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Application of Koopman Mode Analysis in Residential Environments



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# Application of Koopman Mode Analysis in Residential Environments

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requirements for the degree

Master of Science
in
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by

Ljuboslav Boskic

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**Abstract**

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As a step towards our goals of energy efficiency, we investigate data-driven, simple-to-implement residential environmental models that can serve as the basis for energy saving algorithms in both retrofits and new designs of residential buildings. We find that currently used models of thermal behavior of buildings are lacking in a fundamental way associated with the thermal mass of buildings. Despite the nonlinearity of the underlying dynamics, in this study we show that a linear second order model embedding, that captures the physics that occur inside a single or multi zone of a space does well in comparison with data. In order to validate our model, we used EnergyPlus to simulate indoor air temperature. The error ranges from 3.3% to 7.2% according to different thermal mass properties of the residential building. Using data-driven methods such as Koopman mode decomposition we analyze thermal data from a single zone space. With this analysis we are able to identify stability or instability of the modes present in the dynamics. Using data we were able to find the mode that corresponds to heating and cooling control, as well as identify the location this control originated from and it's period of occurrence to be every 1.5 hours.

# Contents

# Chapter 1

# Introduction

The "House as a System" approach is gaining traction as a protocol to gain deep energy efficiency in residential buildings [1]. However, the current approach is focused on scheduling the order of retrofits (insulation first, replacement of furnace second, etc.) and thus high capital expenditure actions. In commercial buildings, the cost of such retrofits has led to development of strategies for optimizing operations of existing systems, focusing first on fault detection and returning the building operation to a "healthy" state [2]. Beyond the fault detection methodologies, model-based approaches lead to optimization of existing systems and potential of deep energy savings for new commercial builds [3], and even US Army facilities [4]. However, these gains are not currently utilized in the context of residential buildings.

Therefore, residential buildings have recently gained more attention within the topic energy efficiency. There are about 136.5 million residential buildings in the United States [5], creating a large opportunity for energy savings via retrofits and new designs, to create more efficient homes. Through addition of sensing, communication and actuation of components, devices are made "smart", such that they communicate wirelessly with each other and transmit data to help reduce use during peak demand periods. With energy monitoring and cost savings, smart home technologies have potential to deliver benefits such as convenience, control, security and monitoring, environmental protection, and simply enjoyment from engaging with the technology itself [6]. In order for retrofits and newly designed systems to work properly, smart technology must be introduced and implemented. Smart technology incorporates sensors, actuators and algorithms. Here we present a reduced order model (ROM) for indoor temperature of a single zone and multi-zone, with the goal of improving energy efficiency for residential buildings that can serve as a basis for all energy saving algorithms which requires no cloud computing.

## 1.1 Zero Net Energy

Many states in the USA are advancing very aggressive goals towards commercial and residential building energy efficiency. As spelled out in the California Energy Efficiency Strategic Plan, the state has ambitious goals for the development of Zero Net Energy buildings. These include:

- All new residential construction will be zero net energy (ZNE) by 2020.
- All new commercial construction will be ZNE by 2030
- 50% of commercial buildings will be retrofit to ZNE by 2030
- 50 % of new major renovations of state buildings will be ZNE by 2025.

In 2016, the Department of General Services issued these definitions of zero net energy:

- ZNE building - An energy-efficient building where, on a source energy basis, the actual annual consumed energy is less than or equal to the on-site renewable generated energy.

- ZNE campus - An energy-efficient campus where, on a source energy basis, the actual annual consumed energy is less than or equal to the on-site renewable generated energy.
- ZNE portfolio - An energy-efficient portfolio in which, on a source energy basis, the actual annual consumed energy is less than or equal to the on-site renewable generated energy.
- ZNE community - An energy-efficient community where, on a source energy basis, the actual annual consumed energy is less than or equal to the on-site renewable generated energy.

In regards to the ZNE plan, the aim of this study is to create a simple ROM for the residential setting that can be easy to implement in energy saving algorithm retrofits. In conjunction it would serve as a basis for energy saving for new design. Furthermore, we investigate how changing the thermal properties of a house such as a structural change like wood to brick would affect certain coefficients of our model. Having a ROM that can capture all the physics like previous models but without the complexity would be critical in saving computational time.

## 1.2 Methodology

Building energy management systems (BEMS) have gained much awareness from the energy efficieny community to act as a standard for controlling buildings [7]. BEMs are computer-based systems that help control and monitor the indoor climatic conditions while still maintaining optimal operational performance and safety comfort levels for occupants [8]. BEMs usually use classical control algorithms such as on-off control, PI control, PID control and optimum start-stop loop routines. These algorithms are generally very good for single-input-single-output (SISO) systems but buildings usually have multi-input behaviors. Buildings have interesting heat dynamics due to thermal interactions between different thermal zones and heating, ventilation and air-conditioned (HVAC) systems [9]. Therefore buildings are complex, not SISO, and sustainability goals would benefit from advanced sensing, actuation, and control.

Large advances have been achieved in commercial building sector research for energy efficiency. The commercial sector is of interest as 5% of the largest buildings consume 50% of energy in commercial buildings. One of the simplifying feature of this sector is predictable work schedules. With predictable work schedules, it is easier to control HVAC units that have adjustable air flow, providing solutions to one of the biggest energy consumption factors in buildings. For example, it was shown in [10] that having variable air volume (VAV) during the day and constant volume (CV) throughout the night time has potential to save over 25% in climate areas like New York and well over 35% in Phoenix. In this work, peak power demand was controlled by switching on the HVAC system prior to peak demand and this offsetting the demand. A lot of work has been done to implement occupancy sensors in order to help control the HVAC unit and also control the lights. While there is much room for further gains, with the combination of control techniques with predictable work schedules, energy savings procedures in the commercial sector have been a success.

Unlike the commercial sector, the residential sector does not have have predictable work schedules so it is not a trivial solution to improve efficiency. In the residential environment, as stated above you have a multi-input-single-output (MISO)system which is hard to model yet alone control. Recent research has developed models that consist of as much as 6 coupled ordinary differential equations (ODEs) [11, 12]. Other work has consisted of trying a transfer function approach of identifying a model [13]. All of these mathematical models include lots of complex factors which do not capture all of the physics of the most important coefficient, thermal mass.

We strongly believe a ROM is needed for these energy savings to transition from the commercial to the residential sector. Having to many complex models, not only drive computational time up but also might not be the dominant coefficients that are needed to capture the most important thing, indoor air temperature. For a residential environment, people along with their comfort level are the most important aspect. We need a ROM that will capture indoor air temperature and then this kind of model can be implemented in the latest control algorithms and sensing.

# Chapter 2

# Smart Sensing Technologies and Techniques

Figure 2.1 shows energy use for residential buildings in the United States. It is clear that most of the energy (over 50%, which constitutes 10% of all the energy consumed in the united states!) is consumed by space heating and cooling, which makes this sector even more important than in commercial buildings, where it varies between 30% and 40%.

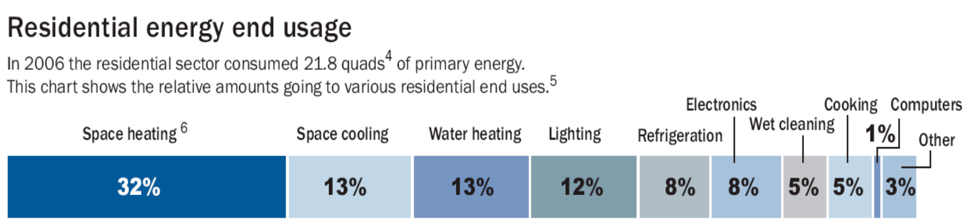



Figure 2.1: Energy use for residential buildings.

## 2.1 Technologies

### 2.1.1 Occupancy Sensors

Occupancy sensors are of major interest due to past research showing the use of these sensors for controlling lighting and HVAC could potentially save 30% and more of electrical energy used for lighting [14]. Residential code reuquires at least one light fixture in every room, hallway, stair way, attached garage, and outdoor entrance [15] getting the use of occupancy sensors a natural value proposition. Occupancy sensors combined with algorithms and actuators can be used to control HVAC and artificial light devices to maximize energy savings. Smart occupancy sensors are distinct from occupancy sensors with preset time delays (the time after the device will switch off) or schedules. A common problem with such sensors is unwanted switching of a particular integrated device/section or misprogrammed schedules.

Smart occupancy sensors [14] are adaptive to changing activity levels and are in effect a closed-loop devices, in contrast with the open loop discussed before. With adaptive control, the change-of-operation time delay is adapted to different activity levels. Savings will be dependent on both the space type and schedule of resident's activities. The savings are highly dependent on peak occupancy e.g. the kitchen, living room and bathroom are rooms the people in the house usually interact with the most while other rooms might see increased energy savings due to occupancy sensors. Roughly, the cost benefit of an occupancy sensor is inversely proportional to occupancy. The issue of "false alarms" is important, in the sense that fast off and on switching is undesirable. Smart occupancy sensors provide savings in both in energy and maintenance. First, the device (lights or HVAC) turns off adaptively causing decreased electricity consumption; secondly, it extends the overall life of devices such as light bulbs.

In order to maintain a good indoor-air-quality (IAQ) and thermal comfort, devices like the smart thermostat can control the HVAC directly based e.g. on the learned user behavior or feedback. For example, the '$CO_2$ meter' [16] device works based on a feedback response where it shows the IAQ in three colors; green, orange and red. This was done in early development to help the user implement energy saving techniques that can either be done through the HVAC unit or done by ventilation techniques; that are discussed later in the paper. This device is very important to have due to always needing to maintain an acceptable IAQ according to ASHRAE Standard 62.2 [17]. Many sensors have now been integrated with technology similar to this but no longer have a color indicator. Most $CO_2$ sensors have now been put into the automatic feedback loop and are used as or in conjunction with other sensors to detect occupancy.

### 2.1.2 Thermostats

Heating, ventilation and cooling (HVAC) is the single largest contributor to home energy bills and carbon emissions, accounting for 45% of residential energy consumption in the U.S., as seen in Figure (2.1). Studies have shown that 9-18% of this energy can be saved by simply turning down the HVAC unit when the residents are sleeping or the space is unoccupied [18]. However, residents do not typically manually adjust thermostat set points multiple times of the day to account for the conditions outside and inside their house. In addition, many residents find programmable thermostats difficult to use effectivity. If programmed poorly, these thermostats can cause energy consumption increase rather than decrease, as seen in [19, 20]. The problem is that programmable thermostats are open loop. In contrast, the closed loop thermostat (smart thermostat [21]) could resolve some of the problems stated above and provide a substantial amount of savings. This thermostat uses occupancy sensors to automatically turn off the HVAC system when the residents are sleeping or the space is vacant. Then, the problem reduces to smart thermostat placement and algorithms for feedback control.

The new Nest Learning thermostats have generated a lot of interest. However, we find their algorithms inadequate as they 1) do not take into account multiple zones of the building and 2) do not rely on occupancy sensors. Still, Nest learning thermostats showed that energy savings could equal $10\% - 12\%$ of heating usage and about 15% of cooling usage [22], indicating that substantial additional savings, for electricity and natural gas are still to be obtained by better control algorithms. The potential impact of smart thermostats is large, but they need to be coupled with better control algorithms in order to achieve them.

## 2.2 Techniques

As argued above, smart feedback control algorithms are needed to fulfill the potential of occupancy-based and smart thermostat technologies. However, energy savings are impacted by the combination of, weather, occupancy patterns and home equipment schedules that can be complex and have uncertainties associated with them. In order to maximize energy savings, there are both manual and automation techniques that can be implemented using feedback from sensors as discussed above.

### 2.2.1 Shock ventilation

Shock ventilation (SV) was first introduced in connection with the '$CO_2$ meter' which provided the user feedback in order to preform a task. SV is a simple energy saving technique where the resident opens the windows for 5 minutes two to four times a day. This kind of ventilation in current research has shown to save up to 25% of heating energy [16]. SV is done based on the provided feedback the user gets based on the IAQ measurement. IAQ can be affected by the $CO_2$, mass or energy stressor, and microbial contaminants (i.e. mold) that may be present in the living space. We will mostly focus on the $CO_2$ and air temperature in the room. The possibility of detecting bacterial contamination in home automation is interesting and we looked as some aspects of biological material sensing for those. We concluded that the maturity of the technology is too early for implementation but it is worth trying to develop better, faster sensors in this direction. This could be of huge consequence for in-home health diagnostics.

### 2.2.2 Night ventilation

Passive energy saving techniques have been developed to maximize energy savings through cooling. Night ventilation is used primarily for areas that may not be occupied during the night time. Night ventilation is defined as a set of procedures using either natural or mechanical ventilation to cool the structure of the building at night and is most effective if the building has a high thermal mass. The cooled structure can then absorb the heat the following day and provide comfortable living conditions. It can affect internal conditions during the day in four ways: 1) reducing peak air-temperatures, 2) reducing air temperatures throughout the day, 3) reducing slab temperatures, and 4) creating a time lag between the occurrence of external and internal maximum temperatures. [23]

### 2.2.3 Night Setback

We are interested in capturing thermal mass effects in a physical model, and have provided the methodology for doing so. Comfortable temperatures along with safe level of IAQ need to be maintained in occupied spaces. Temperatures may be dropped during unoccupied periods and nights which would result in possible energy savings. This strategy of dropping temperature during those periods and then raising it back up during the temperature recovery period is called the night setback illustrated in Figure (2.2). Generally if the temperature is already in the comfort range prior to occupancy, then the startup was too fast and energy is wasted. In contrast, if the temperature reaches the comfort level after the space is being occupied, then this may cause uncomfortable conditions for the occupants. Thus, it is important to find the optimal start time. However, the problem of finding the optimal night setback procedure is difficult, due to HVAC units heating the building depend on changing climate conditions, building structure, HVAC system capacity, etc. [24].

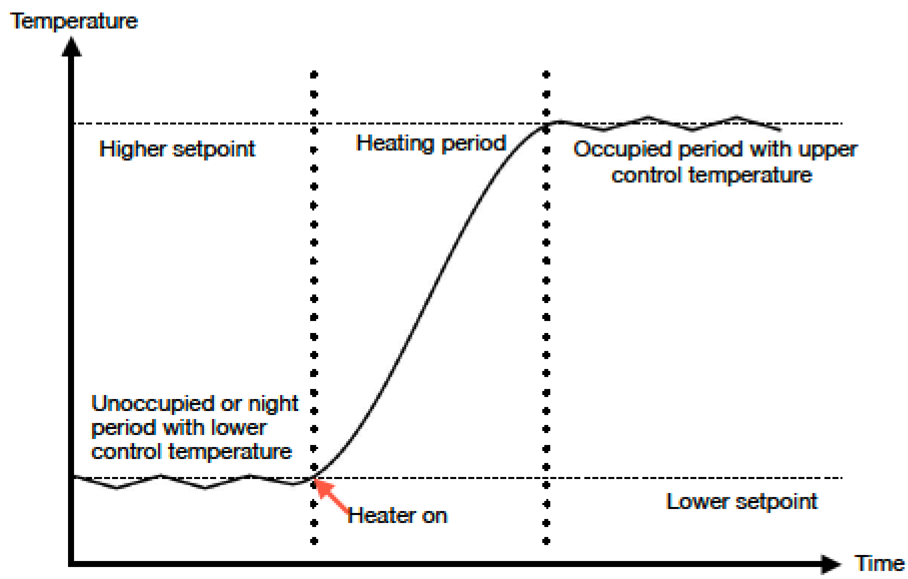



Figure 2.2: The temperature variation in residential buildings with known occupancy.

*The model that we have developed in this research, described below, has a potential to make this optimization procedure easier.*

### 2.2.4 Variable Air Volume

So far we have discussed passive control techniques which may not directly involve the use of the HVAC unit. which we now discuss. Variable Air Volume unit (VAV) is a type of HVAC system. This kind of system varies air flow at a constant temperature unlike most CAV (constant air volume) systems. The advantage of VAV systems is that they include more precise temperature control, reduce wear on the system itself, and lower energy consumption by system fans. In residential sectors most people have a variable schedule where there are periods that the house in unoccupied which is when reducing the ventilation rate is ideal. However, existing residential systems with constant ventilation rate can conflict with ventilation on demand and thus can result in periods with wasting energy on heating unnecessary airflow; hence, to achieve energy-efficient residential buildings, the ventilation rate should be varied according to demand, depending on occupancy level or pollutant emission. [25]. This suggests need for VAV systems where the amount and conditioning of air should be varied based on occupancy levels.

VAV systems have become popular due to improved energy efficiency in comparison with CAV systems [26]. In combination with proper adjustment of setpoints, VAV systems can possibly provide huge savings for the residential sector. In order to implement the demand-controlled ventilation (DCV), there are many variables that come into play for controlling the airflow of VAV systems. In order to have control with acceptable IAQ, $CO_2$ concentration, room temperature, relative humidity, and occupancy level all need to be considered. For high occupancy areas $CO_2$ sensors are needed, while if the building has stable periods of occupancy, then schedule-based control or occupancy detection could do. *Given the cost of implementation, both manual and automatic means of control should be considered.* The data analytic work we present below indicates ways for manual control, and provides data for feedback to automated control.

# Chapter 3

# Commercial Buildings

## 3.1 Implementation

With typical estimated energy saving potential from one-fourth to more than one-half of light energy, occupancy sensors have frequently been promoted as one of the most cost-effective technologies available for retrofitting commercial light systems [27]. In office space buildings the schedule is often set when for example, office spaces are occupied, and delay occupancy sensor based control can be effective. In [27], it was shown that without any occupancy sensor the energy waste for certain areas in a commercial building is tremendously high and the percentage of wasted energy goes up to 67% during the day time for private offices (see Table 3.1). It was also shown that most of the energy waste actually occurred during the weekdays which is when the offices were mostly occupied, as opposed to weekends when the offices are generally empty. Based on the study, the authors were able to integrate occupancy sensors and save large amounts of energy.

More recent work has involved incorporating fine grained occupancy sensors. This has been shown to improve HVAC system control and result in energy savings. Using a low-cost wireless occupancy sensor with an emphasis in accurate detection, reduction

Table 3.1: Percentage of energy waste incurred on weekdays, weekends, and for total monitoring period for each application. [27]

| | Energy Waste (%) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Weekdays | | | Weekends | | | Total monitoring period | | |
| | Day | Night | Total | Day | Night | Total | Day | Night | Total |
| Break room | 50% | 29% | 79% | 12% | 8% | 21% | 63% | 37% | 100% |
| Classroom | 40% | 36% | 76% | 13% | 11% | 24% | 53% | 47% | 100% |
| Conference | 55% | 24% | 80% | 12% | 9% | 20% | 67% | 33% | 100% |
| Private office | 67% | 21% | 87% | 8% | 5% | 13% | 75% | 25% | 100% |
| Restroom | 29% | 41% | 70% | 14% | 16% | 30% | 42% | 58% | 100% |

of 10-15% building energy was shown through simulation [28]. This low-cost occupancy sensor is a combination of magnetic reed switch door and PIR (passive infrared) sensor. This combination enables high accuracy occupancy detection. The reed sensor works as a pressure sensor for doors and the PIR sensor detects occupants. The reed sensor detects when a door is opened or closed and then the PIR sensor detects if the person is there or not. Thus if the space is occupied or unoccupied. This is a way to improve on with false offs or unwanted switching off. With the combination of these two sensors, accuracy is greatly increased and thus energy savings increase.

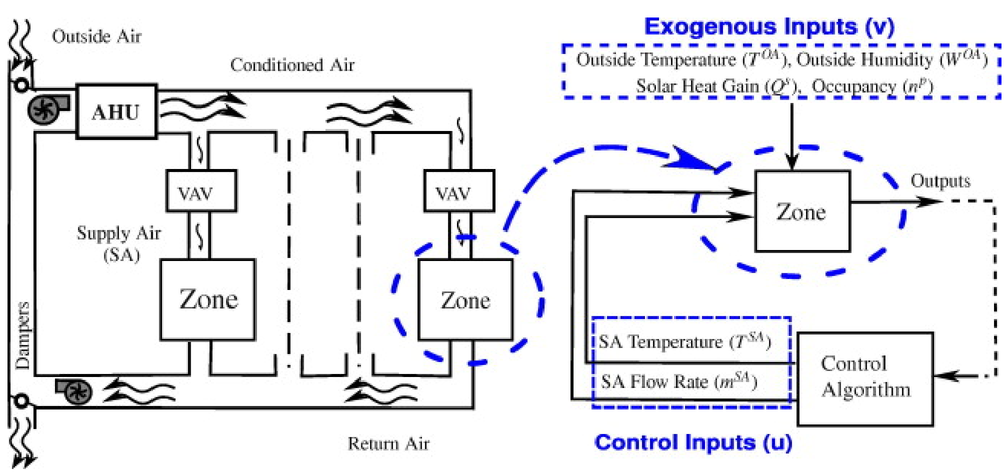



Figure 3.1: Generic scheme for the implementation of a zone-level control algorithm.

Working with commercial buildings tends to be easier due to 30% of them being already equipped with VAV systems [29]. With VAV systems the building can be divided into separate zones where each zone can either be a single zone or having similar characteristics and be grouped. Then in order to improve energy savings, demand control ventilation (DCV) can be implemented which then changes the supply of air flow rate based on observed occupancy. With the combination of the right sensors and thermostat control, near-optimal control is usually obtainable for commercial settings. With the use of DCV for VAV systems, along with control algorithms (see the scheme of VAV control in figure 3.1) it was shown that over-the-baseline energy savings of about 50% on average depending on zone type, weather, climate, design occupancy, ect.) with negligible impact on IAQ or thermal comfort can be achieved [30]. *Thus, residential building space would benefit strongly from implementing VAV systems in new builds.*

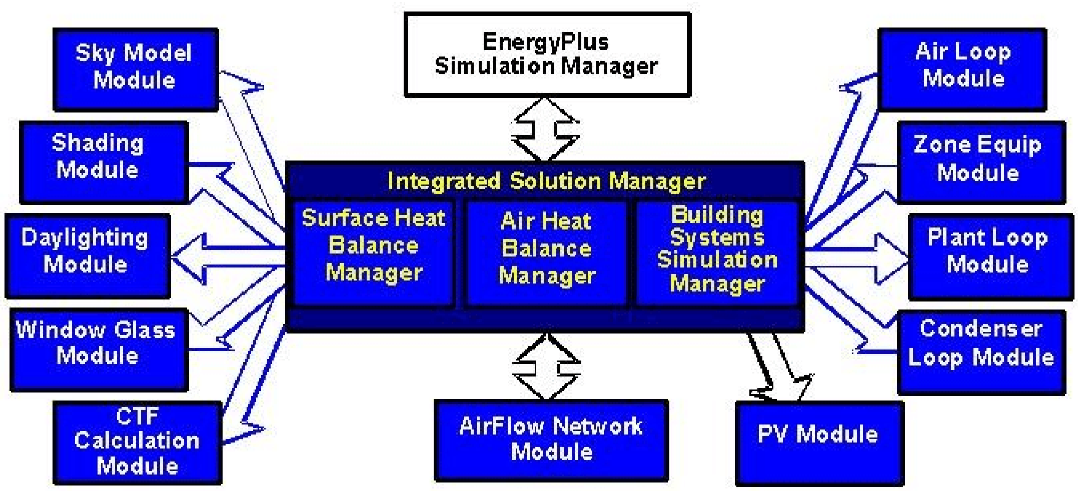



Figure 3.2: EnergyPlus Program Schematic.

In order for commercial buildings to take advantage of these and other various energy saving ideas, they need to be cost-effective. In order for this to be feasible retrofits are preferable to building new structures or buying new equipment. In order to retrofit commercial buildings, studies have been done of using DEEP (database of energy efficiency performance) that provides a direct resource for quick retrofit analysis [31]. DEEP uses data recorded from buildings and includes information like climate zone and building type. With this information, the data is compiled evaluating results from 10 million simulation parametric runs using EnergyPlus [32]. DEEP then serves as a database for screening and retrofitting for commercial buildings. With DEEP it was shown through simulations for small to medium offices that a saving of $8.90 - 18.70\%$ is possible. *However, EnergyPlus type simulations are too expensive to be included in control schemes discussed in the previous section, especially for residential buildings.* Thus, next we discuss development of a cost-effective modeling technology for residential buildings.

# Chapter 4

# Reduced Order Model

In order to fully understand building physics, and enable control, we need to create a model. Building physics is affected by the building interaction with its surroundings through heat transport, with conduction, convection, and radiation. Even more complexity is introduced in mass transport with air, vapour, and moisture. This is where revolutionary programs such as EnergyPlus [32] come into play. The program was developed to combine heat and mass transfer, along with other physical features to provide year long energy simulations. In figure 3.2 we provide a schematic representation of the Energy Plus simulation.

EnergyPlus is the norm when it comes to energy simulating software but its drawback is its complexity. The full development of EnergyPlus scripts that defined everything about the building, HVAC systems, lighting, etc. leads to difficulties. In order to make the Model Predictive Control technology available for modern control systems in residential buildings, there is a need to create a model that is at the simplest form needed to understand and control the building physics. Namely, It is difficult to rely on people's ability to control their environment by monitoring temperatures and opening/closing windows accordingly. Thus, a good physical model is needed to provide such information, and, in conjunction with smart sensors and actuators, could be a revolutionary tool for enabling home energy efficiency.

## 4.1 Model Description

The residential building model used for this analysis was constructed in Sketchup (Computer-aided design (CAD) software) [33] and then applied in combination of OpenStudio [34] and EnergyPlus software to run a year long simulation. The location used in this study is Santa Barbara, California, United States of America. The outdoor temperature for Santa Barbara is obtained from the Department of Energy EnergyPlus website. The data in this study pertains to outdoor temperature readings in the year 2009. The model zone of a building, as seen in figure 4.1 has dimensions of $7.72m \times 7.72m \times 3.046m$ with an approximate volume of $181.5m^3$. The building has 3 windows and one door. All the material used is based on ASHRAE 189.1 standard corresponding to the location of the test area, in our case Santa Barbara, California.

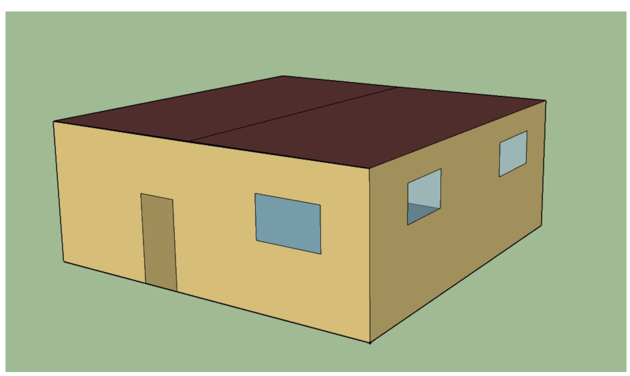


Figure 4.1: Sketchup constructed model of single thermal zone house.

In OpenStudio, we can specify a wide-variety of conditions such as setpoints, occupant schedules, HVAC equipment, loads, and more (see figure 4.1). We first develop the nominal, no-actuation, no-load model, that enables us to parametrize important physical concepts such as the thermal mass. So we first turned off all loads and just had the building take radiation from the sun, and ran a simulation on that in EnergyPlus. This was a way to understand the type of model needed, either linear or nonlinear and possibly what order. Based on work conducted so far, we are planning on designing and implementing a Model-Based Controller of the type we present below.

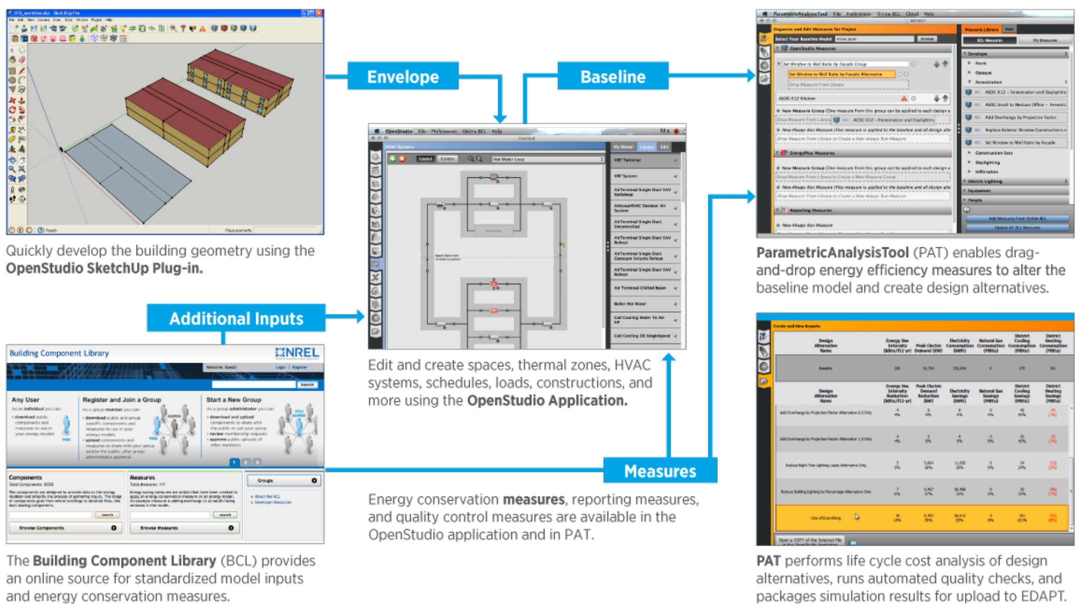



Figure 4.2: Openstudio work flow

## 4.2 Our Model

In order to fully understand the physics of heating and cooling in residential buildings, we first need to develop a model. In our case we will use a model to represent our system which then we can use to describe the phenomena that we can not directly explain at first. Once we have developed a model, we can then start to understand the physics behind it and try to identify the most influential components. The model of building physics we developed is extremely simple, and yet it captures the physical effects necessary for design of new builds and control of old builds. We developed a second order linear differential equation with constant coefficients as our model for temperature inside a particular space/thermal zone. On the right hand side we have the input to the system labeled ($u$) but we also have a constant term ($d$) being added to that in order to model the thermal radiation term, which we discuss later. State $x$ is the temperature. The equation reads,

$$c_1\ddot{x} + c_2\dot{x} + c_3x = u + c_4,$$

or , solving for $\ddot{x}$ and dividing by a,

$$\ddot{x} = -\frac{c_3}{c_1}\dot{x} - \frac{c_2}{c_1}x + \frac{1}{c_1}u + \frac{c_4}{c_1}, \tag{4.1}$$

We rewrite the equation in a state space representation:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ \frac{-c_3}{c_1} & \frac{-c_2}{c_1} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{c_1} \end{bmatrix} u + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{c_4}{c_1} \tag{4.2}$$

The mathematical model is implemented in MATLAB [35] for the model of the single thermal zone house. The model was parameterized by discretizing it and finding the optimal coefficients that best describe the nature of the indoor temperature compared to the known indoor temperature of that thermal zone. We tested the model for the indoor temperature at two different time scales, of two weeks and for six months, the results of which we show below. Our model does have some irregularity in certain regions where the temperatures either spikes up too fast or drops down rapidly but overall it *describes the the behavior of indoor temperature extremely well, thus making it suitable for implementation of optimization and control procedures.*

We now discuss what every term in equation (4.2) represents in terms of thermal physics of the space. The thermal mass influences the $c1$ term strongly. Thermal mass is a very important aspect in buildings due to it being the main source of absorption of outside and inside thermal and passive control of the living space inside. The thermal mass is influenced by the physical structure of the walls of the building, because of the varying ability of the material to absorb and store heat energy. For example, a lot of thermal energy is needed to change the heat inside a building that has been constructed out of brick, due to the fact that the density of the material is high. In fact, any material that has greater thermal mass can store more heat and therefore it will take longer to release the thermal energy after the heat source or the sun is gone.

Thermal insulation affects the "damping term" $c2$. It is used to reduce heat loss or gain by providing a barrier between areas that are significantly different in temperature. Insulation is commonly added between the outside walls and inside walls of the house, this is what provides that barrier of protection from the sun. Insulation and thermal mass both slow down the movement of heat between exterior and interior space. Insulation is used when a desired temperature differential is wanted between the indoor and outdoor space. Thermal mass is inertial, as it involves a substance that will slowly take on heat and then slowly release it over time [36].

Heat conduction affects the $c3$ term. Thermal conduction is when internal energy or heat is transferred by collision of particles and movement of electrons. More easily understood conduction is heat flow within and through a body itself (such as walls). This type of reaction takes place in all phases of matter including solids, more specifically in our case the house and all of it's objects within. All the walls and material within the walls have different properties of how heat is moved from either inside the house or from the outside (sunlight). We noticed adjusting the materials in the walls affected the thermal mass greatly but also affected the thermal conduction term.

We found that thermal radiation affects the term $c4$ in our equation. Thermal radiation is heat transferred by electromagnetic waves such as the visible light or transfer of heat within or through two bodies. It was shown in [37] that radiation heat transfer results in an increase in the heat transfer rate reflecting significant radiation effects that contribute to less thermal resistance.

The other strong effect on the coefficients in the equation is the orientation of the building. Orientation is an important factor when it comes to building design. Proper orientation would allow for passive solar gain and day lighting which would require less use of lighting sources inside the home. The orientation of the building could either provide not enough sunlight or cause overheating. In the northern hemisphere, south facing windows have the greatest exposure to light while west facing windows need to be properly designed, due to the nature of the low angle of the setting can cause overheating. This term is very important in the design aspect of homes and in this case can not really be changed for retrofits, but it is a factor in new designs.

*Thus, roughly, thermal mass affects* $c1$*, insulation affects* $c2$*, heat conduction coefficients* $c3$*, and thermal radiation* $c4$*.* The Reduced Order Model (ROM) (4.1) we have developed reduces computational complexity of modeling problems of indoor temperature, from a very computationally expensive EnergyPlus simulation to the simple model that can be implemented using embedded controllers and has all the essential physics encoded in its coefficients. Having ROM's it is also easier to understand the nature of systems due to its simplicity. In our case, we just have a second order model where we have postulated what each term means and strongly believe have run different analysis that support our assumptions.

# Chapter 5

# Reduced Order Model II

## 5.1 Second Order Model Single Zone

In order to illustrate some of the complexity of temperature changes, we provide the indoor-outdoor temperature plots for one case of 286 operational hours. It can be noticed that there is a shift (delay) between the peaks of temperature between the indoor and outdoor temperature in Figure (5.2). Understanding of this shift affects the old fashioned manual implementation of opening the window before the sun is out and closing it afterwords in order to cool the home. The complexity of the delay timing indicates a machine would be better in determining the exact times for such action. For example, if the outdoor temperature is higher than indoor temperature (as illustrated below), that would result in ventilation doing more harm than good.

The nominal model that we call a ROM is given only input data only (outdoor temperature) and from that we can implement out system identification technique to get the optimal coefficients as described in the previous chapter. After we obtain the optimal values, we plot the modeled indoor temperature compared to the "actual" indoor temperature from that particular zone from the EnergyPlus simulation. Below is our

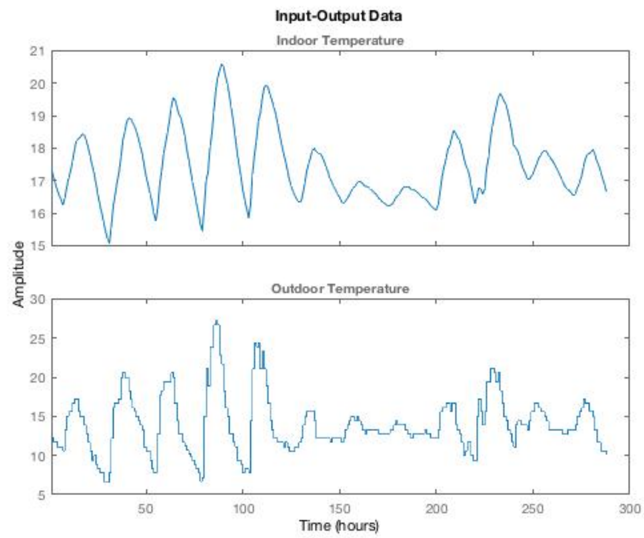



Figure 5.1: Hourly indoor-outdoor temperature plots during 2/26 to 3/09

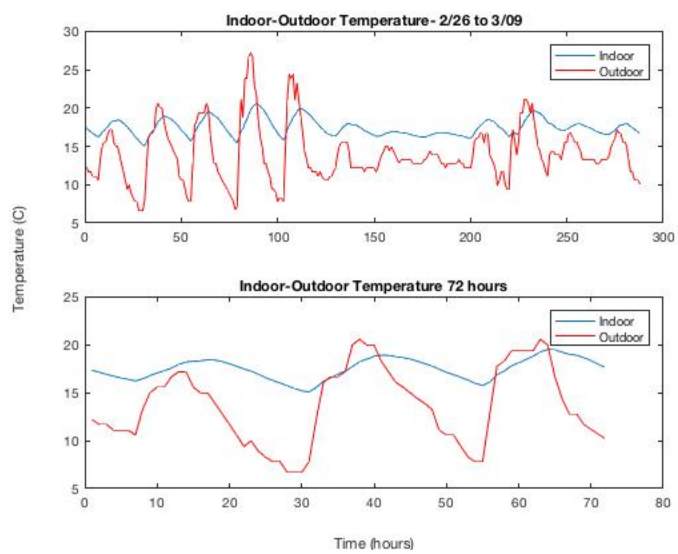



Figure 5.2: Indoor-outdoor temperature plot of 72 hour close up.

model for 286 operating hours 5.3. The percentage error found between the actual indoor temperature from simulation to our model is 6.3512%.

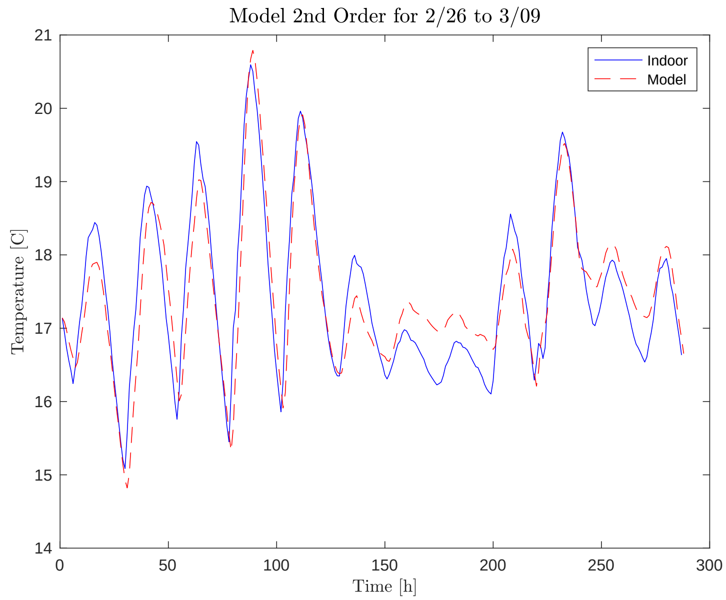



Figure 5.3: ROM for indoor air temperature from 2/26 to 3/09

## 5.2 Second Order Model Multi-Zone

So far we have developed a model that fits well for the physics of a single zone building. A single zone thermal zone does exist when talking about an apartment complex or separate living quarters (by a wall). Now we will investigate how our model holds for a simulation where we may have multiple spaces and thermal zones but also have one thermal zone which encompasses them all. The overall thermal zone of the building is calculate by taking the average temperature of each smaller zone.

From Figure 5.7 we can specifically see that the model holds and even has better results with a 4.1515% difference from the true indoor temperature. Even with having one thermal zone but four different spaces in a house, we see that the model will hold. In EnergyPlus and Openstudios we had four separate spaces along with their own thermal zone and then we added a single thermal zone for the whole house. This single thermal would do the calculations needed to average the thermal properties from each thermal

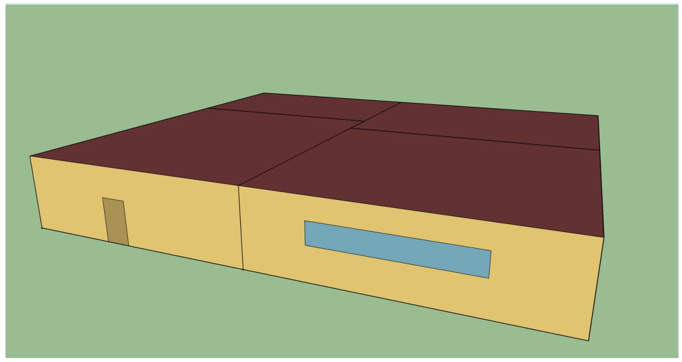


Figure 5.4: Sketchup construction of the multi-zone model used in this section of the study.

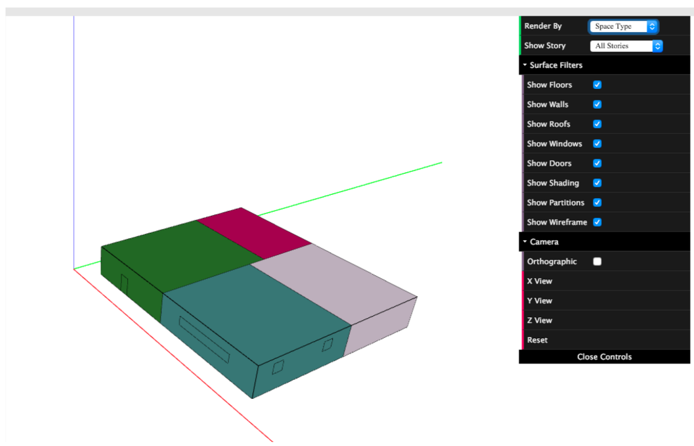



Figure 5.5: Four different space types are illustrated above for the multi-zone model.

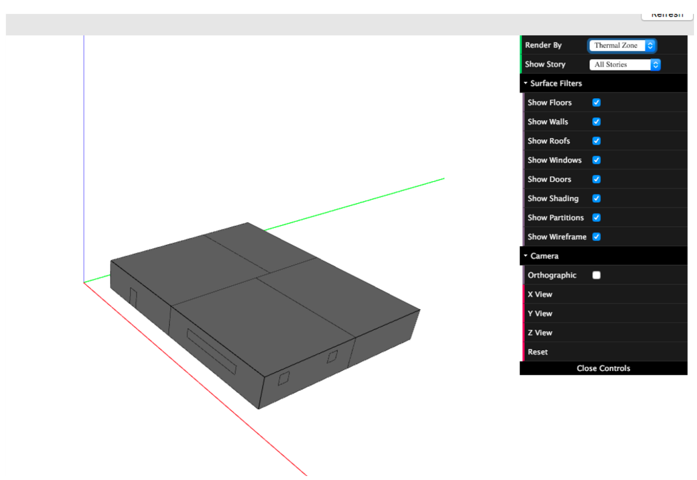



Figure 5.6: Single thermal zone for the four different space type model.

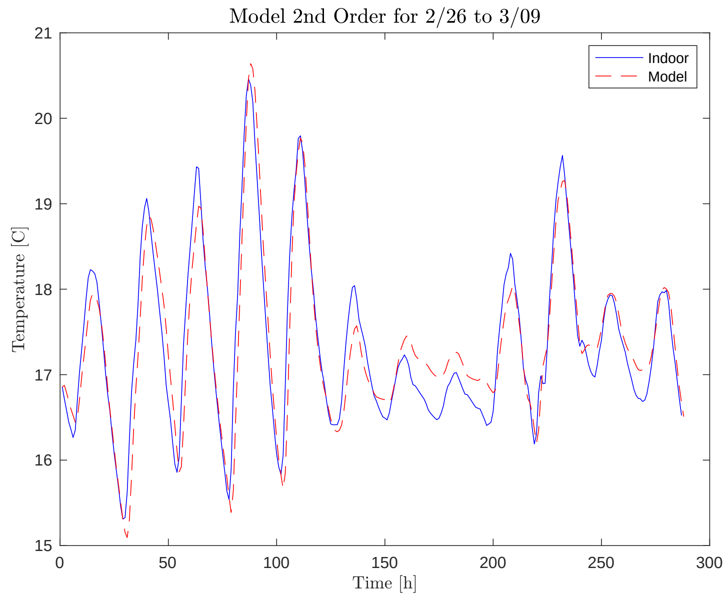



Figure 5.7: MODEL, 4.1515

zone and put four of them into one. Thermal zone averaging and other methods have been used in the past. Zoning based on thermal properties of neighboring spaces was seen to have great success in [3, 38].

## 5.3 Thermal Property Variation and Validation

- Standard Model

From the work done so far we have seen that our model will work for a single space or multi-space residential building. The next step will be for us to analyze and verify the coefficients are what we believe them to be. We will create different test models in OpenStudios which will have different structural builds (steel and brick) and different material that will be in the walls and in between the interior and exterior wall. For this section only the house will be a different size as to the one we have discussed earlier. The size of the house for the following section will be $11.86m \times 13.99m \times 4.57m$ and will have three windows and one door.

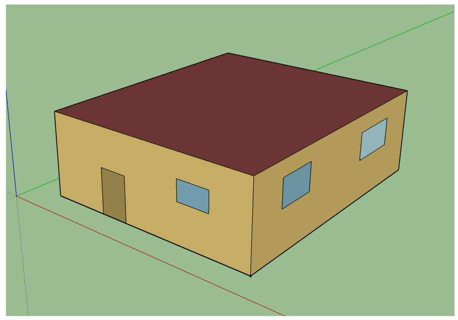


Figure 5.8: The new standard model used in this section to analyze thermal differences based on material adjustments.

In order to maintain the best accuracy possible for these models we did not adjust the frame of observation and kept that fixed. We observed the data at the following dates 2/26 to 3/09 with a total of 288 data points corresponding to hourly data. We also were able to keep the same outdoor temperature fixed for all of the simulations preformed in EnergyPlus.

Our standard model that was developed above used the ASHRAE 189.1 standards and were not adjusted. The specific area of interest we are looking at (the exterior walls) has the following construction corresponding to the standards,

| Name | Material | External Wall Setting |
|---|---|---|
| Standard Model | 1/2in gypsum<br>Wall insulation [39]<br>MAT-sheath | Steel-framed |

Table 5.1: Standard model construction layout.

For both systems we will use this type of matrix analyzing in order to see the differences in the numeric change due to the specific coefficient. Again we fix the thermal radiation term, $c4 = 1.1$. Each system will be displayed prior to discretizing the matrices for easier comparison.

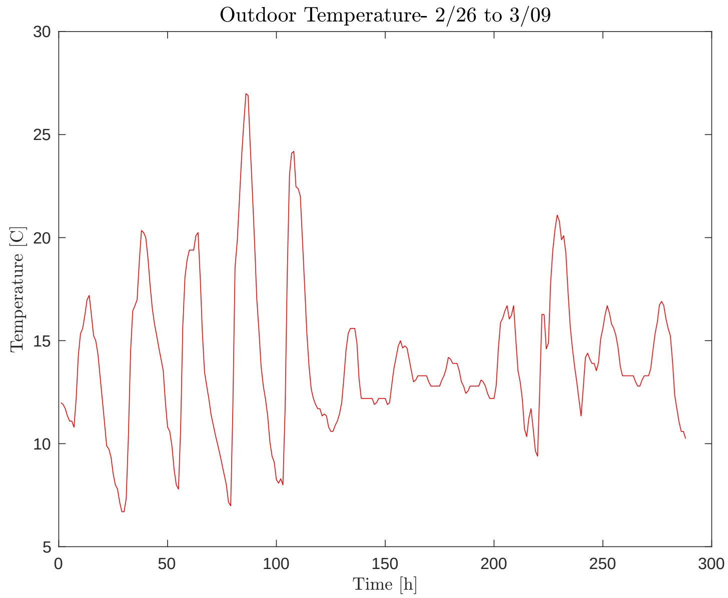



Figure 5.9: The outdoor temperature used for all simulations that is treated as the input variable ($u$).

$$\dot{x} = Ax + Bu + B_1 \frac{c4}{c1} \tag{5.1}$$

When analyzing the standard model, this is what was observed: $c1 = 0.4350$, $c2 = 10.2650$, $c3 = 2.2750$. With an error of 6.1112%.

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -5.2299 & -23.5977 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2.2989 \end{bmatrix} u + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{1.10}{0.4350} \tag{5.2}$$

| Name | Material | External Wall Setting |
| --- | --- | --- |
| Brick Model | 1in stucco<br>8in concrete<br>1/2 gypsum<br>Wall insulation [40] | Brick-framed |
| Brick and Insulation | 1in stucco<br>8in concrete<br>1/2 gypsum<br>Wall insulation [40]<br>**Wall insulation[40]** | Brick-framed |
| Brick, Insulation, and Gypsum | 1in stucco<br>8in concrete<br>1/2 gypsum<br>Wall insulation [40]<br>**Wall insulation[40]**<br>**1/2in gypsum** | Brick-framed |
| Brick, Insulation, and Concrete | 1in stucco<br>8in concrete<br>1/2 gypsum<br>Wall insulation [40]<br>**Wall insulation[40]**<br>**8in concrete** | Brick-framed |

Table 5.2: Brick case study construction layout.

- Brick Model

The brick design will have four different case studies. In each case study, we will keep the same conditions in EnergyPlus for the simulation but the only difference will be the wall material. Table 5.2 has the four case studies we will be investigating and how the wall material will be adjusted. The top row is representing the preset setting for a brick construction of a building in EnergyPlus. With these different case studies, we expect to see the variables in our ROM change accordingly and adjust to the best parameters.

The brick model full identified. The terms were found to be, $c1 = 0.3800$,$c2 = 15.2800$,

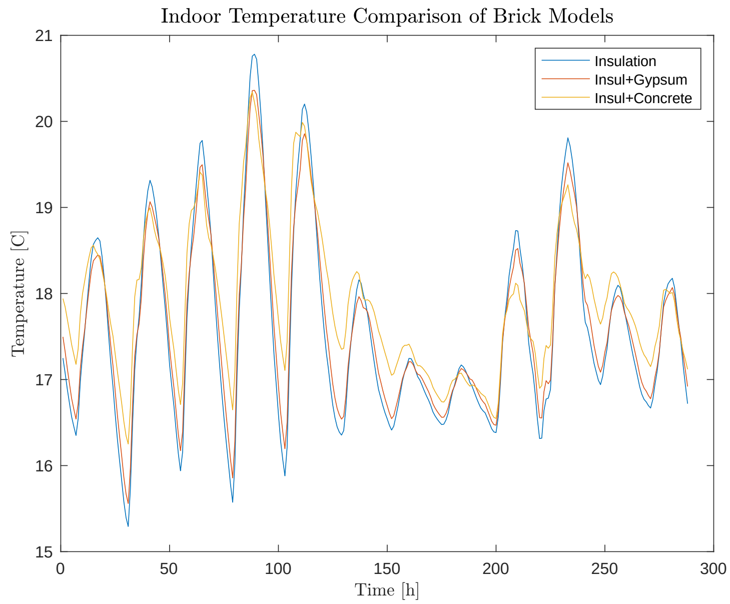



Figure 5.10: Comparison of the indoor temperature for the three different internal structures of the model.

$c3 = 3.2550$ and $c4 = 1.1$. Error=7.7222

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -8.5658 & -40.2105 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2.6316 \end{bmatrix} u + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{1.10}{0.3800} \tag{5.3}$$

The brick mode with added insulation full identified. The terms were found to be, $c1 = 0.1240$, $c2 = 7.7750$, $c3 = 4.6750$ and $c4 = 1.1$. Error=6.0809

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -37.7016 & -62.7016 \end{bmatrix} x + \begin{bmatrix} 0 \\ 8.0645 \end{bmatrix} u + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{1.10}{0.1240} \tag{5.4}$$

The brick mode with added insulation and gypsum full identified. The terms were found

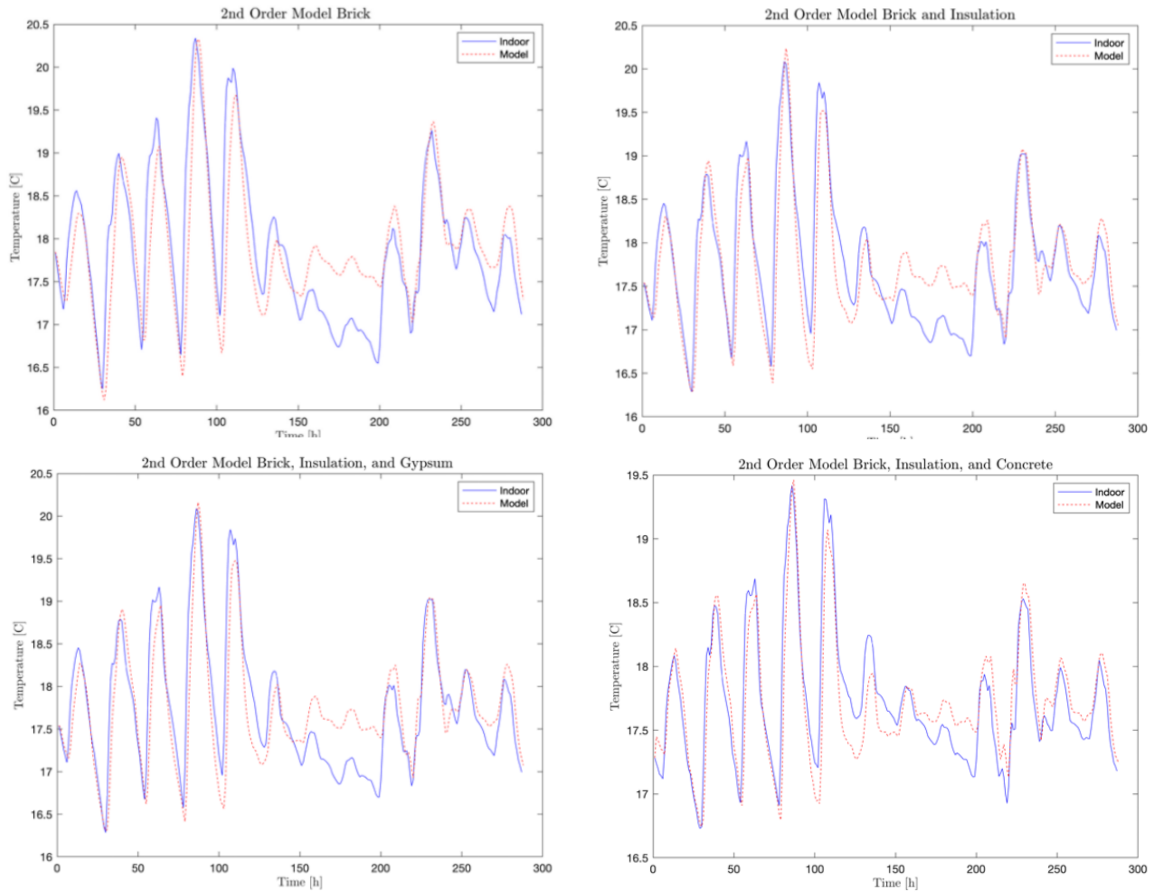



Figure 5.11: ROM plot of the four test cases with a brick construction.

to be, $c1 = 0.1400$, $c2 = 8.950$, $c3 = 4.7650$ and $c4 = 1.1$. Error=5.9672

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -34.0307 & -64.1786 \end{bmatrix} x + \begin{bmatrix} 0 \\ 8.33 \end{bmatrix} u + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{1.10}{0.1400} \tag{5.5}$$

The brick mode with added insulation and concrete full identified. The terms were found to be, $c1 = 0.0600$, $c2 = 6.3350$, $c3 = 7.3400$ and $c4 = 1.1$. Error=3.3882

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -122.3330 & -105.5833 \end{bmatrix} x + \begin{bmatrix} 0 \\ 16.667 \end{bmatrix} u + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{1.10}{0.0600} \tag{5.6}$$

# Chapter 6

# Koopman Mode Analysis

The Koopman operator is an infinite-dimensional, linear operator that acts on a Hilbert space of functions called the space of observables [39, 40]. The eigenvalues and eigenfunctions of this linear operator are capable of capturing key dynamics charactersticis of a linear or nonlinear dynamical system. Additionally, the Koopman modes, corresponding to a particular choice of observable function, allow one to reconstruct and forecast (predict) the observed quantity. Togethere these three values of Koopman eigenvalues, eigenfunctions, and modes yield the Koopman mode decomposition (KMD) of an arbitrary observable [41, 42]. Data-driven algorithms have been created and utilize data or measurements to approximate the KMD of the system. With this analysis in hand, one is able to identify stability or instability of the modes present within the dynamics.

## 6.1 Background on Koopman mode decomposition

In the context of deterministic dynamical systems, state-variables are those variables that if known for a system at a particular time, the dynamics of the system would be determined for all of time (e.g. angle and angular velocity for the mathematical pendulum). The classical approach in dynamical systems theory is to work in a mathematical framework in which variables which describe the current state of the system are the main objects of interest, i.e. state-space. When recording dynamic data from complex systems such as buildings, the state variables which fully describe the system are not known and there may be too many to ever measure. This poses no problem for the operator view of dynamical systems in which the main objects of interest are functions whose domain is the state-space - known as *observables*. In the case of the pendulum, this could be kinetic energy, potential energy, or any function of angle and angular momentum one could think of. In this view, we can think of any measurements (or possibly functions of measurements) we take of a system of interest as a function of some unknown state. In the context of buildings this could be temperature, light, noise, humidity, or another measurement at different locations of the structure.

The simplest context in which to introduce the Koopman view is for discrete-time dynamical systems which can be described by repeated application of a single function $T : M \to M$, i.e.

$$x' = T(x) \tag{6.1}$$

where $M$ is any set whose elements label all possible states (e.g. temperatures) of the system and $x'$ is the updated state, corresponding to $x$, after a single time step. The Koopman operator framework was originally described for time-invariant continuous-time systems [43] and has been extended to the case of stochastic [44, 41] and time-varying [45] systems. For the dynamical system described by equation (6.1) the induced Koopman operator $U$ represents a new discrete-time dynamical system whose state-space is the set of all complex-valued measurables with domain $M$ (denoted by $\mathbb{C}^M$):

$$f' = U(f) = f \circ T \quad (\circ \text{ is function composition}).$$

It is easy to show that $U$ is linear ($c \in \mathbb{C}$ and $f, g \in \mathbb{C}^M$),

$$U(cf + g) = (cf + g) \circ T = c(f \circ T) + g \circ T = cU(f) + U(g),$$

thus we can consider eigenvalues $\lambda$ and eigenfunctions $\phi$,

$$U(\phi) = \lambda\phi.$$

Research into the spectral expansion of the Koopman operator for data-driven analysis of dynamical systems was initiated by [46, 47].

Considering the repeated action of $U$, $p$ times, on a finite collection of $n$ observables $\{f_1, \ldots, f_n\}$ that lie in the span of a finite collection of $m$ eigenfunctions $\{\phi_1, \ldots, \phi_m\}$, with eigenvalues $\{\lambda_1, \ldots \lambda_m\}$ and dual basis $\{\psi_1, \ldots, \psi_n\}$, leads to a special case of the Koopman mode decomposition (KMD):

$$\begin{bmatrix} U^p(f_1) \\ \vdots \\ U^p(f_n) \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{m} \lambda_k^p \phi_k \psi_k(f_1) \\ \vdots \\ \sum_{k=1}^{m} \lambda_k^p \phi_k \psi_k(f_n) \end{bmatrix} = \sum_{k=1}^{m} \lambda_k^p \phi_k \begin{bmatrix} \psi_k(f_1) \\ \vdots \\ \psi_k(f_n) \end{bmatrix}. \tag{6.2}$$

This decomposition allows us to evolve our observables $\{f_1, \ldots, f_n\}$ by simply multiplying each term in the sum above by its corresponding eigenvalue $\lambda_k$. This gives us the intuitve interpretations of the magnitude and complex phase of $\lambda_k$ as corresponding to rate of growth and rate of oscillation, respectivley. If we are only interested in the evolution of the observables along a single trajectory of (6.1) starting at $x$, then the $\phi_k's$ could be chosen so that $\phi_k(x) = 1$, and KMD would take the following simpler form

$$\begin{bmatrix} [U^p(f_1)](x) \\ \vdots \\ [U^p(f_n)](x) \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{m} \lambda_k^p \phi_k(x) \psi_k(f_1) \\ \vdots \\ \sum_{k=1}^{m} \lambda_k^p \phi_k(x) \psi_k(f_n) \end{bmatrix} = \sum_{k=1}^{m} \lambda_k^p \begin{bmatrix} \psi_k(f_1) \\ \vdots \\ \psi_k(f_n) \end{bmatrix}. \tag{6.3}$$

### 6.1.1 Koopman-Dynamic Mode Decomposition

The last column vector appearing in (6.2) and (6.3) is known as the Koopman Mode of the observables $\{f_1, \ldots, f_n\}$ relative to the discrete-time system descibed by (6.1). From (6.3) we see that the magnitude of $\psi_k(f_i)$ tells us how much of a role the growth and oscillations rates given by $\lambda_k$ play a role in evolution of $f_i$ along the single trajectory starting at $x$. Similarly, the complex phase of $\psi_k(f_i)$ gives a relative phase correspsonding to the oscillations give by $\lambda_k$.

Motivated by the fact that several results have been shown regarding the approximation of the Koopman operator and its spectral quantites by dynamic mode decomposition (DMD) [48, 49, 50, 51, 52], we have used it on building data; first we give a brief description of DMD.

Consider the case where we have a discrete-time dynamical system as in (6.1) and we have evaluated a set of observables $\{f_1, \ldots, f_n\}$ along the first $m + 1$ time points of a trajectory starting at $x$. We can put this into a matrix $D$ such that each row corresponds to a different observable and the columns are ordered by time:

$$D = \begin{bmatrix} f_1(x) & f_1(T(x)) & \dots & f_1(T^m(x)) \\ \vdots & \vdots & \ddots & \vdots \\ f_n(x) & f_n(T(x)) & \dots & f_n(T^m(x)) \end{bmatrix}$$
$$= \begin{bmatrix} f_1(x) & [U(f_1)](x) & \dots & [U^m(f_1)](x) \\ \vdots & \vdots & \ddots & \vdots \\ f_n(x) & [U(f_n)](x) & \dots & [U^m(f_n)](x) \end{bmatrix}.$$

We can split $D$ into the matrix $X$ of the first $m$ columns and $Y$ of the last $m$ columns, i.e.

$$X = \begin{bmatrix} f_1(x) & f_1(T(x)) & \dots & f_1(T^{m-1}(x)) \\ \vdots & \vdots & \ddots & \vdots \\ f_n(x) & f_n(T(x)) & \dots & f_n(T^{m-1}(x)) \end{bmatrix},$$
$$Y = \begin{bmatrix} f_1(T(x)) & f_1(T^2(x)) & \dots & f_1(T^m(x)) \\ \vdots & \vdots & \ddots & \vdots \\ f_n(T(x)) & f_n(T^2(x)) & \dots & f_n(T^m(x)) \end{bmatrix}.$$

The main idea in Dynamic Mode Decomposition (DMD) is to find a matrix $A$ such that $AX$ is close to $Y$ in some sense. In this way $A$ would be mapping the vector of samples taken at the point $T^i(x)$ close to those taken at the point $T^{i+1}(x)$, for all $i \in \{0, \dots, m-1\}$. With this, it may seem intuitive to the reader that as our number of time points goes to infinity or as we add more observables (which could just be functions of our original ones), we should better and better approximate the Koopman operator $U$ by the linear operator we are representing by the matrix $A$. One such $A$ we could use is $A = YX^\dagger$, where $X^\dagger$ is the pseudo-inverse of $X$. Such an $A$ happens to satsfy the following [53]

$$\|AX - Y\|_F = \inf_{B \in \mathbb{R}^{m \times n}} \|BX - Y\|_F.$$

If we let $M = \mathbb{R}^n$ and $T(x) = Ax$ as in equation (6.1), then DMD of the data matrix $D$ boils down to a KMD of the functions $\{f_1, \ldots, f_n\}$ with respect to the system descibed by equation (6.1). In paticular, given any basis $\{v_1, \ldots, v_n\}$ of eigenvectors of $A$ (assuming $A$ is diagonalizable as the generic matrix is) with eigenvalues $\{\lambda_1, \ldots, \lambda_n\}$, its dual basis $\{\psi_1, \ldots, \psi_n\}$ is a set of $n$ eigenfunctions of the Koopman operator induced by equation (6.1), also with eigenvalues $\{\lambda_1, \ldots, \lambda_n\}$. Finally, the eigenvectors of $A$ serve as the corresponding Koopman modes.

## 6.2 Sensor Implementation

In order to understand effect of occupancy patterns on temperature variations, we have incorporated Omron Environment Sensors into our laboratory space. In this section we describe their implementation and data analytics. *The use of the sensors in residential building energy savings is quite promising and the current implementation with single thermostats is found woefully inadequate.*

The data is exported from the sensors and stored in a CSV file. The data that we present the analysis for is for the period May $6^{th}$ 0:00 to May $11^{th}$ 0:00. The outdoor temperature was gathered from the National Weather Service, taken by Santa Barbara Municipal Airport, which is right next to UCSB. In Figure (6.4) we show the plot of the indoor and outdoor temperatures. The sensors detect temperature, humidity, light, UV index, pressure, and sound in one compact package. The Figure (6.1) shows the full schematic of the sensor. The sensor uses Bluetooth to sync with a cell phone, as seen in Figure (6.2). We collected the data on 5-minute intervals.

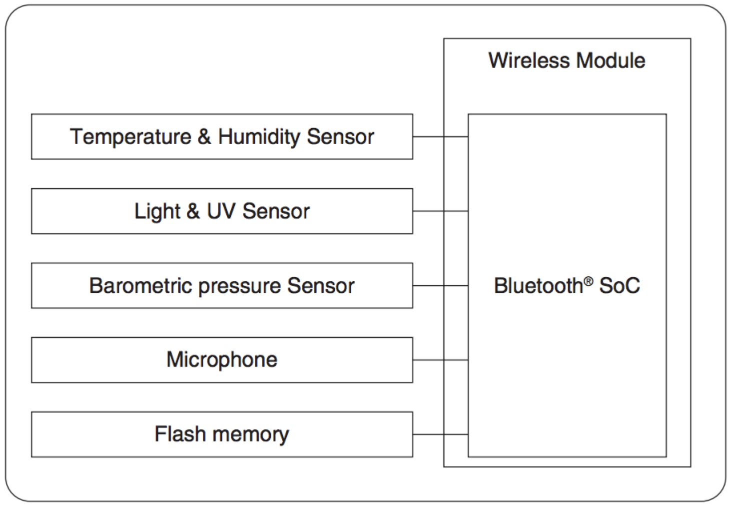



Figure 6.1: Block diagram of the sensor module [54].

We kept the sensors at similar heights and also kept them away from any blowing air from the HVAC systems. We also kept to keep the sensors at a reasonable distance away from the light sources and any pipes that may radiate heat. Figure (6.3) shows the area where the sensors are placed throughout the office area.

The data is exported from the sensors and stored in a CSV file. The data that we present the analysis for is for the period May $6^{th}$ 0:00 to May $11^{th}$ 0:00. The outdoor temperature was gathered from the National Weather Service, taken by Santa Barbara Municipal Airport, which is right next to UCSB. In Figure (6.4) we show the plot of the indoor and outdoor temperatures.

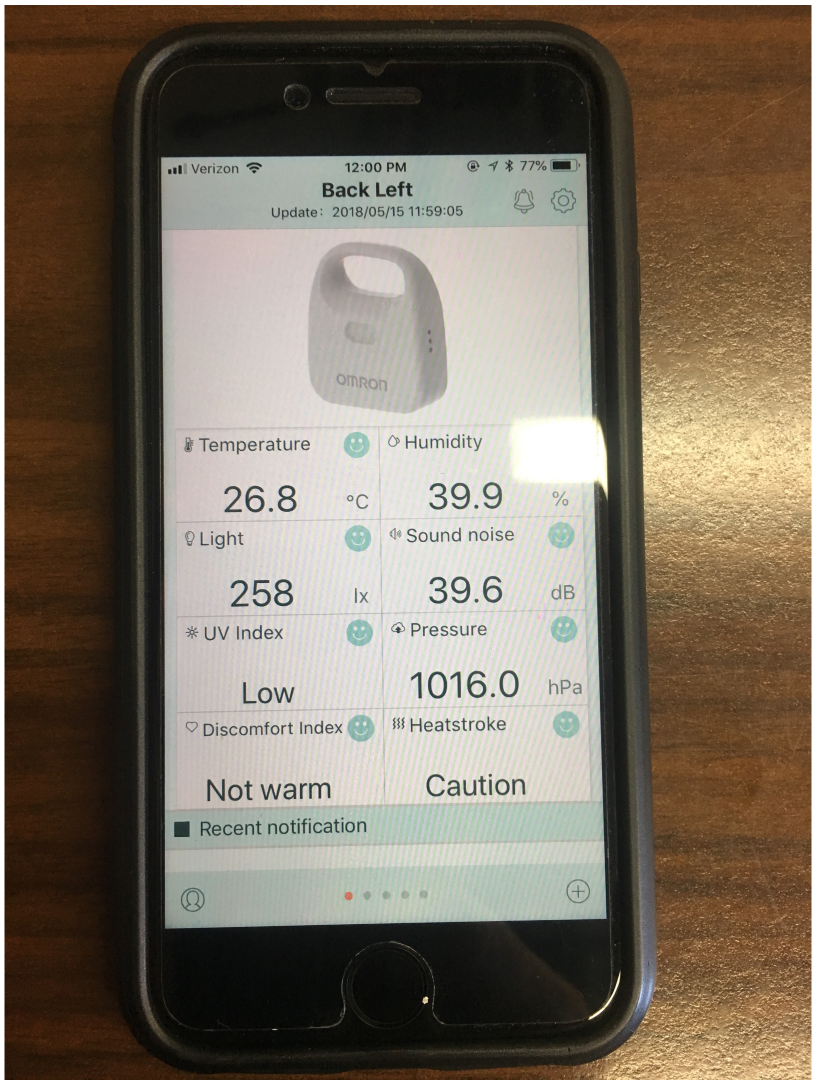
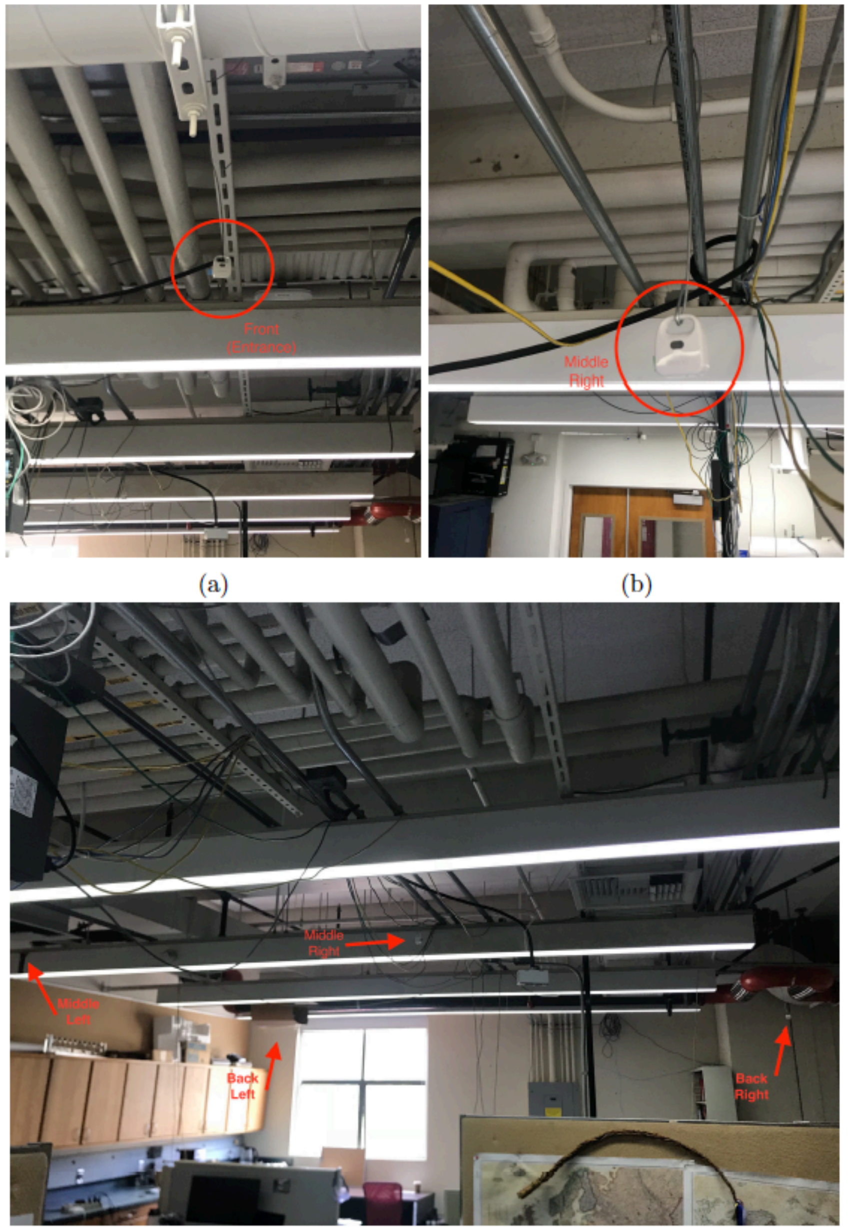



Figure 6.2: Environment sensor application used to export and examine data collection.




(a) (b) (c)

Figure 6.3: Sensor placement throughout the office. (a) shows the front sensor right as one walks into the office. (b) shows the sensor in the middle part of the office labeled as middle right, also can see relation to door from that picture. (c) shows the general placement of the other sensors and their corresponding names.

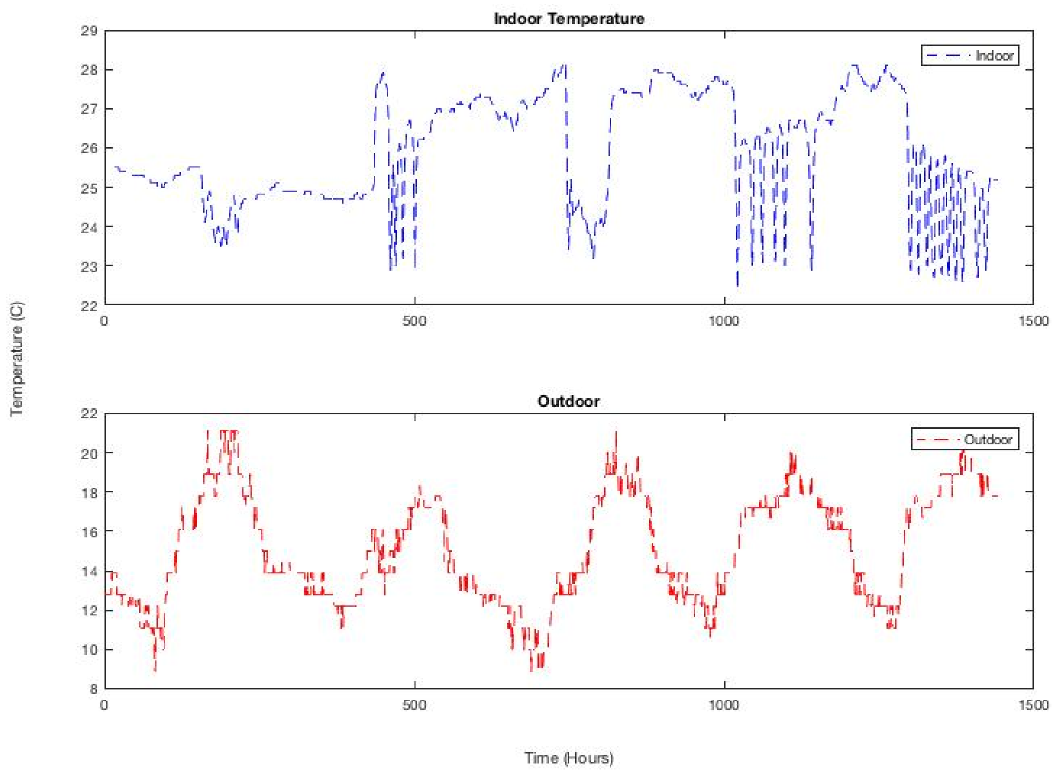



Figure 6.4: Indoor-Outdoor Temperature for May $6^{th}$ 0:00 to May $11^{th}$ 0:00 in our office. This data was taken from the middle right sensor.

## 6.3 Analysis of Temperature Data

We have collected temperature readings at five locations within our Laboratory at UCSB and an outdoor reading from the Santa Barbara Municipal Airport ($\sim 2$ miles away). In addition, we also have humidity, light, pressure, and noise measurments for the indoor sensors. Measurements from these sensors (along with outdoor temperature) taken every 5 minutes over the course of 5 days can be found in figure (6.5). In the following we describe a basic use of Koopman spectral analysis on this data. In the second section we give a simple description of the Koopman view of dynamical systems. In the third section we introduce the basics of dynamic mode decomposition (DMD) and briefly explain its connection to Koopman mode decomposition (KMD). Finally, we apply this framework on the temperature signals we have in figure (6.5).

### 6.3.1 Dynamic Mode Decomposition on Sensor Data

In the following, we perform Koopman spectral analysis on the temperature data in figure (6.5) via the use of DMD. In particular we used the DMD algorthim in [55], which includes a way to assign a "power" that each DMD mode represents in the data matrix $X$. In addition, since the algorithm invovles inverses of the diagonal matrix of singular values of $X$, we treat any singular value less than $\frac{\sigma}{10^6}$ as zero, where $\sigma$ is the largest singular value.

The main assumption here is that there is some underlying dynamical system which describes the real world dynamics of our building which we measure in a localized area of the laboratory, with a measurement rate (every 5 minutes), and that we can represent the restriction of this dynamical system to a discrete time set (with difference between time points corresponding 5 minutes) as in (6.1). We can then use DMD to approximate the Koopman operator $U$ induced by $T$. To do this we need to choose a set of observ-

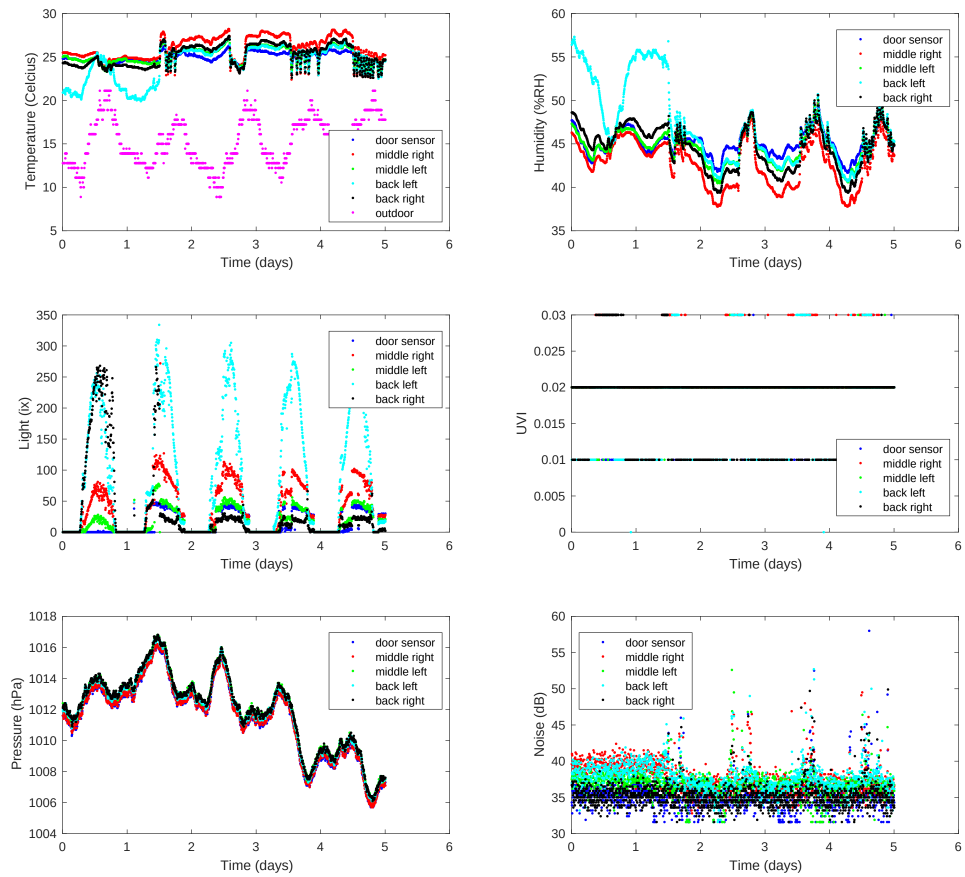

Temperature (Celcius)
Time (days)
door sensor
middle right
middle left
back left
back right
outdoor
Humidity (%RH)
Light (lx)
UVI
Pressure (hPa)
Noise (dB)

Figure 6.5:

ables. Here we use the temperature measured at 5 different points in our office and the local outdoor temperature; even though we do not know the state-space of the underlying system we are observing,i.e. the domain of $T$, we assume that our temperature measurements are a sampling of a set of observables at a single point in state-space and at a single point in time. We denote these corresponding observables by $\{f_1, \ldots, f_6\}$. We also use time shifted versions of these observables to generate new ones, i.e. we could use

$$f_7 = f_1 \circ T.$$

Lastly to compute the optimal amount of time delays used in the construction of the data matrix, we use the following for relative error

$$\frac{\|AX - Y\|_F}{\|Y\|_F} \tag{6.4}$$

Due to our system only consisting of six obseravbles and have data sampled at 5 minute intervals, we have a very 'fat' matrix meaning we have an under determined system.

In the following figures we have several plots representing Koopman spectral quantities related to our chosen obsevables. In figures (6.6) and (6.9) we use 200 and 300, respectively, time shifted observables for each of our six temperature sensors. Both of these figures are structured in the same way:

1. The top left plot contains the frequency of each DMD eigenvalue with its power. Notice the two bumps in the spectrum around a frequency of 15 and 25.

2. The top right plot contain the growth rate of each DMD eigenvalue with its power.

3. The second row contains a representation of the first six components of the highest power DMD mode with frequency close to one. The magnitude (left) and complex

phase (right) of each of the components of this mode are represented using colors over a simplified diagram of our office.

4. The third row is similar to the second, but this row represents the highest power mode with frequency near 15. Notice the similarity across changes in observables.

   The mode with frequency near one has much larger magnitude for the outside component than for any inside component as we would expect since indoor temperature should be more stable and the outdoor temperature has an obvious one day period. The bumps in the frequency plot near frequencies of 15 and 25 are another consistent feature amongst the plots for different numbers of time-shifted observables. The second DMD mode represented is for the dominant frequency in the first of these bumps. This mode, having a period near 1.5 hours, has much larger components indoor than outdoor, and is related to heating and cooling control.

5. Figure (6.12), shows the relative error compared to the number of delays used in the hankel matrix. Relative error was computed using equation (6.4).

The analysis using Koopman modes gives us a remarkable insight into the thermal dynamics of indoor spaces. Namely, the distinction between the zones affected strongly by the outside conditions, near the window, are evident. In addition, there are modes that clearly indicate dynamics of the controllers. In this way, the external influences are separated from control dynamics and the refinement of the analysis using the model in the previous section is possible. *For use in residential buildings, such understanding of thermal behavior is of essence in design and control and we have provided the tools for it.*

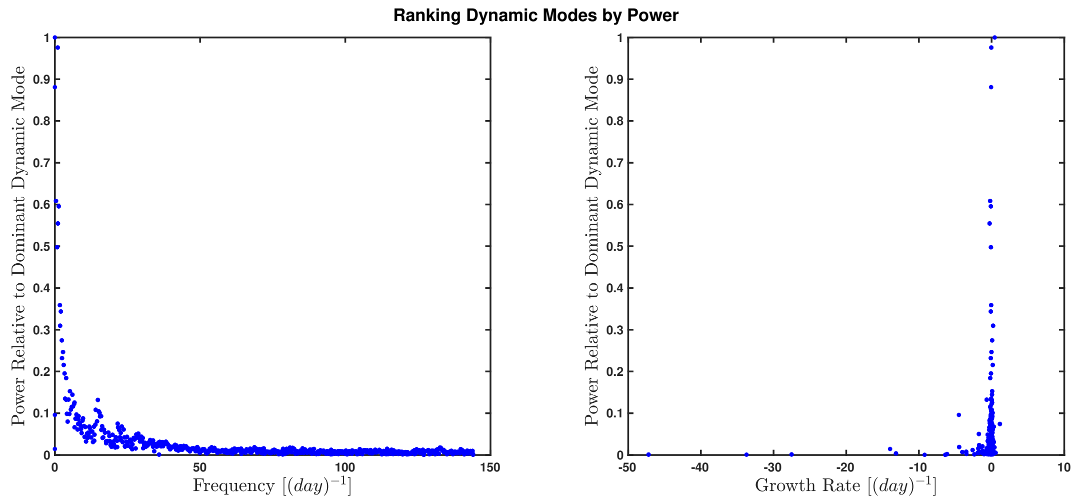



Figure 6.6:

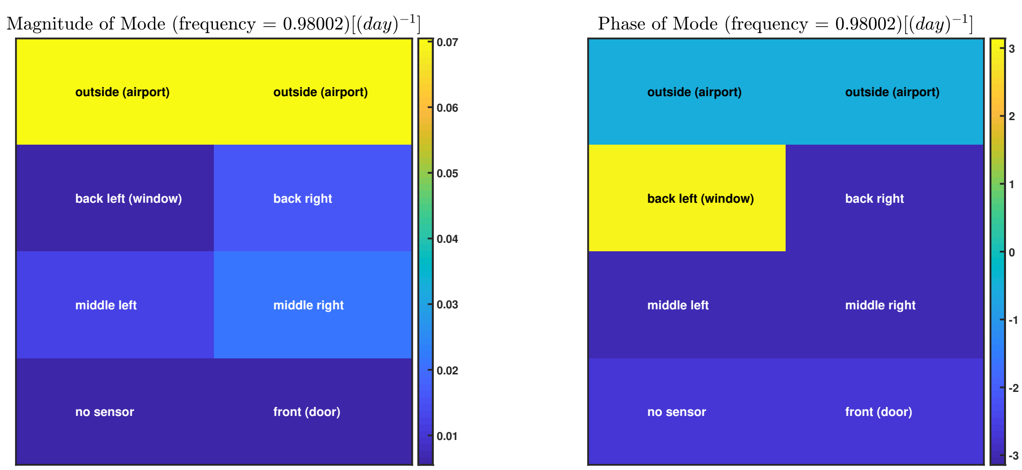



Figure 6.7:

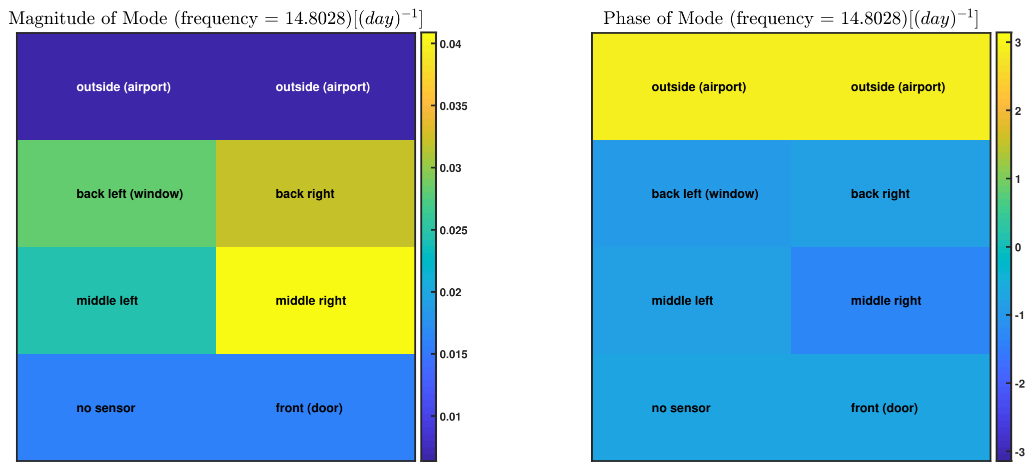



Figure 6.8:

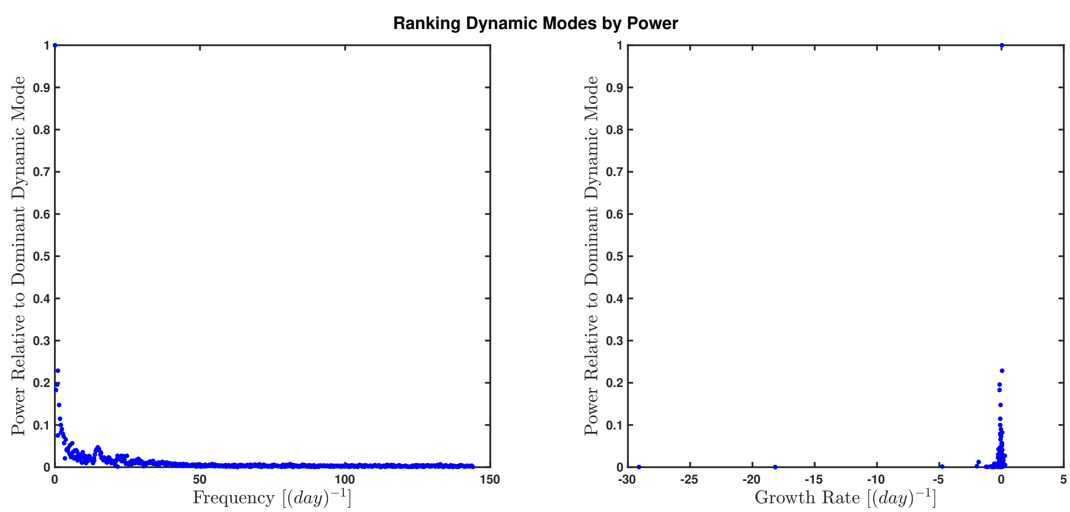



Figure 6.9:

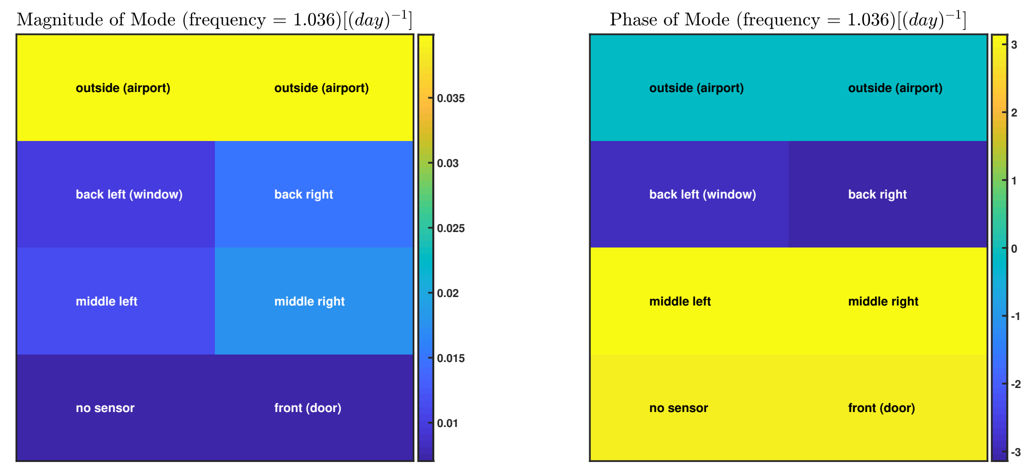



Figure 6.10:

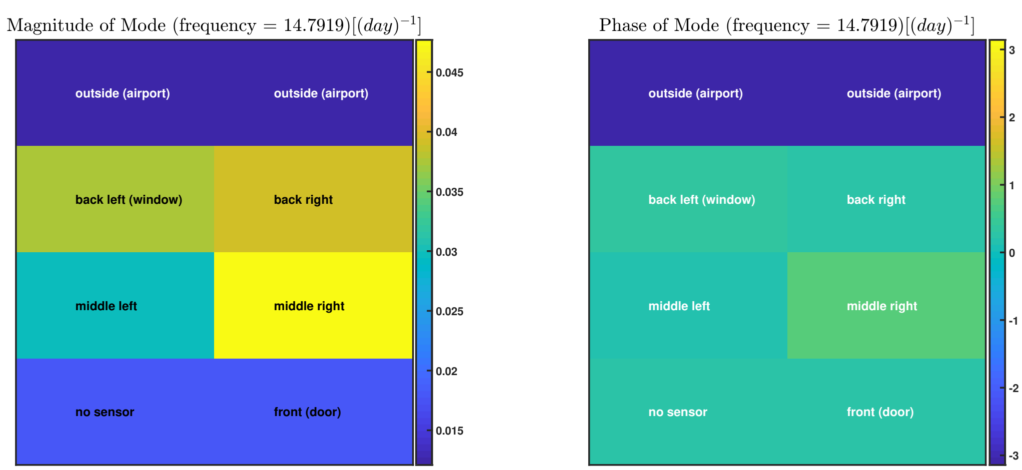



Figure 6.11:

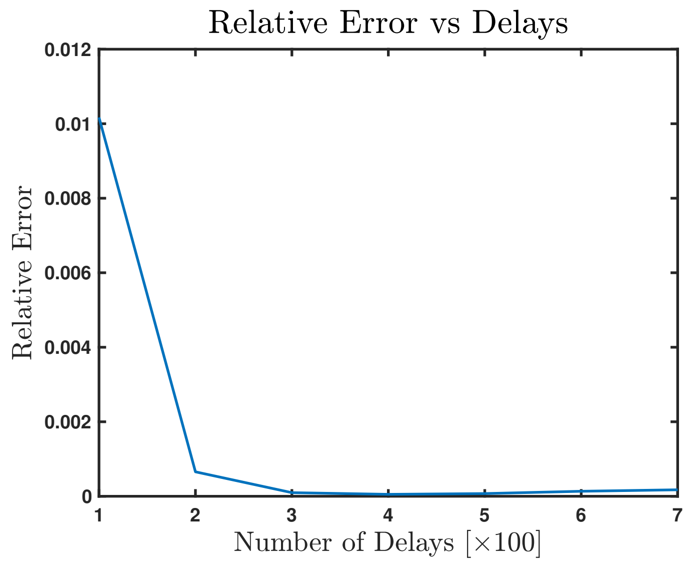



Figure 6.12:

### 6.3.2 Dynamic Mode Decomposition on Simulation Data

We have also run simulations using EnergyPlus and found interesting behavior going on in the dynamics of the system. In these simulations we used the -HouseModelDefine– and created actuation or a simple kick in the system. This kick to the system was to turn off all loads like before but now to add heating or cooling to the system at a particular time. In this case we added heating of $20°C$ to the simulation between the hours of 7am to 8am. This now presents a controlled environment to our system where this will repeat throughout the entire simulation. Figure (6.13) shows the signals analyzed corresponding to 5 minute sampling.

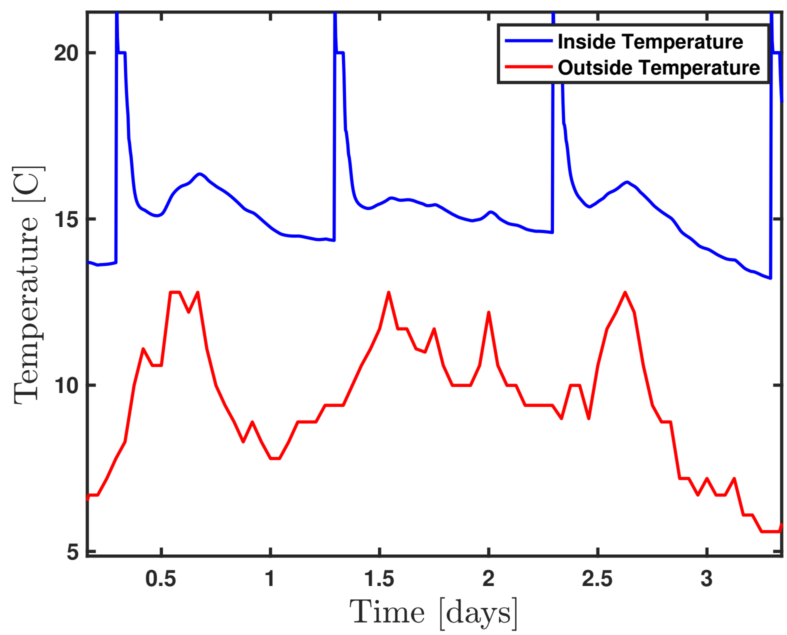



Figure 6.13:

1. The top row of plots in Figure (6.14) contains the frequency of each DMD eigenvalue with its power (left) and the growth rate of each DMD eigenvalue with its power (right). These plots only correspond to the indoor signal with 600 time shifted observables.

2. The second row has figure (6.15) has frequency vs growth rate along with the coloring of each Dynamic Mode relative to it's power.

3. The third row has figure (6.16) has frequency vs growth rate along with the coloring of each Dynamic Mode relative to it's power but is a close up to look at the Dynamic Modes close to the highest power.

4. Last row, shows the relative error compared to the number of delays used in the hankel matrix, again computed from equation (6.4).

When analyzing the system we notice something interesting. We know that we gave this kick to the system to be everyday between the hours of 7am and 8am but that is not what the data analysis is showing us. From the second and third row of figures we notice the highest power Dynamic Mode corresponding to a frequency of 3 which means a period of 8 hours. This is something strange due us knowing we kicked the system between 7am and 8am but then didn't re-kick the system for another 24 hours. This is capturing the normal mode or the daily harmonic in the system which turn out to be that actuation that occurs daily. This is the mode that is also associated with the interaction with natural forcing or in our case heating. Another Dynamic mode that we observe is the growing mode which also corresponds to the heating in the system at a frequency of 1 or once per day. We can see this even from the data due to the indoor temperature rapidly growing once the heating is turned on for the one hour period.

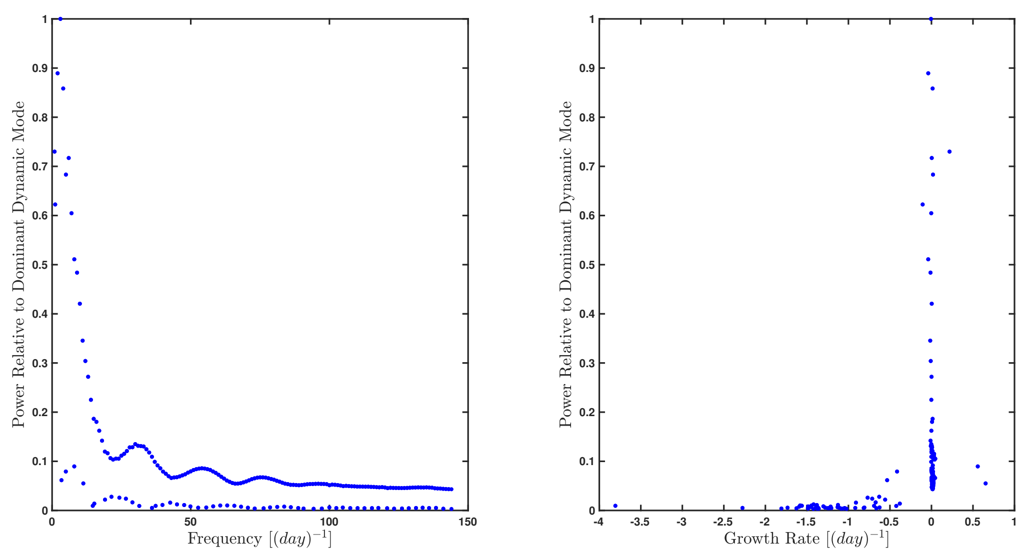



Figure 6.14:

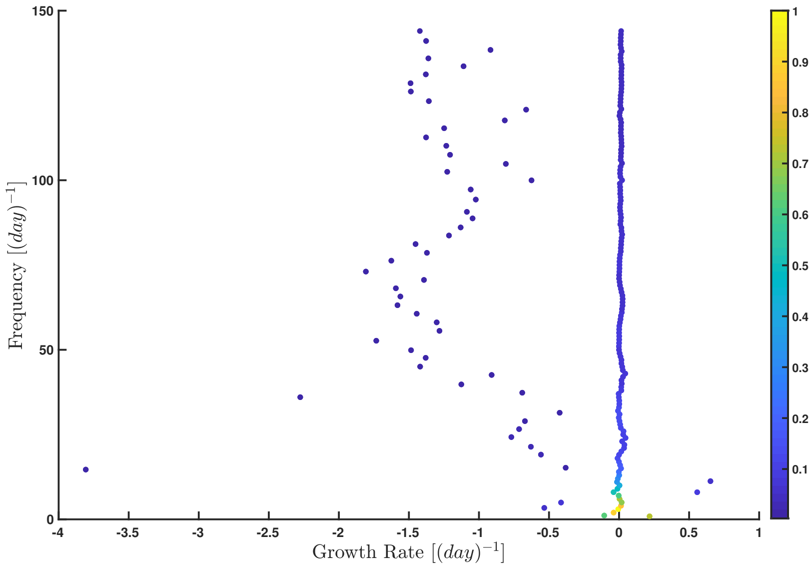



Figure 6.15:

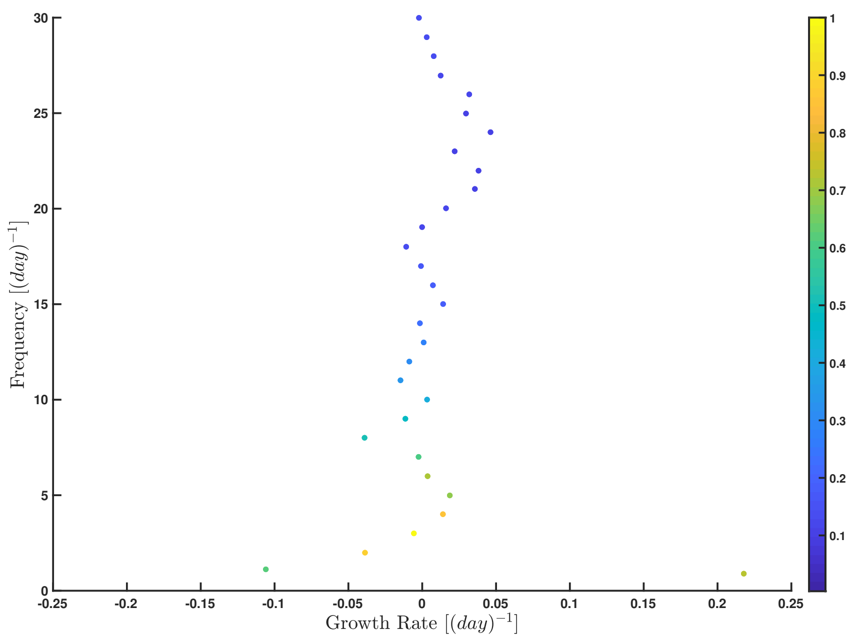



Figure 6.16:

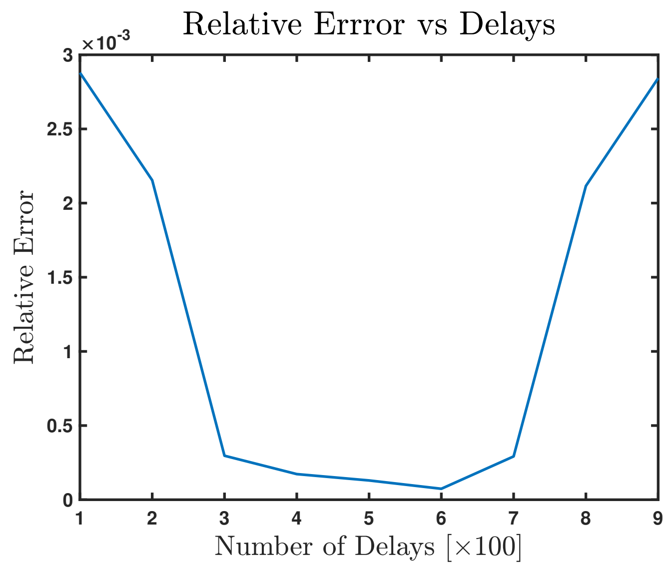



Figure 6.17:

### 6.3.3 Koopman Relation to Reduce Order Model

In order to see if a ROM can fit temperature data of a particular space which also has control (heating or cooling), we can examine the eigenfunctions of the following,

$$\begin{aligned}\dot{\lambda} &= \lambda\phi \\ &= \Big[Re(\lambda) + iIm(\lambda)\Big]\Big[Re(\phi) + iIm(\phi)\Big]\end{aligned}$$

This can now have a further expansion,

$$\Big[Re(\phi) \dot{+} iIm(\phi)\Big] = \Big[Re(\lambda)Re(\phi) - Im(\lambda)Im(\phi)\Big] + i\Big[Im(\lambda)Re(\phi) + Re(\lambda)Im(\phi)\Big]$$

Finally this system can be simplified to something that looks like a second order linear model,

$$\begin{bmatrix} Re\dot{(}\phi) \\ Im\dot{(}\phi) \end{bmatrix} = \begin{bmatrix} Re(\lambda) & -Im(\lambda) \\ Im(\lambda) & Re(\lambda) \end{bmatrix}\begin{bmatrix} Re(\phi) \\ Im(\phi) \end{bmatrix} \tag{6.5}$$

Based on the analysis done above, we can see equation (4.2) and (6.5) relate and both correspond to a second order linear system. We originally derived the second order linear model from simulations and basic physic principals and now we can see even from a data-driven approach we can establish a close connection to that model. This is extremly helpful to have a linear model because we can relate systems and hope to extract more important details of the system such as control or even possibly thermal properties.

# Chapter 7

# Conclusion

Building good energy models is required in order to obtain the energy efficiency standards required. In our work, we have seen good energy models developed and implemented in the commercial sector but not not in the residential areas which make up the majority of our energy use. We have proposed a reduced order model that can serve as a basis for energy saving algorithms and cost-efficient in terms of computation time. Along with implementing smart sensing technologies to build this "house as a system" methodology, we can achieve the Zero Net Energy goal by 2020 and help millions of home owners save money. Our approach of the model proposed here is supporting single or multi-zone buildings and different thermal properties of the structure. We have demonstrated a method based on spectral properties of the Koopman operator to help analyze temperature data from a controlled standpoint. Using KMD we have been able to extract the dynamics of the controllers in that particular zone.

Future work would consist of adding full control to the simulations of not only heating and cooling but variable thermal mass. Variable thermal mass is seen in homes now which have heated floors, i.e pipes under the tiles that get hot/cold water pumped through them when needed. The Koopman model predictive control framework [56] proceeds by constructing a linear model for the building from data. Since the building is nonlinear (e.g. due to the nonlinearities introduced by low-level control), the state of this linear model has evolved to high-dimensional, so called lifted state-space, in order for accurate predictions to be obtained. This linear model is subsequently used within the model predictive control (MPC) framework to obtain high-performance control for the building. The distinctive feature of the Koopman MPC is its ability to handle nonlinear dynamics within a purely convex optimization-based framework, thereby allowing for rapid evaluation of the control as required in practical applications. In particular, the MPC framework allows for a seamless incorporation of constraints (e.g., actuator saturation or room temperature bounds) while optimizing a user-specified objective function (e.g., energy consumption). Also importantly, the linear predictor is constructed purely from data collected from the building without the need for a first-principle model, which dramatically reduces modeling costs.

We believe the use of Model Predictive Control in conjunction with data analytics and embedded model of the type we have discovered *will lead to deep energy savings* of the magnitude that was estimated in the reviewed literature, and go towards the goal of building and retrofitting for Zero Net Energy. With the work we have done so far, we have identified this ROM with percent differences as low as 3.5% and shown KMD analysis can extract the nonlinear behavior from a controlled environment setting.

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