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Scaling Laws in Cognitive Science

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A scaling law (a.k.a. power law) occurs in nature when one observed variable x is a function of another, raised to some power α: $f(x) = ax^{\alpha}$. A well-known example from biology is the *allometric* scaling law relating metabolic rate and body mass across species (Brown & West, 2000). Scaling relations are *scale invariant*, which means that multiplying x by a constant factor c causes only a proportionate scaling of the function itself: $f(cx) = a(cx)^{\alpha}$. This results in a property of *self-similarity* across scale, e.g., a plot of metabolic rate against body mass has the same shape whether the range of species is restricted to small mammals, or covers the entire animal kingdom.

Scaling laws are ubiquitous in physical, chemical, biological, geological, social, and economic systems. However their ubiquity does not mean that scaling laws are trivially mundane, nor does it necessarily mean that all scaling laws emerge from a single, universal principle. A scaling law suggests that some principle or process applies across many orders of magnitude. The particular principle or process may be different from one scaling law to the next. That said, the recurrence of scaling laws across so many different systems has led some researchers to search for unifying principles (e.g. Bak, 1996; West, Brown, & Enquist, 1999); otherwise, one is left wondering why scaling laws are so ubiquitous.

In cognitive science, some of the better-known scaling laws are Weber's law, Steven's law, and power law learning and forgetting curves (Chater & Brown, 2008). The first two laws have played important roles in theories of sensation and perception, the latter two in learning and memory. These laws have been investigated mostly within their respective domains, rather than in the context of other scaling laws in nature (for a notable exception, see Shepard, 2002).

More recently, a number of additional scaling laws have been discovered in studies of language, memory, cognition, and their neural correlates. Evidence is mounting that scaling laws are ubiquitous in cognitive science as in other sciences, which raises the question of why. Are there common principles that apply across different scaling laws, and if so, how do they inform theories of mind and brain?

This symposium brings together seven researchers from a diverse range of disciplines (cognitive science, psychology, computer science, neuroscience) and locations (US, England, Spain, Netherlands) to discuss a range of scaling laws (scaling relations, scale-free networks, power law distributions, long-range correlations a.k.a. 1/f noise, Lévy flights) from a number of empirical sources (episodic recall, language corpora, word naming, electroencephalogram i.e. EEG recordings, category member generation). Accounts of each particular scaling law will be discussed and compared. The overarching aims are to raise awareness of scaling laws in cognitive science, and to discuss their meaning for theories of memory, language, and cognition.

Chris Kello (Moderator) and colleagues have investigated two kinds of scaling laws. One is found in trial-to-trial fluctuations in measurements of repeated behaviors, such as acoustic measures of spoken word repetitions (**Kello**, Anderson, **Holden**, & **Van Orden**, 2008). Measurements are auto-correlated over dozens and even hundreds of trials, and it is now established that their fluctuations follow a pervasive *1/f scaling relation*. Separately, **Kello** and Beltz (in press) analyzed the lexicons of various languages as networks of word nodes whose links are determined by phonemic or orthographic relations. These networks were found to be *scale-free*, in that the number of nodes with N links was a function of N raised to a power. Both findings are interpreted in terms of *criticality* in cognitive and linguistic systems.

Gordon Brown and colleagues have found *scale similarity* over a wide range of time scales for retrospective and prospective memory recall (Maylor, Chater, & **Brown**, 2001). Participants recalled events from the past day, week, or year, and cumulative response probabilities were indistinguishable across time scales. **Brown**, Neath, and Chater (2007) explained these and other results with a *temporal ratio model* of memory. Unlike various models of short-term and long-term memory that assume mechanisms with characteristic time scales, the temporal ratio model holds that a common set of principles governs forgetting and retrieval over the time scales of seconds, minutes, days, and even years.

Ramon Ferrer i Cancho and colleagues have investigated the possibility that *Zipfian scaling laws* in language corpora result from phase transitions between alternate communication modes (the traditional default hypothesis is that Zipfian laws are inevitable and therefore trivial properties of symbol strings; Miller & Chomsky, 1963). Zipf's original law relates the *frequency* F of a word to its *frequency rank* R in a given corpus. **Ferrer i Cancho** and Solé (2003) derived Zipf's law from an information theoretic model that balanced memory needs (speaker's effort) versus disambiguation needs (listener's effort). Zipf's power law distribution was obtained at a critical point in a phase transition. **Ferrer i Cancho**, Solé, & Köhler (2004) have also applied similar analyses and principles to discover and explain scale-free structure in global syntactic dependency networks induced from corpora.

Jay Holden and colleagues have found that distributions of speeded naming latencies to printed words have *power law tails* (**Holden, Van Orden**, & Turvey, in press), and the authors propose that such reaction time distributions can be generally modeled as mixtures of lognormal and power law distributions. Their results are interpreted as evidence for a general principle of *interaction-dominant dynamics*. Cognitive system components must interact to implement cognitive functions. Lognormal-power law mixtures indicate that such interactions are predominantly *multiplicative*, as opposed to additive. The same indication is made by evidence that fluctuations in series of reaction times tend to follow a 1/f scaling relation (**Van Orden, Holden**, & Turvey, 2003).

Klaus Linkenkaer-Hansen and colleagues have found evidence of *long-range correlations* in EEG fluctuations that take the form of a 1/f scaling relation. In an earlier study (**Linkenkaer-Hansen** et al., 2001), intrinsic neural activity was measured while participants remained still but wakeful, and long-range correlations were found in alpha-band (i.e. around 10 Hz) power fluctuations. Such conditions of intrinsic activity are analogous to the repetitive, minimally perturbed conditions used by **Kello, Holden, Van Orden** and colleagues to elicit 1/f fluctuations in behavioral activity. More recently, **Linkenkaer-Hansen** and colleagues have shown *deviations* from scaling relations in EEG fluctuations as well as burst distributions (i.e., *neural avalanches*) measured from Alzheimer's patients (Montez et al., 2009; for analogous behavioral results in attention-deficit disorders, see Gilden & Hancock, 2007). These results indicate that scaling laws are associated with neural and cognitive health.

Theo Rhodes and colleagues have found that generating members of a category (e.g. name all the animals you can think of in twenty minutes) is similar to animal foraging in the wild, in that both activities produce *Lévy distributions* (Rhodes & Turvey, 2007). With respect to animal foraging, *distance intervals* from one foraging event to the next have been found to be power law distributed. Such *Lévy flights or walks* (i.e. a random walk with values sampled from a power law distribution instead of Gaussian distribution) have been shown to be optimal search strategies under sparse resource conditions (Viswanathan et al., 1999). With respect to "memory foraging", **Rhodes** and Turvey found that *time intervals* from one recall event to the next are also power law distributed, suggesting that the ecology of animal foraging shares properties with the ecology of human memory.

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