

# Lawrence Berkeley National Laboratory

## Recent Work

**Title**

COMPOSITE WEAK BOSONS

**Permalink**

<https://escholarship.org/uc/item/7m01m15m>

**Author**

Suzuki, M.

**Publication Date**

1988-04-01

ed



# Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

## Physics Division

Presented at the Symposium on Perspectives in Particle Physics, Tokyo, Japan, March 29-30, 1988

### Composite Weak Bosons

M. Suzuki

April 1988

RECEIVED  
LAWRENCE  
BERKELEY LABORATORY

JUN 3 1988

LIBRARY AND  
DOCUMENTS SECTION



LBL-25086

ed

## **DISCLAIMER**

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

Composite weak bosons

Mahiko Suzuki

Department of Physics and Lawrence Berkeley Laboratory  
University of California, Berkeley, California 94720

Based on a talk given at the Symposium "Perspectives in  
Particle Physics" at University of Tokyo, March 29-30, 1988.

This work was supported in part by the National Science Foundation under Research Grant  
PHY-85-15857 and in part by the Director, Office of Energy Research, Office of High Energy  
and Nuclear Physics of the U.S. Department of Energy under Research Contract DE-AC03-76SF00098.

## COMPOSITE WEAK BOSONS

Mahiko Suzuki

Department of Physics and Lawrence Berkeley Laboratory  
University of California, Berkeley, California 94720  
U.S.A.

## ABSTRACT

Dynamical mechanism of composite W and Z is studied in a 1/N field theory model with four-fermion interactions in which global weak SU(2) symmetry is broken explicitly by electromagnetic interaction. Issues involved in such a model are discussed in detail. Deviation from gauge coupling due to compositeness and higher order loop corrections are examined to show that this class of models are consistent not only theoretically but also experimentally.

## 1. INTRODUCTION

Gauge bosons have a nice geometrical interpretation for their existence. In the standard picture, weak bosons are gauge bosons which acquire mass by spontaneous symmetry breakdown. Experimentally, the most persuasive evidence for this picture is the relation between the weak boson masses and the weak mixing angle. As was emphasized by Hung and Sakurai<sup>1)</sup> and by Bjorken<sup>2)</sup>, however, such a relation can be naturally realized at a phenomenological level even if W and Z are not gauge bosons. A conclusive test must probe the couplings among W, Z and photon in future experiment. From a theoretical viewpoint, renormalizability of the spontaneously broken theory guarantees approximate validity of tree-diagram predictions except for Higgs bosons masses. Elementary Higgs bosons are, until now, an artifice which mars the otherwise beautiful standard theory. By contrast, if W and Z are composite and much lighter than their constituents, they behave just like gauge bosons and Higgs bosons are not needed. There is a long history in construction of gauge-like bosons as composites of fermion-anti-

fermion. I would like to revisit this problem with focus on the following aspects:

- (a) Relation between gauge symmetry and lightness of composites.
- (b) How general the dynamical mechanism of binding can be.
- (c) Loop corrections to the order of W and Z loops.
- (d) How to include quarks and leptons.

## 2. COMPOSITE GAUGE-LIKE BOSONS

The mechanism to generate nonabelian gauge-like bosons from four-fermion interactions of preons was first demonstrated by Eguchi and Sugawara.<sup>3)</sup> It goes as follows: Start with an SU(n) globally symmetric interaction of N families of preons in the fundamental representation,

$$L(\psi) = \sum_{j=1}^N \bar{\psi}_j (i\not{\partial} - M)\psi_j - \frac{2}{v^2} \sum_{j,k=1}^N (\bar{\psi}_j \gamma_\mu \frac{1}{2} \lambda_a \psi_j) (\bar{\psi}_k \gamma^\mu \frac{1}{2} \lambda_a \psi_k). \quad (2.1)$$

Introduce a set of auxiliary vector fields  $A_a^\mu$  to rewrite  $L(\psi)$  into<sup>4)</sup>

$$L(\psi, A) = \sum_j \bar{\psi}_j (i\not{\partial} - M)\psi_j - \frac{2m_0}{v} \sum_j (\bar{\psi}_j \gamma_\mu \frac{1}{2} \lambda_a \psi_j) A_a^\mu + \frac{1}{2} m_0^2 A_a^\mu A_{a\mu}. \quad (2.2)$$

By computing radiative corrections to the leading order in 1/N for  $L(\psi, A)$  and making wave-function renormalization, one finds

$$L_{\text{eff}} = \sum_j \bar{\psi}_j (i\not{\partial} - M)\psi_j - g \sum_j (\bar{\psi}_j \gamma_\mu \frac{1}{2} \lambda_a \psi_j) A_a^\mu + \frac{1}{2} (m^2 + \delta m^2) A_a^\mu A_{a\mu} - \frac{1}{4} G_a^{\mu\nu} G_{a\mu\nu} - i(\lambda/M^2) f_{abc} G_a^{\mu\nu} G_{b\nu}^{\mu\kappa} G_{c\mu\kappa} + \dots, \quad (2.3)$$

where  $G_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - gf_{abc} A_b^\mu A_c^\nu$  and all couplings are evaluated at zero four-momenta. Note that couplings of dimension higher than four are suppressed by powers of the preon mass M. It is remarkable that not only the three-vector and four-vector couplings are given by the same coupling g, but also is the Yukawa coupling to the preons given by g. Realization of gauge symmetry is in fact not surprising if one notices that  $L(\psi, A)$  possesses SU(n) gauge symmetry up to the mass term of  $A_a^\mu$ .

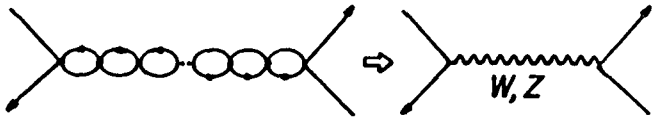
Since the pioneering work of Bjorken<sup>5)</sup> on abelian gauge symmetry, it has been postulated that the mass term  $m^2 + \delta m^2$  can be set equal to zero to turn the vector particles into genuine gauge bosons. This

postulate poses some serious problem.<sup>6)</sup> The bare mass  $m_0$  must not be zero in order for the Lagrangians of (2.1) and (2.2) to be equivalent. On the other hand, the current  $\bar{\psi}_j \gamma_\mu^{1/2} \lambda_a \psi_j$  is conserved, so  $\delta m^2$  must be zero when evaluated at zero four-momentum. Since  $m^2$  is equal to  $m_0^2$  up to the wave function renormalization, it is impossible to require that  $A_a^\mu$  is massless, namely  $m^2 + \delta m^2 = 0$ . Diagrammatically, the procedure from (2.1) to (2.3) is equivalent to summing an infinite series of vector bubbles as shown in Fig.1,

$$\sim (\bar{u} \gamma_\mu^{1/2} \lambda_a u) (\frac{1}{4} v^2 - N \Pi(q^2))^{-1} (\bar{v} \gamma_\mu^{1/2} \lambda_a v), \quad (2.4)$$

where  $(g^{\mu\nu} - q^\mu q^\nu) \Pi(q^2)$  is the vacuum polarization. The mass of  $A_a^\mu$

Fig.1. Formation of vector composites.



is given by  $m_V^2 = (\frac{1}{4} v^2 - N \Pi(0)) / N \Pi'(0)$ , where  $\Pi'(q^2)$  is the first derivative of  $\Pi(q^2)$ . Since  $\Pi(0) = 0$  by global current conservation, one can attain  $m_V^2 = 0$  only by taking the singular limit of  $v = 0$  for a given large finite  $N$ . If one breaks global  $SU(n)$  in (2.3),  $\Pi(0)$  would be no longer equal to zero. But one must make a precise tuning of parameters to achieve  $m_V^2 = 0$ . Besides being artificial, such a tuning involves cancelling of terms of different orders in  $1/N$  in the present parametrization of  $N$ . Therefore, if we insist on that  $v^2$  is  $O(1)$ , the only possibility is to accept  $\Pi(0) = 0$  and give up  $m_V = 0$ . As the effective force  $N/v^2$  of the four-fermion interaction grows stronger, the composite vector boson mass becomes smaller.

This remark on the composite mass is relevant to Higgs boson mass as well. If one introduces an attractive four-fermion force of scalar-scalar type, one can generate spin-0 composites. However, the natural mass scale of such scalar composites is  $O(\Lambda^2)$  where  $\Lambda$  is the cutoff energy of the theory, identified with the scale of the preon mass  $M$ . The scalar force must be fine tuned in order to keep the composite scalar boson mass much lighter than  $M$ . Furthermore, such a fine tuning involves cancellation of terms of different

orders in  $1/N$ . Therefore, we are unable to obtain composite Higgs bosons in a consistent manner, as was emphasized recently by Cohen, Georgi and Simmons.<sup>7)</sup> As we will see later, there is no need for Higgs bosons in composite W and Z models since an explicit global symmetry breaking by electromagnetic interaction generates the correct relation between the weak boson masses and the mixing angle.

### 3. HOW GENERAL IS THE MECHANISM ?

The interaction in (2.1) is carefully set up to guide us to the gauge coupling for the vector composites. The crucial ingredient is conservation of the current  $\partial^\mu (\bar{\psi}_j \gamma_\mu^{1/2} \lambda_a \psi_j) = 0$ . This is in fact much stronger a requirement than just  $SU(n)$  global symmetry. If one replaces the interaction in (2.1) by

$$L_{\text{int}}(\psi) = - \frac{2}{v^2} J_{a\mu}^\mu A_{a\mu}, \quad (3.1)$$

$$J_{a\mu} = \sum_j (\bar{\psi}_j \gamma_\mu^{1/2} \lambda_a \psi_j + \frac{\kappa}{2M} \partial^\nu (\bar{\psi}_j \sigma_{\mu\nu}^{1/2} \lambda_a \psi_j)),$$

this simple dynamical generation of the composite gauge-like bosons fails immediately; the Yukawa coupling with preons, the three-boson coupling and the four-boson coupling are given by  $g(1 + O(\kappa^2 \Lambda^2 / M^2))$ ,  $g(1 + O(\kappa))$ , and  $g^2$ , respectively. The replacement with  $L_{\text{int}}$  of (3.1) does not violate global  $SU(n)$  symmetry. However, once the nonabelian magnetic moment interaction is added, the composite vector boson mass no longer remains light. The origin of this puzzle can be solved when the Lagrangian is written with the auxiliary field  $A_a^\mu$ , as in (2.2):

$$L = \sum_j \bar{\psi}_j (i \not{\partial} - M) \psi_j - (2m_0/v) \sum_j (\bar{\psi}_j \gamma_\mu^{1/2} \lambda_a \psi_j + \frac{\kappa}{2M} \partial^\nu (\bar{\psi}_j \sigma_{\mu\nu}^{1/2} \lambda_a \psi_j)) A_a^\mu + \frac{1}{2} m_0^2 A_a^\mu A_{a\mu}. \quad (3.2)$$

This Lagrangian does not possess local  $SU(n)$  symmetry any longer even after the mass term is removed. The derivative  $\partial^\nu$  in the magnetic interaction destroys the gauge symmetry. A Lagrangian with local  $SU(n)$  symmetry would be obtained from  $\propto (\bar{\psi}_j \sigma_{\mu\nu} D^\mu D^\nu \psi_j)$  with the covariant derivative  $D^\mu$ . Then we can not write any simple Lagrangian like the one in (2.1) in terms of preon fields alone.

In the late 1960's, Kroll, Lee and Zumino<sup>8)</sup> showed in the context

of the field current identity that if massive vector fields  $A_a^\mu$  transform like the adjoint representation of a Lie group and satisfy

$$\partial_\mu A_a^\mu = 0, \quad (3.3)$$

and if the three-body interaction of dimension four obeys global symmetry of the group, the four-body coupling is exactly equal to the one required by local symmetry. According to this theorem, the auxiliary field in (3.2),

$$A_a^\mu = \frac{2}{m_0 v} \sum_j (\bar{\psi}_j \gamma^\mu \frac{1}{2} \lambda_a \psi_j + \frac{\kappa}{2M} \partial^\nu (\bar{\psi}_j \sigma_{\mu\nu} \frac{1}{2} \lambda_a \psi_j)), \quad (3.4)$$

must satisfy (3.3) in order to generate massive gauge bosons. However, the right-hand side of (3.4) is not a conserved current; the Noether current obtained from the Lagrangian (3.1) is

$$J_a^\mu = \sum_j \bar{\psi}_j \gamma^\mu \frac{1}{2} \lambda_a \psi_j + \frac{2\kappa}{v 2M} f_{abc} \sum_{j,k} [\bar{\psi}_j \gamma_\rho \frac{1}{2} \lambda_b \psi_j + \frac{\kappa}{2M} \partial^\nu (\bar{\psi}_j \sigma_{\rho\nu} \frac{1}{2} \lambda_b \psi_j)] (\bar{\psi}_k \sigma_\mu^{\rho} \frac{1}{2} \lambda_c \psi_k) \quad (3.5)$$

Therefore, we do not obtain gauge couplings. We cannot build a composite gauge boson model by replacing  $\bar{\psi} \gamma^\mu \frac{1}{2} \lambda_a \psi$  by the nonminimal current,  $J_a^\mu$  of (3.5); once we introduce  $J_a^\mu J_{a\mu}$  term with  $J_a^\mu$  of (3.5) as an interaction, the Noether current computed from such a Lagrangian would contain six-fermion terms and  $J_a^\mu$  of (3.5) would be no longer a conserved current. We have to keep going to find finally a conserved current which contains infinite-body-fermion terms. Therefore, for any simple calculation to be carried out in such a composite model, the minimal fermionic current is essential.

This embarrassing observation applies to composite vector bosons made of bosonic preons also. For the bosonic preons, the minimal current contains a derivative

$$\phi^\dagger \frac{1}{2} \lambda_a \overleftrightarrow{\partial}_\mu \phi. \quad (3.6)$$

With  $\overleftrightarrow{\partial}_\mu$ , we can not build a simple model Lagrangian of composite gauge-like bosons. The iterative procedure of adding interactions of dimension up to infinity can be worked out in case of SU(2). It gives us a compact nonpolynomial Lagrangian

$$L = \sum_j (|\partial^\mu \phi_j|^2 - M^2 |\phi_j|^2) - \frac{2}{v^2} J_a^\mu J_{\mu a}, \quad (3.7)$$

where

$$J_{\mu a} = i \sum_j \phi_j^\dagger \frac{1}{2} \lambda_a \overleftrightarrow{\partial}_\mu \phi_j / (1 - \sum_j |\phi_j|^2 / v^2). \quad (3.8)$$

Eqs.(3.7) and (3.8) are reminiscent of the  $CP^{N-1}$  model. Here, none of the  $\phi$  fields develop a vacuum expectation value. Unlike the  $CP^{N-1}$  model, our model does not impose a subsidiary condition on  $\sum_j |\phi_j|^2$ .

Does this exercise imply that the dynamically generated gauge interaction is a freak? I do not think so. In the four-fermion model of (2.1), lightness of the vector composites is ensured by global current conservation. On the other hand, in the alternative models like (3.5), there is no reason for vector composites to appear as light particles. Their natural mass scale is the preon mass itself unless one makes a fine tuning of parameters. Though it may sound expedient, we may put forth the conjecture that when vector composites are formed as light particles by dynamical necessity rather than by accident or by fine tuning, such vector particles behave like gauge bosons. Until it is proved in a model in which lightness of vector composites is realized by a mechanism different from the four-fermion model (2.1), the conjecture is not much more than a wishful thinking. In the sense of the Kroll-Lee-Zumino theorem<sup>8)</sup>, we have only to show that an appropriately defined composite wave function  $\Psi_\mu$  obeys the transversality condition

$$\frac{\partial}{\partial X_\mu} \Psi_\mu(X, x_i) = 0 \quad (x_i = \text{relative coordinates}) \quad (3.9)$$

not only on mass shell of composites but also off mass shell. We have to be careful about the ambiguity of the wave functions off shell.

#### 4. COMPOSITE MODEL OF W AND Z WITH EXPLICIT SYMMETRY BREAKING

When we build a concrete model of composite W and Z from our study in the preceding Sections, we must keep in mind the followings:

- (a) To build a simple model, we assume that preons are fermions without derivative interaction. If one chooses bosons as preons, one would have to start with a nonpolynomial effective Lagrangian as was seen in Section 3. Then we would lose calculability of model.

(b) To suppress nongauge couplings of dimension six and higher for vector composites, the preon mass  $M$  must not be smaller than the electroweak scale. When energy approaches  $M$ , nongauge couplings of higher dimensions become important. The larger  $M$  is, the more accurate the dynamical gauge symmetry is for vector composites.

(c) Since the scalar composites of mass much lighter than  $M$  is unnatural or inconsistent with  $1/N$  expansion, the mass splitting between  $W$  and  $Z$  must be realized by explicit breaking. We can introduce an elementary photon and incorporate the  $\gamma$ - $Z$  mixing mechanism proposed by Hlung and Sakurai ten years ago.<sup>1)</sup>

With these reminders, we are ready to write a concrete model.<sup>6)</sup>

It consists of  $N$  fermion doublets as preons and an elementary photon;

$$L = \sum_j \bar{\psi}_j (i\partial - M)\psi_j - \frac{2}{v^2} \sum_{j,k} (\bar{\psi}_j \gamma_\mu \frac{1}{2} \tau_a \psi_j) (\bar{\psi}_k \gamma^\mu \frac{1}{2} \tau_a \psi_k) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - e \sum_j (\bar{\psi}_j \gamma_\mu (\gamma + \frac{1}{2} \tau_3) \psi_j) A^\mu, \quad (4.1)$$

The  $SU(2)$  gauge coupling and the  $W$  mass are obtained by calculating the infinite series of bubbles;

$$g^2 = [N\Pi'(0)]^{-1}, \quad (4.2)$$

$$m_W^2 = \frac{1}{4} v^2 [N\Pi'(0)]^{-1}. \quad (4.3)$$

The neutral component  $W^0$  of the triplet vector composites mixes with photon through a preon loop since electromagnetic interaction is  $SU(2)$  breaking. The mixing is in the form of wave-function mixing

$$\frac{1}{2} (e/g) F_{\mu\nu}(\gamma) F^{\mu\nu}(W^0). \quad (4.4)$$

Since  $g^2$  is  $O(1/N)$ , we regard the electromagnetic coupling squared as also of  $O(1/N)$  for the purpose of power counting of  $1/N$ . Then the weak mixing angle is a well-defined finite number in the large  $N$  limit. The two mass eigenstates of the neutral vector bosons are

$$\begin{aligned} \gamma &= A_\mu, \\ Z &= [W_\mu^0 - (e/g)A_\mu] / [1 - (e/g)^2]^{1/2} \end{aligned} \quad (4.5)$$

with eigenvalues  $m_\gamma = 0$  and

$$m_Z^2 = m_W^2 / [1 - (e/g)^2]. \quad (4.6)$$

The neutral weak current of  $Z$  is given by

$$J_{NC}^\mu = [g/1 - (e/g)^2]^{1/2} [J_3^\mu - (e/g)^2 J_{em}^\mu]. \quad (4.7)$$

By defining the weak mixing angle  $\theta_W$  by  $\sin \theta_W = e/g$ , Eqs.(4.5)-(4.7) are identical with those of the standard electroweak theory. In this model, unlike other dynamical models, there is no need to struggle for making  $\sin \theta_W$  large enough to be compatible with experiment.

## 5. DEVIATION FROM GAUGE COUPLING DUE TO COMPOSITENESS

To the lowest order of electromagnetic coupling  $e$ , all interactions of dimension four are equal to the nonabelian gauge couplings at zero external momenta. Couplings of dimension four defined on the  $W$  and  $Z$  mass shells deviate from the gauge couplings by inverse powers of the preon mass  $M$ . These deviations are effective interactions of dimension six and higher. Such deviations have been computed explicitly for the electromagnetic vertex of  $W$ . The result is as follows:<sup>6)</sup>

$$L_{int} = ief_Y (W_{\mu\nu}^\dagger W^\mu - W_\mu^\dagger W_\nu^\mu) A^\nu + ie\kappa_Y W_\mu^\dagger W_\nu^\mu A^{\mu\nu} + ie\lambda_Y W_{\rho\mu}^\dagger W_\nu^\mu A^{\nu\rho} + ieu_Y^{(1)} (\partial^\lambda W_\mu^\dagger \partial^\mu W_\nu \partial^\nu A_\lambda - \partial^\nu W_\mu^\dagger \partial^\lambda W_\nu \partial^\mu A_\lambda) + ieu_Y^{(2)} [(W_\mu^\dagger \overleftrightarrow{\partial}_\lambda W_\nu) (g^{\mu\nu} \square - \partial^\mu \partial^\nu) A^\lambda + \text{cyclic permutations}], \quad (5.1)$$

where  $f_Y$ ,  $\kappa_Y$ ,  $\lambda_Y$ ,  $\mu_Y^{(1)}$  and  $\mu_Y^{(2)}$  are constants,  $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$  and so forth, and cyclic permutations among  $W_\mu^\dagger$ ,  $W_\nu$  and  $A_\lambda$  are meant in the third line. The constants are given to the first order in  $q_i^2/M^2$  ( $q_i = W$  and  $\gamma$  momenta) by

$$\begin{aligned} f_Y &= 1 + [(1/5)/\ln(\bar{\Lambda}/M)^2] (q_1^2 + q_2^2 + q_3^2)/M^2, \\ \kappa_Y &= 1 + [(1/5)/\ln(\bar{\Lambda}/M)^2] (q_1^2 + q_2^2 + q_3^2)/M^2, \\ \lambda_Y &= -[(9/20)/\ln(\bar{\Lambda}/M)^2] (1/M)^2, \\ \mu_Y^{(1)} &= -[(1/5)/\ln(\bar{\Lambda}/M)^2] (1/M)^2, \\ \mu_Y^{(2)} &= [(1/5)/\ln(\bar{\Lambda}/M)^2] (1/M)^2, \end{aligned} \quad (5.2)$$

where the first terms in  $f_Y$  and  $\kappa_Y$  represent the gauge symmetric terms. The momentum dependent corrections grow with energy to enhance compo-



siteness effect in high energy processes. The sign of the momentum dependent terms in  $f_Y$  and  $\kappa_Y$  are such that it is consistent with form factor damping in the space-like direction. When SU(2) breaking beyond the lowest order  $W^0$ - $\gamma$  mixing is included, the coefficients of  $f_Y \sim \mu_Y^{(2)}$  generally acquire additional correction terms of  $O(e^2)$ .

The characteristics observed in the  $W\bar{W}\gamma$  vertex given above emerge in all other multi-boson vertices. In case of four-point functions, couplings of dimension four receive SU(2) symmetric corrections of  $O(p^2/M^2)$  where  $p^2$  stands for properly symmetrized Lorentz scalar variables made of four external momenta. They are form factor effects. Couplings of interaction operators with dimension  $D (>4)$  are of the order of  $(1/M)^{D-4}$  with SU(2) symmetric coefficients up to the  $W^0$ - $\gamma$  mixing. Symmetry breaking effects are down by a factor of  $O(\alpha/4\pi)$ .

## 6. ABSENCE OF DANGEROUS LOOP CORRECTIONS

The important issue is how large deviation from the standard theory is when one computes loop diagrams of physical W and Z. Since the W and Z propagators obtained from the bubble summation of Fig.1 are those of the unitary gauge, there is a legitimate reason to worry about the potentially dangerous contributions from the longitudinal polarizations of W and Z. On the other hand, the effective Lagrangian of our model is identical to the minimal standard theory with the physical Higgs boson mass let to infinity or set equal to the preon mass M. By this observation, we may feel optimistic to believe that W and Z loop corrections are no worse than those of the standard theory with the Higgs boson mass equal to  $O(M)$ . Demonstrating this expectation is not so trivial because we must deal with quantities in the next leading order in  $1/N$ . We look into this problem in this Section.

The operators which receive potentially the largest correction are those of the lowest dimension, namely the W and Z mass terms. If we compute the difference  $\delta m_Z^2 \cos^2 \theta_W - \delta m_W^2$  from the diagrams of one W, Z or  $\gamma$  loop, we find that the dangerous terms of  $O(1/m_{W,Z}^4)$  and  $O(1/m_{W,Z}^2)$  are cancelled between  $\delta m_W^2$  and  $\delta m_Z^2 \cos^2 \theta_W$  by global SU(2) symmetry and the right mixing relation  $m_Z^2 \cos^2 \theta_W = m_W^2$ . The surviving contribution turns out to be to the lowest order in  $e^2$

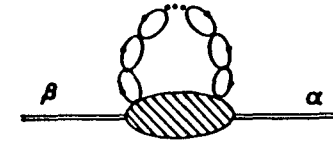
$$\delta m_W^2 - \delta m_Z^2 \cos^2 \theta_W \sim (\alpha/4\pi)(\alpha_g/4\pi) M^2, \quad (6.1)$$

where  $\alpha_g = g^2/4\pi$  and the cutoff is chosen to be equal to M. This correction stays small enough if M is not much larger than  $O(1 \text{ TeV})$ . However, this is not sufficient. The mass shifts themselves must remain small in order for our composite model to be viable. We will see that the potentially dangerous terms proportional to the inverse powers of  $m_{W,Z}$  are cancelled out in  $\delta m_W^2$  and in  $\delta m_Z^2$  separately to the lowest order in electromagnetic coupling and that the explicit symmetry breaking due to electromagnetic interaction generates nonnegligible contributions when the preon mass scale is too high.

In the present parametrization of the four-fermion coupling, which is different from Ref.6 and follows Ref.7, the longitudinal polarization term  $k^\mu k^\nu / m_{W,Z}^2$  of the vector composite propagators is of  $O(N)$ . If this term survives in W and Z diagrams, our model would no longer be consistent with  $1/N$  expansion at higher loop levels and immediately lose its beauty. As we wish, however, such dangerous loop diagram corrections are completely absent thanks to global SU(2) current conservation in the lowest order of electromagnetic interaction beyond the  $W^0$ - $\gamma$  mixing. Once higher order electromagnetic effects are included, the contributions of the longitudinal polarization stay with coefficients of  $O(\alpha/4\pi)$ .

Let us examine a class of diagrams in which a weak boson propagator appears as shown in Fig.2.

Fig.2. Diagrams of physical W or Z loop.



The matrix element can be written as

$$M = i f \langle \beta | T^* (J_\mu^i(x) J_\nu^j(y)) | \alpha \rangle \delta^{ij} \Delta^{\mu\nu}(x-y) d^4x d^4y, \quad (6.2)$$

where

$$\Delta^{\mu\nu}(x-y) = \int \{ (-g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2}) / (i_4 v^2 - N \Pi(k^2)) - \frac{4}{v^2} \cdot \frac{k^\mu k^\nu}{k^2} \} \times \exp(-ik(x-y)) d^4k / (2\pi)^4 \quad (6.3)$$

represents an infinite series of bubbles which includes continuum contributions as well as a W or Z boson pole. The first term of  $\Delta^{\mu\nu}$  is  $g^2(-g^{\mu\nu} + k^\mu k^\nu/k^2)/(m_{W,Z}^2 - k^2)$  near the pole and is of  $g^2 \times O(1)$ . Therefore, it does not generate a harmful contribution in physical W and Z loops. On the other hand, the second term is of  $g^2 \times O(N)$ . If this term stays and makes a full contribution, nice low-energy results of the model would be meaningless once loop corrections are made. The dangerous longitudinal part of  $\Delta^{\mu\nu}$  is separated for W as

$$\Delta_L^{\mu\nu}(x-y) = -(g^2/m_W^2) \partial_x^\mu \partial_y^\nu \int \frac{\exp(-ik(x-y))}{k^2} d^4k/(2\pi)^4. \quad (6.4)$$

Integrating by parts in the matrix element of (6.2), we find the longitudinal contribution as

$$M_L = -i(g^2/m_W^2) \int \langle \beta | T^* (\partial^\mu J_\mu^i(x) \partial^\nu J_\nu^j(y)) - 2ieJ_\mu^3(x) A^\mu(x) \delta(x-y) | \alpha \rangle \times \frac{1}{k^2} \exp(-ik(x-y)) d^4k/(2\pi)^4 d^4x d^4y \quad (6.5)$$

Global current conservation is broken only to  $O(e)$ ,

$$\partial^\mu J_\mu^i = -\epsilon^{ij3} e J_\mu^j A^\mu. \quad (6.6)$$

In the absence of electromagnetic interaction, the matrix element  $M_L$  is completely gone and the correction is under control. If the  $W^0$ - $\gamma$  mixing is included, we obtain for the W and Z mass shifts due to  $\Delta_T^{\mu\nu}$

$$\delta m_W^2 = \delta m_Z^2 \cos^2 \theta_W \sim (\alpha_g/4\pi) M^2. \quad (6.7)$$

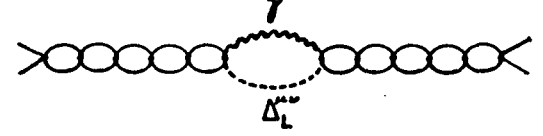
These shifts are of  $O(1/N)$  and do not affect the relation among  $m_W$ ,  $m_Z$  and  $\theta_W$ . The correction to  $m_Z^2 \cos^2 \theta_W = m_W^2$  is similar to that in the standard theory in which the Higgs boson mass is set equal to M.

In the presence of electromagnetic interaction, the matrix element (6.6) acquires a residual contribution of  $O(e^2)$  from the longitudinal polarization. Since  $e^2$  is to be regarded as  $O(1/N)$ ,  $e^2/m_W^2$  is of  $O(1)$ . In the case of W and Z mass shifts, this correction can be evaluated by the infinite series of bubble diagrams shown in Fig.3, where the broken line propagator stands for  $1/k^2$  of  $\Delta_L^{\mu\nu}(k)$ . The result is

$$\begin{aligned} \delta m_W^2 &= (\alpha/2\pi) m_W^2 \ln M^2, \\ \delta m_Z^2 &= 0. \end{aligned} \quad (6.8)$$

This is a small correction.

Fig.3. The contribution to  $\delta m_{W,Z}^2$  in the presence of electromagnetic interaction. See Eq.(6.8).



The argument for suppression of the longitudinal part of  $\Delta^{\mu\nu}$  can be extended to general diagrams where more than one  $\Delta^{\mu\nu}$  are involved. Take diagrams having two of them:

$$M_L = i^2 \int d^4x d^4y d^4z d^4w \langle \beta | T^* (J_\mu^i(x) J_\nu^j(y) J_\kappa^k(z) J_\lambda^l(w)) | \alpha \rangle \times \Delta^{\mu\nu}(x-y) \Delta^{\kappa\lambda}(z-w). \quad (6.9)$$

Separating the longitudinal parts from  $\Delta^{\mu\nu}$  and  $\Delta^{\kappa\lambda}$ , we can show by current algebras that the most singular contribution from  $\Delta_L^{\mu\nu}$  and  $\Delta_L^{\kappa\lambda}$  vanishes when electromagnetic interaction is absent. When we combine this result with the preceding argument in which only one  $\Delta^{\mu\nu}$  was singled out, we can easily show that neither  $\Delta_L^{\mu\nu}$  nor  $\Delta_L^{\kappa\lambda}$  can survive in (6.9) when  $e = 0$ . This argument goes through in diagrams which contain an arbitrary number of  $\Delta^{\mu\nu}$ . Therefore, the  $1/N$  expansion is perfectly consistent for  $e = 0$ . When electromagnetic interaction is present as an explicit breaking of symmetry, the singular terms from  $\Delta_L^{\mu\nu}$  do not go away completely. A factor of

$$(\alpha_g/4\pi) (M/m_W)^2 = (M/2\pi v)^2 \quad (6.10)$$

keeps accumulating for each  $\Delta_L^{\mu\nu}$  in general for  $e \neq 0$ . Therefore, it is necessary to require that

$$M < \sqrt{4\pi/\alpha_g} m_W \quad (6.11)$$

in order for perturbative calculation to be meaningful. This constraint (6.11) suggests that the preon mass M should not be much larger than 1 TeV as long as SU(2) symmetry is broken explicitly by electromagnetic interaction.

Let us turn to another interesting electroweak correction, the anomalous magnetic moment of charged lepton. Since we have not built a model for light fermions, we simply assume that quarks and leptons

interact through V - A and satisfy the weak universality. If they are composites of the same preons as the constituents of W and Z, the weak universality is guaranteed by the gauge coupling of preons. For  $g = 2$ , the relevant matrix element is that of the electromagnetic current which is the very origin of symmetry breaking. The longitudinal part of  $\Delta^{\mu\nu}$  still vanishes by the Ward identity or by an identity

$$\int d^4k/k^2 = 0, \quad (6.12)$$

if we take the lowest order in electromagnetic coupling in the matrix element

$$i\langle \ell | T^*(J_\mu^i(x) J_\nu^j(y) J_\lambda^{\text{em}}(0)) | \ell \rangle \Delta^{\mu\nu}(x-y) d^4x d^4y. \quad (6.13)$$

Therefore, the singular part of the  $\Delta_L^{\mu\nu}$  contribution in the sense of  $1/N$  expansion disappears from loop diagrams. Once higher order electromagnetic effects are included, the situation is the same as in the process  $\alpha \rightarrow \beta$  discussed before. It should be noted that in all cases disappearance of dangerous terms in  $1/N$  occurs for  $e = 0$  by an intricate cancellation between W/Z poles and the continuum part of the infinite series of bubble diagrams.

## 7. WEAK INTERACTION OF QUARKS AND LEPTONS

We have a greater motivation for building composite models of quarks and leptons than W and Z because in the standard theory there are too many parameters associated with Higgs particles whose values can not be determined from a fundamental principle. It is tempting to postulate structure in the light fermions and explain their masses and the quark mixing angles in terms of their underlying structure. Though several attempts have been made in the past, we are still in search of a right direction at present. The issue is how to make composite fermions by orders of magnitude lighter than their constituent particles. Lightness must be based on some dynamical mechanism, not by fine tuning of strength of binding force. The three basic ideas have been so far put forth as promising lines and have been explored.

The first attempt is to introduce a large chiral symmetry at the preon level and to let part of the symmetry survive spontaneous break-

down in order to realize massless composite fermions. Models along this line presume a renormalizable, confining field theory at the preon level. Then the anomaly matching condition of t'Hooft<sup>9)</sup> eliminates a large number of attractive and realistic group theoretical frameworks.<sup>10,11)</sup> It may be a good idea to forget about renormalizability of the preon theory and therefore discard the anomaly matching for a moment. The other line of attempts utilizes supersymmetry. It is possible to generate massless fermions as the Nambu-Goldstone fermions of supersymmetry breaking. Decoupling of the Nambu-Goldstone fermions at zero momenta can be evaded if electroweak and strong gauge interactions break supersymmetry.<sup>12)</sup> To my knowledge, no realistic model has been built successfully along this line. There is another possibility with supersymmetry.<sup>13)</sup> When global symmetry is broken spontaneously in a supersymmetric theory, massless fermions can appear as supersymmetric partners of the Nambu-Goldstone bosons. In some cases where global symmetry and supersymmetry are broken by the same origin, the Nambu-Goldstone fermion mechanism and the super-partner mechanism are intertwined. Explicit computation was done to demonstrate the fermion partner mechanism in the supersymmetric Nambu-Jona-Lasinio model.<sup>14)</sup> However, we have not been able to find a model applicable to the real world. One of the problems in the supersymmetric scenarios is how to keep the light fermion masses as light as several MeV in spite of a large supersymmetry breaking scale indicated by the lower limit on the scalar quark mass  $\geq 100$  MeV. It appears possible to suppress some of the supersymmetric partner masses in the presence of a large supersymmetry breaking.<sup>15)</sup>

When we attempt to introduce composite light fermions in our composite model of W and Z, chirality of the light fermions is a problem. Experiment requires that the weak isospin symmetry is left-handed for quarks and leptons. But it is difficult to build light chiral fermions from the heavy preons. Even if preons interact chirally, there is no guarantee that weak interaction of light composite fermions is also chiral. Only when the preon mass is different from the compositeness scale  $\Lambda_C$  and much smaller than it, can we realize a

chiral weak interaction for quarks and leptons. As we can demonstrate easily in a concrete supersymmetric model of Nambu-Jona-Lasinio type, the V-A interaction of preons generally leads us to only an approximate V-A interaction for composite quarks and leptons with a deviation given by

$$(V/A) - 1 \sim M/\Lambda_c, \quad (7.1)$$

even when all interactions are chiral. It implies that approximate chiral symmetry must be embedded in advance in preon dynamics at the beginning. This observation, among others, makes introduction of our composite W and Z into composite quarks and leptons rather difficult.

Meanwhile, an interesting possibility was recently pointed out by Cohen, Georgi and Simmons.<sup>7</sup> Using the fact that the conjugate of a doublet is an equivalent doublet in SU(2), they give up compositeness of quarks and leptons for a moment and place the light fermions at the same footing as the preons of W and Z.

$$f_L = \begin{pmatrix} \psi \\ i\tau_2 \psi^c \\ q \\ \ell \end{pmatrix}_L \quad (7.2)$$

where  $\psi$  stands for the families of preon doublets and  $\psi^c$  is the charge conjugate. W and Z are made of  $f_L$ . The quarks and leptons inside W and Z do contribute to fast damping of form factors, but such damping can be well hidden when N is large enough. Cohen et al. estimated the magnitude of N from the radiatively generated kinetic energy of W and Z in the effective Lagrangian. They found  $N \approx 60$ . Furthermore, they estimated the dominant contribution to the W mass shift which comes from the wave-function renormalization of W according to their  $1/N$  expansion reasoning. They found that the W mass shift is only 2% even for M as small as  $m_W$  itself. Although their model does not provide a new insight in the long standing problem of light fermion masses and mixing, it circumvents the chirality difficulty of heavy preons and therefore makes the composite W and Z models viable by themselves, leaving the light fermion problem as a separate issue.

## REFERENCES

1. Hung, P.Q. and J.J. Sakurai, Nucl. Phys. 143B, 81, 539(E) (1978).
2. Bjorken, J.D., Phys. Rev. D19, 4257 (1974).
3. Eguchi, T. and H. Sugawara, Phys. Rev. D10, 4257 (1974).
4. Kikkawa, K., Progr. Theor. Phys. 56, 947 (1976).  
Chakrabarti, A. and B. Hu, Phys. Rev. D13, 2247 (1976).  
Eguchi, T. Phys. Rev. D14, 2755 (1976).
5. Bjorken, J.D., Ann. Phys. 24, 174 (1963).
6. Suzuki, M., Phys. Rev. D37, 210 (1988).
7. Cohen, A., H. Georgi and E.H. Simmons, Harvard University Preprint HUTP-88/A004, January 1988 (to be published in Phys. Rev.)
8. Kroll, N.M., T.D. Lee and B. Zumino, Phys. Rev. 157, 1376 (1967).
9. t'Hooft, G., "Recent Developments in Gauge Theories", edited by G. t'Hooft et al. (Plenum Press, N.Y., 1980), p.135.  
Dimopoulos, S., S. Raby and L. Susskind, Nucl. Phys. B173, 208 (1980).
10. Albright, C., Phys. Rev. D24, 1969 (1981).  
Bars, I., Nucl. Phys. B208, 77 (1982).  
Buras, A.J., S. Dawson and A.N. Schellekens, Phys. Rev. D26, 3225 (1982).
11. Peskin, M.E., Proceedings of the 1985 International Symposium on Lepton and Photon Interactions at High Energies, edited by M. Konuma and K. Takahashi (Kyoto Univ., Kyoto, Japan, 1985), p.714.
12. Bardeen, W.A. and V. Visnjic, Nucl. Phys. B194, 422 (1982).
13. Buchmüller, W., S. Love, R.D. Peccei and T. Yanagida, Phys. Lett. 115B, 233 (1982).  
Buchmüller, W., R.D. Peccei and T. Yanagida, Phys. Lett. 124B, 67 (1983).
14. Buchmüller, W. and U. Ellwanger, Nucl. Phys. B245, 237 (1984).
15. Takeshita, S. and M. Yasue, Phys. Rev. D34, 1847 (1986).

*LAWRENCE BERKELEY LABORATORY  
TECHNICAL INFORMATION DEPARTMENT  
UNIVERSITY OF CALIFORNIA  
BERKELEY, CALIFORNIA 94720*