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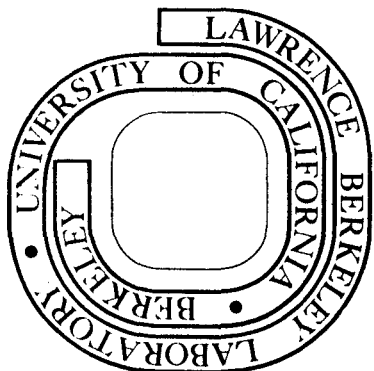
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HIGH-SPIN STATES IN ^{155}Dy AND ^{154}Dy FROM ($^{12}\text{C}, xn, \gamma$) STUDIES [†]

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Abstract: Levels in ^{155}Dy and ^{154}Dy were studied using $^{146}\text{Nd}(^{12}\text{C}, xn, \gamma)$ reactions. Excitation functions between 57 MeV and 109 MeV, γ -angular distributions and γ - γ -time three dimensional coincidence relationships were determined. The most intense feature of the ^{155}Dy spectrum is a cascade of stretched E2 transitions between levels of the strongly mixed positive-parity band arising from the $i_{1/2}^+$, $N = 6$ shell-model state. Several of the strong γ -rays associated with a 6 μs isomer are also observed and placed in this decay scheme with this $1/2^-$ isomeric state of ^{155}Dy at 233.4 keV. A less strongly populated rotational band is based on this $1/2^-$ [505] orbital. The available information on ^{155}Dy from this and previous studies is discussed to arrive at a consistent interpretation. Coriolis mixing calculations are performed for the positive parity band and compared with the experimental data. Additional transitions of the ground rotational band in ^{154}Dy are identified up to the level with spin $14^+(18^+)$. For the spins higher than 14^+ "back-bending" occurs. Transitions from states of a β -vibrational band are confirmed.

NUCLEAR REACTIONS $^{146}\text{Nd}(^{12}\text{C}, 3n\gamma)$, $E = 57-109$ MeV; $^{146}\text{Nd}(^{12}\text{C}, 4n\gamma)$, $E = 57-109$ MeV; measured E_γ , $I_\gamma(\theta)$, $\gamma\gamma$ -coin. Enriched Nd target. ^{155}Dy , ^{154}Dy deduced levels, J, π, K .

1. Introduction

In-beam γ -spectroscopy enables the investigation of high-spin members of the rotational bands of deformed nuclei. For some odd-neutron rare-earth nuclei with neutron number $N = 89$ (i.e., ^{153}Gd and ^{157}Er) a positive-parity band has been identified [refs. 1-3] which is presumably related to Nilsson orbits originating in the $N = 6 i_{1/2}^+$ shell-model state. These rotational bands exhibit a structure which has been interpreted in terms of strong Coriolis coupling.

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The nucleus ^{155}Dy with 89 neutrons represents a missing member of the already studied deformed nuclei with 89 neutrons, ^{153}Gd and ^{157}Er . Levels in ^{155}Dy have been investigated from the radioactive decay ^{4,5}) of ^{155}Ho and by (d, t) reaction studies ⁶). Recently, low-energy γ -rays associated with a 6 μs isomeric state were reported ⁷) but no level assignment was attempted. None of these experiments revealed evidence for the high-spin members of a positive-parity band. We have therefore undertaken this study using the $^{146}\text{Nd}(^{12}\text{C}, xn, \gamma)$ reactions. Preliminary results of our work have been reported earlier ⁸).

TABLE I
Transitions assigned in ^{155}Dy

E_γ (keV)	I_γ (arb. units)	A_2	A_4	Assignment ^{a)}
73.7 \pm 0.4	9 \pm 2			A, $\frac{11}{2}^+ \rightarrow \frac{13}{2}^+$
86.2 \pm 0.4	5 \pm 2			A, $\frac{7}{2}^- \rightarrow \frac{3}{2}^-$
103 \pm 1	\approx 1			B, $\frac{11}{2}^- \rightarrow \frac{13}{2}^+$
109.8 \pm 0.3	2.5 \pm 0.8			B, $\frac{11}{2}^+ \rightarrow \frac{9}{2}^+$
137.9 \pm 0.2	9 \pm 1			A, $\frac{11}{2}^- \rightarrow \frac{9}{2}^+$ ^{b)}
147.2 \pm 0.2	6 \pm 1			A, $\frac{11}{2}^- \rightarrow \frac{7}{2}^-$
162 \pm 1	\approx 2			B, ^{c)}
186.0 \pm 0.2	18 \pm 2			B, ^{d)}
191 \pm 1	6 \pm 2			B, $\frac{15}{2}^+ \rightarrow \frac{17}{2}^+$
202.0 \pm 0.2	26 \pm 3			A, $\frac{13}{2}^- \rightarrow \frac{11}{2}^-$
221.0 \pm 0.3	11 \pm 2			B, $\frac{15}{2}^- \rightarrow \frac{13}{2}^-$
227.3 \pm 0.1	100	0.334 \pm 0.052	-0.129 \pm 0.085	A, $\frac{17}{2}^+ \rightarrow \frac{13}{2}^+$
239.7 \pm 0.4	10 \pm 2			B, $\frac{17}{2}^- \rightarrow \frac{15}{2}^-$
254.2 \pm 0.4	6 \pm 2			B, $\frac{19}{2}^- \rightarrow \frac{17}{2}^-$
268 \pm 1	3 \pm 1.5			B, ($\frac{21}{2}^- \rightarrow \frac{19}{2}^-$)
344.7 \pm 0.2	40 \pm 4			A, $\frac{15}{2}^+ \rightarrow \frac{11}{2}^+$
363.4 \pm 0.1	69 \pm 6	0.253 \pm 0.083	-0.033 \pm 0.093	A, $\frac{21}{2}^+ \rightarrow \frac{17}{2}^+$
380.0 \pm 0.4	4.5 \pm 1.5			B,
423.7 \pm 0.5	6 \pm 2			B, $\frac{15}{2}^- \rightarrow \frac{11}{2}^-$
464.5 \pm 0.2	70 \pm 10	0.270 \pm 0.091	-0.048 \pm 0.125	A, $\frac{25}{2}^+ \rightarrow \frac{21}{2}^+$
495 \pm 2	8 \pm 3			B, $\frac{19}{2}^- \rightarrow \frac{15}{2}^-$
544.7 \pm 0.3	45 \pm 5			A, $\frac{29}{2}^+ \rightarrow \frac{25}{2}^+$
605.3 \pm 0.4	35 \pm 7			A, ($\frac{33}{2}^+ \rightarrow \frac{29}{2}^+$)

^{a)} The assignment of transitions to ^{155}Dy is made with two confidence levels, A or B: A - unequivocal isotopic assignment based on a well-established excitation function, B - assignment with lower confidence.

^{b)} A small admixture of the $2^+ \rightarrow 0^+$ transition of ^{156}Dy may be present.

^{c)} See text. May be depopulating 202.4 keV level of radioactive decay scheme.

^{d)} See text. May be depopulating 224.4 keV level of radioactive decay scheme.

For ^{154}Dy , an even nucleus with 88 neutrons, states of the collective sequence based on the ground state have already been reported through the 10^+ member as well as several members of a β -band. We have attempted a search for even higher states of these sequences.

2. Experimental procedures

For these studies a self-supporting metallic target (10 mg/cm^2) enriched in ^{146}Nd was used. It was bombarded with ^{12}C ions from the Yale Heavy Ion Linear Accelerator. The γ -rays were detected with a 40 cm^3 Ge(Li) detector having a resolution (FWHM) of 2 keV at 660 keV. Spectra taken at various energies between 57 MeV and 109 MeV of the incident ^{12}C beam enabled assignment of the strong γ -rays to either the $^{146}\text{Nd}(^{12}\text{C}, 3n)^{155}\text{Dy}$ or the $^{146}\text{Nd}(^{12}\text{C}, 4n)^{154}\text{Dy}$ reaction. The γ -radiations assigned on this basis to ^{155}Dy and ^{154}Dy are marked in table 1 and table 2

TABLE 2
Transitions assigned in ^{154}Dy

E_γ (keV)	I_γ (arb. units)	A_2	A_2	Assignment ^{a)}
334.9 ± 0.1	100	0.198 ± 0.046	0.045 ± 0.048	A, $2^+ \rightarrow 0^+$
406.8 ± 0.3	10 ± 3			A, $6^+ \rightarrow 4^+$
412.5 ± 0.1	102 ± 10	0.216 ± 0.050	-0.012 ± 0.051	A, $4^+ \rightarrow 2^+$
440.7 ± 0.3	16 ± 6			B,
477.1 ± 0.1	77 ± 8	0.302 ± 0.069	0.050 ± 0.071	A, $6^+ \rightarrow 4^+$
504.8 ± 0.6	10 ± 3			B, $4^+ \rightarrow 4^+$
523.8 ± 0.2	65 ± 6	0.245 ± 0.065	0.080 ± 0.097	A, $8^+ \rightarrow 6^+$
557.6 ± 0.3	42 ± 5	0.340 ± 0.146	0.032 ± 0.095	A, $10^+ \rightarrow 8^+$
579 ± 1	10 ± 4			B,
583 ± 1	11 ± 4			B, $18^+ \rightarrow 16^+$
589.9 ± 0.3	37 ± 5			A, $12^+ \rightarrow 10^+$
597 ± 1	17 ± 5			B, $16^+ \rightarrow 14^+$
616.0 ± 0.4	28 ± 4			A, $14^+ \rightarrow 12^+$
637.5 ± 0.6	10 ± 4			B,

^{a)} The assignment of transitions to ^{154}Dy is made with two confidence levels, A or B: A - unequivocal isotopic assignment based on a well-established excitation function, B - assignment with lower confidence.

with confidence symbol A. The weaker lines marked in these tables with confidence B have only been observed in a high statistics singles spectrum (fig. 1) which was recorded simultaneously with our coincidence measurements (see below). Accurate energy and intensity calibrations were made by recording γ -rays of standard radioactive sources before and during the accumulation of the in-beam spectra.

At ^{12}C beam energies of 67 MeV and 87 MeV angular distributions of the γ -radiations relative to the direction of the incident beam were measured. The areas of the Dy K X-rays were used for normalization purposes. Values of A_2/A_0 for the stronger γ -lines were obtained by least square fits of the observed angular dependences to the

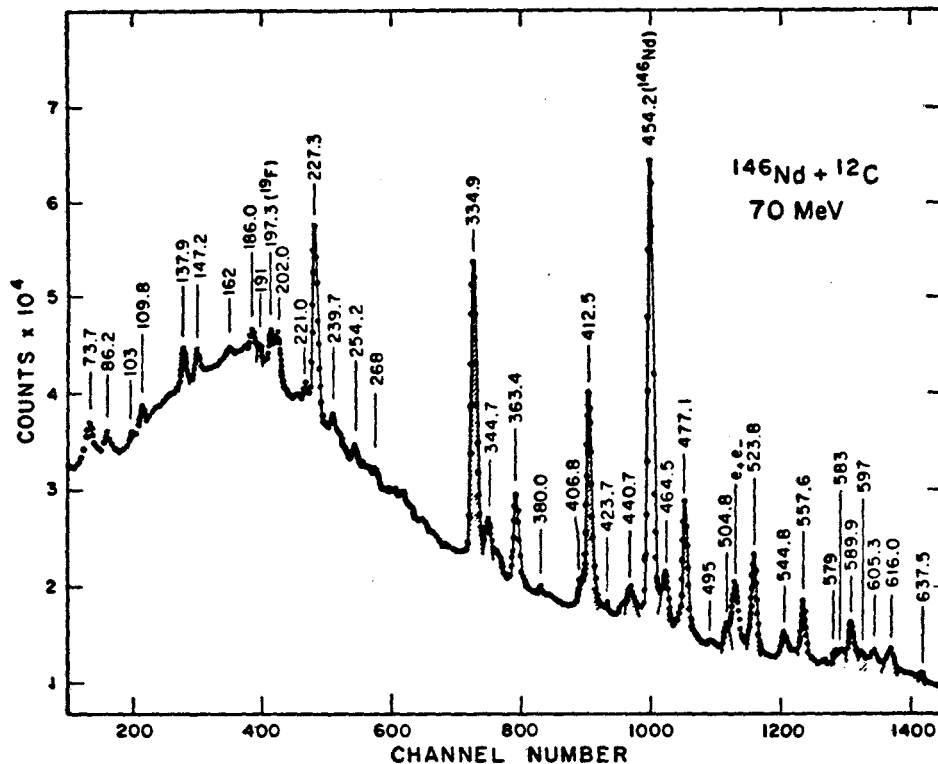


Fig. 1. Gamma-ray singles spectrum from 70 MeV ^{12}C ions on enriched ^{146}Nd target. Shaded lines are assigned to ^{154}Dy . Other lines are in ^{155}Dy except where noted.

equation

$$I(\theta) = \sum_{n=0,2,4} A_n P_n(\cos \theta)$$

and are included in tables 1 and 2.

At a beam energy of 70 MeV a ($\gamma\gamma$) coincidence experiment was performed. Two Ge(Li) detectors positioned at 180° to each other and 90° relative to the beam direction were used. Standard fast-slow electronics was employed resulting in a time resolution of about 30 nsec. All coincidence events were recorded serially on a magnetic tape. After completion of the experiment coincidence spectra were obtained by sorting the stored events with appropriate energy and time gates. The coincidence relationships are summarized in table 3. Searches for isomeric states in the nanosecond range as well as in the millisecond range were unsuccessful.

3. The level scheme of ^{155}Dy

The proposed level scheme for ^{155}Dy , shown in fig. 2, is based on all information now available for this nucleus. Borggreen and Sletten ⁷⁾ have reported five strong

γ -transitions in ^{155}Dy associated with a $6\ \mu\text{s}$ isomer. We also observe the five strong transitions. Guided by similarities in the level schemes proposed ^{1,2)} for the neighboring isotope ^{153}Gd , we associate this isomer with an $\frac{11}{2}^-$ state at 233.4 keV. We place all delayed γ -lines observed by Borggreen and Sletten in the decay scheme except

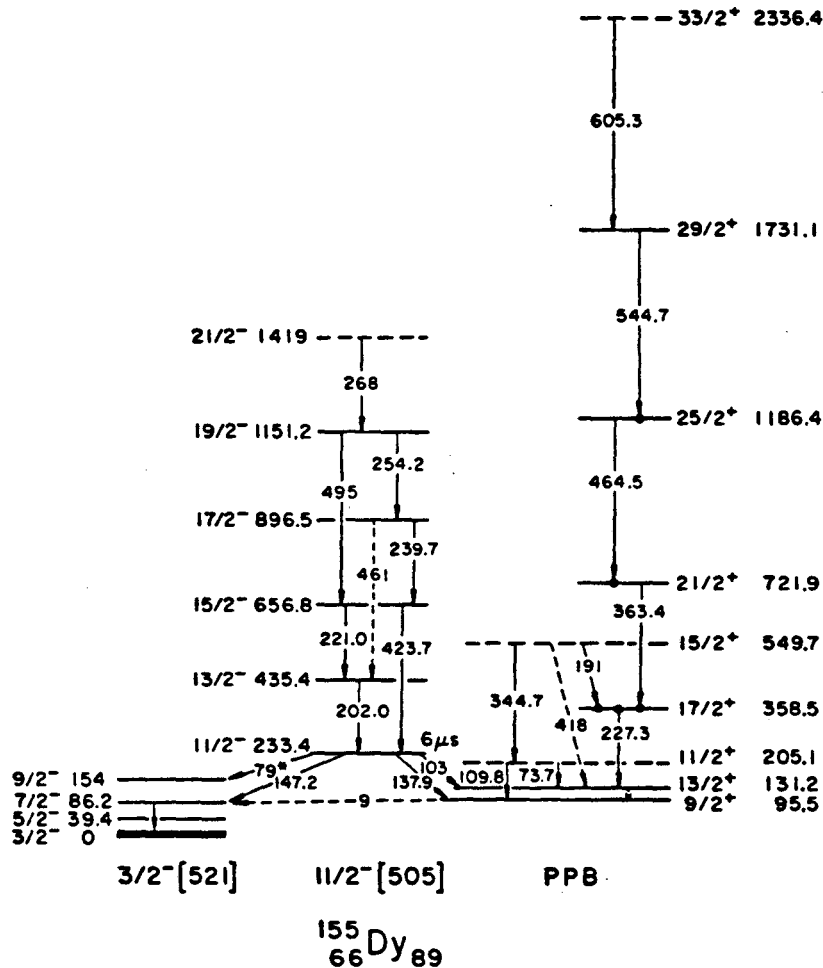


Fig. 2. Proposed ^{155}Dy scheme of levels populated by the in-beam gamma experiments. Nilsson band assignments $\Omega\pi(Nn, A)$ are indicated for the two negative-parity bands. The positive-parity band (labeled PPB) is believed to be so strongly mixed by the Coriolis interaction that labeling by a single Nilsson band is inappropriate. Note that the in-beam work emphasizes higher spin levels.

the 186 keV transition which was reported to have a different half-life. Our assignment is in agreement with the observed levels in ^{155}Dy in the (d, p) reaction ⁶⁾ and in the decay ⁵⁾ of ^{155}Ho . The measured multipolarities, particularly the E2 character of the 147.2 keV and the M2(E1) character of the 137.9 keV transition, are in agreement

with our spin-parity assignments. The hindrance of the E1 component of the 137.9 keV transition can be understood in terms of a K -forbiddenness, as the $\frac{3}{2}^+$ level is composed primarily of the lower K -values, $K = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$. (See later discussion on Coriolis band mixing.)

TABLE 3
Coincidences in $^{134}, ^{133}\text{Dy}$ (the energies are rounded off to 1 keV)

Gate (keV)	Coincident γ -ray (keV)
^{133}Dy	
227	86, 191, 363, 464
363	86, 227, 464
464	227, 363
227+363+464 *)	86, 191, 227, 363, 464, 545, (605)
^{134}Dy	
335	412, 477, 524, 558
412	335, 477, 524, 558
477	335, 412, 524, 558
524	335, 412, 477, 558
558	335, 412, 477, 524, 590, (616)
$\Sigma 335-558$ *)	335, 412, 477, 524, 558, 583, 590, 598, 616

*) In order to detect coincidences with weaker high-lying lines, the coincidence spectra are summed for the whole already established band.

According to the coincidence relationships (table 3) the 227.3 keV, 363.4 keV, 464.5 keV, 544.8 keV and probably the 605.6 keV transitions form a cascade. The observed coincidences between the 227.3 keV and the 86.2 keV gamma lines exclude the possibility that this cascade feeds the $\frac{1}{2}^-$ isomeric state. Because of its large intensity this series cannot feed odd-parity levels above the 86.2 keV level. The energy spacing suggests a stretched quadrupole character for this cascade (i.e., $I \rightarrow I-2$) in agreement with the angular distributions. By analogy this suggests that these transitions occur between levels of a positive-parity band. We therefore assign the cascade to de-excite levels with spins and parities $\frac{3}{2}^+, \frac{2}{2}^+, \frac{2}{2}^+, \frac{2}{2}^+$ and $\frac{1}{2}^+$. The $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$ transition of 35 keV is strongly converted and was not observed due to its low energy. In this scheme the de-excitation of the $\frac{3}{2}^+$ level occurs mainly through a 9 keV E1(M2) unobserved transition to the 86.2 keV $\frac{3}{2}^-$ level, thus accounting for the observed coincidences of the 227.3 keV and 86.2 keV γ -rays. The main de-excitation of the 86.2 keV level, strongly populated by the 9 keV transition, proceeds through 47 keV and 39 keV intraband transitions. These are expected to be highly converted and also masked by the X-ray lines. Based on the observed coincidences between the 227.3 keV and the 191 keV transitions and on the energy fit, two other levels of the positive parity band with spins $\frac{1}{2}^+$ and $\frac{1}{2}^+$ are proposed. The 418 keV transition from $\frac{1}{2}^+$ to $\frac{1}{2}^+$ levels may be present, but it is masked by the tail of the 412.5 keV transition

in ^{154}Dy (cf. fig. 1) and we can give only the upper limit of its intensity, 10 % of the 344.7 keV $\frac{1}{2}^+$ to $\frac{1}{2}^{1+}$ line intensity, or 4 in units of table 1. A similar effect has been observed in another 89-neutron nucleus, ^{153}Gd , where the $\frac{1}{2}^+$ to $\frac{1}{2}^{1+}$ transition was found ²⁾ to be weak compared to the $\frac{1}{2}^+$ to $\frac{1}{2}^{1+}$ line intensity.

Most of the remaining weaker lines have been assigned according to their energy and intensity relationships within the $\frac{1}{2}^-$ band, depopulating states of this band with spins up to $\frac{7}{2}^-$. In these assignments we have been guided by analogy with the ^{153}Gd level spectrum. Expected crossover transitions at 461 keV and 522 keV are masked by the strong γ -lines of ^{154}Dy (cf. fig. 1).

The ^{155}Dy level scheme from this study is shown in fig. 2. In its main features it is in agreement with the band assignment independently proposed in ref. ⁹⁾ although the placement of band heads is different.

There is not much overlap between the levels populated in the beta decay of ^{155}Ho and those populated by our in-beam studies, as can be seen from fig. 3, taken from

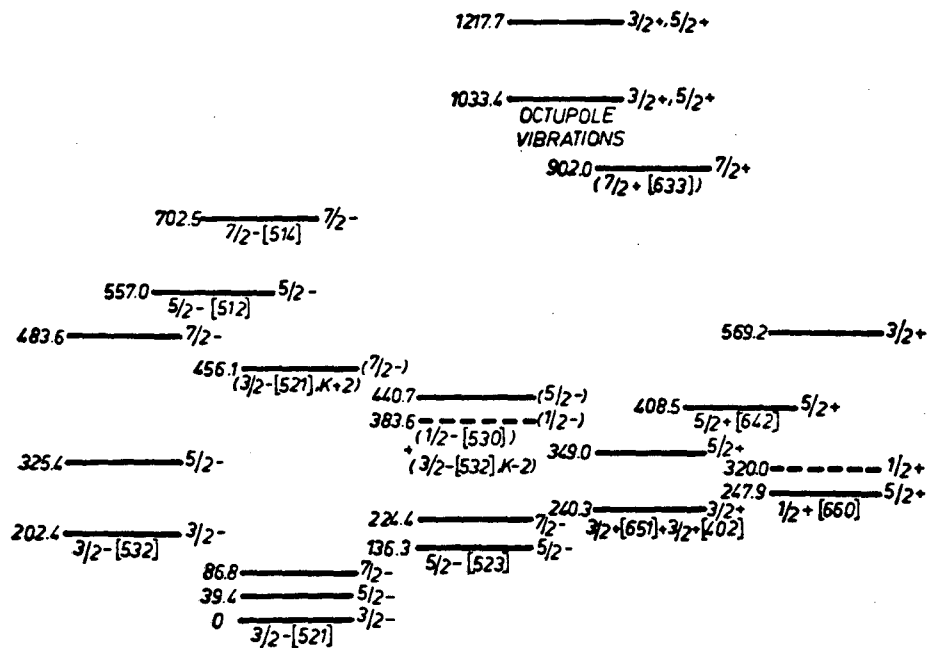


Fig. 3. Level scheme of ^{155}Dy derived from ^{155}Ho decay studies by Torres *et al.* ⁵⁾. Radioactive decay selectively populates lower spin levels.

the work of Torres *et al.* ⁵⁾. Only the ground band levels $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}^-$ are in common between their independently derived level scheme and ours. This lack of commonality is to be attributed to the tendency of the beta decay to populate lower spins and the in-beam excitation to populate high spins near the yrast limit.

Consider now the γ -rays of table 1 to see which other levels of the Torres level

0 0 0 0 4 7 0 5 5 5 3

scheme may be populated weakly by our in-beam gamma experiment. Torres *et al.* ⁵⁾ assign a level at 136.3 keV of spin $\frac{3}{2}$ or $\frac{5}{2}$.⁻ The decay is predominantly by a gamma to ground with $\approx \frac{1}{2}$ intensity going by a 96.9 keV gamma. The level could be weakly populated in-beam, but a 136.3 keV gamma would be unresolved from the 137.9. Comparing our relative intensities with Borggreen and Sletten's ⁷⁾ for depopulation of the $\frac{1}{2}^{1-}$ isomer, we seem to have about 10% extra intensity in the 138 relative to the 147. The presence of some 136.3 in-beam would also help explain the difference in energies determined [by Borggreen and Sletten (139 keV) and us (137.9)]. Regarding intensity comparisons with Borggreen and Sletten, our 103 keV transition is too weak relative to the 147, but our 103 is situated in a high background part of the spectrum, and our intensity determination is thus less certain than theirs.

Is the 202.4 keV $\frac{3}{2}^{-}$ level of Torres *et al.* ⁵⁾ populated in-beam? Their main depopulating gammas, of nearly equal intensity, are 202.4 and 163. In table 1 we list an unassigned 162 keV γ -ray of intensity 2. The 202.4 from the level would be masked by our stronger 202.0 cascade in the $\frac{1}{2}^{1-}$ band. Thus, this 202.4 level is probably weakly populated in-beam, and we have so assigned the 162 transition in table 1.

What about in-beam population of Torres' 224.4 keV level of spin $\frac{3}{2}$, $\frac{5}{2}$?⁻ By their scheme it decays by 185.0 keV and 137.6 keV photons with intensity ratio 22 : 5.0. In table 1 we list a fairly strong unassigned γ -ray of 186.0 keV. This could be assigned to the 224.4 keV level, but if so, nearly half of our 137.9 keV photon intensity must also be assigned to depopulating the 224.4, leaving less intensity for the $\frac{1}{2}^{1-}$ isomer. Thus, it is not reasonable to assign our 186.0 keV transition to their 224.4 keV level. We note that Borggreen and Sletten observed a longer-lived isomeric γ -transition of 186 keV and it may be that our gamma is also the same isomer, unplaced in the level scheme.

Consider Torres' 240.3 keV level with spin $\frac{3}{2}^{+}$. More than 90% of the depopulation is by a gamma to ground. This gamma would be unresolved from the 239.7, assigned to a cascade transition in the $\frac{1}{2}^{1-}$ band. Thus, it is possible that the 240.3 keV level could be weakly populated in-beam.

Consider Torres' 247.9 keV level with spin $\frac{3}{2}^{+}$. We do not observe either of the principal depopulating γ -rays of 247.9 and 208.5 keV. Neither do we see radiation from his 325.4, 349.0, or 408.5 keV levels.

4. Discussion of the nucleus ¹⁵⁵Dy

In the nucleus ¹⁵⁵Dy, the two well-developed rotational bands are quite different. The negative-parity band interpreted as an $\frac{1}{2}^{-}$ [505] Nilsson orbital is very regular and can be well understood in terms of Bohr and Mottelson's scheme between the rotational and single-particle motions. This regular behavior occurs in spite of the fact that the nucleus ¹⁵⁵Dy with 89 neutrons is in the transitional region between the deformed and spherical nuclei.

On the other hand, the level order in the positive-parity band, the lowest member

of which has been assigned in this study as a $\frac{3}{2}^+$ level, is of a different character resembling in its γ -decay pattern a spectrum of a doubly even nucleus. For nuclei in this mass region, such positive-parity levels originate in the $i_{1/2}$ shell-model state. The Coriolis interaction among Nilsson states of this shell-model origin is usually so strong that the coupling scheme between particle motion and the deformed field is greatly altered. Although such deviations from the Bohr-Mottelson rotational strong coupling have been observed for a number of unique-parity bands, the 89-neutron systems seem to be the cases where this breakdown of the strong coupling between the single-particle and rotational motions is most prominent.

Turning to more empirical considerations, a systematics of these positive-parity bands in 89-neutron nuclei can be made for the three nuclei ^{153}Gd , ^{155}Dy , and ^{157}Er for which experimental data are now available. From this systematics, shown in fig. 4,

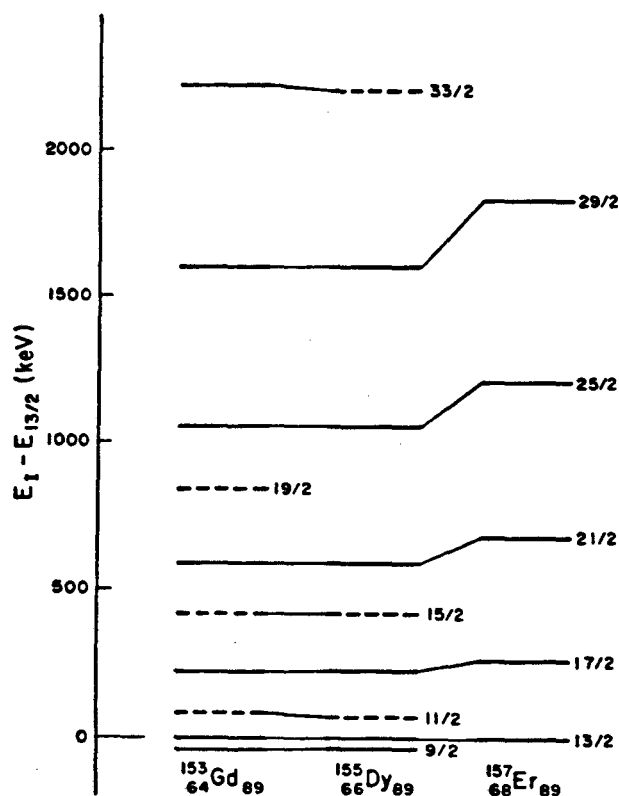


Fig. 4. Comparison of the higher spin members of positive-parity bands of 89-neutron isotones. Note the close correspondence of ^{153}Gd and ^{155}Dy , with larger spacing for ^{157}Er .

the similarity of these bands is quite apparent. There is a trend of the level spacings to increase with increasing atomic number. Such a systematics may be helpful for extending the investigation of 89-neutron nuclei to higher atomic number, more neutron-

deficient species where reliable information about the lowest nuclear levels from earlier studies is scarcer.

The positive-parity band is of the class that is very strongly perturbed by Coriolis band mixing. The band spacing in ^{155}Dy is nearly identical to that in ^{153}Dy . Løv-høiden, Hjorth, Ryde and Harms-Ringdahl²⁾ have performed Coriolis band fitting calculations for ^{153}Gd . These calculations should be quite applicable to analogous ^{155}Dy . They show two theoretical calculations, case A and case B. Case A is a normal calculation in the BCS framework. Case B, the better fit, involves letting the moments of inertia of the $K = \frac{1}{2}$ band and other bands vary separately, the best fit being found with rotational constant A_K for $K = \frac{1}{2}$ being 12.60 keV and for all other K -bands, $A_K = 28.74$. We are interested in using the theoretical predictions to ascertain the nature of the low-spin positive-parity levels of Torres *et al.* Theoretical case B predicts the lowest $\frac{3}{2}^+$ level to be close to the $\frac{1}{2}^+$ level. The experimental $\frac{3}{2}^+$ level at 247.9 keV is not far from the $\frac{1}{2}^+$ at 205.1.

The position of the $\frac{3}{2}^+$ level seems quite sensitive to the differences between cases A and B, but in the better case B it is predicted to lie close to the $\frac{1}{2}^+$ level. The higher $\frac{3}{2}^+$ level of Torres at 569.2 keV may fill this role, the $\frac{1}{2}^+$ being at 549.7 keV. The other positive-parity levels of Torres do not seem to correspond to the Stockholm theoretical $i_{1/2}$ band mixing calculation.

Indeed, the (d, t) reaction work of Grottdal, Nybø, and Elbek⁶⁾ constitutes strong evidence that the 240 keV $\frac{3}{2}^+$ level of Torres *et al.*⁵⁾ is of quite different character from the family of levels based on the odd $i_{1/2}$ neutron. The (d, t) reaction tends only weakly to excite such high- j orbitals as $i_{1/2}$. What strength there is will go mainly to the lowest of the spin- $\frac{1}{2}$ levels, since Coriolis band mixing concentrates the strength there. It may be that our $\frac{1}{2}^+$ level at 131.2 keV and the $\frac{3}{2}^-$ level at 136.3 keV of Torres *et al.*⁵⁾ are populated by the (d, t) and reported⁶⁾ as an unresolved line of 134 keV. However, the 240 keV line, the strongest in the (d, t) spectrum, is probably to be associated with the $d_{3/2}$ orbital of $\frac{3}{2}^+$ [402] or $\frac{1}{2}^+$ [400] from the shell below 82 neutrons¹⁾. We would agree with Grottdal *et al.*⁶⁾ that the 320 keV level strongly populated by (d, t) but, not seen either by in-beam gamma nor radioactive decay, could have predominant character of the $\frac{3}{2}^+$ level of the $\frac{1}{2}^+$ [400] band. It seems somewhat surprising that these $N = 4$ orbital states should lie as low as they do. Theoretical considerations of equilibrium deformation would suggest that hole states involving the 4th shell orbitals here could be somewhat more deformed than other states. There may, of course, be $\Delta N = 2$ mixing, so that a mixed $N = 4, 6$ description is necessary for positive-parity states. The level diagram of fig. 2b of Nilsson *et al.*¹¹⁾ indicates strongest $\Delta N = 2$ mixing near deformation values of 0.27 and 0.29. However, calculations with a deformed Woods-Saxon well by Ford, Hoffman, and Rost¹²⁾ show a somewhat

¹⁾ J. P. Torres has kindly read our manuscript and commented on this point. He and his colleagues believe that beta-decay intensities argue against there being a large component of $\frac{3}{2}^+$ [402] in the 240.3 keV level. They prefer an assignment of $\frac{3}{2}^+$ [402] to ^{155}Ho , and beta decay would go faster than observed to the 240.3 keV level if it were mainly $\frac{3}{2}^+$ [402]. Further studies are needed to unravel the character of the several low-spin positive-parity levels.

larger range ($0.27 \leq \beta \leq 0.4$) of deformation over which $\Delta N = 2$ mixing might occur. From lifetime measurements in the ground rotational band Kilcher *et al.*¹³⁾ derive a deformation $\epsilon = 0.2$, though the positive-parity levels need not have exactly the same deformation.

We have concurred with Grotdal *et al.*⁶⁾ on their assignments for their 239 keV and 320 keV levels. In light of all the additional subsequent experimental information, one should re-examine their other level assignments in their table 1. We concur in the assignment of the ground level as $\frac{3}{2}^-$ [521], mainly on the basis of supporting work of refs. ^{5, 12)}. The weak (d, t) group to a "201" keV state is consistent with the Torres *et al.*⁵⁾ assignments of $\frac{3}{2}^-$ levels, since there is more $p_{3/2}$ character in the $\frac{3}{2}^-$ [521] orbital than in the $\frac{5}{2}^-$ [532].

The analysis of the (d, t) reaction signature to odd-parity levels with spin greater than $\frac{3}{2}$ will evidently be complicated by the extensive band-mixing of $\frac{3}{2}^-$ [521], $\frac{5}{2}^-$ [532], and $\frac{7}{2}^-$ [523] levels, as calculated by Torres *et al.*⁵⁾. Such a signature analysis will need to explain the lack of (d, t) population of the $\frac{5}{2}^-$ level of the ground band at 39.4 keV, and the strong population of $\frac{7}{2}^-$ at 86.2 keV (a large $f_{7/2}$ character evidently). The (d, t) line at 153 keV we would reassign to the $\frac{9}{2}^-$ level of the ground band, substantial $h_{7/2}$ character being likely. The (d, t) line at 223 keV goes with the 224.4 keV $\frac{7}{2}^-$ level of ref. ⁵⁾. From the radioactivity studies of ref. ⁵⁾ there are suitable candidates for assignment to the higher (d, t) levels within an uncertainty of 4 keV, but none of these levels appear in our in-beam gamma work, since they lie well above the yrast region. Of our $K = \frac{1}{2}^-$ band levels, one might expect weak population of the $I = \frac{1}{2}^-$ level at 233.4 keV. Such a line would be unresolved from the line to neighboring 239 and 223 keV levels; hence such (d, t) population could be present.

With the preceding analyses as a guide to positive-parity level character, we have attempted Coriolis band-fitting including all our positive-parity levels plus one or two low-spin levels from the radioactive decay work. As an alternative to many-band mixing theory we have lately been exploring the analytical energy formulas from two-band mixing. With Coriolis mixing of two bands the energies are the solution of 2×2 matrices, expressible as simple roots of quadratic equations. The matrix elements are as usual except that we introduce a parameter β to take into account the intrinsic change of effective moment of inertia for high angular momentum. Thus, letting

$$A(I, K^2) = \alpha \{1 + \beta [I(I+1) - K^2]\}^{-1}, \quad (1)$$

$$H_{K,K} = E_K + A(I, K^2) [I(I+1) - K^2] \\ + (-)^{I+K} \frac{(I+K)!}{(I-K)!} \alpha_{2K} \{1 + \beta [I(I+1) - K^2]\}^{-2K}, \quad (2a)$$

$$H_{K+1,K+1} = E_{K+1} + A(I, (K+1)^2) [I(I+1) - (K+1)^2], \quad (2b)$$

$$H_{K,K+1} = -A(I, K(K+1)) [I(I+1) - K(K+1)]^{\frac{1}{2}} \langle K | j_- | K+1 \rangle. \quad (2c)$$

Using the least squares minimization routine BKY LSQVMT at the LBL Computer

0 0 0 0 4 7 0 5 5 5 7

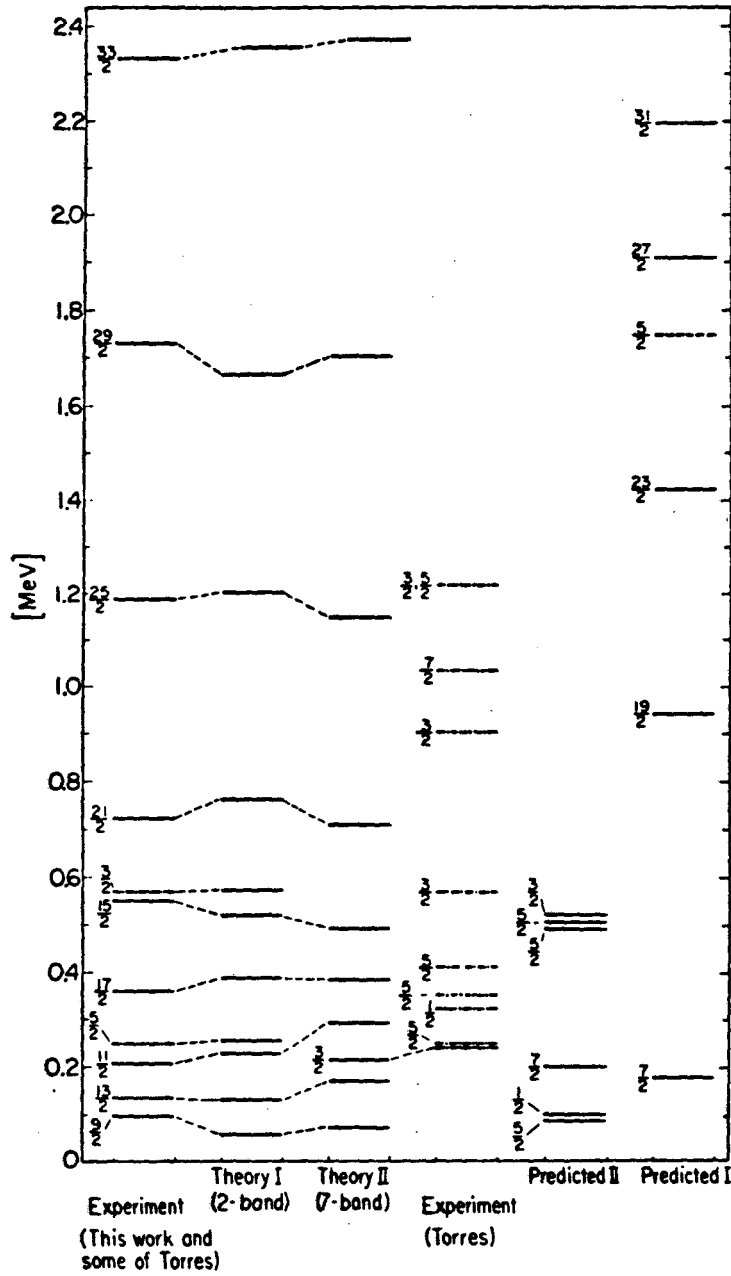


Fig. 5. Comparison of positive-parity band energies in ¹⁵³Dy with Coriolis band mixing theory least squares fits. Theory I is a two-band theory and theory II a more conventional seven-band (all projections of $i_{1/2}$) mixing theory. Note the difference that theory II includes the lowest $3/2^+$ level of Torres *et al.*⁵ in the fit and theory I includes the second $3/2^+$ and the lowest $3/2^+$ in the fit. See text for parameters and discussion.

center, Mr. Herbert Massmann developed a program permitting adjustment of several parameters. Fig. 5 shows as theory I an 11-level, six-parameter fit for ^{155}Dy , where the basis involves a $K = \frac{3}{2}$ band lower and a $K = \frac{1}{2}$ band upper.

It is not easy to compare exactly the parameter values to those of a conventional band mixing treatment. Certainly more than two bands are heavily involved in mixing, and the parameter values represent some kind of renormalized effective values.

The off-diagonal matrix element is above the unattenuated single-particle value. The sign of β is the same as that of an even-even rotor. The parameters for the theory I fit shown in fig. 5 are as follows:

$$\begin{array}{ll}
 K_L = \frac{3}{2}, & \alpha_{2K} = 0.970 \text{ keV}, \\
 K_U = \frac{1}{2}, & \langle J_+ \rangle \text{matrix elem.} = 10.3, \\
 \alpha = \hbar^2/2\mathcal{I} = 30.38 \text{ keV}, & E_{K_U}^0 - E_{K_L}^0 = 717 \text{ keV}, \\
 \beta = \text{softness parameter} = 0.0044 \text{ keV}, & E_{K_U}^0 + E_{K_L}^0 = 1756 \text{ keV}.
 \end{array}$$

Theory II is a more conventional multi-band Coriolis mixing calculation carried out by Dr. Carol Alonso. Here she has used the lower energy $\frac{3}{2}^+$ level of Torres *et al.* [ref. ⁵)] plus the nine levels of the in-beam work to make a three-parameter fit. The optimum attenuation factor for bare off-diagonal matrix elements (no pairing reduction) is 0.77 with a decoupling parameter for the $K = \frac{1}{2}$ band of half the theoretical value. The zero-order separation of $K = \frac{1}{2}$ and $\frac{3}{2}$ bands was set to zero and not allowed to vary. That is, the chemical potential was frozen to a value half-way between the Nilsson energies of $K = \frac{1}{2}$ and $K = \frac{3}{2}$ bands, and the diagonal moments of inertia of all bands were equal. Neither of these constraints were relaxed, as in the Stockholm band fitting of ^{153}Gd . Like the Stockholm calculations and the Dubna calculations ¹⁰), (for ^{159}Er , a similar case), our calculations show the "favored" family of levels ($\frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \dots, \frac{1}{2}(4n+1), \dots$) to have wave functions predominantly $K = \frac{1}{2}$, but with appreciable $K = \frac{3}{2}$ and $\frac{5}{2}$ components, whereas the "unfavored" levels ($\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots, \frac{1}{2}(4n-1), \dots$) have $K = \frac{3}{2}$ and $\frac{5}{2}$ components larger than $K = \frac{1}{2}$. This mismatch of wave functions between "favored" and "unfavored" families tends to reduce the E2 transition probabilities between families relative to those within families. However, the effect is not large enough to quantitatively explain the weakness of the $\frac{1}{2}^{5+}$ to $\frac{1}{2}^{3+}$ transition, for which we only have an upper limit, relative to the $\frac{1}{2}^{5+}$ to $\frac{1}{2}^{1+}$ transition. See table 6 and associated text of Løvholden *et al.* ²) for the analogous problem in ^{153}Gd .

For the $\frac{1}{2}^{1-}$ [505] band, the cascades to cross-over ratio (1.8) from the $\frac{1}{2}^{5-}$ level in ^{155}Dy is the same as that (2.2) in the analogous level in ^{153}Gd . Furthermore, the transition energies are only about 5% greater in the ^{155}Dy case. The net effect is that the deduced $g_K - g_R$ factor should be about the same as reported in ref. ¹), namely, -0.4. The Nilsson model predicts -0.5 for $g_K - g_R$ in the $\frac{1}{2}^{1-}$ [505] band.

5. The level scheme of ^{154}Dy

The levels of ^{154}Dy were studied before ¹⁴), and a level scheme consisting of a ground-state quasi-rotational band up to 10^+ and a β -vibrational band was proposed. We do observe, however, additional transitions of the ground state band. The β -vibrational band is less intensely populated in the $^{146}\text{Nd}(^{12}\text{C}, xn, \gamma)$ reaction. However, we were able to verify the previously proposed 4^{++} and 6^{++} members of this band by observing the transitions depopulating these levels. The observed coincidence relationships (table 3 and fig. 6) and the measured angular distributions (table 2) are in agreement with proposed level scheme, fig. 7.

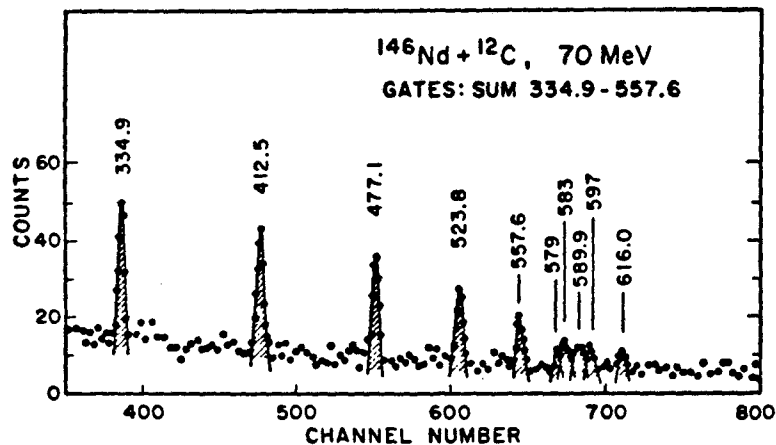


Fig. 6. Spectrum of γ -rays in coincidence with gates from five lowest members of the ^{154}Dy ground rotational band. Note the presumed back-bending transitions at 597 keV and 583 keV.

The placement of transitions up to the 14^+ level is clear, but there are two extra transitions, 597 and 583 keV, which are in coincidence with lower cascade transitions and clearly belong to the band. In view of the common occurrence of "back-bending" behavior of rotational bands in this region, we believe the 597 and 583 keV transitions are just above the 14^+ level, probably in that order since the 597 is more intense in the singles. However, the intensities seem nearly equal in the coincidence spectrum of fig. 6, so the 583 keV transition could, alternatively, be assigned to $16 \rightarrow 14^+$. There may also be a weak 579 keV transition in the coincidence spectrum. Thus, we have in fig. 7 drawn the levels above 14^+ as dashed lines to indicate their tentative nature. Whichever of these transitions is the $16 \rightarrow 14$, the back-bending rotational behavior is clearly present.

The dependence of the moment of inertia on the square of the angular velocity ω is shown in fig. 8. The method denoted "Brookhaven approximation" by Sheline was used here ¹⁵). For the calculation of the rigid-body moment of inertia the value of $\beta = 0.17$ was used.

Table 4 gives a comparison of rotational transition energies in this region. Note

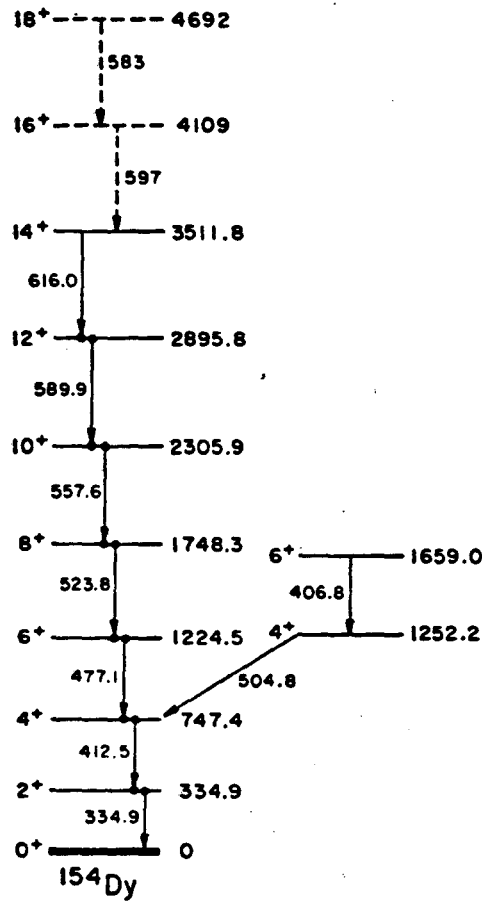


Fig. 7. Proposed level scheme of ^{154}Dy from this work. Additional beta-band members were identified by Neiman and Ward ¹⁴). The 16^+ and 18^+ levels are shown dashed, since the order of 597 and 583 keV transitions is not certain.

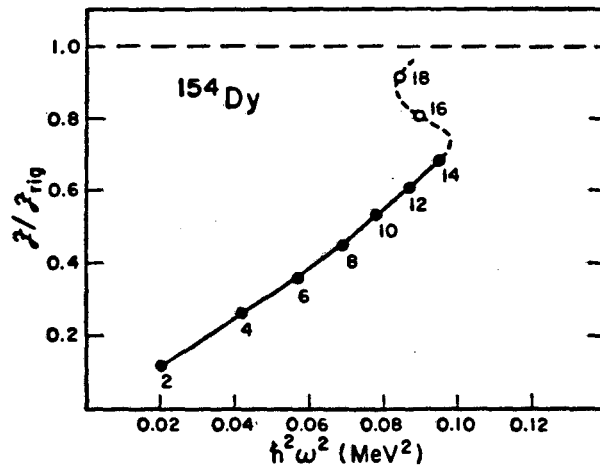


Fig. 8. Plot of moment of inertia versus angular velocity squared for ground band of ^{154}Dy . The Brookhaven approximation was used to calculate $\mathcal{I}\omega$ from the data.

TABLE 4
Ground band transition energies (keV) in ^{154}Dy and neighbors

	^{152}Gd	^{154}Dy (this work)	^{156}Er	^{156}Dy	^{158}Er
2 \rightarrow 0	344.267	334.9	344.4	137.7	192.0
4 \rightarrow 2	411.071	412.5	452.9	266.3	335.1
6 \rightarrow 4	471.9	477.1	543.2	366.4	443.1
8 \rightarrow 6	519.1	523.8	618.2	445.6	523.0
10 \rightarrow 8	553.4	557.6	\approx 675	509.8	578.9
12 \rightarrow 10	584.0	597		561.1	608.1
14 \rightarrow 12		616.0		602.3	510.0
16 \rightarrow 14		597 or 583		611.1	472.8
18 \rightarrow 16		(583)			566.3

Except for ^{154}Dy , energies are taken from ref. 17).

that ^{154}Dy has a lower 2^+ energy than either nearest isotone, but above the lowest transition the energy tend is monotonic with ^{152}Gd lowest, followed by ^{154}Dy and ^{156}Er . The nearest established 16^+ "back-bender" is ^{158}Er , with back-bending setting in also around spin 14. In ^{156}Dy the transition energies highest in the band are approaching constancy at spin 18.

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