Title
Opportunistic Routing in Wireless Networks

Author
Van Buhler, Eric Avery

Publication Date
2015

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA, SAN DIEGO

Opportunistic Routing in Wireless Networks

A Thesis submitted in partial satisfaction of the
requirements for the degree
Master of Science

in

Electrical Engineering (Communication Theory and Systems)

by

Eric Van Buhler

Committee in charge:

Professor Tara Javidi, Chair
Professor Massimo Franceschetti
Professor Bhaskar Rao

2015

The Thesis of Eric Van Buhler is approved, and it is acceptable in quality and form for publication on microfilm and electronically:

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Chair

University of California, San Diego

2015

DEDICATION

This Thesis is first and foremost dedicated to my parents, Gary and Bernice, who raised me to always be curious and have a continuous hunger for knowledge. I would also like to dedicate this Thesis to my brother, Evan, who is always there to push me to be better, and my girlfriend, Mary, who put up with me more than anyone during this journey and gave me much needed pep talks and inspiration.

EPIGRAPH

What you get by achieving your goals
is not as important as what you become
by achieving your goals.
—Zig Ziglar

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ACKNOWLEDGEMENTS

I would like to acknowledge Professor Tara Javidi for her continuous support as my advisor. She knew from the beginning that, as a part-time student, my progress would at times be slow moving, but she patiently provided feedback and guidance, and was a major factor for my reaching this milestone.

Chapters 1 to 4 are adapted from Javidi, T; Van Buhler, E. "Opportunistic Routing in Wireless Networks," currently being prepared for submission to Foundations and Trends in Networking.

ABSTRACT OF THE THESIS

Opportunistic Routing in Wireless Networks

by

Eric Van Buhler

Master of Science in Electrical Engineering (Communication Theory and Systems)

University of California, San Diego, 2015

Professor Tara Javidi, Chair

Wireless multi-hop networks have become an important part of many modern communication systems. Opportunistic routing aims to overcome the deficiencies of conventional routing on wireless multi-hop networks, specifically, taking advantage of wireless opportunities and receiver diversity. This Thesis provides an overview of optimal opportunistic routing and compares its performance to that of several routing algorithms from the literature. The performance is examined first in analytical examples, then via simulation to identify the strengths of the optimal opportunist routing algorithm. The performance of a distributed implementation of the optimal opportunistic routing algorithm is also examined via simulation.

Chapter 1

Introduction

Wireless multi-hop networks have become an important part of many modern communication systems. Some of the earliest examples were military communication networks utilizing wireless relays in remote areas. More recently, many industries have used wireless multi-hop networks to achieve a multitude of fascinating tools and systems. Take, for example, the health-care industry. Body-area networks utilize many small sensors that transmit data wirelessly from node to node until it reaches a data collection node. This design allows for a robust low power network, keeping the sensors small and low cost. The same goes for environmental monitoring, such as distributed water quality sensing. The ever-growing Internet of Things (IoT) brings mesh networks into the home with products such as ZigBee and many others. As data collection and communication grows, it will be increasingly important to maximally utilize the wireless resources.

Motivated by classical routing solutions in the Internet, conventional routing attempts to find a fixed path along which the packets are forwarded [1]. Such fixed path schemes fail to take advantage of the broadcast nature and opportunities provided by the wireless medium, and result in unnecessary packet retransmissions. To the best knowledge of the author, the first articles that noticed the benefits of opportunistic receiver selection and selection diversity were those of Lott and Teneketzis [2] and Larsson [3]. Much research interest followed and several opportunistic routing algorithms were developed [4, 5, 6]. Later, in [7], Lott and Teneketzis further developed their framework which unified many of the algorithms. In opportunistic routing, decisions are made in an online manner by choosing the next relay based on the actual transmission outcomes as well as a rank ordering of neighboring nodes. In other words, opportunistic routing mitigates the impact of poor wireless links by exploiting the broadcast nature of wireless transmissions and path diversity.

Chapter 1 is adapted from Javidi, T; Van Buhler, E. "Opportunistic Routing in Wireless Networks," currently being prepared for submission to Foundations and Trends in Networking.

Chapter 2

Opportunism and Receiver Diversity

To understand the benefits of opportunistic routing, it is important to first consider a single-hop wireless transmission. While it is common to conceptualize the nodes' neighbor relations through the notion of a link, the transmission of information over a wireless medium is subjected to stochastic variations in channel as well as a broadcast (overhearing) phenomenon. In this work, a simple probabilistic model of packet transmissions is considered which unifies the probabilistic broadcast model popular in the literature [7, 8]. This model captures some of the important aspects of wireless transmission: it 1) provides an intimate coupling between lower network layers through careful probabilistic modeling of the key channel characteristics, 2) allows for routing decisions to account for broadcast nature of the medium and consequently utilize the phenomenon of overhearing, and 3) provides a clear control for trading off complexity/overhead for performance.

First, a precise and detailed characterization of a (single-hop) probabilistic model is provided, where each node's transmission, given the choice of physical layer parameters, is modeled as a simple erasure broadcast channel model. Following the model description is a discussion on how the model captures important aspects of wireless transmission. This allows for a brief review the notions of opportunistic scheduling, multi-user diversity gain, and receiver diversity, and consequently motivate the advantages of opportunistic routing.

2.1 Local Broadcast Model

Let $\mathcal{U}(i)$ indicate the (finite) collection of physical and MAC layer (TX) parameters available to node i, and Ω to be the set of all nodes in the network. The wireless transmission is modeled as a local erasure broadcast channel from node i to its neighbors.

Definition 2.1. *The packet transmission by a node i, utilizing TX parameter $u \in \mathcal{U}(i)$, can be received by the subset S of nodes with a given probability $P(S|i,u)$.*

The choice of TX parameter $u \in \mathcal{U}$ abstracts the choice of many physical and MAC layer parameters such as the choice of the transmission power, coding and modulation, as well as the transmission rate. For instance, at an increased energy cost, a packet can be transmitted with higher power, ensuring a higher reliability in reaching a particular neighbor and increasing the number of nodes that can be reached.

Definition 2.2. *The u-neighbor set of node i is defined to include all nodes that can be reached via node i under the TX parameter u, i.e. $\mathcal{N}(i,u) := \cup_{S:P(S|i,u)>0} S$. Furthermore, the neighbor set of nodes i is defined to include all nodes that can be reached via node i, i.e. $\mathcal{N}(i) := \cup_{u\in\mathcal{U}} \mathcal{N}(i,u)$.*

Note that in general this model allows for the transmission success at two neighboring nodes to be correlated. However, when the link success probabilities exhibit statistical independence, the model takes a simpler form:

Remark 2.1. *If the link success probabilities are statistically independent, then for all $S \subseteq \Omega$, $P(S|i,u) = \prod_{k\in S} \prod_{l\notin S} p_{ik}(1 - p_{il})$ where p_{ij} denotes the success probability of the link between nodes i and j with $p_{jj} = 1$ for all j.*

The successful reception of packets at a neighbor node, in general, may depend also on the receiver's explicit participation in decoding. The set of intended receivers of a packet from node i is donated by $R \subseteq \mathcal{N}(i)$. Set R is chosen by node i and is specified at the IP header of the transmitted packet. Inclusion in this set is a necessary condition for a node to attempt the decoding of the packet. The wireless transmission is modeled as a local erasure broadcast channel from node i to its intended receivers $R \subseteq \Omega$:

Definition 2.3. *We somewhat abuse the notation and set* $P(S|i, u, R) = \sum_{S'} P(S'|i, u)\mathbf{1}_{\{S=S'\cap(R\cup\{i\})\}}$ *to denote the probability of packet reception by a subset* S *of the intended receivers* $R \subseteq \Omega$*.*

Note that $P(S|i, u, R) = 0$, if $i \notin S$ or $S \not\subseteq R \cup \{i\}$. Furthermore, with the above notation, $P(S|i, u)$ can also be interpreted as shorthand for $P(S|i, u, \mathcal{N}(i))$ or $P(S|i, u, \Omega)$.

2.2 Opportunistic vs. Conventional Routing

In conventional network routing, every packet transmitted from a node is decoded by a maximum of one neighbor node. In other words, the selection of the next hop is predetermined prior to transmission. In contrast, opportunistic routing allows for more than one neighbor to decode the packet. The subsequent relay hop can then be selected (opportunistically) based on the outcome of the original transmission.

Remark 2.2. *If the intended receiver set* R *is always chosen to be a singleton, then the problem of opportunistic routing reduces to that of conventional routing protocol design. In other words, during every transmission, node* i *specifies a particular receiver* $j \in R = \{j\}$ *to decode its packets, and hence, every transmitted packet is decoded by a maximum of one neighbor node.*

In this section, two main advantages of opportunistic operation at the routing layer[1] are discussed. It is well known that knowledge of the local channel realization leads to improved performance by allowing the decision making to rely on receiver diversity and rare, yet opportune, events. Below these are further discussed and examples are used to illustrate the main idea.

[1]Note that this Thesis focuses on opportunistic routing solutions in which only a single copy of any packet will be traversing the network, in contrast to prior work on delay tolerant networking which the local broadcast nature of wireless transmission is utilized by multiple copies of each packet to be *sprayed* towards the destination [9].

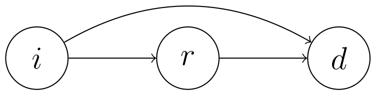

Figure 2.1: An example of opportunism with a relay node r between the start node i and the destination node d.

2.2.1 Opportunistic Receiver Selection

Consider the simple network model in Figure 2.1. Let node i transmit while utilizing TX parameter u and let set S be the local realization of nodes that successfully received and decoded the packet. The probabilities of successful transmission from node i at the relay node r and destination node d are given by

$$\begin{aligned} P(S = \{i, r\}|i, u) &= 0.9 \qquad & P(S = \{i\}|i, u) &= 0.05 \\ P(S = \{i, r, d\}|i, u) &= 0.04 \qquad & P(S = \{i, d\}|i, u) &= 0.01 \end{aligned} \tag{2.1}$$

In this example, it is clear that the event the destination receives the packet directly is a rare, yet opportune outcome. More precisely, while reaching the destination directly is a rare enough outcome, insistence on which would be inefficient and costly, recognizing and utilizing this rare, yet opportune, outcome can significantly reduce cost by allowing node i to reduce the number of transmission attempts. In some of the literature [10], this is referred to as taking advantage of "overhearing" phenomenon, where the protocol allows nodes who have overhead a packet to participate in relaying and transmission of the packet down the stream. In other words, given the availability of information regarding channel realization, i.e. knowing S, it is beneficial to design protocols which fully utilize such rare nonetheless opportune outcomes.

2.2.2 Receiver/Path Diversity

Perhaps the earliest wireless solution that capitalized on receiver selection diversity is that of soft handoff in CDMA cellular networks, where the better of two (or more) sectors are assigned to the mobile device in an opportunistic manner. To underline the concept of receiver diversity, consider the simple example

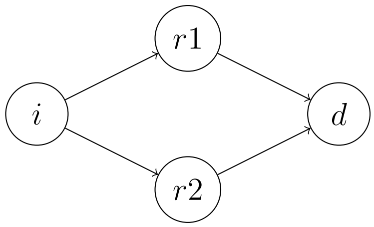

Figure 2.2: An example of receiver diversity with two relay nodes, $r1$ and $r2$, between the start node i and the destination d.

in Figure 2.2 where the probability of successful transmission from node i at the relay nodes r_1 and r_2 are given as follows:

$$P(S = \{i, r_1\}|i, u) = P(S = \{i, r_2\}|i, u) = 0.4$$
$$P(S = \{i, r_1, r_2\}|i, u) = 0.16 \qquad P(S = \{i\}|i, u) = 0.04$$
$$P(\{d\} \subset S|i, u) = P(\{i\} \not\subset S|i, u) = 0.$$

Reaching any one of the relays occurs with the probability of 0.96, which is significantly larger than the probability of successful transmission to either relay specifically, which occurs with probability 0.56. In other words, while each link has a significant chance of being in a bad state, the chance of having bad channels to both relays is of low probability. Hence, when both nodes can advance the packet towards the destination, it is desirable to design network protocols that incorporate and utilize the receiver/path diversity as much as possible.

Note that the gains associated with opportunistic receiver selection and receiver/path diversity strongly depend on the nature of the local broad cast model. More precisely, the significant receiver diversity gain in the above example has been obtained by constructing (for illustration purposes) a model in which the channel realizations are statistically independent across various links. Given this strong dependency, one might ask whether such gains are valid/feasible in practice. A simple experiment conducted at UCSD confirmed the link independence assumption by randomly placing nodes and measuring the success probabilities and confirming this statistical independence. It was found that, except for the hidden terminal cases, the statistical independence assumption largely holds.

2.3 Opportunistic Single-Copy Relay Selection Model

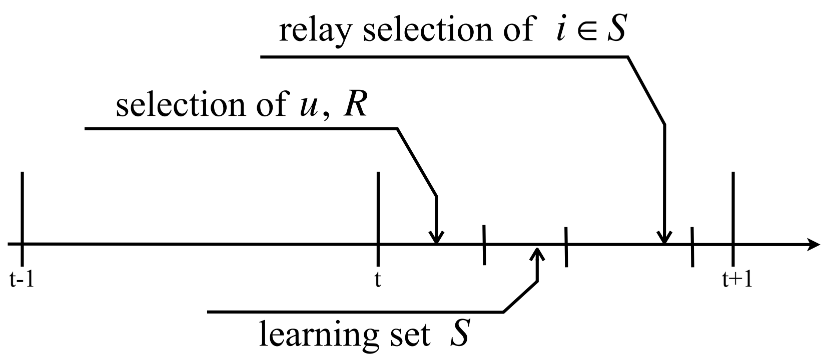

Figure 2.3: In opportunistic relay selection the transmission parameters as well as the specification of the intended receiver set are set ex-ante, while the relay selection decision is made ex-poste.

The local broadcast nature of wireless transmission allows for solutions where multiple copies of each packet are *sprayed* towards the destination [9]. It is clear that in the case of a lightly loaded network, the increased number of packets traversing the network increases the reliability and/or decreases the delay in delivery of packets. However, this is only achieved at the cost of an increase in the total transmissions and traffic across the network. This study, in contrast, focuses on opportunistic routing solutions in which only a single copy of any packet will be traversing the network. In other words, the destination is expected to receive no more than one copy of each packet and unless a packet is being transmitted and/or the next relay is being selected, each packet is uniquely stored at one of the nodes in the network. On the other hand, the routing protocol can include those protocols that take advantage of the opportunism and receiver selection diversity by utilizing the broadcast nature of the wireless transmission.

In the opportunistic relay selection framework, as shown in Figure 2.3, the protocol's decisions can be divided into two sets depending on whether they are made before or after learning the realization of the channel state S at a given time (knowledge about the statistics/distribution of channel state is assumed). The TX parameter u and the list of intended receivers R are set ex-ante and only (possibly) using channel distribution information $P(\cdot|\cdot,\cdot,\cdot)$, while the ex-poste selection of the next relay is done with perfect knowledge of the channel state realization, i.e. set S, and involves selection of one member of S as the next step.

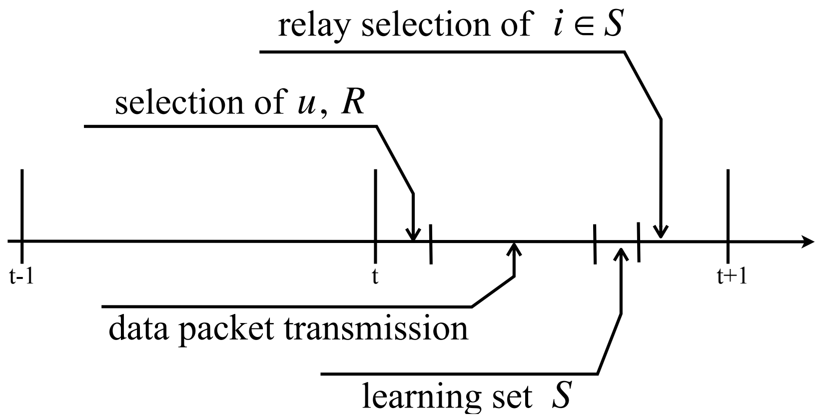

Figure 2.4: In Type-A opportunistic routing protocols the receiver diversity is achieved via a three-way handshake.

The above model captures the following two classes of protocols and scenarios:

Type-A Protocols: The first class of protocols that are well abstracted by the above model utilize a *three way relay selection handshake.* In these protocols, 1) a node transmits a given packet (specified by its sequence number as well as its destination) utilizing a given set of physical layer parameters u and with a given set of intended receivers R specified in its header. 2) Upon successful reception and decoding of the packet by an intended receiver, the receiver transmits an acknowledgment with its ID. 3) The original transmitter, upon the receipt of all acknowledgments from subset $S \subseteq R$, decides and broadcasts the ID of the next relay (in case of retransmission it broadcast its own ID); all nodes in S but that which has received its broadcast ID expunge the packet. The timeline of control decisions are shown in Figure 2.4.

Type-B Protocols: The second type of protocols well abstracted by the above model are those that *probe/estimate/train* the channel from the transmitter node i to a set of potential receivers $R \subseteq \mathcal{N}(i)$, assuming TX parameter u. This initial probing provides the transmitter with channel state information (CSI) which identifies the subset S of potential receivers for which the channel is in good state. The transmission of the data packet, subsequently, follows a conventional packet transmission over the air where the identity of the selected relay will be included in the IP header. The timeline of control decisions are shown in Figure 2.5.

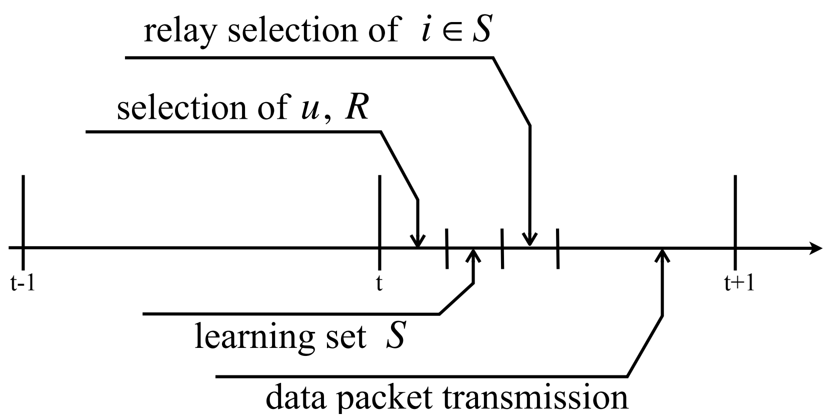

Figure 2.5: In Type-B opportunistic routing protocols the receiver diversity is achieved via an initial estimation and probing of the channel state information.

2.4 Multi-Hop Network Model

Next, the single-hop network model previously described is expanded to a multi-hop network model. An expression for the per-packet reward is provided, associated with each packet successfully arriving at the destination. The main objective of opportunistic routing is to optimize this reward.

Consider the problem of routing packets from a source node 0 to a destination node d in a wireless ad-hoc network of $d + 1$ nodes denoted by the set $\Omega = \{0, 1, 2, \ldots, d\}$. The time is slotted and indexed by $n \geq 0$ (this assumption is not technically critical and is only assumed for ease of exposition).

Each node i can transmit packets utilizing a set of physical layer parameters indicated by variable $u \in \mathcal{U}_i \subset \mathcal{U}$. Furthermore, while relying on the broadcast transmission mode, a node i also is to indicate the potential set of receivers R for each transmission in the packet's header. As detailed in Chapter 2, both u and R are set prior to observation the channel state realization. Furthermore, the model adopts the local broadcast model introduced in Definition 2.1, and considers an opportunistic routing setting with no duplicate copies of the packets. Given a set of nodes S which successfully decode the packet, the next (possibly randomized) routing decision is for node i to either retransmit the packet, relay the packet to a node $j \in S$, or to drop the packet altogether. If node j is selected as a relay, then it proceeds with relaying the packet at the next slot, while other nodes $k \neq j, k \in S$, expunge that packet.

Definition 2.4. *A fixed transmission cost $c_i(u, R) > 0$ is incurred upon a transmission from node i broadcasting to the set of nodes R and when it employs TX parameter u.*

Transmission cost $c_i(u, R)$ can be considered to model the amount of energy used for transmission, the expected time to transmit a given packet, the overhead associated with gathering information about the channel state realization to all nodes in R, or the hop count when the cost is set to unity.

The termination event for a packet is defined to be the event for which that packet is either received at the destination or is dropped by a relay before reaching the destination. The action of terminating the packet is denoted by T, with termination time τ. Upon the termination of a packet at the destination (successful delivery of a packet to the destination), a fixed, known, and positive delivery reward χ is obtained, while no reward is obtained if the packet is terminated before it reaches the destination. Let X_τ denote this reward obtained at the termination time τ, i.e.

$$X_\tau = \begin{cases} \chi & \text{if packet is received at the destination} \\ 0 & \text{if packet is terminated before reaching the destination} \end{cases}$$

Definition 2.5. *The expected per packet reward associated with routing a packets along a sequence of* $\{i_n\}$ *nodes with TX parameters* $\{u_n\}$ *and intended receiver sets* $\{R_n\}$ *is:*

$$J(i_0) = \mathbb{E}\left[X_\tau - \sum_{n=1}^{\tau_T - 1} c_{i_n}(u_n, R_n)\right], \tag{2.2}$$

where the expectation is taken over the realization of the channel state ($S_n \subset R_n \subset \mathcal{N}(i_n)$) *and consequently the conditional distribution of* i_n*, and* (u_n, R_n)*.*

The decomposability of the local broadcast model leads to the following remark:

Remark 2.3. *The problem of opportunistic routing for multiple source-destination pairs, without loss of generality, can be decomposed to the single source-destination problem described above (Problem 1 is solved for each distinct source* i_0 *and each destination* d*).*

Let $\mathcal{H}_n^- = \sigma(S^{n-1}, u^{n-1}, R^{n-1})$ and $\mathcal{H}_n^+ = \sigma(S^n, u^{n-1}, R^{n-1})$ be the history of the observations and actions before and after the realization of the channel at time n. An opportunistic routing strategy, $\mathfrak{c}$, is more than a single sequence of $\{i_n\}$ nodes with TX parameters $\{u_n\}$ and intended receiver sets $\{R_n\}$, it is the sequential rule that dictates the choice of these sequences depending on the past observations and past selected actions prior to the stopping time. The expected per packet reward from a node i to the destination node d under routing strategy $\mathfrak{c}$ is denoted by $J^{\mathfrak{c}}(i_0)$

Definition 2.6. *An opportunistic routing strategy,* $\mathfrak{c}$*, is said to be* feasible *in real-time if the chosen sequences* $\{u_n, R_n\}_{n=1}^{\tau-1}$ *and* $\{i_n\}_{n=1}^{\tau-1}$ *are measurable with respect to* $\mathcal{H}_n^-$ *and* $\mathcal{H}_n^+$*, respectively.*

The problem of opportunistic routing is nothing but optimizing the above expected per packet delivery reward.

Problem P 1. *Find a feasible opportunistic routing strategy* $\mathfrak{c}$ *such that* $J^{\mathfrak{c}}(i_0)$ *is maximized.*

Example 2.1. *Note that when* $c_i(u, R) = 1$ *for* $i = 0, 1, \ldots, d$*,* $u \in \mathcal{U}$*, and* $R \subset \Omega$*, the network is connected, and* χ *is sufficiently large*[2]*, maximizing the expected per packet reward,* J*, corresponds to minimizing the expected per packet delivery time. This is because there is a one-to-one mapping between these two performance measures when* $c_i = 1$*.*

Chapter 2 is adapted from Javidi, T; Van Buhler, E. "Opportunistic Routing in Wireless Networks," currently being prepared for submission to Foundations and Trends in Networking.

[2]The worst case routing cost can be determined by taking supremum over ETX metrics for all source-destination pairs. So long as the network is connected and χ is larger than this worst case routing cost, then it is optimal to have every packet delivered to the destination.

Chapter 3

Opportunistic Multi-Hop Routing

Before proceeding with the problem of optimal opportunistic routing (selection of TX parameters, intended receivers, and relays), the popular paradigm of distance-vector routing is briefly covered. Later, it will be shown that the optimal opportunistic routing is of a distance-vector nature!

3.1 Decentralized Distance-Vector Routing

The problem of opportunistic multi-hop routing is that of determining the node's best TX parameters, $u^*(i)$, and an optimized intended recipient list, $R^*(i)$ (made prior to a transmission) as well as the next best hop to relay the packet (chosen on a sample-path basis as a function of the observed realization of the channel, S_t).

One class of routing protocols, known as *distance-vector* protocols, implement the routing decisions based on a routing table at each node. The routing table at node i consists of a list of neighbors $\mathcal{N}(i)$, and a structure consisting of distance-vector information $\tilde{D}_{\mathfrak{c},t}^{(i,d)}(k)$ for all neighbors $k \in \mathcal{N}(i)$ associated with each destination d. In general, this distance vector can be a function of time t. While the design of distance-vector routing relies on the network designers' ability to devise, compute, and sustain an appropriate estimate of each node's distance to the destination, the distance structure allows for decentralized implementation of routing decisions. In other words, independent of the actual value and method to compute such distance metric, the routing table (distance-vector) allows for a decentralized routing decision implementation.

For a packet at node i destined for node d, upon the observation of the successful transmission outcome $S_t \subseteq R$, the next-hop $j^{(d)}_{\mathfrak{c},t}(i, S_t)$ in S_t is chosen as the neighbor with the minimum "distance". After each successful three-way handshake, the routing responsibility is transferred to the next hop.

Next, a few solutions in the literature are described. In particular, the following candidate policies are considered: Bellman-Ford with hop count measure (BF-HC), shortest path routing protocol (SRCR) [11], and extremely opportunistic routing (ExOR) [5]. For each algorithm the distance vectors $D^i_{\mathfrak{c}}(k)$, and specific operation to compute them, are specified,[1] where $\mathfrak{c}$ is the protocol of interest in the set {BF-HC, SRCR, ExOR}.

BF-HC This routing strategy does not utilize the wireless opportunities and operates in a conventional fashion. It ignores the statistics of the channels when computing the distance metric, instead limiting neighbors of any given node to those that can be reached with sufficiently high probability. More precisely, let $\gamma < 1$ be a scalar indicating a sufficiently high transmission probability; the reliable neighborhood of node i under TX parameter u is defined to be:

$$\tilde{\mathcal{N}}(i,u) \triangleq \{j \in \Omega : \exists S, u \text{ for which } j \in S \text{ and } P(S|i,u) > \gamma\}.$$

Under BF-HC routing strategy, the next hop is selected prior to transmission according to the minimum distance to the destination using the following rule:

$$j_{\text{BF-HC}}(i) = \underset{k \in \tilde{\mathcal{N}}(i,u)}{\arg\min} \left[c_i(u,k) + D^i_{\min}(k) \right] \tag{3.1}$$

where the minimum distance solves the following Bellman fixed point equa-

[1] Considering each destination separately and time invariant protocols, the notation d is suppressed whenever there is no ambiguity.

tion:

$$D_{\min}(d) = 0;$$
$$D_{\min}(i) = \min_{u}\left[\min_{k\in\tilde{\mathcal{N}}(i,u)} c_i(u,k) + D^i_{\min}(k)\right]. \quad (3.2)$$

In particular, the routing table at each node i' is populated with the minimum distance of each neighbor node:

$$D^{i'}_{\text{BF-HC}}(i) = D^{i}_{\text{BF-HC}}(i) = D_{\min}(i) \quad (3.3)$$

SRCR This routing strategy does not utilize the wireless opportunities and operates in a conventional fashion. However, the distance metric $D^i_{\text{SRCR}}(i)$ as well as the choice of next relay account for the reliability of wireless transmissions. More specifically, the next hop is selected and set prior to observing the channel state information by the following rule:

$$j_{\text{SRCR}}(i) = \arg\min_{k\in\mathcal{N}(i,u)}\left[\frac{c_i(u,k)}{P(\{i,k\}|i,u,\{k\})} + D^i_{\text{SRCR}}(k)\right] \quad (3.4)$$

In other words, the intended next relay is selected such that it optimally trades off the distance of the relay $D^i_{\text{SRCR}}(k)$ against the expected cost of reaching the relay reliably (the cost of transmissions multiplied by the number of expected transmission attempts.

The distance metric of node i is the minimum expected airtime (ETX) for the packets routed and relayed via node i, where ETX is the solution to the following fixed-point equation:

$$ETX(d) = 0;$$
$$ETX(i) = \min_{u}\left[\min_{k\in\mathcal{N}(i,u)} \frac{c_i(u,k)}{P(\{i,k\}|i,u,\{k\})} + ETX(k)\right] \quad (3.5)$$

Recall from Definition 2.3 that $P(\{i,k\}|i,u,\{k\}) = \sum_{S':\{i,k\}\subset S'} P(S'|i,u)$ is the probability that transmission of node i reaches node k. The routing table at each node i' is populated with the minimum distance of each neighbor node:

$$D^{i'}_{\text{SRCR}}(i) = D^{i}_{\text{SRCR}}(i) = ETX(i). \quad (3.6)$$

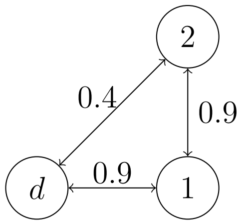

Figure 3.1: A simple network to demonstrate distance-vector computations. The link success probabilities are independent and shown in the figure. Consider a packet at node 2 destined for node d.

and the next hop is chosen to be the minimizer of (3.4). Note that the i^{th} elements in the routing tables of all nodes $i' \in \{j : i \in \mathcal{N}(i)\}$ are consistent and equal to the average expected air time associated with relaying packets via node i.

ExOR This routing strategy does utilize the wireless opportunities. In other words, the selection of the next relay depends on the instantaneous channel state realization, $S_t \subseteq \mathcal{N}(i)$:

$$j_{\text{ExOR},t}(i, S_t) = \arg\min_{k \in S_t} D^i_{\text{ExOR}}(k), \tag{3.7}$$

where the distance-vector computations following that of SRCR, i.e.

$$D^{i'}_{\text{ExOR}}(i) = D^i_{\text{ExOR}}(i) = ETX(i). \tag{3.8}$$

Example 3.1. *To more clearly illustrate the differences between these algorithms, the distance metrics for each are computed and the relay selection is described for the simple network topology in Figure 3.1. For simplicity of computation, keep the TX parameter u constant ($\mathcal{U} = \{u\}$), let TX cost $c_i(u, i)$ at each node be equal to* 1*, and assume successful decoding to be independent across links (see Remark 2.1), with success probabilities shown. In general, upon computation of the distance metrics the routing decisions are conducted in a decentralized manner at each node.*

BF-HC *The BF-HC algorithm is parameterized by threshold γ on the link quality. First, consider the case $\gamma = 0.8$, meaning node* 2 *will not consider node d as a potential receiver. The algorithm computes the distance for each node* *and chooses the next relay prior to transmission. The distances and next hop decisions are as follows:*

- *The distance metrics:*

$$\begin{aligned} D_{BF\text{-}HC}(0) &= D_{HC\text{-}min}(0) = 0 \\ D_{BF\text{-}HC}(1) &= D_{HC\text{-}min}(1) = 1 \\ D_{BF\text{-}HC}(2) &= D_{HC\text{-}min}(2) = 2 \end{aligned}$$

- *Node* 1 *next relay selection:*

$$\begin{aligned} \tilde{\mathcal{N}}(1,u) &= \{d, 2\} \\ j_{BF\text{-}HC}(1) &= \arg\min_{k \in \tilde{\mathcal{N}}(1,u)} [1 + D_{min}(k)] = d \end{aligned}$$

- *Node* 2 *next relay selection:*

$$\begin{aligned} \tilde{\mathcal{N}}(2,u) &= \{1\} \\ j_{BF\text{-}HC}(2) &= \arg\min_{k \in \tilde{\mathcal{N}}(2,u)} [1 + D_{min}(k)] = 1 \end{aligned}$$

In summary, under BF-HC Algorithm with $\gamma = 0.8$*, every single packet originating at node* 2 *destined for node* d *is transmitted until node* 1 *successfully decodes the packet.*

Now consider the case $\gamma = 0.38$ *where node* d *is in the neighbor set of node* 2*.*

- *The distance metrics:*

$$\begin{aligned} D_{BF\text{-}HC}(0) &= D_{HC\text{-}min}(0) = 0 \\ D_{BF\text{-}HC}(1) &= D_{HC\text{-}min}(1) = 1 \\ D_{BF\text{-}HC}(2) &= D_{HC\text{-}min}(2) = 1 \end{aligned}$$

- *Node* 1 *next relay selection:*

$$\begin{aligned} \tilde{\mathcal{N}}(1,u) &= \{d, 2\} \\ j_{BF\text{-}HC}(1) &= \arg\min_{k \in \tilde{\mathcal{N}}(1,u)} [1 + D_{min}(k)] = d \end{aligned}$$

- *Node 2 next relay selection:*

$$\begin{aligned}\tilde{\mathcal{N}}(2,u) &= \{d,1\}\\ j_{BF\text{-}HC}(2) &= \arg\min_{k\in\tilde{\mathcal{N}}(2,u)}[1+D_{min}(k)] = d\end{aligned}$$

In this case, every single packet originating at node 2 destined for node d is transmitted until node d successfully decodes the packet, which has a relatively small probability of success.

Note that this algorithm not only neglects the wireless opportunities, but is also sensitive to the choice of parameter γ and may in general result in significant loss of performance.

SRCR *The SRCR algorithm computes the ETX for each node, and then chooses the next relay prior to transmission (without knowledge of channel realization) so as to balance the immediate transmission cost versus the relay's distance. In particular,*

- *The distance metrics:*

$$\begin{aligned}D_{SRCR}(0) &= ETX(0) = 0\\ D_{SRCR}(1) &= ETX(1) = \frac{1}{0.9} = 1.11\\ D_{SRCR}(2) &= ETX(2) = \min\left[\frac{1}{0.4}, \frac{1}{0.9} + ETX(1)\right] = 2.22\end{aligned}$$

- *Node 1 next relay selection:*

$$\begin{aligned}\mathcal{N}(1,u) &= \{d,2\}\\ j_{SRCR}(1) &= \arg\min_{k\in\mathcal{N}(1,u)}\left[\frac{1}{P(\{1,k\}|1,u,\{k\})} + ETX(k)\right] = d\end{aligned}$$

- *Node 2 next relay selection:*

$$\begin{aligned}\mathcal{N}(2,u) &= \{d,1\}\\ j_{SRCR}(2) &= \arg\min_{k\in\mathcal{N}(2,u)}\left[\frac{1}{P(\{2,k\}|2,u,\{k\})} + ETX(k)\right] = 1\end{aligned}$$

ExOR *The ExOR algorithm computes the ETX for each node (same as SRCR), but operates in an opportunistic manner by choosing the next relay only after the packet is broadcast to all neighbors and a subset of them have acknowledged the successful decoding of the packet. Denote* $S_t \subseteq \mathcal{N}(i,u)$ *as the set of neighbor nodes that correctly decode the packet in time slot* t. *Note that here the intended receiver set* R *for each node is chosen to coincide with the neighbor set of node* i.

- *The distance metrics:*

$$\begin{aligned} D_{ExOR}(0) &= ETX(0) = 0 \\ D_{ExOR}(1) &= ETX(1) = 1.11 \\ D_{ExOR}(2) &= ETX(2) = 2.22 \end{aligned}$$

- *Node 1 next relay selection (post-transmission):*

$$\begin{aligned} R(1,u) &= \{d,2\} \\ j_{ExOR,t}(1,S_t) &= \arg\min_{k\in S_t} ETX(k) \\ P\{j_{ExOR,t}(1,S_t) = d\} &= 0.9 \\ P\{j_{ExOR,t}(1,S_t) = 1\} &= 0.1 \end{aligned}$$

- *Node 2 next relay selection (post-transmission):*

$$\begin{aligned} R(2,u) &= \{d,1\} \\ j_{ExOR,t}(2,S_t) &= \arg\min_{k\in S_t} ETX(k) \\ P\{j_{ExOR,t}(2,S_t) = d\} &= 0.40 \\ P\{j_{ExOR,t}(2,S_t) = 1\} &= 0.54 \\ P\{j_{ExOR,t}(2,S_t) = 2\} &= 0.06 \end{aligned}$$

Note that while both SRCR and ExOR rely on ETX as the distance metric, ExOR utilizes the wireless opportunities e.g. 40% of transmissions from node 2 *can directly reach the destination, which is strictly better than relaying packets through node* 1. *In other words, ExOR strictly outperforms SRCR despite their similarly relying on ETX metric.*

In summary, for a packet arriving at node 2 and destined for node d the following are the expected values of the number of per-packet transmissions, denoted as N_{TX}, for each algorithm:

$$\begin{aligned}
\mathbb{E}\left[N_{TX}\left(BF\text{-}HC_{\gamma=0.8}\right)\right] &= 2.22 \\
\mathbb{E}\left[N_{TX}\left(BF\text{-}HC_{\gamma=0.35}\right)\right] &= 2.50 \\
\mathbb{E}\left[N_{TX}\left(SRCR\right)\right] &= 2.22 \\
\mathbb{E}\left[N_{TX}\left(ExOR\right)\right] &= 1.70
\end{aligned}$$

3.2 Optimal Opportunistic Multi-hop Routing

In this section, the problem of optimal opportunistic routing (Problem 1) is further examined. It is shown that the optimal routing strategy also has a distance-vector structure, where the optimal distance-vector solves a Bellman-type fixed point equation. Polynomial complexity (centralized) methods are provided to compute this optimal distance metric.

3.2.1 Optimal Distance-Vector

Consider the fixed-point equation below:

$$\begin{aligned}
V^*(d) &= \chi \\
V^*(i) &= \max_{u,R}\left\{-c_i(u,R) + \sum_{S\subset\Omega} P(S|i,u,R)\max_{j\in S} V^*(j)\right\}
\end{aligned} \qquad (3.9)$$

The following theorems show that $V^*(i)$ is nothing but the maximum expected reward of routing a packet through node i and, hence, can be used as an optimal distance-vector for decentralized routing decisions:

Theorem 3.1. *The functional solution of the fixed-point (3.9) is unique. Furthermore, for any feasible routing strategy $\mathfrak{c}$ and any node $i \in \Omega$, $V^*(i) \geq J^{\mathfrak{c}}(i)$ where $J^{\mathfrak{c}}(i)$ denotes the expected reward of routing packets from node i under the routing strategy $\mathfrak{c}$ and is given in Definition 2.5. Furthermore,*

1. *there is an optimal history-invariant, deterministic and stationary policy that selects the TX parameter and the intended receiver set for node i -denoted as $u^*(i)$ and $R^*(i)$- to be the maximizers of the left hand side of (3.9); and*

2. *an optimal relay selection policy is to select among the potential relays $S \subset R^*(i)$ the node with the highest expected reward to the destination.*

$$j^*(i, S) = \arg\max_{j \in S} V^*(j). \tag{3.10}$$

Theorem 3.1 allows us to restrict our search for optimal routing strategies to the simpler class of history-invariant (Markov) deterministic and stationary distance-vector strategies. In particular, given a full characterization of V^* (associated with each transmitting node and the destination), the optimal routing strategy can be implemented in a local and decentralized fashion following a simple distance-vector protocol. It is clear that any monotonically strictly decreasing function of $V^*(k)$, $k \in \mathcal{N}(i)$, can be used in the routing table at node i to denote each node's opportunistic "distance" to the destination. This allows for a decentralized implementation of the optimal opportunistic routing of packets across the network, albeit assuming that each node has access to the optimal expected per packet reward $V^*(k)$, $k \in \mathcal{N}(i)$, and thus gives rise to the construction of the optimal stochastic routing (OSR) algorithm:

OSR: This routing strategy uses the increasing function of $(\chi - V^*(k))$ as a distance measure:

$$D^j_{\text{OSR}}(i) = D^i_{\text{OSR}}(i) = (\chi - V^*(i)) \tag{3.11}$$

and operates in an opportunistic fashion. In other words, the selection of the next relay depends on the instantaneous channel state realization,
$S_t \subseteq R \subseteq \mathcal{N}(i)$:

$$j_{\text{OSR},t}(i, S_t) = \arg\min_{k \in S_t} D^i_{\text{OSR}}(k), \tag{3.12}$$

Following Theorem 3.1, in order to arrive at the optimal opportunistic routing strategy, all we need is a method to compute, for each given destination, the optimal distance vector associated with maximum expected reward, i.e. the solution to the fixed-point equation (3.9). Next, one such method is provided.

3.2.2 Centralized Computation

The main focus of this subsection is to provide a specific algorithm to compute the solution to the fixed-point equation (3.9). Inspired by the well known Dijkstra method to solve the Bellman equation associated with the classical problem of minimum distance routing over a graph [12], consider the following algorithm (due to Lott and Teneketzis [7]):

Algorithm 3.1. ***Dijkstra-Lott-Teneketzis Algoritm***

Step 0 *Initialize* $\mathcal{O} = \{d\}$, $V^*(d) = \chi$, *and* $\mathcal{A} = \Omega - \{d\}$.

Step 1 *For every* $i \in \mathcal{A} \cap \mathcal{N}(\mathcal{O})$ *compute*

$$V(i) = \max_{u,R} \frac{-c_i(u,R) + \sum_{S:S\cap\mathcal{O}\neq\emptyset} P(S|i,u,R) \max_{j\in S\cap\mathcal{O}} V^*(j)}{\sum_{S:S\cap\mathcal{O}\neq\emptyset} P(S|i,u,R)}. \tag{3.13}$$

Step 2 *Let* $u'(i)$ *and* $R'(i)$ *be the maximizers of (3.13) and let* $l = \arg\max_{i\in\mathcal{A}} V(i)$;

- *Set* $\mathcal{O} = \mathcal{O} \cup \{l\}$ *and* $\mathcal{A} = \mathcal{A} - \{l\}$.
- *Set* $V^*(l) = V(l)$, $u^*(l) = u'(l)$ *and* $R^*(l) = R'(l)$.

Step 3 *Repeat Steps 1-3 until* $\mathcal{A} = \emptyset$.

Lemma 3.1. *The Dijkstra-Lott-Teneketzis Algorithm described above results in* $V^*(i)$ *which is the maximum expected reward of routing a packet via node* i, *for* $i = 0, 1, \ldots, d-1$.

Proof. To arrive at Equation 3.13, we split the summation in Equation 3.9 over two disjoint groups of sets:

- S_1: all sets where $V^*(j) > V^*(i)$ for some $j \in S \subset \Omega$.
- S_2: all sets where $V^*(j) \leq V^*(i)$ $\forall j \in S \subset \Omega$.

Further, since $S_1 \cup S_2$ is the entire sample space, Ω, the sum of probabilities over S_1 is the compliment of the sum of all probabilities in S_2 and vice versa.

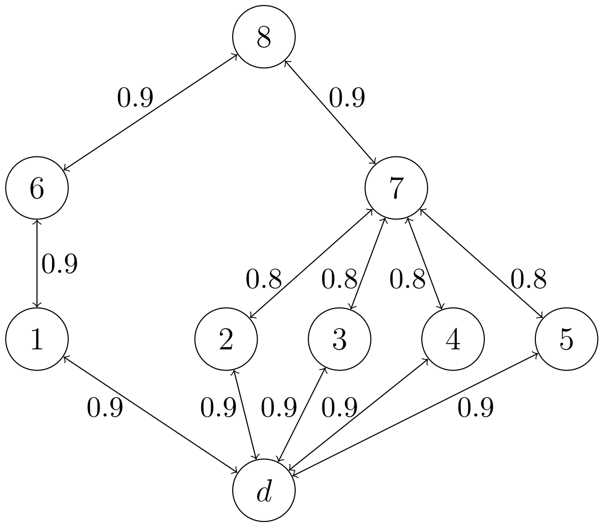

Figure 3.2: A network topology with two paths, one with receiver diversity and one with better link quality.

$$
\begin{aligned}
&\sum_{S\subset\Omega} P(S|i,u,R)\max_{j\in S} V^*(j)\\
&= \sum_{S_1} P(S|i,u,R)\max_{j\in S} V^*(j) + \sum_{S_2} P(S|i,u,R)V^*(i)\\
&= \sum_{S_1} P(S|i,u,R)\max_{j\in S} V^*(j) + \left(1 - \sum_{S_1} P(S|i,u,R)\right)V^*(i)
\end{aligned}
$$

By substituting this into Equation 3.9 and solving for $V^*(i)$, we obtain Equation 3.13. □

Example 3.2. *To clearly see the benefits of OSR over the previously described algorithms (ExOR in particular) consider the network topology shown in Figure 3.2. Notice that this topology contains two paths to the destination, one which has a sequence of high quality links ($P(\{i,j\}|i,u,\{j\}) = 0.9$), and one which contains a section with worse link quality ($P(\{i,j\}|i,u,\{j\}) = 0.8$), but receiver diversity. It will be shown that the receiver diversity makes up for the worse link quality, but only OSR takes advantage of it. The TX parameter is kept constant and equal between nodes ($\mathcal{U} = \{u\}$), TX cost is set to 1, and link probabilities are independent. For BF-HC, we set $\gamma = 0.85$, meaning that the path with receiver diversity will not be* *considered for that algorithm. For OSR, we set $\chi = 10$. Table 3.1 shows the results of the distance computations at each node.*

Table 3.1: The distance metrics computed by each algorithm. The relationship between the metrics at nodes 6 and 7 determines whether the algorithm chooses the left or right path.

node	$D_{\text{HC-min}}$	ETX	D_{OSR}
d	0	0	0
1	1	1.11	1.11
2	1	1.11	1.11
3	1	1.11	1.11
4	1	1.11	1.11
5	1	1.11	1.11
6	2	2.22	2.22
7	4	2.36	2.11
8	3	3.33	3.13

Observe the relationship between the computed distance of node 6 and node 7 to understand how the different algorithms will treat this topology. For BF-HC, node 7 can't use the links to nodes $2, 3, 4$ and 5, and therefore must go through node 8 to reach the destination. Base on the hop count metric, BF-HC will strictly choose to send to node 6. SRCR finds that $ETX(6) < ETX(7)$, and since it chooses the next relay prior to transmission, always chooses to route to node 6. ExOR uses the same ETX metrics, but takes advantage of the link from node 8 to node 7 when the link to node 6 fails. For OSR, $D_{OSR}(7) < D_{OSR}(6)$, so OSR will prefer to send to node 7, only sending to node 6 when the link to node 7 fails.

For a packet arriving at node 8 and destined for node d the following are the expected average number of transmissions for each algorithm:

$$
\begin{aligned}
\mathbb{E}\left[N_{TX}\left(\textit{BF-HC}\right)\right] &= 3.33 \\
\mathbb{E}\left[N_{TX}\left(\textit{SRCR}\right)\right] &= 3.33 \\
\mathbb{E}\left[N_{TX}\left(\textit{ExOR}\right)\right] &= 3.22 \\
\mathbb{E}\left[N_{TX}\left(\textit{OSR}\right)\right] &= 3.13
\end{aligned}
$$

By favoring the path with greater receiver diversity, OSR achieves the fewest number of expected transmissions.

3.3 Distributed Optimal Opportunistic Routing

In this section, a distributed algorithm to compute the expected packet reward is introduced. This algorithm is known as the Distributed Dijkstra-Lott-Teneketzis Algorithm, and is simply a distributed method of computing the weights based on the DLT algorithm.

Algorithm 3.2. ***Distributed Dijkstra-Lott-Teneketzis Algorithm***
At each event time, any number of the following two events can occur:

Event 1 *A node i receives $V_{DDLT}(j)$ from a neighbor j, $j \in \mathcal{N}(i)$, and updates its routing table, i.e. distance metric $V^i_{DDLT}(j)$, with this most recent value.*

Event 2 *A node $i \neq d$ recomputes $V_{DDLT}(i)$ using the current $V^i_{DDLT}(j)$ values and the set $\mathcal{H}^i = \mathcal{N}(i) - \{i\}$ via an intermediate value J:*

$$J(i) = \max_{H \in \mathcal{H}^i, u, R} \frac{-c_i(u,R) + \sum_{S:S\cap H \neq \emptyset} P^i(S|i,u,R) \max_{j \in S\cap H} V^i_{DDLT}(j)}{\sum_{S:S\cap H\neq\emptyset} P^i(S|i,u,R)} \tag{3.14}$$

which is subsequently used to update $V_{DDLT}(i)$:

- *If $J(i) \geq 0$ then set $V_{DDLT}(i) = J(i)$.*
- *If $J(i) < 0$, then set the decision at node i to be early termination; reset $V_{DDLT}(i) = 0$.*

It is assumed that events 1 and 2 occur infinitely often.

This algorithm does not require *a priori* time ordering on the above events, nor on the nodes where they are occurring. At times when neither of the above events is taking place, the system can be viewed in a frozen state, with all system parameters remaining unchanged. An event which occurs at some event time can have no effect on other events at the same time. Hence, an arbitrary order for all events occurring at a given time can be chosen without affecting the outcome.

The following theorem is the main result for Algorithm 3.2, summarizing its convergence properties.

Theorem 3.2. *For Algorithm 3.2 with any initial estimate* $0 \leq V^i_{DDLT}(j) \leq \chi$ *for all* $i, j \in \Omega$, *we have*

1. $$\lim_{t \to \infty} V_{DDLT}(i) = V^*(i), \ \forall i \in \Omega \tag{3.15}$$

2. *There exists an event* $n_p < \infty$ *after which each node* i *agrees with the optimal routing ranking. In other word, after finite steps, Alogrithm 3.2 results in an optimal distributed policy*

3. *If*
$$V^*(i) \neq V^*(j), \forall i \in \Omega, j \in \mathcal{N}(i) \tag{3.16}$$
then there exists an event $n_p < \infty$ *after which*
   (a) $V_{DDLT}(i) = V^*(i), \forall n \geq n_p, \forall i \in \Omega$
   (b) *Algorithm 3.2 results in a globally consistent metric.*

This theorem is proved in the work of Lott and Teneketzis [7], and in the simulation results it will be shown that in realistic scenarios this algorithm does converge to the optimal packet reward in finite time.

Chapter 3 is adapted from Javidi, T; Van Buhler, E. "Opportunistic Routing in Wireless Networks," currently being prepared for submission to Foundations and Trends in Networking.

Chapter 4

Opportunistic Routing Simulation Results

In this chapter, the previously described algorithms are simulated and their performance is compared. The first group of simulations examines the average per-packet transmission count on various network topologies, following a method similar to the earlier examples (3.1 and 3.2). The results will show that OSR results in the fewest transmissions per packet on average. Then, the performance related to maximizing the TX parameter and receiver set is analyzed. In the final simulation, the DDLT algorithm is tested on a dynamic network topology to observe its convergence properties.

4.1 Examining the Transmission Count

In this section, the performance metric examined is the number of per-packet transmissions. This is a good way to compare the algorithms discussed, since the distance metrics of computed by SRCR, ExOR, and OSR try to minimize the number of transmissions when the TX parameter u is kept constant, as in the examples. This performance metric also influences other metrics, such as end-to-end delay and power usage, since fewer transmissions means that packets are leaving the network quicker, and potentially using less power.

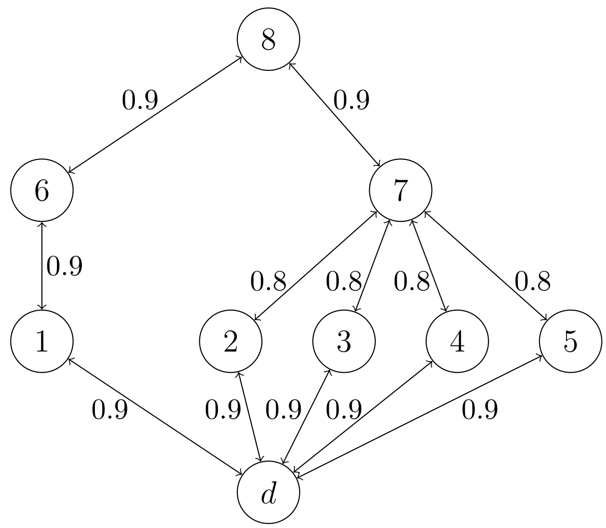

Figure 4.1: A network topology with two paths, one with receiver diversity and one with better link quality.

4.1.1 Diversity vs. Higher Quality Links

This simulation uses the same network used in Example 3.2, shown again here for convenience in Figure 4.1. Packets start at node 8 and are destined for node d. The routing algorithm will either choose the left path with higher link qualities, or the right path with greater receiver diversity. In the example, we found that BF-HC and SRCR would strictly take the path on the left. ExOR would favor the path on the left, sending that way with a probability of 0.9. In the case where the link from node 8 to 7 fails but 8 to 6 succeeds, with probability 0.09, the packet is sent to the path on the right. OSR favors the path on the right, sending that way with probability 0.9 and to the left with probability 0.09. Recall from the example the expected values for the number of transmissions (shown in Table 4.1).

Table 4.1: The expected number of transmissions for a packet starting at node 8 and destined for node d (computed in Example 3.2).

	BF-HC	SRCR	ExOR	OSR
$\mathbb{E}[N_{\text{TX}}]$	3.33	3.33	3.22	3.13

Figure 4.2 shows the number of transmissions as a moving average for each algorithm. The expected values are plotted for each algorithm as well. It's clear

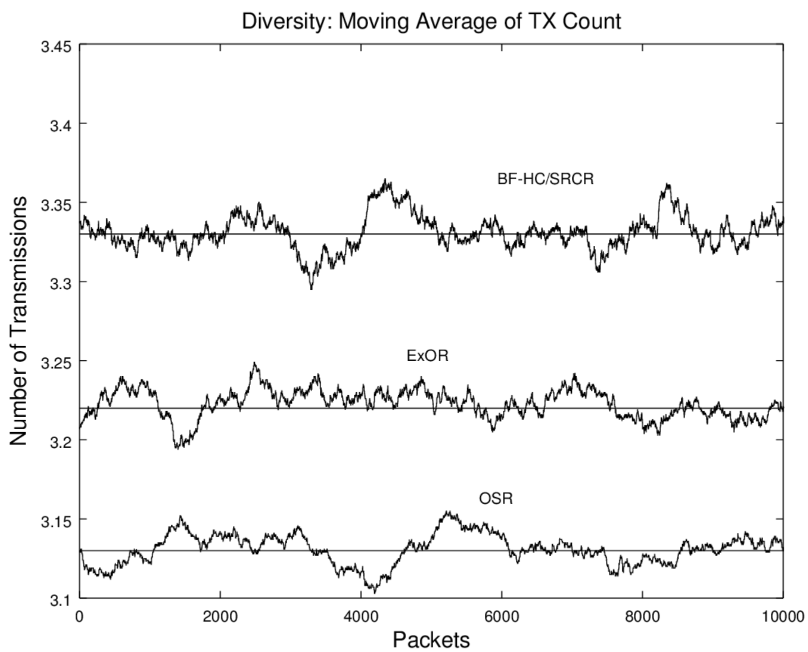

Figure 4.2: The number of transmissions logged in the diversity simulation, shown as moving averages (filter length 1000) to more clearly represent the trends. The expected value of the number of hops is overlaid for each algorithm.

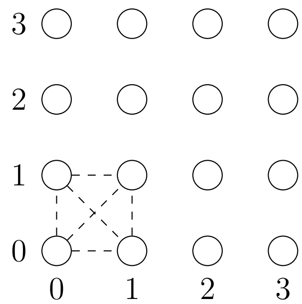

Figure 4.3: The layout of the wireless mesh network. Links to adjacent neighbors have a success probability of 0.9 and links to diagonal neighbors have a success probability of 0.4.

that the actual outcomes line up closely with the expected outcomes. OSR has the best overall performance due to taking advantage of the receiver diversity.

4.1.2 Well-Connected Mesh Network

In the following simulations we examine the average number of transmissions for each algorithm in several different mesh network topologies. These topologies will show the results of both opportunistic routing decisions and receiver diversity. The TX parameter u is chosen to be constant and equal for all nodes, so that their range and link probabilities are equal. The nodes are strongly connected to their adjacent neighbors ($P(\{i,j\}|i,u,\{j\}) = 0.9$), and have weaker links to their diagonal neighbors ($P(\{i,j\}|i,u,\{j\}) = 0.4$). For BF-HC, we choose $\gamma = 0.85$ so that the diagonal links will always be ignored.

In this simulation, the performance of the algorithms is examined on a well-connected mesh network constructed as a 4x4 grid, shown in Figure 4.3. Node $(3,3)$ is the start and node $(0,0)$ is the destination.

Figure 4.4 shows the moving average of the transmission count for each algorithm on the mesh network. Since the expected number of transmissions wasn't computed for this topology, we can use the computed distances in the simulation to estimate the expected number of transmissions. The distance metrics for node $(3,3)$ are listed in the Table 4.2, and the ETX and OSR distance metrics are overlaid on the plot.

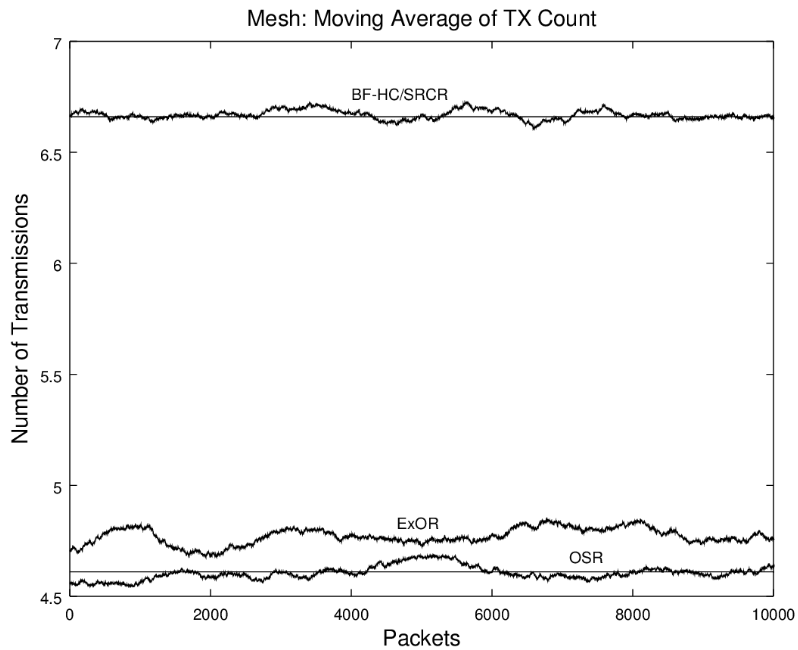

Figure 4.4: The number of transmissions logged in the mesh simulation, shown as moving averages (filter length 1000) to more clearly show the trends. The ETX and OSR weights for node $(3,3)$ are overlaid on the plot.

Table 4.2: The distance metric computed at node $(3,3)$ in the simulation on the mesh network topology.

	BF-HC	SRCR	ExOR	OSR
$D_{\mathfrak{c}}$	6	6.67	6.67	4.61

We can see that BF-HC and SRCR both had about the same performance as estimated by the ETX value. ExOR outperformed the ETX metric, just as we saw in Example 3.2, by taking advantage of wireless opportunities as well as receiver diversity, even though the distance metrics do not fully reflect the advantages. OSR performed very close to the estimated number of transmissions, and outperformed the other three algorithms.

To better understand why OSR has fewer average transmissions per packet than ExOR, which both take advantage of the diagonal links, we can examine the distance metrics computed at each node. Figure 4.5 shows the normalized distance metrics per node so they can be directly compared. Notice first that the hop count computed by BF-HC, and ETX computed by SRCR and ExOR are identical when normalized. This is because the link probabilities of the longer links cause them to be excluded from the shortest path in the ETX computation, leading to an ETX value that is just a scaled version of the hop count metric. The optimal distance metric computed by OSR favors the central nodes over the edge nodes since they have greater receiver diversity. It is this difference that gives OSR the reduced average number of transmissions compared to ExOR.

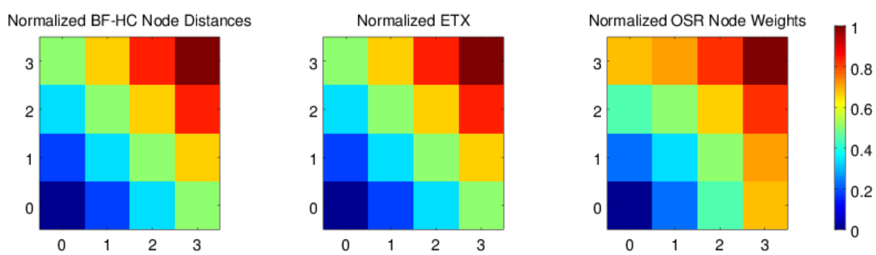

Figure 4.5: The normalized distance metrics for the wireless mesh network. OSR favors the internal nodes to the edge nodes due to the increase in receiver diversity.

4.1.3 Mesh Network With Blockages

For this simulation we constructed a wireless mesh network with some blockages, shown in Figure 4.6. The blockages disconnect some of the links and create two paths on the right side with no receiver diversity or wireless opportunities. For example, nodes $(2, 0), (2, 1)$, and $(2, 2)$ can no longer reach node $(1, 1)$. Once

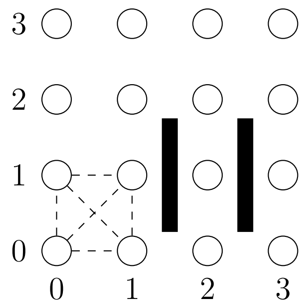

Figure 4.6: The layout of the wireless mesh network with blockages. Links to adjacent neighbors have a success probability of 0.9 and links to diagonal neighbors have a success probability of 0.4. The blockages create some broken links.

again, node $(3,3)$ is the start node and node $(0,0)$ is the destination.

The moving average of the transmission count is shown in Figure 4.7. Table 4.3 shows the computed distance metric at node $(3,3)$, which we can once again use to estimate the number of expected transmissions, and the ETX and OSR distance metrics are overlaid on the plot.

Table 4.3: The distance metric computed at node $(3,3)$ in the simulation on the mesh network with blockages topology.

	BF-HC	SRCR	ExOR	OSR
$D_{\mathfrak{c}}$	6	6.67	6.67	4.90

BF-HC and SRCR both perform around the ETX value, and ExOR outperforms the ETX value again, but by a smaller margin this time. OSR significantly outperforms the other algorithms on this topology. Since the ETX computation doesn't take the longer links into account, SRCR and ExOR evenly weight the left and right side of the grid, while OSR recognizes the increased path diversity and wireless opportunities of the left side of the grid and routes more packets that direction. This relationship is shown in Figure 4.8. Once again, the hop count computed by BF-HC and the ETX computed by SRCR and ExOR match when normalized, and both fail to recognize the increased path diversity and wireless opportunities of the left side of the grid. The distance metric computed by OSR weights the left side of the grid better, resulting in the reduction of the average number of transmissions per packet.

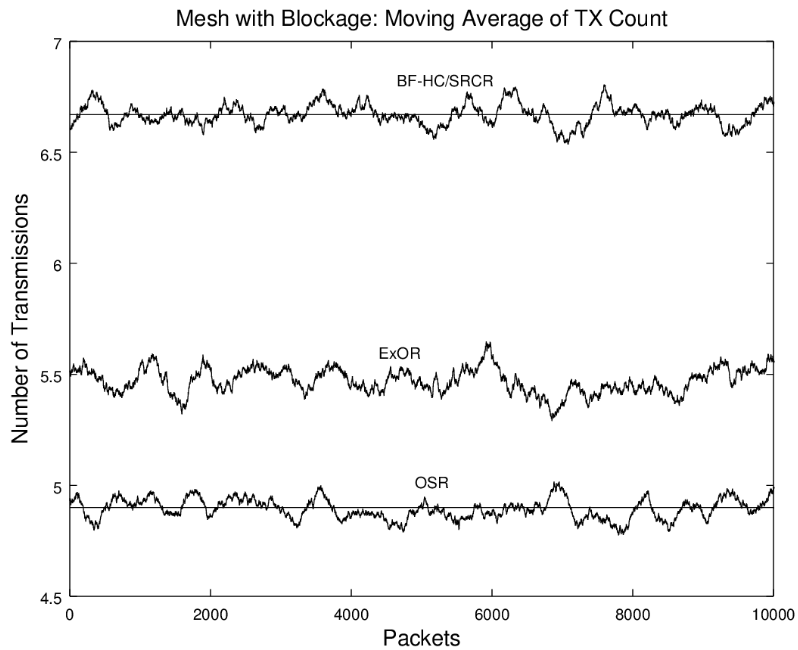

Figure 4.7: The number of transmissions logged in the mesh with blockages simulation, shown as moving averages (filter length 1000) to more clearly show the trends. The ETX and OSR weights for node $(3, 3)$ are overlaid on the plot.

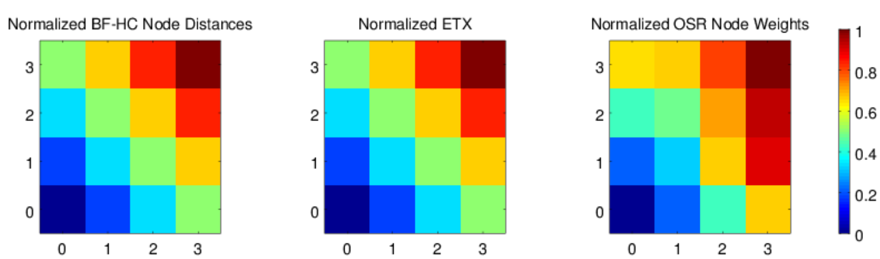

Figure 4.8: The normalized distance metrics for the wireless mesh network with blockages. OSR recognizes the increase in receiver diversity on the left side of the network and favors that side.

4.1.4 Mesh Network With Random Blockages

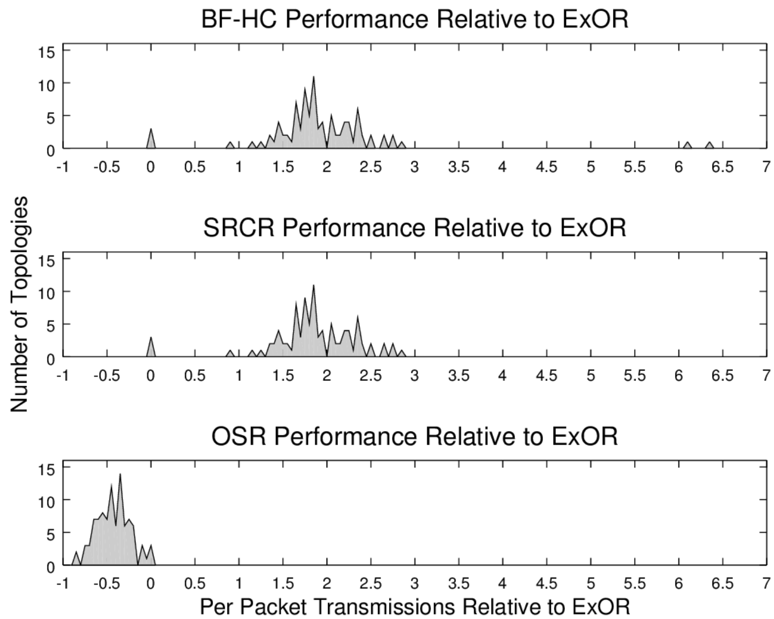

Figure 4.9: The performance of the algorithms relative to ExOR on random topologies.

Now that the transmission count performance of the algorithms has been examined in a few specific mesh network topologies, the question remains: does the advantage of OSR hold over all topologies? In this simulation, this question is tested by generating random mesh topologies based on a 6x6 grid, and measuring the average transmissions per packet across all the networks. Networks which had no path to the destination were thrown out, and ExOR was chosen as the base algorithm for comparison. Figure 4.9 shows the relationship of the average number of transmissions per packet relative to ExOR. For most of the topologies, BF-HC and SRCR had a higher number of average transmissions than ExOR. OSR had fewer average transmissions than ExOR on almost all the topologies, having the same performance on a few and never performing worse.

These simulation results clearly show the benefits of taking both receiver diversity and wireless opportunities into account when computing the distance metric.

4.2 Receiver Set and TX Parameter Optimization

To determine the maximum reward, the algorithms perform an optimization over the TX parameter[1], as well as the receiver set in the case of OSR. In the previous examples the TX parameter was kept constant to simplify the analysis, and the receiver set for ExOR and OSR was chosen to be the entire neighbor set. In the following simulations the TX parameter and the receiver set are made variable and the optimizations are simulated. The TX cost is defined as:

$$C_{TX}(u, R_n) = 1 + \alpha u + \beta R_n$$

where $R_n = |R|$ is the size of the receiver set, and α and β define the increase in cost by moving to a higher TX parameter or making the receiver set larger. By moving to a higher TX parameter, more power could be used in many ways, for instance slower data-rate resulting in longer TX time, or a higher transmit power. By increasing the size of the receiver set, the number of ACKs that must be decoded is increased, resulting in greater power usage. Due to the variable TX parameters, the channel now has different success probabilities, defined by the following function:

$$P(\{i,j\}|i,u,\{j\}) = P(\{i,j\}|i,0,\{j\}) + g(u)(1 - P(\{i,j\}|i,0,\{j\}))$$

where $P(\{i,j\}|i,0,\{j\})$ is the success probability of the channel with the lowest TX parameter, $u = 0$, and matches the probability shown in the previous topology diagrams. The function $g(u)$ determines the decoding improvement associated with moving to a higher TX parameter.

4.2.1 Parameter Optimization in OSR

In the following simulation, the path diversity topology (Figure 4.1) is once again examined, but this time using the new TX cost and channel success probabilities, with $P(\{i,j\}|i,0,\{j\})$ being the probability shown in Figure 4.1. Two

[1] The optimizations over u in the computations of BF-HC (Eq. 3.1) and ETX (Eq. 3.5) did not come directly from their original work, but were added to give the algorithms a consistent structure and allow for this performance comparison.

distinct TX parameters are allowed, $\mathcal{U}(i) = \{0, 1\}$, and the TX cost parameters are set to be $\alpha = \beta = 0.01$. The function $g(u)$ to be used in the simulation is chosen to be $g(u) := \frac{u}{1+u}$. The results of the simulation for OSR are shown in Table 4.4.

Table 4.4: The TX parameter and size of the receiver set for each node.

node	u	$\|R\|$	D_{OSR}
d	0	0	0
1	1	1	1.07
2	1	1	1.07
3	1	1	1.07
4	1	1	1.07
5	1	1	1.07
6	1	1	2.14
7	0	3	2.11
8	0	2	3.14

The results show that all nodes without receiver diversity towards the destination choose to transmit with the higher TX parameter, increasing the success probability of the channel to their single receiver. On the other hand, the nodes with receiver diversity, 7 and 8, choose to expand their receiver set as opposed to increasing the TX parameter. For node 7, a randomized set of three of the four neighbors towards the destination is chosen to maximize the expected reward. After the optimization, the path through node 7 is still a better choice as seen by the distance metrics. This simulation is a good example of the tradeoffs considered by OSR.

4.2.2 TX Cost vs. Number of Transmissions

Following the results of the previous subsection, the same simulation was run with the other three algorithms, each performing the optimizations given in their respective distance metric computations. The average transmission cost per packet and average number of transmissions per packet were logged and plotted in Figure 4.10. The figure shows the results of both the original case ($\mathcal{U}(i) = \{0\}$) and the optimized case ($\mathcal{U}(i) = \{0, 1\}$).

Even with the TX parameter optimization, BF-HC wasn't able to get any

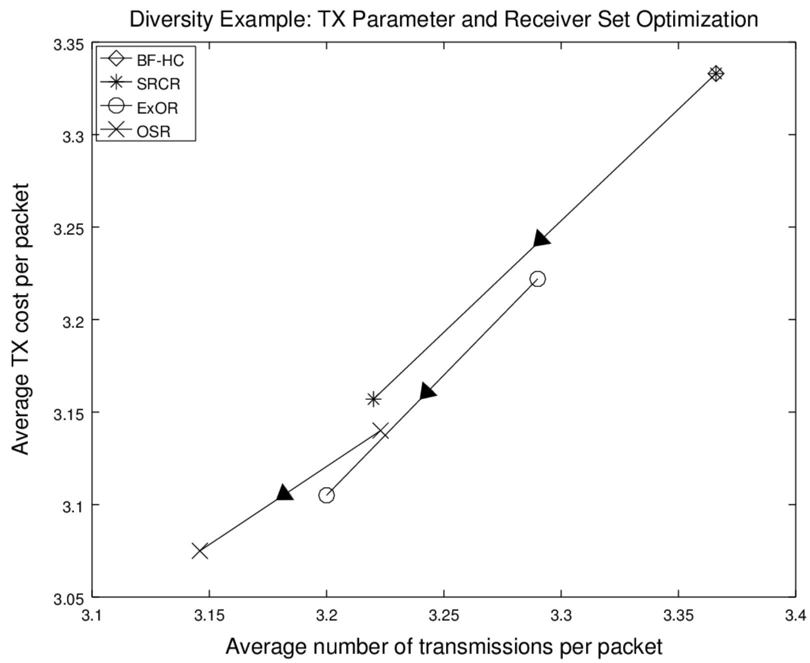

Figure 4.10: A comparison of the algorithms' performance with respect to average TX cost per packet and average number of transmissions per packet.

improvement. This is because even by moving to a higher TX parameter for node 7, BF-HC still chooses the left path as the lower cost path. Both SRCR and ExOR could optimize over TX parameters resulting in a significant reduction in transmission cost per packet, but a less significant impact on the average number of transmissions. OSR, on the other hand, could optimize over both the TX parameter and receiver set, resulting in a similar improvement in both transmission cost per packet and average number of transmissions, and having the best overall performance with regards to both parameters.

4.3 Distributed Distance-Vector Computation

The previous simulations used a static network topology and data was recorded only during steady state for the distance-vector computations. In the next simulation the performance of the Distributed Dijkstra-Lott-Teneketzis (DDLT) Algorithm (3.2) is examined. The weights are transmitted between nodes by adding

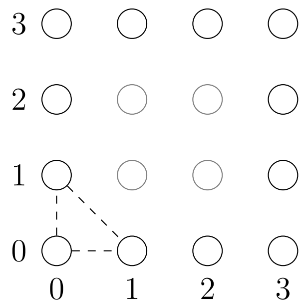

Figure 4.11: Layout of the 4x4 mesh network, where at slot 600 the four center nodes, $(1,1),(1,2),(2,1)$, and $(2,2)$ go to sleep, leaving only the edge nodes as relays.

them to the ACK and data packets as additional payload. The simulation begins with the well-connected mesh network in Figure 4.3, but after 600 slots the four center nodes go to sleep, and therefore no longer relay packets. Figure 4.12 shows the expected number of transmissions over time for both the centralized case and the DDLT case. Also, the actual transmission outcomes from the simulation are overlaid. The settling time of DDLT is clearly related to the amount of traffic in the network, since the weights are only sent to neighbors when a node is sending a packet or responding to an ACK. In this simulation, traffic was very light, with an arrival rate $\lambda = 0.05$ so there would be a clear difference between the centralized and DDLT results. After about 200 slots (about 8 packets), DDLT has settled to the new value. In this simulation, it looks like there are two packets that suffered more transmissions than expected (8) possibly due to the changing weights.

Chapter 4 is adapted from Javidi, T; Van Buhler, E. “Opportunistic Routing in Wireless Networks,” currently being prepared for submission to Foundations and Trends in Networking.

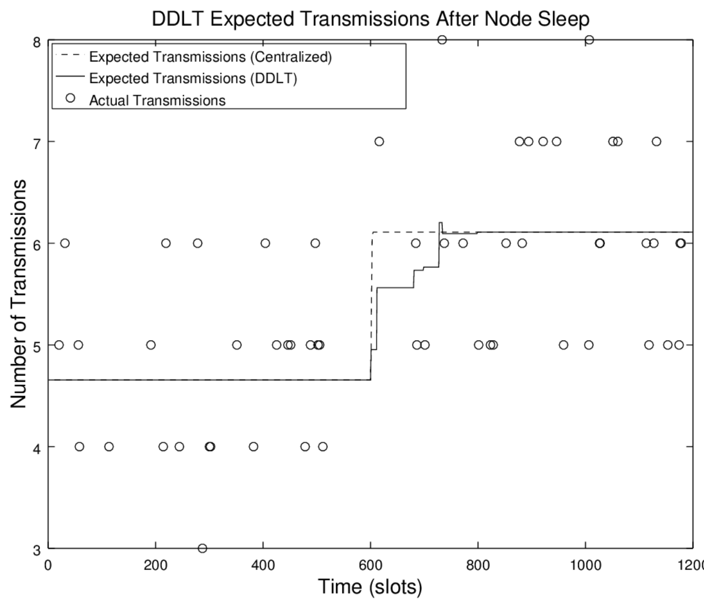

Figure 4.12: The performance of the DDLT algorithm on a dynamic network topology. At time 600, the center nodes of the mesh network go to sleep and no longer relay packets.

Chapter 5

Conclusion

In summary, this Thesis showed, following the work of Lott and Teneketzis [7], that the optimal distance metric for a wireless multi-hop network should take both wireless opportunities and receiver diversity into account when computing distance metrics and choosing the next relay. The steps to compute the optimal distance metric were shown in a centralized manner (DLT Algorithm) and a distributed manner (DDLT Algorithm). A few non-optimal algorithms from the literature were examined and compared to the optimal algorithm in analytical examples. The performance of each algorithm, with respect to average number of transmissions, was tested via simulation over many different topologies. It was shown that OSR results in the fewest number of average transmissions overall. Next, a TX cost function was defined and each algorithms was tested to measure the power performance as it relates to average number of transmissions. It was shown that by optimizing over the TX parameter as well as the receiver set, OSR outperformed the other algorithms in terms of power usage and average number of transmissions. Finally, a simulation was run to test the DDLT algorithm in a dynamic network setting. It was found that the distance computed by the distributed algorithm converged to the centralized solution in finite time when the distance metrics were communicated to neighbors as additional payload in ACK and data packets.

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