ is positive, which is just another illustration of the similar roles that growth and convexity play. Last, the sensitivity of the half-life to changes in the flow-performance sensitivity parameter, $\theta$, is ambiguous. In particular, if $c < \Sigma$ then the half-life if rising in $\theta$ as the convexity effect dominates, and vice versa.

One special case is of particular interest, namely the case of zero growth and zero flow sensitivity. In this case $V_t = M_t$, as shown above (see equation (21)). The only thing that prevents a mutual fund from owning all the assets in a present value sense is the finite horizon or declining growth rates in flows as the fund's size increases. This latter feature will be built into our model of fund flow valuation in Sections 3 and 4. In this case, the half life is simply $T^* - t = \frac{1}{c}\ln(2)$. This is similar to the standard half-life formula because the constant percentage fee $c$ is taken from a depleting pot.[10] Consider, for example, an annualized fee of 2% (hence $c = 0.0198$). One half of the funds will go to management fees in 35 years. That is, even without asset growth, much of the value is generated fairly quickly. The case is different for low-fee funds. In particular, for a fund with one tenth of the fees (i.e., 0.20% p.a.), half the value will be obtained in 347 years.

In terms of the comparative statics results with respect to $M_{t,T}$, the finite horizon setting tends to mitigate some of the sharper effects analyzed for $M_t$, the infinite horizon value. For example, consider the surprising result that increasing the fee lowers mutual fund valuation (see Section 2.1.2). The intuition is that higher fees hurt the growth rate in net flows, thus lowering the present value of all future revenue streams. For shorter horizons, however, this may no longer be true. Note that the second component of equation (28) always works in the opposite direction. Thus, whether the value of the mutual fund increases or decreases as a function of the fee depends on the horizon length. A similar argument can be made with respect to both a change in $\Sigma$ and $\theta$. The derivative of the second term in equation (28) is again of the opposite sign to the first term. Thus the overall effect of the comparative static

[10]Recall, in present value terms, the risk free rate disappears in the risk neutral economy and we can think of the fees being a percentage of a decreasing number of shares in the fund.

is diminished at shorter horizons. This is important because the evidence in the next section shows that the effective horizon is shorter because both $\nu$ and $\theta$ tend to decline with size and age.

# 3 Fund Flows

The theoretical model in Section 2 captures the important stylized fact that net flows into the fund depend on the underlying fund's performance. This stylized fact induces a convex relation between the mutual fund's value and its net asset value. Figures 1A - 1B illustrate the effect of this convexity on the mutual fund's value and the dampening of this effect when the fund is judged against a benchmark. While these results capture the most important component of the valuation, the specification was fairly simple in order to generate closed-form solutions.

Because there has been a considerable effort in the literature to document the relation between fund flows and performance, it is worthwhile incorporating these results into our framework.[11] Most important, this literature identifies a number of characteristics that render closed-from solutions intractable. Fortunately, as in Schwartz and Torous (1989), we can perform a Monte Carlo simulation of the possible paths of the fund's NAV and keep track of important features such as the fund's size and its most recent performance. We can build quite complex, and empirically realistic, path-dependencies and nonlinearities into the relation between flows and performance. In the remainder of this section we develop a model of empirical fund flows and estimate the model on the universe of equity mutual funds over the period 1970 to 2002.

## 3.1 Specification for Fund Flows

The first step in generating an empirical specification for the net flow of mutual funds is to break up the universe of equity mutual funds into separate classes. Motivated by the theory, our primary reason for doing this is to analyze the empirical model for funds with different asset volatilities. That is, it may not be sensible to pool estimates of $\nu$ and $\theta$ across, for example, growth and value funds. A secondary reason is that we would like to be able to comment on a mutual fund company's business which includes a cross-section of portfolios within different equity asset classes. To the extent the empirical model does not capture all the important features of net flows an estimation by asset class may be more

[11]See, for example, Patel, Zeckhauser, and Hendricks (1991), Ippolito (1992), Warther (1995), Gruber (1996), Chevalier and Ellison (1997), Remolana, Kleiman and Gruenstein (1997), Sirri and Tufano (1998), Edelen (1999), Bergstresser and Poterba (2002), and Del Guercio and Tkac (2002), among others.

appropriate. In this section, we consider three classes of equity funds, namely all-equity, equity-growth, equity-value and equity-international. The all-equity class includes equity-growth, equity-value and all the remaining domestic equity funds. The previous literature has looked at the relative performance of asset classes but less so the relation between fund flows and performance. An exception is Bergstresser and Poterba (2002) who look at regression dummy effects for mutual fund investments in small versus large firms, as well as growth versus value. No pattern emerges from their results.

The second step is to choose a tractable model for the empirical relation between net flows and fund performance. We measure fund performance relative to a benchmark defined as the median return from the asset class (e.g., Sirri and Tufano (1998)). Consistent with the literature, we look at the recent performance of the fund over the past year.[12] The reason for looking at lagged one-year returns is the belief in the literature that investor behavior is somewhat sticky as it applies to investing in or removing monies from mutual funds.

We consider the following econometric specification:

$$\begin{aligned} \frac{\Delta N_{i;t,t+1}}{N_{it}} &= \left[\nu_i + \sum_m \nu_{im} d_{imt}\right] + \left[\theta_i + \sum_m \theta_{im} d_{imt}\right]\left(\frac{\Delta S_{i;t-1,t}}{S_{it-1}} - \frac{\Delta I_{I;t-1,t}}{I_{it-1}}\right) + \epsilon_{it} \\ \epsilon_{it}^2 &= \left[\upsilon_i + \sum_m \upsilon_{im} d_{imt}\right] + \eta_{it}, \end{aligned} \tag{30}$$

where $i$ subscripts the fund, $m$ the fund characteristic, $\Delta N$ is the fund's net flow, $\Delta S$ is the change in the fund's NAV, $\Delta I$ is the change in the index's value, and $d_{imt}$ represents a dummy variable equal to one if the fund $i$ at time $t$ has characteristic $m$. For example, $d_{imt}$ might represent the dummy variable for a Fidelity growth fund at time $t$ with the characteristic that it is between 7 and 10 years old. The second equation takes into account the fact that there is heteroskedasticity across the funds that depends on the characteristics. For example, the variance of the random flow is probably less for large, old funds than for emerging, small ones.

With respect to the fund's characteristics, we consider the following variables that can affect the fund's growth rate, $\nu$, the fund's sensitivity, $\theta$, and the volatility of flow shocks:

- Size of the fund (e.g., Chevalier and Ellison (1997), Sirri and Tufano (1998) and Del Guercio and Tkac (2002)). Of some interest, Sirri and Tufano (1998) look at how both

[12] We also analyze the current performance and long-run performance of the fund to better understand the fund-flow dynamics. We find that both the recent performance and long-term performance contribute to the regression's explanatory power. The effects, however, are second order to the recent lagged return. Moreover, to the extent fund performance is not highly autocorrelated, these additional effects do not have much impact on that of the one-year lagged return. Since these effects only serve to increase the sensitivity of net flows to fund performance, we consider only the lagged return results to coincide with previous analyses.

$\nu$ and $\theta$ change with size. Consistent with the rest of the literature, they generally find that the growth rate in net flows declines in the size of the fund. However, they find that the sensitivity to performance does not change much at all. In our analysis we break fund size into four categories, small, small-medium, medium-large and large. The categories are chosen to roughly approximate size quartiles, for want of a better rule. A small-medium equity fund, for example, has assets under management that fall between the 25th and 50th size percentiles of equity funds.

- Annual percentage fees, i.e., $c$ (e.g., Sirri and Tufano (1998) and Bergstresser and Poterba (2002)). Sirri and Tufano (1998) find that the sensitivity of net flows to performance depends on the magnitude of the fees. In particular, higher fees tend to decrease net flow growth but increase the sensitivity to performance. This result is potentially important as the manager's choice of fees can have varied effects on the valuation of the mutual fund. We consider two levels of fees within each asset class. The categories are chosen to roughly approximate fees below or above the 50th percentile of the sample.
- Age of the fund (e.g., Chevalier and Ellison (1997), Bergstresser and Poterba (2002) and Del Guercio and Tkac (2002)). Chevalier and Ellison (1997) investigate the effect of age on estimates of both $\nu$ and $\theta$. Their general finding is that, as the age of the fund increases, both the growth rate and sensitivity tend to decline. This is an important result in the context of our valuation model. In particular, this explains why the mutual fund business is not quite as profitable as implied by equation (20). In the infinite horizon case, the decline in NAV due to fees will eventually drown out the growth in future fund flows as this growth disappears as time passes. We consider four different age classes, again reflecting the quartiles of the sample.

Note that there are several features of fund flow dynamics not employed in this paper, and these omissions warrant further discussion. First, we do not take into account nonlinearities in the net flow-fund performance relation in equation (30). Chevalier and Ellison (1997) and Sirri and Tufano (1998) both document a nonlinear relation between fund flows and fund performance. In particular, they find that the sensitivity to flows tends to be greater for strong performance while fairly flat for weak fund performance. This result has led them, as well as others, to analyze the incentives of mutual fund managers to take riskier investments. As described in Section 2, this incentive exists even without this nonlinear relation due to the convexity of the payoff as a result of $N$'s dependence on $S$. We ignore this nonlinearity for a few reasons. First, and most important, the result appears to be somewhat sample specific. In particular, while the nonlinear relation was in evidence during our sample period of the early 80's to the early 90's, it does not show up in the full 1970-2002 period. It would be interesting to evaluate this issue more closely given the focus of the literature; however, we feel this point is beyond the scope of the paper. Second, it is important to document the effect of the fund's characteristics and resulting nonlinearities between $M$ and $S$ without the added complication of specifying a general, nonlinear relation between fund performance and net flows. With the interaction terms, this would add another dimension to the estimation.

Second, we do not incorporate investor rationality into our framework beyond what is implied by the empirical model for net flows. An analogous debate can be found in the mortgage-backed security literature with respect to empirical prepayment and rational prepayment models. Though we discuss this issue briefly in the conclusion, two potential areas seem potentially important. The first is the tax overhang issue looked at by Barclay, Pearson and Weisbach (1998) and Bergstresser and Poterba (2002). They show that the flow of monies into and out of a fund are affected by its tax overhang. To the extent that tax overhang might be related to the fund's age or size, this could impact our coefficient estimates. That stated, Bergstresser and Poterba (2002) find that the size and age variables are statistically important regardless of the tax burden. The second is the market's "assessment" of a fund's alpha. Recent theoretical work by Berk and Green (2002) provides a rationale for fund flows in a world where funds may generate positive alphas. Empirically, there is some evidence to suggest that a fund's Morningstar rating, or similar measure of the fund's ex ante alpha, may be important (e.g., Del Guercio and Tkac (2002) and Bergstresser and Poterba (2002)). This paper abstracts from the debate about the alpha of the fund and how it impacts pricing.

Lastly, there is evidence of macro factors affecting fund flows (e.g., Warther (1995), Chevalier and Ellison (1997) and Sirri and Tufano (1998)). While the choice of variables differs across these studies, there is a general idea that a flow variable either for the fund's asset class or the market in general has explanatory power. If this variable is not correlated with the characteristics of the fund, or that fund's performance, then our estimates will be unbiased.

Though we view these issues as potentially interesting extensions, their introduction would unnecessarily complicate the valuation framework. In contrast, our focus is on developing better understanding of how mutual fund valuation depends on properties of the underlying assets under management when flows themselves also depend on these assets.

## 3.2 Data

The data employed in this study covers annual data on mutual funds over the period 1970-2002. The source of the data is CRSP's survivorship-free database on US mutual funds. The advantage of this database is that it includes funds that have disappeared during the sample period, such as through mergers or liquidations. It was originally compiled by Mark Carhart and is described in detail in his studies on mutual fund performance (Carhart (1995,1997)). The file includes a number of items of interest for each mutual fund. In particular, the fund's return, total assets under management, fee structure, age, and description are provided on a quarterly basis.

As has been pointed out by others, the database is not free of errors. For example, Elton, Gruber and Blake (2001) argue that, though in principle the data is survivorship-free, missing observations lead to some similar problems. Moreover, they find other errors, including those associated with mergers and splits. This is potentially important as mergers and splits will affect the measure of net flows into the funds. As a result, we employ several filters to address these concerns.

First, we only consider funds with a beginning of year assets under management of $10 million or greater. Second, we filter out observations with either extreme net flows or extreme returns. We filter out observations in the 5% tails of the net flows distribution and the 1% tails of the return distribution. A close examination of a subset of these filtered observations indicates that many are either recording errors or errors resulting from mishandling of fund mergers and splits. Third, we eliminate observations for which the data on the year in which the fund was organized is clearly incorrect, e.g., data for the fund exists in years prior to the recorded fund organization year. Finally, we eliminate observations that have missing data for one or more of the variables–flows, returns, age, size, and fees.

Tables 1A and 1B provide a summary of the main characteristics of the four equity asset classes studied in this paper. Specifically, we look at the distribution of returns, flows, size, fees, and age. We document the mean and standard deviation of these characteristics, as well as the various percentiles, within each asset class. We also report the volatility of the benchmark for each class and its correlation with the funds' NAV return. (Note that we define the benchmark as the median of the class per period (e.g., Sirri and Tufano (1998).) Consistent with the model in Section 2 and Section 4 to follow, these are the parameters that define the economic setting of mutual fund valuation.

The key variable for determining the net flow dynamics is the age of the fund. For the four classes – equity growth, equity value, all-equity, and international equity – the median ages are 9, 6, 7 and 5 years, respectively. To the extent that fund age tends to diminish both the net flow growth rate and sensitivity to fund performance (e.g., Chevalier and Ellison (1997)), the distribution of a fund's age across the sample will be an important determinant of value. In this case, equity growth funds are almost twice as old as international equity funds.

For the same asset classes above, the median asset sizes over our sample period are 93.78, 133.12, 113.14, and 89.44 million dollars, respectively. The distribution is heavily skewed for all of these classes, with the respective means being 505.7, 708.9, 641.5, and 479.7 million. The results in the next section show that growth rates and flow-performance sensitivity vary across fund size. These stylized facts, therefore, are especially relevant for understanding mutual fund valuation. The fee distribution across funds is similarly interesting. For the growth, value and all-equity funds, the median fee is 1.04%, 1.28% and 1.16%, respectively. The 95% tails are 2.29%, 2.27% and 2.20%. In contrast, the international funds have significantly higher fees, with a median of 1.67% and tail of 2.68%. Since fees are one of the parameters at the discretion of the mutual fund manager, it will be important to analyze how mutual fund values and risks change with fees. The international equity fund provides a useful basis for comparison.

The final set of parameters governing the four classes come from the variance-covariance matrix of the NAV returns and the corresponding benchmark returns. The volatilities of the growth, value, all-equity and international classes are 17.2%, 18.4% 19.0% and 22.3%, respectively, with corresponding benchmark volatility of 15.1%, 11.8%, 14.5% and 16.9%. Under the assumption that the NAVs of the funds have a beta of 1 against their benchmarks (e.g., growth funds against the growth benchmark), a common assumption in much of the empirical work, we can derive the correlation of the funds within the four classes with their respective benchmarks, which are 0.88, 0.64, 0.76 and 0.76, respectively.

In the analysis to follow in Section 4, these parameters and the median values above form the typical mutual fund that we analyze. We then vary these parameters across classes to better understand the interaction of the fund's characteristics with mutual fund's value and risk.

## 3.3 Empirical Results

Tables 2A and 2B provide the regression results for equation (30) for each of the four asset classes. Table 2A reports the results for net flows, while Table 2B reports the results for the variance of the random shocks to these flows. For each asset class the tables provide the parameter estimates, standard errors, number of fund observations and $R^2$s.[13]

As an illustration of how to understand the results, first consider Table 2A and the all-equity asset class. The growth rate estimate, $\hat{\nu}$, without the dummy variables is 0.366. This

[13]Note that the standard errors reported are derived under standard OLS assumptions. These standard errors are most probably understated due to the correlation across contemporaneous fund flow residuals arising from macro factors, amongst other variables. To the extent that this paper uses only the point estimates in deriving the mutual fund's valuation and risk properties, this issue is peripheral to our analysis.

number represents the growth rate for funds in the lowest 25% of age (i.e., $< 4$ years) and size (i.e., $<$ \$40.07 mm), and lowest 50% of fees (i.e., $< 1.16\%$). (Note that these percentile values are presented in Table 1.) If the fund grows in size, however, then this growth rate changes by the amount provided in the table. For example, if the fund size reaches a value between the 50th and 75th percentiles (i.e., $113.4 <$ \$ size $< 384.8$), its growth rate declines by 0.089 to 0.277. In fact, once the size of the fund increases over \$40.07 million, the decrease in growth rate does not vary much across size, namely 0.073, 0.089 and 0.106 in each percentile range. By far the largest effect is with respect to the age of the fund. Given that the fund grows older each period, these effects are important for effectively shortening the fund life as discussed in Section 2.3. Consider a large fund over \$384 mm, charging high fees. Its growth rate in flows starts at 0.249, but then declines over the years to 0.073, then 0.003 and finally -0.048. Thus, a fund that started at a growth rate of 24.9% per year ends up at a negative growth rate of 4.8% by its sixteenth year.

For this same equity class, consider the flow-performance sensitivity parameter, $\theta$. The estimate for a fund in the lowest group across all three characteristics is 1.363, which implies that for each 1% in excess return performance, there is an increase of 1.36% in net flows. In terms of the theoretical model of Section 2, and results of Section 2.2, the nonlinearity between the fund's value and its NAV is economically significant. However, similar to the growth rate effects above, this sensitivity tends to diminish both in the size of the assets and especially in the age of the fund. For example, with respect to size, $\theta$ drops to 1.352, then 1.266 and finally 1.186 as the assets increase in size. This parameter falls even more with age, namely to 1.049, then 0.725 and eventually 0.672. Thus, old funds tend to have negative growth rates and much less sensitivity to fund performance.

Table 2B reports results for the regression of the squared residuals from the net flow regression on a constant and dummy variables for size, fees and age. The breakpoints used to define the dummy variables are identical to those used in Table 2A. The key feature of the results is that the variance of idiosyncratic flow shocks is decreasing in both fund size and age. Consider, for example, the all-equity asset class. For small, young, low fee funds, the volatility of annual flow shocks is 61.4%, i.e., the square root of the constant 0.377. Holding the other factors constant, an increase in size to the upper quartile reduces the volatility to 47.5%. An increase in age to the upper quartile reduces volatility even more to 39.0%. Combining these two effects, large, old funds have a flow volatility of only 3.2%. In general, these same patterns hold across all four asset classes. In other words, net flow shocks may induce large amounts of volatility in the value of small and/or young funds, but as these funds age and grow, flows become much more predictable and the volatility will be determined principally by return shocks.

Given the results of Tables 2A and 2B, several observations are in order. First, though the magnitudes differ across the various equity asset classes, the same pattern emerges for each. Specifically, both the growth rate and sensitivity parameters decline with fund size and fund age. Age is by far the most important characteristic in determining the net flow dynamics. Note also that the fee parameter, though small in magnitude, is similar across the classes. Fees tend to have a negative effect on growth but a positive effect on the sensitivity to fund performance. This latter result is consistent with Sirri and Tufano (1998) who argue that sensitivity should be greater for higher fees because these funds tend to be more actively managed and thus marketed to investors.

Second, along these lines, the results in this paper are consistent with those in the literature (e.g., Chevalier and Ellison (1997), Sirri and Tufano (1998) and Bergstresser and Poterba (2002)). Thus, even though our sample length is much longer, and the data source is different (i.e., CRSP versus Morningstar), the key findings with respect to age and fees carry through. Our results, however, are a little stronger with respect to size. While this may reflect the different specification and nonlinear interaction terms of our empirical model, it may well be due to the fact that we simply have more evidence on large funds. Nevertheless, the results here show that it is important to take account of the fund characteristics and to do this in a complex way. As an aside, and a comment on the existing literature, the results of this section lead us to believe that there is room for additional research at both the theoretical and empirical level on the determinants of fund flows.

Third, in comparing across the four asset classes, some interesting results do emerge. In the highest percentile ranges (i.e., large, old funds with high fees), one can compare both the fund flow growth rates and sensitivities across the classes. For example, the growth rates are -0.037, -0.020, -0.048, and -0.107 for the growth, value, all-equity and international funds, respectively. (Note, however, that while fees play little role for most of the classes, the growth rate is only marginally negative, i.e., -0.005, for international funds with low fees.) Similarly, the sensitivities are 1.094, 0.862, 0.672 and 0.512, respectively. These parameters are especially relevant for valuation purposes because, in terms of the horizon, they most probably govern the net flow dynamics if the fund survives and prospers.[14]

In the next section, we evaluate how changes in the fund's characteristics impact the valuation and risk of the mutual fund. As a preview, we find that changing fund character-

[14] In the model that follows, it is possible that, with a large enough random shock to net flows or poor enough performance, the fund's size can go negative. In our model, we treat this as a fund death. The majority of poorly performing funds, however, probably do not die since the variance of the random shock to flows is close to zero for old funds (see Table 2B). These funds, however, hover around low values of assets under management. Presumably, these are the types of funds that get merged into other funds (e.g., Gruber (1996)).

istics and the corresponding changes to $\nu$ and $\theta$ are consistent with the theoretical analysis of Section 2.1 with respect to comparative statics.

# 4 Valuation

## 4.1 Methodology

Valuing the stream of cash flows to the fund manager is equivalent to calculating the expectation in equation (14), which can be written as

$$M_t = \int_t^\infty E_t\left[e^{-r(\tau-t)} c V_\tau\right] \, d\tau.$$

Intuitively, this says that the value of the asset equals the sum of expected values of discounted cash flows that it produces.[15] The dynamics of $V_t$ are driven by the dynamics of $S_t$ and $I_t$ (given in equations (1), (2) and (3)), and by the performance related fund flows given in equation (4). We can estimate this expectation numerically by simulating (discrete) paths for $S_t$ and $I_t$, calculating the fee paid and the change in assets each period, discounting the fees back along each path, and finally averaging over a large number of paths. This is the basis of all Monte Carlo valuation techniques (see, for example, Boyle (1977) or Jacob, Lord and Tilley (1987)), and allows us to value the fund taking into account an arbitrary dependence of cash in- and outflows on the history of asset returns. Monte Carlo simulation has been used many times in the academic literature, including the mortgage valuation papers of Schwartz and Torous (1989), McConnell and Singh (1994), Patruno (1994), and Hayre and Rajan (1995).

To implement the valuation procedure, we simulate each path using a discretized version of equations (1), (2) and (3)),

$$S_{t+(i+1)\,\Delta t} = S_{t+i\,\Delta t} \exp\left[\left(r - c - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\sqrt{\Delta t}\,\tilde{\epsilon}_i^S\right], \tag{31}$$

$$I_{t+(i+1)\,\Delta t} = I_{t+i\,\Delta t} \exp\left[\left(r - \frac{1}{2}\sigma_I^2\right)\Delta t + \sigma_I\sqrt{\Delta t}\,\tilde{\epsilon}_i^I\right], \tag{32}$$

where $\tilde{\epsilon}_i^S$ and $\tilde{\epsilon}_i^I$ are normal random variables with mean 0, variance 1, and correlation $\rho$.[16]

[15] Note that all expectations are taken relative to the risk-neutral probability distribution.

[16] To generate these, we generate two independent standard normal random variables, $\tilde{e}_i^S$ and $\tilde{e}_i^I$, and then calculate

$$\begin{aligned}\tilde{\epsilon}_i^S &= \tilde{e}_i^S, \\ \tilde{\epsilon}_i^I &= \rho\tilde{e}_i^S + \sqrt{1-\rho^2}\,\tilde{e}_i^I.\end{aligned}$$

Given this path, we use equation (30) to determine the value of $N_t$ at each time step, then calculate the cash flow each period as a multiple of the current assets under management, $N_t S_t$. We repeat this many times, and average across paths.

To improve the accuracy of our estimate of $M_t$, we use the antithetic variate approach. Instead of drawing each path independently, we draw paths in pairs. The first path in each pair is determined by a sequence of i.i.d. standard normal random numbers $\tilde{\epsilon}_1, \tilde{\epsilon}_2, \ldots$. The second path is generated using the random numbers $-\tilde{\epsilon}_1, -\tilde{\epsilon}_2, \ldots$. The random numbers that generate each path are from the correct distribution,[17] but this procedure generates negative correlation between the present values obtained from each path, lowering the standard error of the estimate of $V_t$ (see Boyle (1977)).

## 4.2 Results

For each asset class, we first consider the fund with the median characteristics of its class (Table 1A) and the corresponding variance-covariance matrix parameters (Table 1B). We report the present value of the fund as a fraction of current assets under management, i.e., $\frac{M_t}{V_t}$, the length of time it takes to reach half the present value of the revenue stream, and the distribution of the realized present values across the simulated paths. All simulations involve 10,000 paths, and these results are provided in Table 3A.

Fund value, $\frac{M_t}{V_t}$, varies substantially across the four asset classes, from as low as 0.227 and 0.456 for value and all-equity funds to as high as 0.971 and 1.060 for value and international funds. One of the main characteristics explaining the difference between these classes is their median ages, 9 and 7, versus 6 and 5 years. Young funds tend to grow faster in our sample. Moreover, even when they reach old age, the growth rates tend to be less negative for value and international funds (at the lower fee schedule). This can be seen from the fact that the half-lives are almost double for these fund families within our model. In other words, the parameters governing the younger funds' fund flow dynamics are such that their growth opportunities last longer. Moreover, the variance-covariance matrix parameters, $\Sigma$, are also larger for these two classes. While $\Sigma$ may not literally be the correct parameter due to the nonlinear interaction terms, it clearly is positively associated with the true combined effect of $\sigma$, $\sigma_I$ and $\rho$. Because it is difficult to disentangle all the differences across fund types, we investigate the effect on each class of a change in the relevant fund characteristics and parameters.

One startling fact is that although the present values differ across classes, their median $\frac{M_t}{V_t}$ values are more similar, namely 0.174, 0.291, 0.239 and 0.39, respectively. This suggests that

[17] If $\tilde{\epsilon}$ is $N(0,1)$, so is $-\tilde{\epsilon}$.

not only is the distribution of mutual fund values highly skewed, but that this skewness is the source of the difference in value across classes. For example, the 95% tails of these classes are 0.57, 3.32, 1.47 and 3.37. In order to understand why the value funds and international funds are so valuable, note that they are relatively young with high growth rates. At the tail of the distribution, extraordinary high excess return performance will lead to a quick build-up of assets under management. This coupled with naturally high growth in its formative years adds tremendous value. In contrast, even with the same tail behavior in excess returns, the other asset classes have much lower growth rates, so that the quick build-up does not follow. For international and value funds, once the assets are in place, the funds have a cash cow for a number of years until the eventual negative growth finally overwhelms the future values of the fees. Interestingly, Table 1A shows that the actual distribution of asset size across funds within each class is highly skewed. The model suggests this may be a natural outcome of the distribution of asset returns and fund flow dynamics.

Table 3B looks at the valuation and distribution of $\frac{M_t}{V_t}$ assuming that all the fund flow sensitivity parameters, $\theta$, are zero. We hold the growth parameters as a function of the characteristics the same for comparison purposes. Comparing the results here to Table 3A might suggest that the sensitivity parameter is not that important. This conclusion, however, would be misleading. Recall that the parameter $\theta$ has two offsetting effects in the net flow dynamics. The first effect arises from NAVs falling over time due to fees; thus, the sensitivity drives down flows and $\frac{M_t}{V_t}$s. The second effect leads to a convex payoff between fund values and fund NAVs, leading to higher $\frac{M_t}{V_t}$s. For the parameter values estimated here, these effects are similar in magnitude.

There are several interesting points that show the potential importance of the $\theta$ parameter across classes. First, note that growth funds are actually less valuable when $\theta$ is positive as estimated from the data. That is, the first effect described above dominates for growth funds, while the convexity effect has greater impact for the value, all-equity and international funds. Second, across all the asset classes, the median values are somewhat higher for the $\theta = 0$ case. This is because the value of the convexity effect is most powerful for large moves in NAVs, which naturally occurs at the tails of the distribution. Third, below, we confirm this intuition about $\theta$'s importance by varying $\sigma$ and show that values can change considerably. (Note that the Table 3B values would not change as we change $\sigma$.)

### 4.2.1 Valuation

For each asset class, keeping the fund characteristics and variance-covariance matrix parameters given by Table 3A fixed, we vary one characteristic of interest. Specifically, Figures 2 – 5 plot how $\frac{M_t}{V_t}$ changes as $\sigma$, $c$, fund age and fund size varies.

First consider Figure 2 depicting the effect of $\sigma$, which we vary from zero to 30%. Recall that the volatility across asset classes varies from 17.2 to 22.3%. The value of the funds increases in volatility of the assets, though at different rates depending on the type of fund. For example, the effect on growth funds is very little compared to that on value funds. The reason is that the higher volatility of asset returns leads to the convexity effect being much larger. As described above, the convexity effect is most relevant when it can be combined with funds that have natural growth in order to create the "cash cow". Of course, this is necessary but not sufficient as seen by comparing the relation between $\frac{M_t}{V_t}$ and $\sigma$ for value funds against those of international funds. Figure 2 shows that value funds have mcuh greater convexity. Note that the sensitivity parameter $\theta$ for the median value fund is 1.260 versus 0.679 for the median international fund. Thus, along with having high asset return volatility, and growth possibilities, the net flows must also be sensitive to the fund's performance.

Figure 3 graphs the impact of changing fees on the $\frac{M_t}{V_t}$s across asset classes. We look at the range of 10 to 300 basis points as this covers the distribution of fees described in Table 1. There are several interesting features to this graph. First, outside of the breakpoints across which fees affect the growth rate and sensitivity of flows, the values tend to increase in fees. Recall the differences of the fee effect between the infinite horizon and finite horizon models of Section 2; the results here serve to show that the changes in parameter values are paramount to shortening the life of the fund. Second, the impact of the breakpoint, i.e., switching to high fees, has a negative impact on value for most of the asset classes. The most profound effect occurs for international equity funds. Recall that its growth rate drops by -0.102 when it starts charging fees above the median value. Thus, increasing fees has a fairly complex effect. On the one hand, high fees produce higher short-term revenues, against (i) lower NAVs due to fees, (ii) lower estimated growth rates in flows due to higher fees, and (iii) lower growth rates due to the flow-performance sensitivity's implicit effect of fees driving down NAVs. For the sample studied here, it appears that the most important effects are the higher revenues against the change in the dynamics of fund flows.

Figure 4 shows the importance of age on $\frac{M_t}{V_t}$ for the median fund across the asset classes. All the asset classes' values decline with age. Even more important is the finding that $\frac{M_t}{V_t}$ is many times greater for new versus old funds. In other words, the decline in value is rapid as the age of the fund increases. As mentioned above, most of the effects that lead to large tail values (and thus large fund valuation) require (i) a quick growth in assets under management via NAV increases, and (ii) high natural growth rate in flows. As seen from Table 2A, value funds tend to have much lower growth rates than the other asset classes. This also comes through in the figure as it has the weakest age effect. A similar effect for these classes can be found with respect to the fund's size. These results are given in Figure 5. $\frac{M_t}{V_t}$ declines in size; this decline is most rapid as the size of the fund initially increases; and this decline can be attributed to the parameter estimates described in Section 3.3 and reported in Table 2. Of some note, however, the size effect seems to be of second order importance to that of age.

### 4.2.2 Risk

The results above show how values vary as a function of the parameters. We now turn to studying how the distribution of the values changes. Table 4 provides a first pass at the fund's risk characteristics by reporting the percentiles of the realized present values across the 10,000 simulated paths for low and high values of the parameters. For most of the fund characteristics, we look at parameter values at the 25th and the 75th percentile across the various classes as representative of small and large parameter values. For example, the lowest 25th percentile for age is 3 years (i.e., international funds) and the highest 75th percentile is 20 years (i.e., growth funds). The other parameters we use are $\sigma$ which we range from 5% to 30%, and $k$, where the variance of random flow shocks is defined as $k * \epsilon_{it}^2$ and $\epsilon_{it}^2$ is estimated using the specification in equation (30). The idea behind varying $k$ is to consider the effect on the distribution of $\frac{M_t}{V_t}$ of the magnitude of flow shock uncertainty.

Comparing the means of the simulated $\frac{M_t}{V_t}$s across parameter values confirms our discussion of Figures 2-5. Fund values for all classes are rising in $\sigma$ and fees, while falling in size and age. However, this rise in value is accompanied by higher volatility in all pairwise comparisons (sixteen in all - four types times four parameter pairs). Moreover, the rise in volatility is multiplefold for many of these comparisons. This is important because to the extent fund managers care about their distribution of $\frac{M_t}{V_t}$s, the higher $\frac{M_t}{V_t}$ comes at a cost not priced by the market. For example, all-equity funds have twice the value if they are young versus old, but also three times the volatility. In theory, the manager's compensation contract with the mutual fund company should possibly take this feature into account.

Note that higher values and higher volatilities are, in all cases, also accompanied by higher 95% tail values. For each asset class, however, the skewness coefficient is similar across the parameter and fund characteristics. The one exception is with respect to the volatility of the NAV return, $\sigma$. Both the volatility and the skewness of the mutual fund's valuation are sharply increasing in $\sigma$, and at a rate much greater in magnitude than the corresponding increase in value. While this result may not be surprising per se, it does suggest the importance of treating asset classes separately.

With respect to the random shocks to flows, we vary $k$ between 1, the full estimated fund flow variance, and 0.1, a 90% smaller flow variance. Several interesting points can be made. First, as the theoretical model of Section 2 predicts, large differences in flow risk have only a small effect on fund values. The values of $\frac{M_t}{V_t}$ are slightly lower for $k = 1$. This is because the higher variance in flows leads to more fund deaths.[18] Second, and related, the high volatility case produces much lower 5th percentiles and 50th percentiles for $\frac{M_t}{V_t}$. Nevertheless, in spite of the fact that the median values are between 10% lower for value funds and 67% lower for growth funds, the fund values are marginally different. This latter point illustrates that most of the value of mutual fund revenue streams derives from the tail of the distribution. The risk of mutual fund valuation is therefore substantial as most of the payoff occurs in a small number of states. Third, a pairwise comparison of the volatility of $\frac{M_t}{V_t}$ across asset classes shows that the volatility is higher for $k = 1$. This is not surprising as the variance of the flow's random shocks ties directly into the variance of the assets under management and therefore the revenue stream. In fact, equation (25) shows that the volatility of returns on mutual fund value is proportional to the variance of the random shock to flows.

The results of Table 4 provide an analysis of the risk of mutual fund values across 10,000 simulated paths. However, it may be the case that this risk is idiosyncratic and gets diversified away across many funds. In order to address more systematic risks, and the importance of the nonlinearity induced by the flow-performance sensitivity, we calculate for each of the 10,000 paths the next period's present value of the revenue streams for each fund. This next period's value requires us to replicate the original procedure along each path, that is, to estimate the expected value using 10,000 possible paths starting from each of the 10,000 realizations next period, for a total of 10 million paths. We then calculate the return on the fund's value, the NAV and the benchmark and estimate the relation between these variables using our 10,000 return values. If we were to uncover the true relation, then the residual variance would reflect the variance of the random flows.

Table 5 reports results from the regression of the fund value returns ($R^M$) on contemporaneous returns on the NAV ($R^S$) and the benchmark ($R^I$), including nonlinear terms. The regressions use 10,000 simulated returns for the median characteristics from the all-equity class with the exception of age, which takes on three different values–zero years, seven years (the median age), and fourteen years. For each regression, we report the coefficients (with corresponding standard errors below) and the $R^2$. The results illustrate two important features of the simulated data.

First, asset returns exhibit nonlinearity that decreases with age. The convex relation between fund value returns and NAV returns ($R^S$) is seen most clearly in the first two regressions for each age. Squared fund returns enter with a positive and statistically significant coefficient that decreases from 0.43 for young funds to 0.18 for old funds. When controlling for the effect of the index return in the final three regressions, this convexity is more pro-

[18] Note that unlike our closed-form solution, we treat $N_t$, the number of shares, as an absorbing barrier at zero. That is, any fund hitting zero shares dies.

nounced, with coefficients as large as 0.78 for young funds. The reduction in convexity as funds age is expected since flow-performance sensitivity also decline with age. While this nonlinear effect is economically and statistically significant, it does not provide a dramatic increase in explanatory power, as is demonstrated by the small increase in $R^2$s for the regressions that include nonlinear terms. There is no apparent nonlinearity in the relation between fund value returns and the index return when not controlling for the NAV return, as shown in regressions three and four. Recall, however, that the benchmark has a correlation of 0.76 with the NAV return. Holding everything else constant, the fund value return is negatively related to the benchmark return, but the linear coefficient on $R^I$ is positive and significant because it serves as a proxy for $R^S$, which is the more important variable in determining the fund value return. When controlling for $R^S$, the coefficient on $R^I$ is negative and significant, as expected, and there is a concave relation, with a negative and significant coefficient on the squared term as well. As before, these effects decrease with age.

Second, asset returns exhibit a substantial amount of risk, some of which can be hedged with positions in securities correlated with either NAV returns or benchmark returns. In regressions with only benchmark returns, the coefficient on $R^I$ is 0.92 for young funds, decreasing to 0.77 for old funds. Therefore, the sensitivity of the fund value to NAV returns, as proxied for by the benchmark return, leads to a level of "market" risk slightly lower than that of the benchmark. However, the $R^2$s of these regressions are low, ranging from 7.6% to 11.8% for young and old funds, respectively, indicating a large amount of residual risk. This risk comes primarily from the component due to NAV returns that are uncorrelated with the benchmark and shocks to net flows. NAV return risk is higher, with linear coefficients on $R^S$ ranging from 1.37 to 1.01. This risk falls as funds age due to a decline in the sensitivity of flows to performance. The $R^2$s are substantially higher than for the benchmark return, and they again increase with age because the variance of idiosyncratic net flow shocks falls as funds get older. Hedging with instruments correlated with both factors would require taking a short position in the security correlated with NAV returns, to hedge return and flow risk, and a long position in the asset correlated with the benchmark, to hedge further flow risk. Appropriate positions hedge about one third of the risk, assuming perfect hedge instruments, but the residual risk due to idiosyncratic net flow shocks would diversify away across funds under our assumptions.

# 5 Concluding remarks

We develop a valuation methodology for the fee flow generated by asset managers (i.e., mutual funds) using a contingent claims approach. A closed form formula shows the dependence of fund value not only on the usual suspects such as fees, current asset value and expected asset growth, but also on asset return volatility and the flow- performance sensitivity. Motivated by economic intuition developed from theory we estimate typical fund parameters for various asset classes. Using Monte Carlo simulation, we can go outside the confines of the closed form formula and value funds where such critical parameters as growth and flow-performance sensitivity are age and size dependent. We show the critical importance of fund characteristics for fund valuation in an economically meaningful environment.

This valuation framework and corresponding results suggest several fruitful areas of future research. First, Sections 2 and 4 provide a framework for analyzing the optimal fee structure of a fund, both through time and across different states of nature. This fee structure will depend on important parameters such as the variance-covariance matrix of the fund and benchmark, the benchmark rule, the fund's age, its size and the type of fund. Since the fees themselves affect both the growth rate and sensitivity to fund performance, the optimization will be a complex function of these variables. Aside from a theoretical analysis of the fee structure, it would be interesting to (i) test whether funds follow such a strategy, and (ii) gather evidence (if available) on whether the "optimizing" funds produce more revenues (and growth) through time. One interesting feature of such an analysis is the issue of how flexible a fund can be in terms of choosing fee structures in a competitive environment. To the extent the database includes fee changes for funds through time, this point could be partially addressed.

Second, the valuation framework in Sections 2 and 4 provide predictions on the relative value of particular funds (or more generally, fund families) and on how these values change through time as a function of the important variables. We have identified three sources of variation: the return on NAV, the excess return on NAV relative to a benchmark, and random shocks to net flows. Moreover, these variables enter nonlinearly and in a state-dependent way as a function of the fund's current characteristics. It would be interesting to examine the sales of mutual fund companies or the behavior of publicly traded fund companies to see how well the model captures these basic elements cross-sectionally. For example, do we find that most of the return variation can be explained by fund performance and the underlying asset movements?

Third, this paper has focused entirely on the revenue stream of mutual funds. To complete the picture, one would like to build costs into the analysis.[19] In particular, to what extent are mutual fund families subject to fixed versus variable costs? The degree to which mutual

[19]In this paper we have not addressed the revenues earned from load charges, such as those found with A, B and C shares. Many of these revenues offset marketing costs paid directly to the brokers selling the funds. Nevertheless, Boudoukh, McCallister, Richardson and Whitelaw (2000) show how to value the revenue streams from these charges albeit in a setting in which net flows do not depend upon fund performance.

funds face operating leverage, and the returns to scale from operations, will be important determinants of the underlying risk of the mutual fund business. Sections 2 and 4 show that funds already act like a levered position on the underlying NAV through the relation between fund flows and fund performance.

Fourth, the model of Section 4 imposes particular assumptions on how fund flows relate to fund performance. While this model was justified via an estimated empirical relation, similar in spirit to the empirical prepayment models in the mortgage-backed literature (e.g., Schwartz and Torous (1989)), it may be interesting to develop quasi-rational models of fund flows, again similar in spirit to those developed in the mortgage-backed literature (e.g., Dunn and McConnell (1981a, 1981b), Stanton (1995), and Downing, Stanton and Wallace (2003)). This problem is considerably more difficult, however, as the theory of rational fund flows is less clear than that of rational prepayments. Nevertheless, several recent papers of Berk and Green (2002) and Bergstresser and Poterba (2002) would appear to be useful starting points (e.g., Berk and Green (2002), Bergstresser and Poterba (2002) and Lynch and Musto (2003)).

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Table 1: Descriptive Statistics

A: Fund Characteristics

| | | Mean | SD | Percentile 5 | 25 | 50 | 75 | 95 |
|---|---|---|---|---|---|---|---|---|
| Growth | Fees (%) | 1.14 | 0.47 | 0.53 | 0.80 | 1.04 | 1.40 | 2.01 |
| | Age (yrs) | 14.42 | 13.05 | 2.00 | 5.00 | 9.00 | 20.00 | 44.00 |
| | Size ($M) | 505.71 | 2034.39 | 13.78 | 37.36 | 93.78 | 309.97 | 1653.21 |
| | Returns (%) | 11.25 | 16.68 | -19.43 | -0.18 | 12.80 | 23.40 | 35.15 |
| | Flows (%) | 6.27 | 34.90 | -23.76 | -12.50 | -4.40 | 11.23 | 74.27 |
| Value | Fees (%) | 1.38 | 0.52 | 0.67 | 1.00 | 1.28 | 1.76 | 2.27 |
| | Age (yrs) | 10.08 | 11.11 | 2.00 | 4.00 | 6.00 | 11.00 | 34.00 |
| | Size ($M) | 708.87 | 3045.00 | 14.30 | 44.44 | 133.12 | 437.73 | 2667.51 |
| | Returns (%) | 5.84 | 21.90 | -29.16 | -11.97 | 7.18 | 21.87 | 38.38 |
| | Flows (%) | 18.32 | 56.07 | -26.45 | -11.10 | 1.29 | 26.24 | 118.45 |
| All | Fees (%) | 1.25 | 0.54 | 0.48 | 0.88 | 1.16 | 1.59 | 2.20 |
| | Age (yrs) | 13.39 | 14.51 | 2.00 | 4.00 | 7.00 | 16.00 | 48.00 |
| | Size ($M) | 641.51 | 2779.40 | 14.00 | 40.07 | 113.14 | 384.76 | 2312.96 |
| | Returns (%) | 7.67 | 19.43 | -25.42 | -6.07 | 9.35 | 21.63 | 36.54 |
| | Flows (%) | 12.79 | 46.47 | -24.99 | -11.81 | -1.36 | 19.16 | 99.09 |
| Int'l | Fees (%) | 1.69 | 0.62 | 0.78 | 1.24 | 1.67 | 2.12 | 2.68 |
| | Age (yrs) | 7.31 | 6.96 | 2.00 | 3.00 | 5.00 | 8.00 | 20.00 |
| | Size ($M) | 479.72 | 1795.42 | 13.66 | 34.20 | 89.44 | 276.91 | 1677.05 |
| | Returns (%) | 3.64 | 23.69 | -28.09 | -15.22 | 1.94 | 16.79 | 47.90 |
| | Flows (%) | 13.11 | 52.81 | -32.01 | -15.78 | -1.25 | 20.90 | 112.97 |

B: Benchmark Returns

| | SD | Corr. |
|---|---|---|
| Growth | 0.151 | 0.883 |
| Value | 0.118 | 0.640 |
| All | 0.145 | 0.762 |
| Int'l | 0.169 | 0.758 |

Panel A presents means, standard deviations and percentiles for the fees, age, size, returns and flows for the funds within each of the four asset classes. Panel B presents the standard deviation of the benchmark return and the correlation of this return with the NAV returns for each of the four asset classes.

Table 2: Flow Regression Results

A: Flow Regression

| | | | Growth | Value | All | Int'l |
|---|---|---|---|---|---|---|
| Const | | | 0.253 | 0.407 | 0.366 | 0.481 |
| | | | 0.016 | 0.014 | 0.008 | 0.020 |
| | Size | $> 25, < 50$ | -0.050 | -0.078 | -0.073 | -0.077 |
| | | | 0.016 | 0.014 | 0.008 | 0.019 |
| | Size | $> 50, < 75$ | -0.072 | -0.112 | -0.089 | -0.115 |
| | | | 0.016 | 0.015 | 0.008 | 0.020 |
| | Size | $> 75$ | -0.082 | -0.130 | -0.106 | -0.130 |
| | | | 0.017 | 0.016 | 0.008 | 0.021 |
| | Fees | $> 50$ | 0.004 | -0.012 | -0.011 | -0.102 |
| | | | 0.013 | 0.011 | 0.006 | 0.014 |
| | Age | $> 25, < 50$ | -0.161 | -0.170 | -0.176 | -0.224 |
| | | | 0.016 | 0.015 | 0.008 | 0.019 |
| | Age | $> 50, < 75$ | -0.212 | -0.244 | -0.246 | -0.313 |
| | | | 0.015 | 0.014 | 0.007 | 0.019 |
| | Age | $> 75$ | -0.226 | -0.285 | -0.297 | -0.356 |
| | | | 0.017 | 0.015 | 0.008 | 0.020 |
| Return | | | 1.346 | 1.832 | 1.363 | 0.916 |
| | | | 0.199 | 0.102 | 0.063 | 0.141 |
| | Size | $> 25, < 50$ | 0.498 | -0.185 | -0.011 | 0.023 |
| | | | 0.188 | 0.100 | 0.061 | 0.130 |
| | Size | $> 50, < 75$ | -0.119 | -0.358 | -0.096 | 0.003 |
| | | | 0.190 | 0.101 | 0.061 | 0.135 |
| | Size | $> 75$ | 0.292 | -0.428 | -0.177 | -0.073 |
| | | | 0.204 | 0.114 | 0.064 | 0.140 |
| | Fees | $> 50$ | 0.596 | 0.243 | 0.189 | 0.149 |
| | | | 0.157 | 0.076 | 0.047 | 0.100 |
| | Age | $> 25, < 50$ | -0.897 | -0.387 | -0.314 | -0.260 |
| | | | 0.192 | 0.100 | 0.058 | 0.130 |
| | Age | $> 50, < 75$ | -1.127 | -0.529 | -0.638 | -0.551 |
| | | | 0.188 | 0.097 | 0.058 | 0.126 |
| | Age | $> 75$ | -1.138 | -0.785 | -0.691 | -0.480 |
| | | | 0.217 | 0.109 | 0.071 | 0.147 |
| | | $R^2$ | 0.181 | 0.197 | 0.165 | 0.125 |
| | | Obs | 3267 | 9948 | 25860 | 5469 |

B: Variance Regression

| | | Growth | Value | All | Int'l |
|---|---|---|---|---|---|
| Const | | 0.191 | 0.499 | 0.377 | 0.584 |
| | | 0.018 | 0.028 | 0.013 | 0.038 |
| Size | $> 25, < 50$ | -0.035 | -0.078 | -0.064 | -0.095 |
| | | 0.017 | 0.030 | 0.013 | 0.037 |
| Size | $> 50, < 75$ | -0.029 | -0.158 | -0.101 | -0.124 |
| | | 0.017 | 0.031 | 0.013 | 0.038 |
| Size | $> 75$ | -0.057 | -0.232 | -0.151 | -0.183 |
| | | 0.018 | 0.033 | 0.014 | 0.040 |
| Fees | $> 50$ | 0.036 | 0.015 | 0.014 | -0.030 |
| | | 0.013 | 0.022 | 0.010 | 0.028 |
| Age | $> 25, < 50$ | -0.085 | -0.173 | -0.123 | -0.234 |
| | | 0.017 | 0.030 | 0.013 | 0.037 |
| Age | $> 50, < 75$ | -0.120 | -0.196 | -0.170 | -0.314 |
| | | 0.016 | 0.029 | 0.013 | 0.037 |
| Age | $> 75$ | -0.131 | -0.248 | -0.225 | -0.359 |
| | | 0.018 | 0.031 | 0.013 | 0.039 |
| | $R^2$ | 0.043 | 0.022 | 0.025 | 0.029 |
| | Obs | 3267 | 9948 | 25860 | 5469 |

Results from the estimation of equation (30). Panel A presents results from the regression of annual net flows on a constant and lagged annual returns with dummy variables for size, fee and age classifications. Panel B presents results from the regression of the squared fitted residual from Panel A on a constant with dummy variables for size, fee and age classifications. The percentile breakpoints for the dummy variables are given in Table 1A.

Table 3: Summary statistics for Monte Carlo simulations

A: Using full fund-flow regression results

| Type | M/V | Half Life | 5% | 25% | 50% | 75% | 95% |
|---|---|---|---|---|---|---|---|
| Growth | 0.227 | 14.5 | 0.0753 | 0.124 | 0.174 | 0.265 | 0.567 |
| Value | 0.971 | 34.2 | 0.0284 | 0.105 | 0.291 | 0.847 | 3.32 |
| All | 0.456 | 18.8 | 0.0442 | 0.118 | 0.239 | 0.527 | 1.47 |
| Int'l | 1.06 | 31.4 | 0.0584 | 0.183 | 0.39 | 0.883 | 3.37 |

B: With $\theta = 0$

| Type | M/V | Half Life | 5% | 25% | 50% | 75% | 95% |
|---|---|---|---|---|---|---|---|
| Growth | 0.237 | 14.8 | 0.0832 | 0.134 | 0.187 | 0.276 | 0.572 |
| Value | 0.866 | 31 | 0.0407 | 0.154 | 0.395 | 0.969 | 2.94 |
| All | 0.447 | 18.5 | 0.053 | 0.139 | 0.271 | 0.558 | 1.35 |
| Int'l | 1.05 | 31.2 | 0.0662 | 0.209 | 0.437 | 0.948 | 3.28 |

Summary statistics for Monte Carlo simulations. For each fund type, 10,000 Monte Carlo simulations were performed. The table shows the resulting estimate of $M/V$, the half-life of the fund (defined as the time to maturity of a fund whose value equals 50% of the value of a fund with a life of 100 years), and the 5%, 25%, 50%, 75% and 95% quantiles of the total present values of cash flows for each path. In each case, funds are assumed to have the median age and size for their type. Panel a. uses the full fund-flow regression estimates for each type, while in panel b. the dependence on realized returns is set to zero.

## Table 4: Comparative Statics

| Growth | $\sigma = 0.05$ | $\sigma = 0.3$ | Fee = 0.0025 | Fee = 0.015 | Age = 3 | Age = 20 | Size = 30.3 | Size = 437.73 | $K = 0.1$ | $K = 1$ |
|---|---|---|---|---|---|---|---|---|---|---|
| M/V | 0.227 | 0.285 | 0.0648 | 0.285 | 0.361 | 0.207 | 0.346 | 0.183 | 0.227 | 0.227 |
| Vol | 0.112 | 0.81 | 0.0569 | 0.35 | 0.375 | 0.147 | 0.274 | 0.131 | 0.125 | 0.178 |
| Skew | 1.47 | 28.3 | 4.08 | 10.5 | 3.99 | 3.09 | 3.75 | 3.4 | 2.86 | 3.23 |
| Half | 14.1 | 17.9 | 16.6 | 12.6 | 14.6 | 14.1 | 17.9 | 11.8 | 14.7 | 14.5 |
| 5% | 0.102 | 0.0409 | 0.0203 | 0.0643 | 0.0715 | 0.0758 | 0.0852 | 0.0618 | 0.107 | 0.0753 |
| 25% | 0.149 | 0.0846 | 0.0335 | 0.124 | 0.148 | 0.121 | 0.185 | 0.102 | 0.148 | 0.124 |
| 50% | 0.197 | 0.14 | 0.0477 | 0.195 | 0.24 | 0.165 | 0.285 | 0.151 | 0.194 | 0.174 |
| 75% | 0.273 | 0.25 | 0.0748 | 0.324 | 0.443 | 0.241 | 0.417 | 0.219 | 0.265 | 0.265 |
| 95% | 0.459 | 0.855 | 0.166 | 0.806 | 1.03 | 0.493 | 0.817 | 0.413 | 0.461 | 0.567 |

| Value | $\sigma = 0.05$ | $\sigma = 0.3$ | Fee = 0.0025 | Fee = 0.015 | Age = 3 | Age = 20 | Size = 30.3 | Size = 437.73 | $K = 0.1$ | $K = 1$ |
|---|---|---|---|---|---|---|---|---|---|---|
| M/V | 0.635 | 2.54 | 0.403 | 0.697 | 1.4 | 0.847 | 1.6 | 0.803 | 1.05 | 0.971 |
| Vol | 0.742 | 28.5 | 2.14 | 2.54 | 5.16 | 3.06 | 6.37 | 2.59 | 2.71 | 3.47 |
| Skew | 3.6 | 37.5 | 21 | 25.6 | 18.2 | 19.2 | 18.2 | 15.3 | 17.9 | 17.3 |
| Half | 24.9 | 52.8 | 53.2 | 23.2 | 33.6 | 35 | 37.4 | 32.2 | 36.3 | 34.2 |
| 5% | 0.0399 | 0.0171 | 0.00615 | 0.0264 | 0.0249 | 0.0355 | 0.0221 | 0.0409 | 0.168 | 0.0284 |
| 25% | 0.148 | 0.0606 | 0.0259 | 0.0855 | 0.128 | 0.109 | 0.0941 | 0.13 | 0.314 | 0.105 |
| 50% | 0.384 | 0.183 | 0.0847 | 0.221 | 0.401 | 0.276 | 0.331 | 0.312 | 0.537 | 0.291 |
| 75% | 0.901 | 0.665 | 0.263 | 0.614 | 1.15 | 0.741 | 1.16 | 0.686 | 0.985 | 0.847 |
| 95% | 1.94 | 5.03 | 1.28 | 2.46 | 4.84 | 2.84 | 5.97 | 2.6 | 3.16 | 3.32 |

| All | $\sigma = 0.05$ | $\sigma = 0.3$ | Fee = 0.0025 | Fee = 0.015 | Age = 3 | Age = 20 | Size = 30.3 | Size = 437.73 | $K = 0.1$ | $K = 1$ |
|---|---|---|---|---|---|---|---|---|---|---|
| M/V | 0.38 | 0.74 | 0.14 | 0.425 | 0.719 | 0.335 | 0.674 | 0.377 | 0.474 | 0.456 |
| Vol | 0.314 | 3.68 | 0.283 | 0.697 | 1.4 | 0.444 | 1.16 | 0.562 | 0.514 | 0.731 |
| Skew | 1.84 | 26.1 | 10.7 | 8.22 | 8.45 | 6.6 | 6.44 | 7.18 | 6.18 | 6.98 |
| Half | 16.7 | 26.2 | 24.6 | 14.9 | 19.3 | 18.4 | 21.8 | 17.2 | 19.4 | 18.8 |
| 5% | 0.0592 | 0.0278 | 0.0106 | 0.0422 | 0.0389 | 0.0502 | 0.0366 | 0.0518 | 0.13 | 0.0442 |
| 25% | 0.15 | 0.0798 | 0.0305 | 0.111 | 0.142 | 0.112 | 0.135 | 0.121 | 0.214 | 0.118 |
| 50% | 0.275 | 0.186 | 0.0655 | 0.221 | 0.329 | 0.202 | 0.331 | 0.229 | 0.337 | 0.239 |
| 75% | 0.563 | 0.49 | 0.153 | 0.479 | 0.783 | 0.393 | 0.739 | 0.422 | 0.546 | 0.527 |
| 95% | 0.939 | 2.26 | 0.461 | 1.42 | 2.56 | 1.01 | 2.38 | 1.15 | 1.21 | 1.47 |

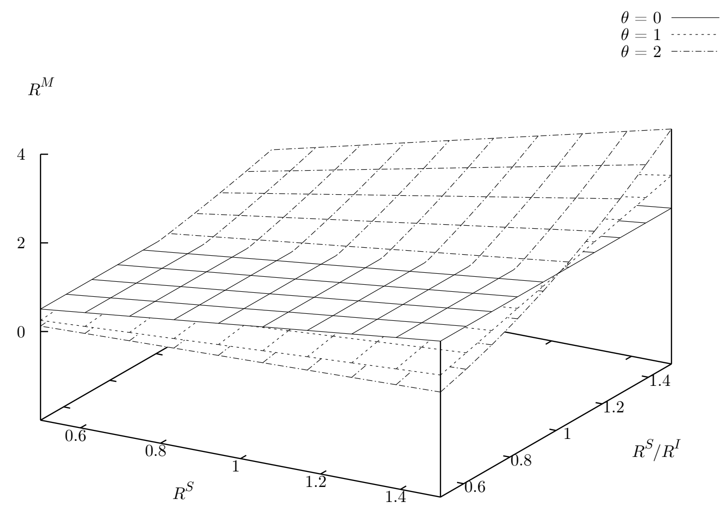
| Int'l | $\sigma = 0.05$ | $\sigma = 0.3$ | Fee = 0.0025 | Fee = 0.015 | Age = 3 | Age = 20 | Size = 30.3 | Size = 437.73 | $K = 0.1$ | $K = 1$ |
|---|---|---|---|---|---|---|---|---|---|---|
| M/V | 0.815 | 1.46 | 0.359 | 1.04 | 1.26 | 0.982 | 1.58 | 0.819 | 1.12 | 1.06 |
| Vol | 1.18 | 12.4 | 2.67 | 4.78 | 5.45 | 4.3 | 6.64 | 3.27 | 2.54 | 4.56 |
| Skew | 8.46 | 47.5 | 34.6 | 27.7 | 25.3 | 28.9 | 23.2 | 24 | 18.2 | 26.8 |
| Half | 25.6 | 39 | 53 | 33.6 | 30.7 | 31.9 | 34.4 | 28.4 | 32.9 | 31.4 |
| 5% | 0.0806 | 0.0448 | 0.0101 | 0.0534 | 0.049 | 0.0633 | 0.0462 | 0.0712 | 0.242 | 0.0584 |
| 25% | 0.25 | 0.141 | 0.035 | 0.169 | 0.193 | 0.177 | 0.208 | 0.169 | 0.399 | 0.183 |
| 50% | 0.487 | 0.325 | 0.0826 | 0.363 | 0.442 | 0.363 | 0.548 | 0.327 | 0.627 | 0.39 |
| 75% | 0.99 | 0.8 | 0.211 | 0.833 | 1.03 | 0.813 | 1.28 | 0.674 | 1.07 | 0.883 |
| 95% | 2.45 | 4 | 1.01 | 3.27 | 4.13 | 3.02 | 5.16 | 2.5 | 3.21 | 3.37 |

The table shows how the results of the Monte Carlo simulations vary with the input parameters. For each valuation, 10,000 Monte Carlo simulations were performed. The table shows the resulting estimate of $M/V$, the half-life of the fund (defined as the time to maturity of a fund whose value equals 50% of the value of a fund with a life of 100 years), the standard deviation, the skewness, and the 5%, 25%, 50%, 75% and 95% quantiles of the total present values of cash flows for each path. In each case, apart from the parameter listed, all other parameters are assumed to take their median value for a fund of that type.

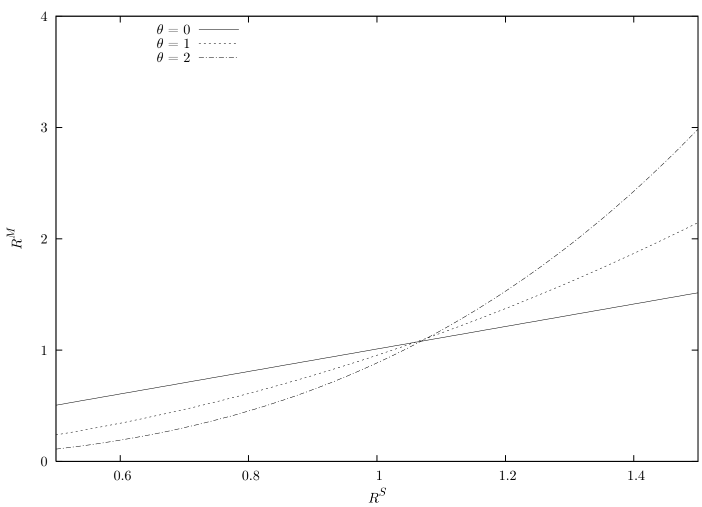
Table 5: Simulated Return Regression Results

| Const | $R^S$ | ${R^S}^2$ | $R^I$ | ${R^I}^2$ | $R^S R^I$ | $R^2$ |
|---|---|---|---|---|---|---|
| | | | Age=0 years | | | |
| 0.0444 | 1.3697 | | | | | 0.2780 |
| 0.0032 | 0.0156 | | | | | |
| 0.0299 | 1.2878 | 0.4313 | | | | 0.2803 |
| 0.0036 | 0.0186 | 0.0535 | | | | |
| 0.0505 | | | 0.9218 | | | 0.0755 |
| 0.0037 | | | 0.0228 | | | |
| 0.0502 | | | 0.9187 | 0.0181 | | 0.0755 |
| 0.0041 | | | 0.0286 | 0.1008 | | |
| 0.0658 | 1.9005 | 0.5341 | -0.9676 | -0.5438 | | 0.3223 |
| 0.0037 | 0.0273 | 0.0652 | 0.0364 | 0.1081 | | |
| 0.0615 | 1.9196 | 0.7781 | -1.0056 | | -0.7365 | 0.3223 |
| 0.0037 | 0.0264 | 0.1029 | 0.0329 | | 0.1480 | |
| 0.0639 | 1.9070 | 0.6728 | -0.9793 | -0.3104 | -0.3728 | 0.3223 |
| 0.0040 | 0.0277 | 0.1250 | 0.0375 | 0.2095 | 0.2866 | |
| | | | Age=7 years | | | |
| 0.0234 | 1.0393 | | | | | 0.3311 |
| 0.0021 | 0.0104 | | | | | |
| 0.0167 | 1.0015 | 0.1991 | | | | 0.3322 |
| 0.0024 | 0.0125 | 0.0359 | | | | |
| 0.0229 | | | 0.7974 | | | 0.1169 |
| 0.0025 | | | 0.0155 | | | |
| 0.0227 | | | 0.7953 | 0.0124 | | 0.1169 |
| 0.0028 | | | 0.0194 | 0.0685 | | |
| 0.0357 | 1.3340 | 0.2366 | -0.5282 | -0.2433 | | 0.3570 |
| 0.0025 | 0.0185 | 0.0442 | 0.0246 | 0.0732 | | |
| 0.0338 | 1.3432 | 0.3322 | -0.5471 | | -0.3069 | 0.3570 |
| 0.0025 | 0.0179 | 0.0697 | 0.0223 | | 0.1002 | |
| 0.0353 | 1.3354 | 0.2669 | -0.5307 | -0.1923 | -0.0815 | 0.3570 |
| 0.0027 | 0.0188 | 0.0847 | 0.0254 | 0.1419 | 0.1941 | |
| | | | Age=14 years | | | |
| 0.0333 | 1.0080 | | | | | 0.3335 |
| 0.0020 | 0.0101 | | | | | |
| 0.0273 | 0.9741 | 0.1790 | | | | 0.3343 |
| 0.0023 | 0.0120 | 0.0346 | | | | |
| 0.0328 | | | 0.7748 | | | 0.1181 |
| 0.0024 | | | 0.0150 | | | |
| 0.0328 | | | 0.7742 | 0.0032 | | 0.1181 |
| 0.0027 | | | 0.0188 | 0.0661 | | |
| 0.0456 | 1.2953 | 0.2127 | -0.5107 | -0.2281 | | 0.3591 |
| 0.0024 | 0.0178 | 0.0426 | 0.0238 | 0.0707 | | |
| 0.0439 | 1.3040 | 0.3006 | -0.5287 | | -0.2848 | 0.3590 |
| 0.0024 | 0.0172 | 0.0672 | 0.0215 | | 0.0967 | |
| 0.0453 | 1.2965 | 0.2372 | -0.5128 | -0.1869 | -0.0658 | 0.3591 |
| 0.0026 | 0.0181 | 0.0817 | 0.0245 | 0.1369 | 0.1873 | |

The table presents results from regressions of fund value returns on returns, squared returns and an interaction term of the NAV and the benchmark. Returns are from 10,000 Monte Carlo simulations for a fund from the all-equity class with median characteristics with the exception of age, which is set to 0 years, 7 years and 14 years.




(a) $R^M$ vs. $R^S$ and $R^S/R^I$




(b) $R^M$ vs. $R^S$ keeping $R^I = e^r = 1.0513$

Figure 1: Expected return on fund's value versus NAV return and return on index
The figures show how the expected return on the fund's value varies with its NAV return and the return on the index, assuming fund flows in and out of the fund are governed by equation (4), with parameters $\sigma = 0.2, \sigma_I = 0.15$, $\rho = 075$, $r = 0.05$, $\nu = 0$.

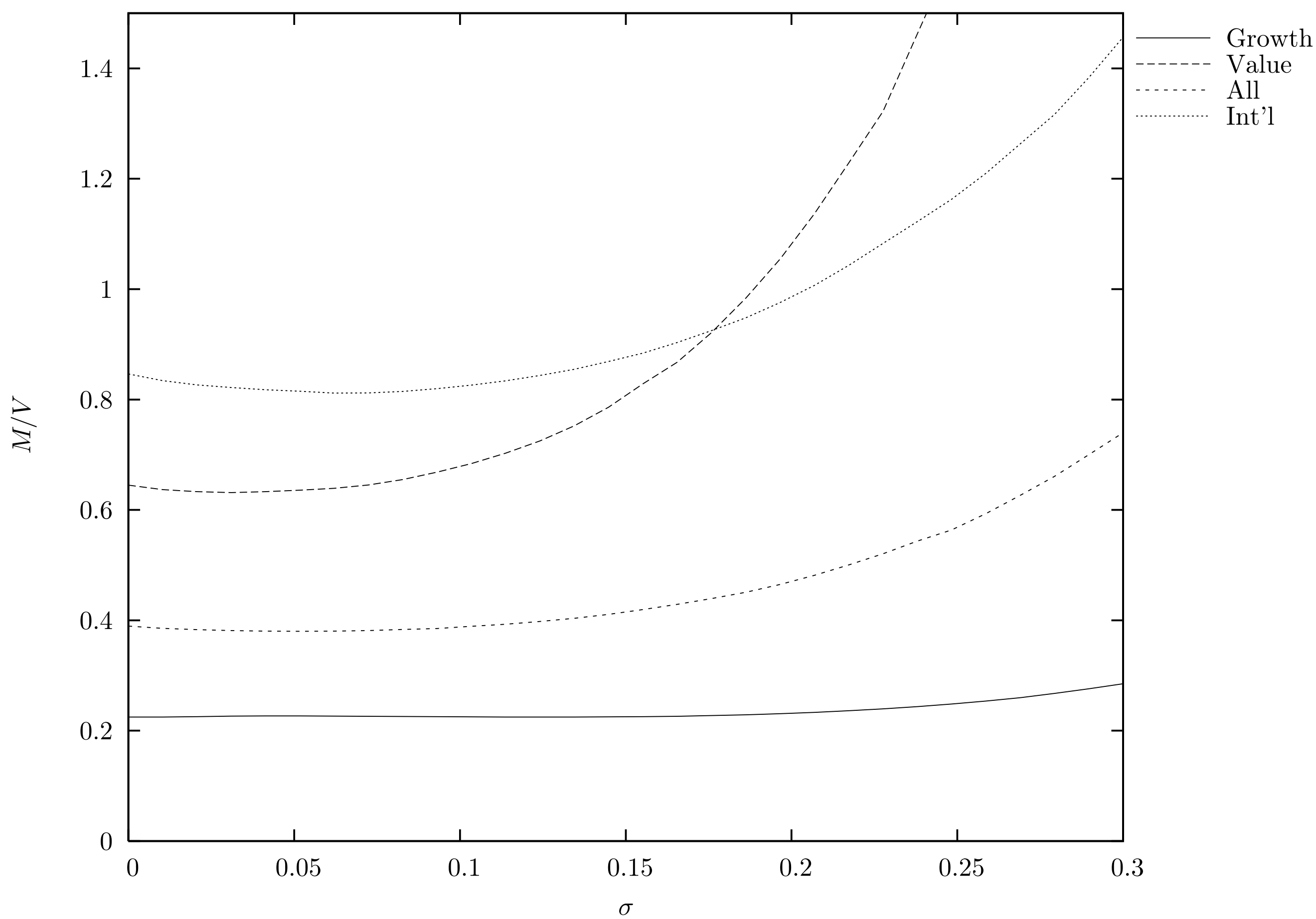



Figure 2: $M/V$ versus $\sigma$

The figure shows the fund's value as a fraction of NAV for different assumptions about $\sigma$, the volatility of the assets in the fund. All other parameters are assumed to take the median value for the appropriate fund type, and valuations are done using 10,000 Monte Carlo simulations.

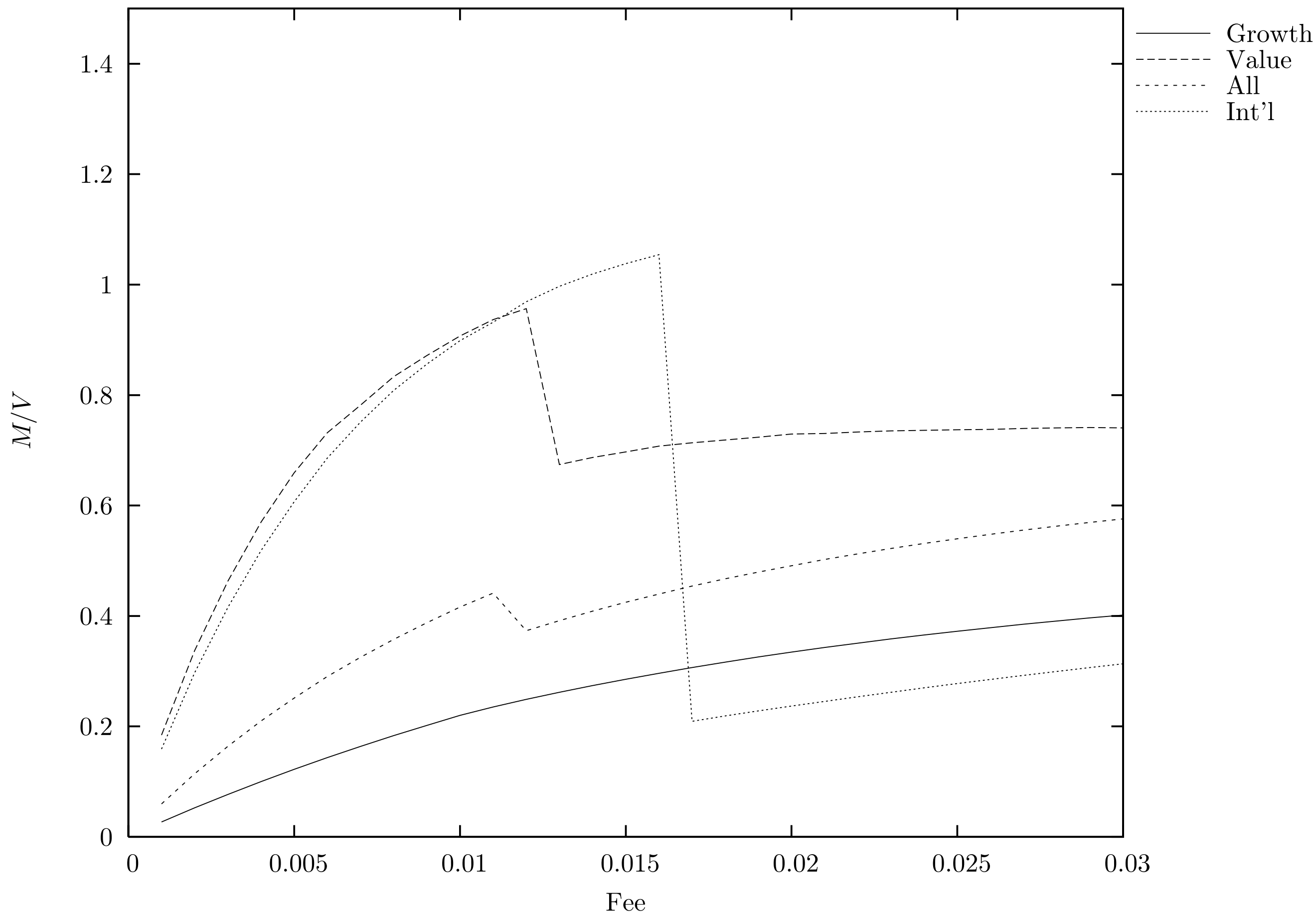



Figure 3: $M/V$ versus fee

The figure shows the fund's value as a fraction of NAV for different assumptions about the fund's annual fee. All other parameters are assumed to take the median value for the appropriate fund type, and valuations are done using 10,000 Monte Carlo simulations.

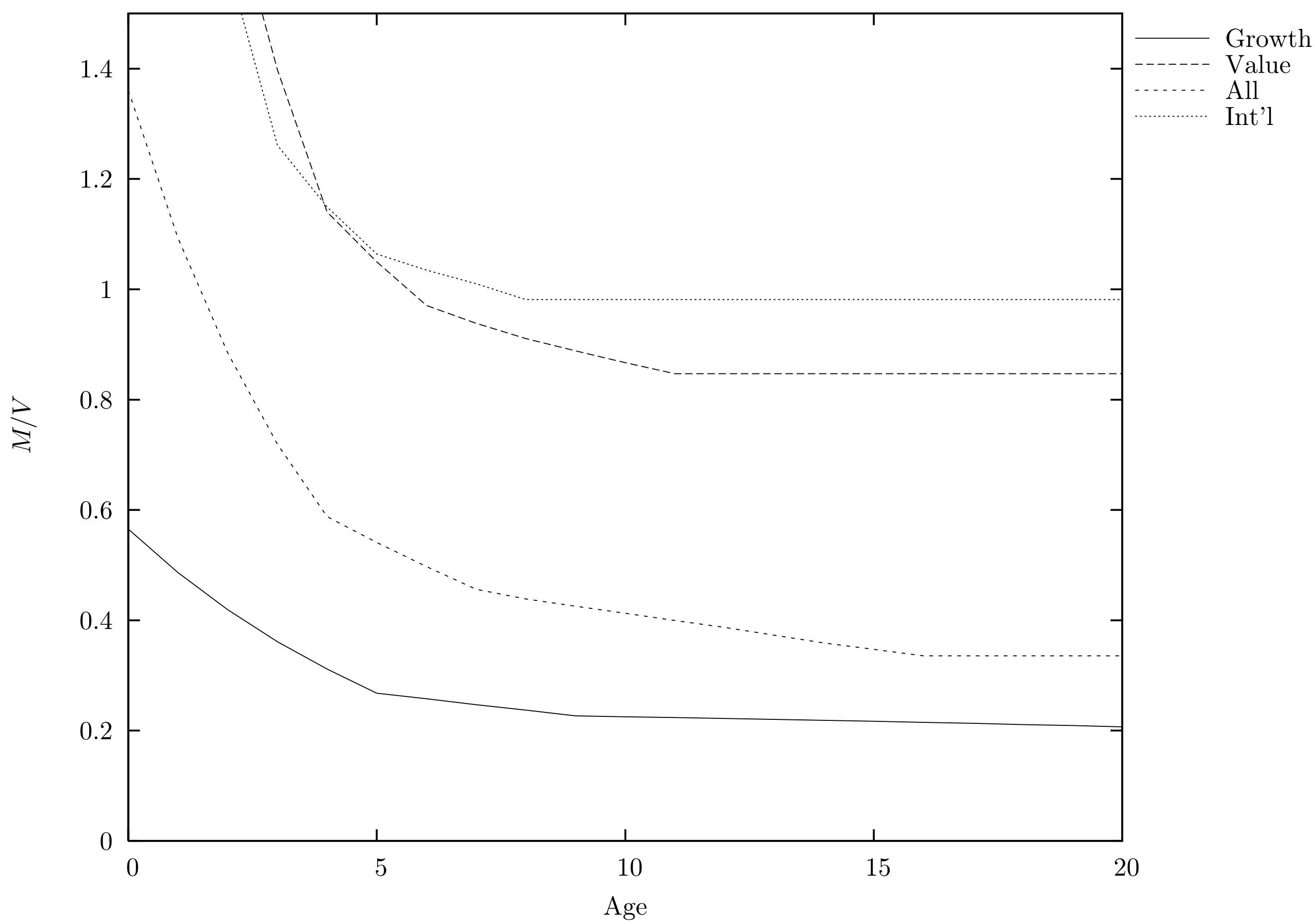



Figure 4: $M/V$ versus age

The figure shows the fund's value as a fraction of NAV for different assumptions about the fund's age. All other parameters are assumed to take the median value for the appropriate fund type, and valuations are done using 10,000 Monte Carlo simulations.

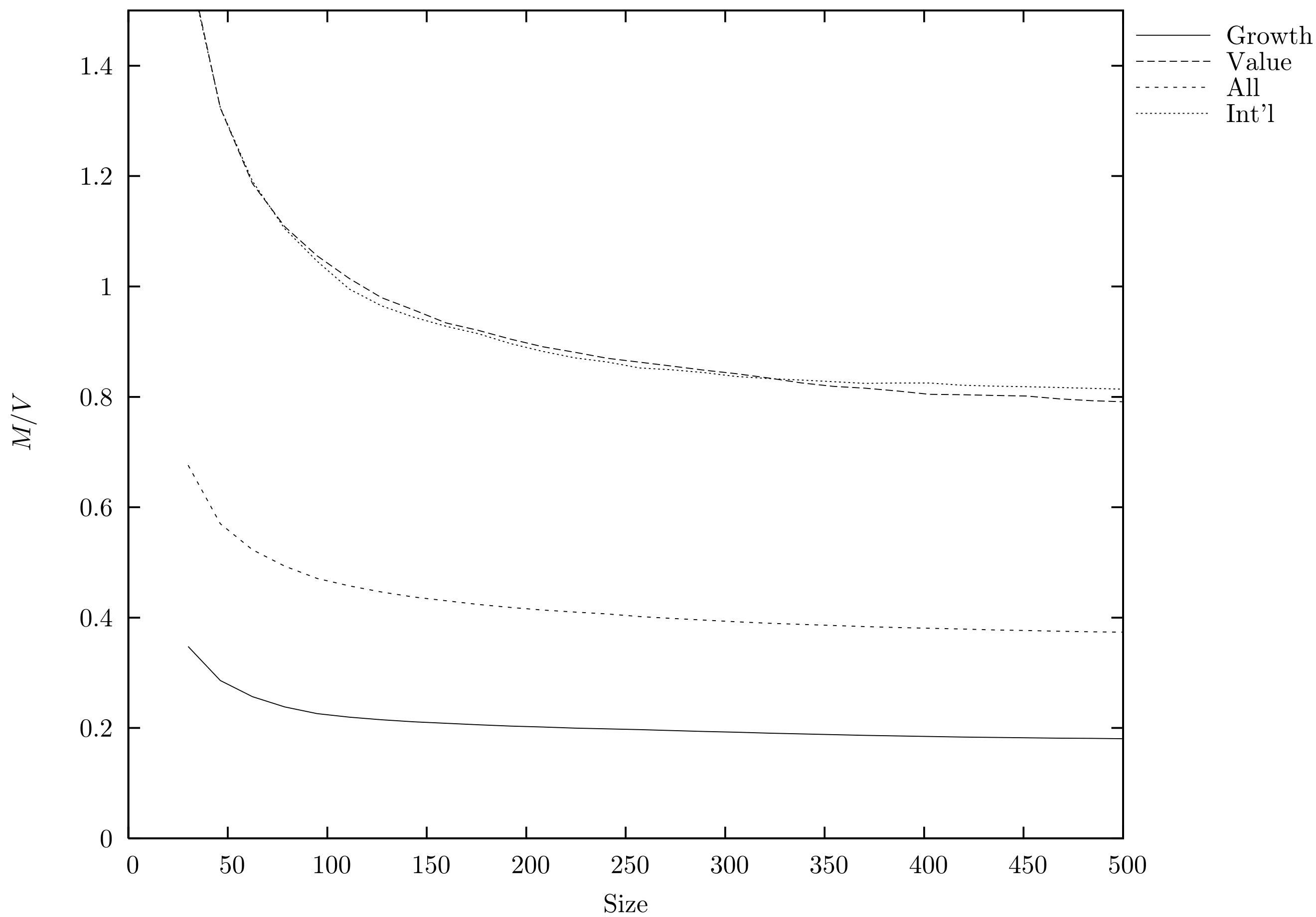



Figure 5: $M/V$ versus size

The figure shows the fund's value as a fraction of NAV for different assumptions about the size of the fund. All other parameters are assumed to take the median value for the appropriate fund type, and valuations are done using 10,000 Monte Carlo simulations.