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Career-Hopping: Learning and Turnover in an Imperfect Labor Market

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Abstract

This paper studies a two-sector model of learning-by-doing that is partially transferable between sectors. There is a potential efficiency gain from intersectoral turnover when the sectors have different complementary production costs or learning curves of different steepness. If workers are liquidity constrained then there is a bias towards increased intersectoral turnover, resulting in socially inefficient career patterns. Excess turnover can even result in lower average productivity of workers in both sectors. If individual productivity is decreasing towards the end of the career, then a liquidity constraint on the young workers will also cause retirement to be delayed beyond the socially efficient retirement age. (JEL codes D31, J62)

1 Introduction

It is well known that common market imperfections tend to reduce investment in human capital. For example, if young workers have limited ability to borrow then there will be insufficient training for general skills (i.e., skills that are transferable across firms), and worker productivity will be inefficiently low. The possibility of insufficient learning-by-doing has not been raised, possibly because it may seem impossible—learning-by-doing being by definition an unavoidable side-effect of working. This paper will show that inefficient learning-by-doing, although

less obvious than low levels of education and job training, belongs to the same list of potential problems arising from individual liquidity constraints. I use a two-sector model of learning-by-doing that is partially transferable across sectors to study the accumulation of human capital, turnover, and wages. I show that a liquidity constraint causes increased turnover between sectors, as workers are attracted to careers that result in flatter wage profiles. The loss in workforce productivity compared to the efficient benchmark is partly due to excessive wastage of accumulated skills, and partly to a distorted allocation of labor across sectors.

The efficient rationale for intersectoral turnover is that it allows a kind of "trade" in human capital between sectors. If one sector has lower complementary costs of production (e.g., cheaper machines) then it can make sense to produce there some of the human capital used in the "more expensive" sector, even if the resulting learning is not quite as effective. Also, if the learning curve is relatively shallow in one sector, then inexperienced workers have a comparative advantage there and it can be efficient to have some turnover, again, if not too much of the learning is lost in the switch between sectors. There is inevitably a trade-off as one sector ends up with a smaller proportion of experienced workers, and thus with a lower average worker productivity, than it would without turnover. A liquidity constraint distorts workers' turnover decisions, because it directs them to obtain job experience where it requires a lower initial outlay, not where the return to investment would be highest. I find that the turnover induced by the liquidity constraint can lower the average productivity of workers in both sectors, and in some cases it can even reverse the direction of turnover.

The model in this paper combines a competitive equilibrium in one labor market and in two product markets. (There is also a large "outside" sector that fixes the lifetime utility of workers so this is not a general equilibrium model). To keep the model as simple as possible in other respects, the setup has completely predictable learning-by-doing and perfect competition. Workers are ex ante homogeneous with differences between workers only arising from differences in job experience. The equilibrium level of turnover, as well as the prices and wages in the two sectors, depend on the combination of the production costs and the slopes of the learning curves in the two sectors, and the degree of portability (how useful is experience in one sector in increasing productivity in the other). The analysis compares the equilibrium under a worker liquidity constraint with the efficient benchmark of unconstrained worker financing.

The nature of distorted learning-by-doing is simplest to understand by first considering a single isolated sector. Under lifelong wage contracts workers would be willing to work at the constant outside wage, and the equilibrium price of output—the price at which firms break even—would only depend on workers' average lifetime productivity. (This would be the efficient solution). In the absence of long-term contracts, firms must break even from every worker every period, so even while employing a novice. But novice-hiring firms cannot break even

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at the efficient price unless the novices "subsidize" them—if novices are unable to do so, then the equilibrium price of output will have to rise. This distortion is increasing in the steepness of the learning curve. With multiple sectors, the distorted prices also distort individual career decisions (where to start and whether to switch sectors). The effect on wages turns out to be somewhat complicated because it depends also on the relative demand for the output of the two sectors; in most cases one of the sectors absorbs the change in prices and wages and the other merely ends up with a different age and skill distribution among its workers.

Why should workers have to pay for job experience that accrues without any active investment by the firm? Think of a simple job where a worker operates a machine. If there is substantial learning-by-doing, then most of the value created while a novice worker is operating the machine comes in the form of increased productivity for that same worker—and she is, inexorably under modern labor market institutions, the owner of her own human capital. If the machines are very expensive, then the market value of this "side-benefit" may be so large that a mere wage cut (relative to the outside wage) is not enough to pay for the efficient market price of this learning opportunity and first best efficiency requires novices to literally pay to work. However, this does not imply that a binding liquidity constraint results in a zero starting wage (unless workers have linear utility—I assume a utility function with the standard properties) but merely that the starting wage is not as much below the outside wage as it would be in the efficient solution.

The model suggests that turnover would flow from sectors with lower capital intensity and less steep learning curves to those with higher capital intensity and steeper learning curves. The reason is that, in the absence of turnover, the former would allow for higher starting wages than are possible in the latter, precisely because of the smaller difference in the market value of output between novices and experienced workers. Liquidity constraints cause excessive turnover because workers put more weight on avoiding low starting wages and less weight on the effectiveness of learning. The resulting inefficient turnover can also be interpreted as cross-industry poaching. For example, it could be efficient for the investment banking sector to "grow" its own quantitative analysts on the job. In the absence of hiring from other sectors, some novices would inevitably be hired into investment banking in equilibrium, despite the mutual poaching problem among the employers. However, if it is possible to hire mid-career physicists as "somewhat qualified" analysts, then they can displace novice-hiring by investment banks. If young workers are liquidity constrained, then this can happen even if the job experience of physicists is not very useful in investment banking, as long as the switching physicists are more productive than complete novices. The task of novice-hiring gets dumped to another sector, not necessarily because it is efficient, but because hiring novices has an element of a public good from the point of view of the banking industry. If novices were able to pay the efficient market price for their

job experience, then they would choose a switching career only if it maximized their lifetime income, which in turn happens only if doing research in physics is actually the socially efficient way to produce human capital for investment bankers.

The modern literature about on-the-job investment into skills was started by Becker (1962) who made the point that only the enhancement of general skills needs to be paid by the worker—with the implication that borrowing constraints reduce industry-specific training. Ben Porath (1967) presented the life-cycle model of human capital where workers face a trade-off between "activities that produce earnings and additions to the stock of human capital." The classic setup that the current paper is most closely related to is Rosen (1972), where worker productivity increases purely through learning-by-doing, i.e., skill enhancement is not rivalrous with current output in any given job.[1] There firms create jobs with various levels of learning content, workers build their career ladder from a succession of jobs with decreasing emphasis on learning, and there is an implicit market for learning opportunities where young workers "buy jobs" by accepting lower wages.[2] However, the creation of high-learning positions is not inevitable in the setups of Rosen and Ben Porath: if one were to add a worker liquidity constraint to these models, then the most expensive jobs—those with the highest learning content and aimed at the youngest workers—would simply disappear. One way to interpret why this is so is that in these models every type of a job produces the same exogenously priced output: if we thought of types of jobs as "sectors" then they would all be facing a perfectly elastic demand curve. In the current paper the jobs with different learning content produce output for different product markets with their own downward sloping demand curves. (Thus the demand for investment banking services inevitably results in the creation of investment banking jobs, no matter how much learning these entail). In this setup, the implicit prices of jobs result from an equilibrium that combines free entry conditions in both product and labor markets. Market imperfections raise the price of output and can reduce employment in a sector, but could not completely wipe out a type of a job, no matter how valuable the learning that it contains, except if the imperfections push the price of output above the choke price.[3]

I also consider two extensions to the model, in order to study the efficiency of the timing of entry and exit of workers in and out of the labor force. In the first, one of the sectors has

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[1]Killingsworth (1984) made this point to distinguish on-the-job training from learning-by-doing.

[2]Park (1997) adds a more detailed learning process by assuming that it is the older co-workers in the same firm that do the teaching; then part of the returns to learning come in the form of earning "tuition income" from the next generation. Sicherman and Galor (1990) analyze the effect of initial education on the probability of subsequent "occupational upgrading." Both models assume perfect capital markets.

[3]The effect of borrowing constraints on occupational choice has also been studied in development economics, with emphasis on the implications for the distribution of income; see Banerjee and Newman (1993) and Ghatak, Morelli and Sjöström (2001).

a production cost but no current output—this is interpreted as education. For education to be efficient it has to either result in faster learning than actual job experience or require fewer resources than learning-by-doing (as in operating a training machine as opposed to a real machine). I show that moderately liquidity constrained workers obtain too much education, when job experience would result in a steeper income profile than a (socially) inefficient education. In the other extension, individual productivity is decreasing near the end of the career; this means that it would be socially efficient for workers to retire (and to vacate their "machines") strictly before their productivity has fallen to the level of a novice, because the latter can also benefit from the learning opportunity that comes with the job. In this case a liquidity constraint results in a bias towards late retirement because novice workers are unable to pay the efficient price for jobs that is needed to induce older (and contemporaneously more productive) workers to retire from the industry.

I begin by introducing the one-sector "workhorse" model in Section 2; the intuition and results developed in this simple setup are needed to understand the more complex two-sector case. Section 3 characterizes the equilibrium with two sectors. The extensions to education and retirement are analyzed in Sections 4 and 5, and the paper is concluded with a discussion of the results in Section 6.

2   The one-sector model

The main building block for the analysis in this paper is the model of a single isolated sector. The assumptions of the one-sector model are as follows.

A1. There is an unconstrained supply of identical inexperienced workers with the outside opportunity of a constant wage of $\omega > 0$ per period.

A2. Workers have two-period careers: a worker produces $1 - \delta$ units of output in her first period and $1 + \delta$ in her second period, where $\delta \in (0, 1)$.

A3. There is free entry by firms and a fixed production cost of $\phi \geq 0$ per worker.

A4. Workers cannot commit to long-term wage contracts.

A5. Workers have a time-separable utility function in consumption, $u(x^1) + u(x^2)$, where u is a concave function that is twice differentiable for $x > 0$ and has $\lim_{x->0}^{+} u(x) = -\infty$.

The demand curve is not explicitly needed to derive the equilibrium price in this single-sector setup, beyond the implicit assumption that there is always some demand at the prices that are considered.[4] Due to Assumptions 3 and 4, firms make zero profits regardless of who they

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[4]Higher price also leads to lower welfare via lower consumer surplus—the only welfare effect in the model—although this not explicitly considered.

employ, so equilibrium wages are equal to the contemporaneous value of output created by the worker less the production cost:

$$w(y) = py - \phi, \tag{1}$$

where y is the worker's current period output.

There is no uncertainty, strategic interaction, effort, or discounting in the model.[5] The units of output are chosen so that the average output per period over the career is one. The parameter capturing the steepness of the learning curve, δ, is unit-free (so it will later be comparable across sectors). It gives the proportion of lifetime output that is due to learning-by-doing: $\delta \equiv (y^2 - y^1) / (y^1 + y^2)$, where the superscripts refer to the career period.[6] Workers in their first period will be referred to as "novices" and those in their second period as "veterans."

Considering the case where novices have the *unconstrained* ability to pay for jobs provides the efficient benchmark. Unconstrained workers would consume half of their lifetime income in each career period, and choose between careers purely based on the lifetime income they offer. If workers are liquidity *constrained* then they consume their current wage in each period (due to learning, the novice wage is never higher than the veteran wage). The efficient prices will be denoted by stars and the equilibrium prices (with the liquidity constraint) by tildes. Only stationary equilibrium is considered, which implies that there is an equal mass of novices and veterans in the workforce.

General setup. Combining the zero-profit wage equation (1) with the lifetime profile of output gives the wages for novices and veterans as

$$
\begin{aligned}
w^1 &= p(1 - \delta) - \phi, \\
w^2 &= p(1 + \delta) - \phi.
\end{aligned}
\tag{2}
$$

The price of output p is the equilibrating variable determining the wages. The equilibrium price must result in wages that leave entering workers just indifferent between a career in this sector and the outside opportunity of earning ω in both periods. Denoting consumption in career period t by x^t, this condition is

$$u(x^1) + u(x^2) = 2u(\omega). \tag{3}$$

In the constrained case $x^t = w^t$ and the solution to (3) depends on the shape of the utility function. However, in the absence of a liquidity constraint, consumption equals half of lifetime

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[5]The interaction of imperfect information and labor turnover (with risk neutral workers) has spawned a large literature since Johnson (1978) and Jovanovic (1979), with asymmetric information added by Greenwald (1986).

[6]While the exogenous level of learning is a convenient simplification, it can also be interpreted as learning that depends on hours worked when the hours worked per period are essentially fixed.

income in both periods, so the utility function can be simplified out of (3). The marginal utility of unconstrained workers is the same in both periods, and their career choice problem is equivalent to that of a constrained worker with a linear utility function. Thus the efficient solution can be solved in closed form.

Efficient solution.   The equilibrium requirement is that lifetime wages $(w^1 + w^2)$ add up to 2ω. Using (2), this determines the efficient output price as

$$p^* = \omega + \phi \tag{4}$$

which is, of course, just the average cost of production.[7] The wage profile over the career is then

$$
\begin{aligned}
w^{*1} &= p^*(1-\delta) - \phi = \omega - \delta(\omega + \phi), \\
w^{*2} &= p^*(1+\delta) - \phi = \omega + \delta(\omega + \phi).
\end{aligned}
\tag{5}
$$

For workers to get the outside level of lifetime utility from these wages, they must be able to borrow $\delta(\omega + \phi)$.

Proposition 1 *Efficient solution requires that young workers finance a fraction of the total costs of production that corresponds to the share of lifetime output due to on-the-job learning.*

The required financing by the young workers is by no means bounded by ω, the opportunity cost of their own labor. According to (5), the young worker is required to finance a proportion δ of the total opportunity costs of production, which she then earns back as a wage premium when old. This is the proportion of a young worker's "output" that comes in the form of her increased future output. For moderate levels of learning and production costs the financing required from the worker would only involve a wage discount. However, if the effect of learning on lifetime output is higher than the labor's share of the costs of production, then novices would literally have to pay to work: $w^{*1} < 0$ if $\omega/(\omega + \phi) < \delta$.

In the efficient solution, the age profile of output, captured by the learning parameter δ, is irrelevant for the price of output—all that matters is the average output per period, which was normalized at unity. The shape of the learning curve obviously matters for the shape of the wage profile over the career, but so does the complementary cost of production: the higher the costs (ϕ), the steeper the wage profile ($w^2 - w^1 = 2\delta(\omega + \phi)$). In professions with costly complementary inputs and substantial on-the-job learning the production of human capital is

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[7]Efficiency would also be achieved by binding long-term wage contracts. Firms would offer, and the entering workers would agree to, career-long contracts at the outside wage ω.

expensive: young workers should pay a lot to get access to a job, from which they later reap large benefits. If workers can painlessly finance the production when young, then it all adds up to no premium at all over the lifetime.

Constrained solution. Suppose now that the young workers are liquidity constrained, and, for simplicity, incapable of borrowing at all.[8] They are willing to work for below their outside wage in return for higher wages in the future, but unable to actually pay to work. The need for positive consumption and the preference for smooth consumption means that the workers require a premium in terms of the lifetime income if they are to bear any of the costs of production when young. The wage is higher in the second period due to learning, so constrained workers simply consume their wage in both periods. The equilibrium price of output is defined implicitly by the indifference condition of entering workers:

$$\tilde{p} = p \ \text{ st } \ u\left(p\left(1-\delta\right)-\phi\right) + u\left(p\left(1+\delta\right)-\phi\right) = 2u\left(\omega\right), \tag{6}$$

where the wages are from (2).[9]

Proposition 2 *The price of output and wages in both career periods are higher if workers are liquidity constrained.*

Proof. The proof of the first part ($\tilde{p} > p^*$) will be implied by Proposition 3 and $\delta > 0$. For the second part, combine (2) with $\tilde{p} > p^*$ to see that $\tilde{w}^t > w^{*t}$ for $t = 1, 2$. ∎

In the efficient solution the lifetime wages were 2ω and obviously unaffected by the age profile of output and by the cost of production; in the presence of a liquidity constraint the lifetime wages depend on both. The inability of workers to finance a sufficient portion of the production costs at the discount rate (set at zero here) causes real distortions, which are increasing in the amount of financing that the workers would provide in the efficient solution.

Proposition 3 *In an isolated sector with liquidity constrained workers, the price of output and lifetime wages are increasing in the steepness of the learning curve. The efficient price and lifetime wages are independent of the shape of the learning curve.*

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[8]If workers can borrow up to some $b < \delta\left(\omega + \phi\right) \equiv b^*$ then $c^1 = w^1 + b$ and $c^2 = w^2 - b$ in (3). It is straightforward to show that, for all results that follow, the distortions in prices are continuously decreasing in $b \in [0, b^*)$.

[9]There is typically no closed-form solution to (6), but with log-utility it can be solved for

$$\tilde{p} = \frac{\phi + \sqrt{\left(1 - \delta^2\right)\omega^2 + \delta^2\phi^2}}{1 - \delta^2}. \tag{7}$$

Proof. Solving the equilibrium condition (6) at $\delta = 0$ results in $\tilde{p} = p^* = \omega + \phi$. By differentiating (6) with respect to p and δ at $\delta > 0$ we get

$$\frac{\partial \tilde{p}}{\partial \delta} = \frac{u'(w^1) - u'(w^2)}{(1 - \delta)\,u'(w^1) + (1 + \delta)\,u'(w^2)}\tilde{p} > 0. \tag{8}$$

The inequality follows from $w^1 < w^2$ and concave utility, so $\tilde{p} > p^*$ for all $\delta \in (0, 1)$ and lifetime wages $2\tilde{p} - 2\phi$ are increasing in δ. ■

While the wages are higher than efficient both for young and old, the young consume less than in the efficient solution. The distortion in prices and wages is due to the workers' inability to absorb the path of lifetime income implied by the learning curve and the efficient price of output. In the absence of learning the prices would be unaffected by the liquidity constraint as there would be no need for worker financing.

Proposition 4 *In an isolated sector with liquidity constrained workers, the price of output increases more than proportionally in the production cost, and lifetime wages are increasing in the production cost.*

Proof. By differentiating the equilibrium condition (6) with respect to p and ϕ we get

$$\frac{\partial \tilde{p}}{\partial \phi} = \frac{u'(w^1) + u'(w^2)}{(1 - \delta)\,u'(w^1) + (1 + \delta)\,u'(w^2)} = \frac{u'(w^1) + u'(w^2)}{u'(w^1) + u'(w^2) - \delta\,(u'(w^1) - u'(w^2))} > 1, \tag{9}$$

where again $u'(w^1) - u'(w^2) > 0$ due to concave utility and learning. It follows that lifetime income $(2\tilde{p} - 2\phi)$ is increasing in the production cost ϕ. ■

This is in contrast to the efficient solution, where the price increases one-for-one in production costs, by (4), and lifetime wages are not affected by the cost of other inputs. With a liquidity constraint, not only does the wage profile get steeper as the role of learning or production costs are increased; lifetime wages are also increased. The reason is that a higher cost of production increases the value of future output that can be attributed to current on-the-job learning, but the novices are not able to fully pay for this increase in their productivity. To attract workers to a sector where first period wage is lower than the constant outside wage, the second period wage premium must more than match the initial discount to equalize the lifetime utility with the outside opportunity.[10]

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[10]In another paper (Terviö 2005) I explored this one-sector model when workers can choose their (contractible) effort level. In addition to competing for the positions by accepting wages lower than the outside opportunity, the novices can also offer to work harder. As a result, the young work inefficiently hard, but the wage profile is not as skewed as with a fixed effort level. There even the novice wage can be above the outside wage, if the efficient level of effort is relatively high.

3   Intersectoral Turnover

To determine whether or not there would be turnover between any two sectors requires only the additional specification of the effectiveness of cross-sector learning. However, to also model the level of turnover and its effect on prices and wages requires also the specification of demand curves facing the two sectors. The two-sector model is a combination of two one-sector models from the previous section, with subscripts $i = 1, 2$ added where necessary, and the following additional assumptions.

A6. If a worker switches mid-career from sector i to sector j, then her output as a veteran is $(1 + \delta_j)\beta$, where $\beta \in [0, 1]$.

A7. The demand curves facing the two sectors are $D_1(p_1) = \sigma p_1^{-1}$ and $D_2(p_2) = (1 - \sigma) p_2^{-1}$, where $\sigma \in (0, 1)$.

Initially I also assume

A8. Sector 1 has less learning and lower production costs than sector 2: $\delta_1 \leq \delta_2$ and $\phi_1 \leq \phi_2$, with at least one of the inequalities holding strictly.

Assumption 6 defines β as the effectiveness of cross-sector learning, with $\beta = 1$ referring to fully transferable learning. Assumption 7 defines, in a sense, the relative size of the sectors: in the absence of turnover the average output per worker would be one in both sectors, and so the size of the workforce would be $D_i(p_i)$ in sector i. It also implies that the demand for the output of the two sectors is unrelated.[11] Assumption 8 simplifies the analysis by restricting any possible turnover to be from sector 1 to 2: slower learning and cheaper production costs are the two reasons why one sector might produce job experience for the other and this guarantees that they both go the same way. However, this simplification is not without loss of generality, so the precluded case, where one sector has a steeper learning curve and the other higher production costs, is analyzed separately in Section 3.5. In any case there would never be turnover in both directions.

Table 1.   Summary of parameters

$\delta_i \in [0, 1)$	within-sector learning in sector i
$\beta \in [0, 1]$	transferability of learning across sectors
$\omega > 0$	outside wage
$\phi_i \geq 0$	production cost in sector i
$\sigma \in (0, 1)$	relative demand for sector-1 good

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[11]This demand would result from a representative consumer with Cobb-Douglas preferences and exogenous income, with σ being the relative expenditure share for sector-1 output. The exogenous income is justified if these two sectors comprise only a small part of total spending in the economy.

3.1 To turnover or not?

When is there intersectoral turnover? The absence of intersectoral turnover would require that the equilibrium prices associated with sectoral isolation do not give incentives for workers to switch sectors mid-career. Using the convention that the direction of possible turnover is from sector 1 to 2, this would mean that a worker who worked in sector 1 when young would earn less if she switched to sector 2 than if she stayed in sector 1. From this condition we can solve the threshold value for the effectiveness of cross-sector learning above which there is switching in equilibrium:

$$
\begin{aligned}
p_1 \left(1 + \delta_1\right) - \phi_1 \ &< \ p_2 \left(1 + \delta_2\right) \beta - \phi_2 \iff \\
\beta \ &> \ \frac{p_1 \left(1 + \delta_1\right) + \phi_2 - \phi_1}{p_2 \left(1 + \delta_2\right)} \equiv \beta_A.
\end{aligned} \tag{10}
$$

The prices p_i stand for either (4) or (6), depending on the presence of the liquidity constraint. In either case, this is the necessary and sufficient condition for intersectoral turnover.

Proposition 5 *Worker liquidity constraint causes a bias towards increased intersectoral turnover: the level of cross-sector transferability of learning that induces turnover is decreased ($\tilde{\beta}_A < \beta_A^*$).*

Proof. Rearrange the definition of β_A from (10):

$$
\beta_A = \frac{1 + \delta_1 + \frac{\phi_2 - \phi_1}{p_1}}{\frac{p_2}{p_1} \left(1 + \delta_2\right)}. \tag{11}
$$

First consider the numerator. It is smaller for $\tilde{\beta}_A$ than for β_A^* because $\phi_2 \geq \phi_1$, and $\tilde{p}_1 > p_1^*$ by Proposition 2. Then consider the denominator: it is larger for $\tilde{\beta}_A$ than for β_A^* if $\tilde{p}_2/\tilde{p}_1 > p_2^*/p_1^*$. This holds because $\tilde{p}_i$, unlike p_i^*, is increasing in δ_i (Proposition 3), and $\tilde{p}_i$ also increases faster than p_i^* in ϕ_i (Proposition 4), and both of the inequalities $\delta_2 \geq \delta_1$ and $\phi_2 \geq \phi_1$ hold, and at least one of them holds strictly (by Assumption 8). ■

While the liquidity constraint increases the output price in both sectors, this distortion is larger in sector 2 where the learning curve is steeper, because $\tilde{p}_i$ is increasing in the steepness of the learning curve δ_i. By contrast, the efficient prices are independent of δ_i. Therefore, in the constrained case, the transferability of skills (β) has to be lower before switching wipes out enough of the skills learned in sector 1 to deter turnover to sector 2.

3.2 Equilibrium in the Two-Sector Model

Three more variables are needed to fully characterize the equilibrium prices and the level of turnover in the two-sector model. Use $s_i \geq 0$ to denote the mass of workers entering sector

i each period, and $m \in [0, 1]$ as the proportion of sector 1 starters switching to sector 2 mid-career. Taking into account both the young and the old cohorts, this translates into $s_1 (2 - m)$ workers in sector 1 and to $s_1 m + 2 s_2$ in sector 2 in stationary equilibrium. Total outputs in the two sectors are

$$Y_1 (s_1, m) = s_1 (2 - m (1 + \delta_1)) \tag{12}$$

$$Y_2 (s_1, s_2, m) = 2 s_2 + s_1 m (1 + \delta_2) \beta \tag{13}$$

There are two types of equilibrium conditions. First, there are the product market equilibrium conditions—equality of demand and supply of output for both sectors from assumption 1:

$$Y_i = D_i (p_i), \text{ for } i = 1, 2. \tag{14}$$

Second, there are the labor market equilibrium conditions. The exact form of these conditions depends on which career paths are part of the equilibrium. There are three possible career paths: a sector-specific career for each sector, and a switching career from 1 to 2. I will refer to the combination of career paths that are chosen by some individuals in equilibrium as the "career regime."

Note that there will always be some workers who start in sector 1 ($s_1 > 0$ always holds) since that is the only way to have any workers in sector 1, and there is always some demand no matter how high the price. Similarly, there will always be some workers in sector 2, so either some workers start there ($s_2 > 0$) or switch from sector 1 ($m > 0$) or both. This allows in principle four qualitatively different career regimes, each with a different combination of binding constraints.

A. No switching. All careers sector-specific: $s_2 > 0, m = 0$.

B. Dispersed veterans. Some sector-1 careers, some switchers: $s_2 = 0, 0 < m < 1$.

C. Total switching. All are switchers: $s_2 = 0, m = 1$.

D. Dispersed novices. Some sector-2 careers, some switchers: $s_2 > 0, m = 1$.

Which career regime is selected in equilibrium depends on the parameters and on the possible liquidity constraint.[12] Workers' lifetime utility must be equal to the outside opportunity in all career paths that are part of the regime, and less for the path(s) that are not. The exact regime (B, C, or D) will determine how the prices and wages adjust to the presence of intersectoral turnover.

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[12]It would also be feasible to have an interior solution ($s_1 > 0, s_2 > 0, 1 > m > 0$). However, it turns out that not all three types of careers can be part of an equilibrium at the same time. Intuitively, one of the ways of producing human capital for sector 2 will be more effective, and that method is used to the full before the less effective method is utilized.

[Figure 1: Schematic representation of the career regimes]

Regime A—sectoral isolation—is selected if the level of spillover learning is below the threshold β_A from (10). Above that threshold, the other parameters will determine which of the regimes B, C, or D holds in equilibrium. Since we are holding $\delta_1 < \delta_2$ constant, the choice between these regimes depends on the relative "size" of the sectors, captured by σ. Loosely, "size" here refers to the efficient size of a sectors workforce in the absence of switching. If sector 1 is relatively large then it can "train" all of the workers for sector 2 on the side, and still absorb some of the veterans (regime B); in this case there are no novices in sector 2 and it is the sector 2 prices and wages that are affected by intersectoral turnover. On the contrary, if sector 1 is relatively small, then even though all of the workers there switch to sector 2, that alone is not enough to supply sector 2 with labor and some will start their careers in sector 2 (regime D); in this case it is the sector-1 price and wages that are affected by turnover. In the in-between case of everyone starting in sector 1 and switching (regime C) the prices and wages in both sectors are affected by turnover. (This will turn out to be more than just a knife-edge case).

It simplifies the analysis considerably to note that whenever a sector has at least some workers who stay there for the whole career then the equilibrium price and wages in that sector must be the same as if it were an isolated sector: the price analysis of the one-sector model still applies because it was based on the indifference between the one-sector career and the outside opportunity. (However, the age structure and the amount of workers in such a sector are affected by turnover). In what follows, the prices and wages associated with regimes B, C, and D will be denoted by the appropriate superscripts, but those associated with an isolated sector continue to be denoted without superscripts. To prepare for the analysis of the equilibrium, let's first consider each of these regimes in isolation without a concern for when they would hold in equilibrium.

B. Dispersed veterans   Under this regime there are novices only in sector 1, and veterans in both sectors. Since some workers spend their whole career in sector 1, the price there must be as if it were isolated: $p_1^B = p_1$. Sector 2 price must adjust to keep the switchers indifferent with non-switchers, given the sector 1 price p_1. Considering the problem of a worker who has just spent the first period in sector 1, the sector 2 wage for switchers must then equal the wage for those who continue in sector 1. This pins down the relation of the prices in the two sectors.

$$
\begin{aligned}
p_1\left(1+\delta_1\right) - \phi_1 &= p_2^B\left(1+\delta_2\right)\beta - \phi_2 \Longrightarrow \\
p_2^B &= \frac{p_1\left(1+\delta_1\right) + \left(\phi_2 - \phi_1\right)}{\left(1+\delta_2\right)\beta}
\end{aligned} \tag{15}
$$

Mirroring the switching threshold in (10), regime B cannot hold in equilibrium if $p_2^B > p_2$. Given that there are no novices in sector 2 ($s_2^B = 0$) by the definition of regime B, conditions (12)-(14) can be solved to yield

$$s_1^B = \frac{1}{2p_1}\left(\sigma + \frac{(1-\sigma)(1+\delta_1)}{(1+\delta_1)+(\phi_2-\phi_1)}\right), \tag{16}$$

$$m^B = \frac{2(1-\sigma)}{(1+\delta_1)+\frac{\sigma}{p_1}(\phi_2-\phi_1)}. \tag{17}$$

Regime B is consistent only with parameters that imply $m^B \in (0,1)$.

C. Total switching   Now all novices work in sector 1 and all veterans in sector 2, so the mass of workers in both sectors must be the same in equilibrium. By definition, $s_2^C = 0$ and $m^C = 1$, while equilibrium prices and cohort size s_1^C are defined as the solution to

$$u\left(p_1^C(1-\delta_1)-\phi_1\right) + u\left(p_2^C(1+\delta_2)\beta-\phi_2\right) = 2u(\omega), \tag{18}$$

$$p_1^C s_1^C(1-\delta_1) = \sigma, \tag{19}$$

$$p_2^C s_1^C(1+\delta_2)\beta = 1-\sigma. \tag{20}$$

By solving for the prices in (19) and (20), and plugging these into (18), these can be combined to a condition in terms of the cohort size only.

$$s_1^C = s \quad \text{st.} \quad u\left(\sigma/s - \phi_1\right) + u\left((1-\sigma)/s - \phi_2\right) = 2u(\omega) \tag{21}$$

This in turn determines the prices via (19) and (20). The unconstrained case is equivalent to having linear utility.

D. Dispersed novices   Under this regime there are novices in both sectors, but veterans only in sector 2. Since some workers stay in sector 2 for the whole career, the output price there must be that of an isolated sector: $p_2^D = p_2$. Sector 1 price is the solution to keeping entering workers indifferent between careers

$$p_1^D = p \quad \text{st.} \quad u\left(p(1-\delta_1)-\phi_1\right) + u\left(p_2(1+\delta_2)\beta-\phi_2\right) = 2u(\omega). \tag{22}$$

Again, the unconstrained case corresponds to using linear utility in (22). Note that p_1^D is decreasing in β: sector-1 novices are willing to accept a lower wage when the payoff they can expect from switching becomes larger. The equilibrium has $m^D = 1$ by definition and (12)-(14) result in

$$s_1^D = \frac{\sigma}{p_1^D(1-\delta_1)}, \tag{23}$$

$$s_2^D = \frac{1}{2}\left(\frac{1-\sigma}{p_2} - \frac{\sigma(1+\delta_2)\beta}{p_1^D(1-\delta_1)}\right). \tag{24}$$

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Regime D is consistent only with parameters that imply $s_2^D > 0$.

Equilibrium Career Regime The parameter space $\{\sigma, \beta\} \in (0,1)^2$ can be partitioned into regions according to which of the four career regimes holds in equilibrium, holding as constant the other parameters ($0 < \delta_1 < \delta_2 < 1$, $0 \leq \phi_1 \leq \phi_2$, and $\omega > 0$). To fully characterize the effects of a liquidity constraint on intersectoral turnover, we need to analyze its effects on the shape of this partition.

First, regime A is just the absence of intersectoral turnover, and as we saw in (10), that results if the effectiveness of cross-sector learning is below a certain threshold. This leaves three possible regimes for $\beta > \beta_A$. Figure 2 shows the partition of the parameter space by equilibrium regime, where the switching threshold β_A shows up as the horizontal border between regime A and the rest.[13] Intuitively, the threshold for intersectoral turnover is independent of the relative size of the sectors, because whether the output prices that arise under sectoral isolation would attract switchers or not is independent of the relative size of the sectors.

[Figure 2]

As for the case when there actually is turnover ($\beta > \beta_A$), it turns out that there is a threshold σ_B, independent of β, such that regime B is selected if and only if $\sigma > \sigma_B$. This is the vertical border between regimes C and B in the figure; it is solved from setting $m^B = 1$ in (17):

$$\sigma_B = \frac{1 - \delta_1}{2 + \frac{\phi_2 - \phi_1}{p_1}}. \tag{25}$$

Intuitively, if sector 1 is sufficiently large then it will retain some veterans ($m < 1$) in addition to "training" all of sector 2 workforce ($s_2 = 0$). Inside region B, the proportion of switchers is declining in σ, as there is less and less demand for sector 2 output—this is clear by inspection of (17). The remaining region (upper left corner in the figure) is split by a decreasing curve $\beta_C(\sigma)$ such that regime D is selected if $\beta < \beta_C(\sigma)$. This curve is implicitly defined by setting $s_2^D = 0$ in (24), while taking into account that p_1^D depends on β:

$$\beta_C(\sigma) = \beta \ \text{ st } \ \frac{\beta}{p_1^D(\beta)} = \left(\frac{1 - \sigma}{\sigma}\right) \frac{(1 - \delta_1)}{p_2 (1 + \delta_2)}. \tag{26}$$

To see that $\beta_C(\sigma)$ is strictly decreasing, note that the right hand side of (26) is strictly decreasing in σ, while the left side is strictly increasing in β. It is also straightforward to check that $\beta_C(\sigma_B) = \beta_A$, so that all regions really are "neighbors" at point $\{\sigma_B, \beta_A\}$.

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[13]Figures 2 and 3 are drawn with parameter values $\delta_1 = 0.3, \delta_2 = 0.6, \omega = 1, \phi_1 = 1, \phi_2 = 3$, and with log-utility.

Intuitively, the difference between regimes C and D is that, in addition to everyone switching out of sector 1, there are also some sector-2 novices under regime D. The need to also have some sector-2 careers arises from a combination of sector 1 being small (low demand parameter σ) and the switchers being relatively unproductive (low transferability of skills β).

The effect of the liquidity constraint on the partition into regions by equilibrium regime is shown in Figure 3. The benchmark case of no liquidity constraint (in fact, Figure 2) is superimposed in gray dashed lines. Recall that the proportion of switchers m is, by definition, zero in region A, and one in regions C and D. The union of the regions where $m = 1$ swells due to the liquidity constraint, while the region where $m = 0$ contracts. Finally, for parameters that have regime B both with and without a liquidity constraint, the proportion of switchers $m \in (0, 1)$ is weakly higher under a liquidity constraint. This cannot be seen in Figure 3, but it is shown in the proof to the following Proposition.

[Figure 3]

Proposition 6 *There is a higher proportion of switchers in the liquidity constrained case ($\tilde{m} \geq m^*$). A sufficient condition for the proportion to be strictly higher is that there are some (but not all) switchers in the unconstrained case and that the production cost in sector 2 is strictly higher than in sector 1.*

Proof. This is to show that $\tilde{m} \geq m^*$, with a strict inequality guaranteed if $m^* \in (0, 1)$ and $\phi_2 > \phi_1$. First consider $\beta > \tilde{\beta}_A$, so that $m > 0$ is guaranteed. Now recall the equation for the proportion of switchers m^B from (17). If $\phi_2 = \phi_1$ then $\tilde{m}^B = m^{*B}$; and if $\phi_2 > \phi_1$ then $\tilde{m}^B > m^{*B}$ because $\tilde{p}_1 > p_1^*$. This also implies that the border between regions where $m \in (0, 1)$ (regime B) and $m = 1$ (regimes C and D) is defined by $\tilde{\sigma}_B \geq \sigma_B^*$, where the inequality is strict if $\phi_2 > \phi_1$. Second, consider $\tilde{\beta}_A \geq \beta > \beta_A^*$. This implies that $\tilde{m} > 0 = m^*$ for all σ (and even if $\phi_2 = \phi_1$). Third, if $\beta \leq \beta_A^*$ then $\tilde{m} = m^*$ since there is no turnover in either case. ∎

The bottom line is that there are relatively more workers switching out of the source sector in the liquidity constrained case as long as the constraint has any effect on turnover. If everyone is already switching in the unconstrained case then any further increase is of course impossible, and also if transferability of skills is sufficiently low then there is no effect because there is no turnover in either case.

3.3   Use and Misuse of Intersectoral Turnover

Job experience in sector 1 and sector 2 are two possible ways of producing human capital for sector 2. We have seen that a liquidity constraint causes a bias towards producing job experience

for both sectors in whichever sector this requires less financing from the worker. The reason is that the efficient way of producing human capital may be too expensive for the young workers to finance, and in addition to distorting prices and wages (which it would do even in the absence of turnover) the liquidity constraint can distort the career paths along which human capital is acquired. This leads to lower worker productivity as constrained workers choose their career paths, instead of by simply maximizing lifetime income, in a way that puts more weight on the income earned as a novice. This inevitably means putting less weight on the cost of later wasting some of the novice job experience in a switch to another sector.

The opportunity cost of production per job is fixed within each sector, so in terms of productive efficiency, the trade-off involved in intersectoral turnover concerns the average productivity of workers in the two sectors. In the absence of switching the output per worker is (normalized at) unity in both sectors. The downside of intersectoral turnover is the decrease in average output per job in sector 1 (as it loses experienced workers) down at most to $1 - \delta_1$ if everyone switches. The potential upside of switching is that it can increase the average output of workers in the receiving sector, at most up to $(1 + \delta_2)\beta$ if there are no novices in sector 2. Therefore a necessary condition for switching to increase the average output in sector 2 is that the productivity of a switcher there must be higher than the average output of non-switchers: $(1 + \delta_2)\beta > 1$. However, in the presence of a liquidity constraint it is possible to have counterproductive turnover which reduces the average output per worker in both sectors! This requires that the threshold value for the transferability of learning that induces switching is sufficiently low: $\tilde{\beta}_A < 1/(1 + \delta_2)$. Using the definition of the threshold $\tilde{\beta}_A$ from (10), this is equivalent to $\tilde{p}_1(1 + \delta_1) - \phi_1 < \tilde{p}_2 - \phi_2$. This merely states that, under the prices associated with sectoral isolation, the veteran wage in one sector would be lower than the average lifetime wage of a worker in the other sector. If the disparity in the learning curves is large enough then there is nothing to prevent this, because $\tilde{p}_2$ increases without limit as δ_2 approaches one.[14]

Learning-by-doing in another sector can be a poor substitute for long-term wage contracts or financing by the workers. If the young worker is unable to rent a machine, she may have to start in a job with a cheap "toy" machine. However, even counterproductive turnover is constrained efficient in the sense that it would not be helpful to prevent it while doing nothing to address the actual imperfections behind the inefficiency. Voluntary turnover between sectors reduces the lifetime wages at which liquidity constrained workers accept to enter this two-sector labor market because it makes possible career paths (whether technically inefficient or not) that result in less steep wage profiles over the career.

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[14]See equation 6 and Assumption 5.

3.4   Prices and Wages

Now let's consider the effect of the liquidity constraint on prices and wages, while holding constant all the parameters of the model. Part of this effect can be due to the change in career patterns, as the equilibrium career regime can be affected, although there are differences in prices and wages even under parameter values for which the career regime is not affected by the liquidity constraint. We already saw in Section 2 that the liquidity constraint distorts the price of output, and therefore also the wages, upwards in any sector that has single-sector careers. This result generalizes to the two-sector model, with one exception.

Proposition 7 *The liquidity constraint increases the price and wages in the receiving sector for sure. It increases the price and wages in the sending sector unless it is relatively small ($\sigma < \sigma_B^*$) and the price distortion there in isolation ($\tilde{p}_1 - p_1^*$) is sufficiently small.*

The proof is in the Appendix. (Recall that the price distortion under isolation is small when the learning curve is not very steep and when production costs are low). The intuition is for the most part the same as in the one-sector case: constrained individuals have to be compensated for the uneven consumption by higher lifetime earnings, which implies a higher price of output in at least one of the sectors. However, this still allows in principle for the higher price in one sector to be partly offset by a lower price in the other. And this is indeed what happens in the exceptional case.

To understand how the exceptional case arises, recall that the real benefit of a switching career is that learning in sector 1 is a cheaper way to acquire skills for sector 2 than actually working in sector 2, while the cost is that the cross-sector learning does not increase a worker's productivity as much as same-sector learning. The benefit accrues to workers when they are young, so its value is enhanced in the liquidity constrained case, where workers care about smoothing their income. Now imagine a situation with sectoral isolation, but where switching suddenly becomes attractive, for example due to an increase in the transferability of learning. The higher transferability increases the earnings from a switching career, but equilibrium lifetime utility cannot increase as it is pinned down by the outside opportunity, so some price must be "bid down" as a result of switching compared to what the prices were under sectoral isolation. If sector 1 is small, then novices there bid down the wage (and, in effect, the price of output) below the isolation price. If the distortion caused by the liquidity constraint in sector 1 was small to begin with, then this additional bidding for the "income smoothing value" of a sector 1 job can cause the price distortion to become negative in the constrained case. (By contrast, if sector 1 were large, then some workers would still spend there their whole career, so the price there would continue to be the output price of an isolated sector.)

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3.5   Direction of Turnover

Until now I have assumed that sector 2 has both a steeper learning curve and higher production costs than sector 1 (Assumption 8). This simplified the setup by stacking both possible rationales for intersectoral turnover in favor of turnover from sector 1 to 2. In this section I consider the remaining case precluded by Assumption 8 by setting $\delta_1 \leq \delta_2$ and $\phi_1 \geq \phi_2$. It turns out that now the liquidity constraint can affect the direction of turnover. In other words it may be efficient for young workers to get their experience in one sector and switch mid-career to another, even though the market equilibrium result in the exact opposite career pattern.

To understand what determines the direction of turnover, consider for a moment the limiting case where learning is fully transferable between sectors ($\beta = 1$). As the productivity of veterans in a given sector is not affected by where they worked as novices, there is switching whenever the wages associated with isolated sectors would be unequal. The direction of possible turnover is from sector 1 to 2 if the veteran wage under isolation is higher in sector 2:

$$p_2 \left(1 + \delta_2\right) - \phi_2 > p_1 \left(1 + \delta_1\right) - \phi_1, \tag{27}$$

and vice versa if the converse inequality holds. The corresponding equality divides the parameter space into regions by the direction of turnover, so there would be guaranteed to be switching except for a knife-edge set of parameters (which includes the case where the two sectors are identical).[15] Substituting in the efficient prices $p_i^* = \omega + \phi_i$ yields the following result.

Proposition 8 *The efficient direction of possible turnover is to the sector that under isolation would have a higher value for job experience, $\delta_i \left(\omega + \phi_i\right)$.*

This confirms that , in the efficient case, Assumption 8 is a sufficient condition for turnover to be from 1 to 2. To see that it is a sufficient condition also under the liquidity constraint, consider (27) with $p_i = \tilde{p}_i$ from (6), and recall that $\tilde{p}_i$ is increasing in δ_i (Proposition 3) and increases faster than linearly in ϕ_i (Proposition 4). However, in the absence of Assumption 8, things get more complicated.

Proposition 9 *Reversal of turnover. When one sector has a steeper learning curve and the other higher production costs, then the liquidity constraint can cause a reversal in the direction of turnover. For a given difference in learning curves, this happens for intermediate values of the cost difference.*

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[15]A less effective cross-sector learning ($\beta < 1$) just "fattens" the region of no switching from a mere surface: in general the region with no switching can be defined as where $p_1 \left(1 + \delta_1\right) \beta - \phi_1 < p_2 \left(1 + \delta_2\right) - \phi_2$ and $p_2 \left(1 + \delta_2\right) \beta - \phi_2 < p_1 \left(1 + \delta_1\right) - \phi_1$ hold simultaneously.

The proof is in the Appendix. The idea is to define the isoquants for the veteran wages under isolation in (δ_i, ϕ_i)-space both for the efficient and liquidity constrained case. Each isoquant marks the parameter values under which there would be no turnover between the sectors, if both sectors were located on the same isoquant. It can be shown that at almost any point (δ_1, ϕ_1) the isoquants for the two cases must be non-parallel, which in turn implies that there must be a region between the isoquants where, if (δ_2, ϕ_2) were located there, the direction of possible turnover would be the opposite under the two cases.

A steeper learning curve in sector 2 means that young workers are relatively productive in sector 1, which is a good reason for turnover to be from sector 1 to 2, unless sector 1 has a cost disadvantage that is so large as to offset the advantage in relative novice productivity. In the absence of intersectoral turnover, both sectors would be forced to use an even mix of novice and veteran workers. The turnover can be thought of as "trade" in workers in the sense that the sector that loses veterans gains novices, because more workers will want to enter as novices under the current wage (regime B) or at a lower wage than under isolation (regimes C and D) knowing that they can later switch to the other sector. If it weren't for the "transport cost" of imperfect cross-sector learning ($\beta < 1$) then it would be almost never optimal to have an even mix of novices and veterans in both sectors.

The efficient rule for turnover implies that any job experience that is transported between sectors should be created in the sector where the required worker borrowing under efficient prices is the lowest. It may therefore seem puzzling that a borrowing constraint could reverse the direction of turnover. The reason is that the price distortion caused by the liquidity constraint also distorts the desired levels of borrowing (the level needed to equalize lifetime consumption over time, given the market wages) so that job experience can appear cheaper in the sector where the social costs of producing it are actually higher. However, for this to happen, the two sectors have to be relatively similar in the sense that the "gains from trade" from intersectoral turnover would be relatively small to begin with.

4   Education

In some jobs the experience gained by the worker is much more valuable than the immediate output. In terms of the two-sector model, education can be thought as the extreme case of a job with no demand at all for the immediate output. In this section I assume that all that sector 1 provides is enhancement to a worker's second period output in the other sector. (In terms of the model parameters, this means assuming $\sigma = 0$). Now that the learning curve for sector 1 becomes meaningless (it doesn't matter how much better student one would be when old) we can discard the parameter δ_1 and and use simply δ for the steepness of the learning curve at

work, and denote the output of an educated person as $1 + e$. The cost ϕ_1 is now the cost of education and ϕ_2 the cost of production.

The income in the education sector is necessarily negative, consisting only of the cost of education. Without any borrowing or endowment there could be no education at all, since consumption has to be positive. Therefore, in this section, instead of only considering the polar opposite cases of unconstrained borrowing and no borrowing at all, I will now consider the level of borrowing ability as a parameter b (given by institutions). While the introduction of borrowing complicates the model, it is simplified at another margin because there is no longer a price for sector 1 output. The earnings of a worker who chooses to get an education are

$$w^{E,1} = -\phi_1, \tag{28}$$

$$w^{E,2} = p^E (1 + e) - \phi_2. \tag{29}$$

Without education the wages are those of a single-sector career, as seen in (2). Whether or not there is education in equilibrium, the price of output will now also depend on the borrowing ability b, which augments consumption when young and subtracts from it when old. (I assume that borrowing is at the discount rate of interest, which is normalized at zero).

Unconstrained case   Under education and unconstrained borrowing the worker entry condition simply equates lifetime income, $w^{E,1} + w^{E,2}$, with the outside opportunity, 2ω. The first-period income is the cost of education $-\phi_1$, so the wage in the second period must be $2\omega + \phi_1$ in equilibrium. Given zero profits for firms, the efficient price of output under education is

$$p^{E*} = \frac{2\omega + \phi_1 + \phi_2}{1 + e}. \tag{30}$$

There is education in equilibrium if it results in a lower output price than no education. Recall from (4) that the efficient output price in a single sector career—in the absence of education—is $p^* = \omega + \phi_2$. The threshold level of schooling effectiveness above which education is efficient is solved from $p^{E*} < p^* \iff$

$$e > \frac{\omega + \phi_1}{\omega + \phi_2} \equiv e^*. \tag{31}$$

This threshold captures both of the good reasons for education present in the model. One is that education may be so effective that an educated person will produce more output during the career than an uneducated one, despite having a shorter working career due to time spent at school. This corresponds to $e > 1$.[16] The other good reason is that education may be a cheaper way to improve productivity than work experience ($\phi_1 < \phi_2$). If real machines are very

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[16]This possibility that on-the-job learning for one sector could be more effective in a different sector was not allowed earlier as $\beta > 1$ was ruled out.

expensive, then it makes sense to have the inexperienced worker start by working with a toy machine, even if the resulting learning is less effective then working with the real thing.

The unconstrained case is the limiting case when the borrowing ability b is sufficiently high. The level of b required for borrowing to be unconstrained depends on whether or not education is efficient. If education is efficient, then workers need to borrow the opportunity cost of education $\omega + \phi_1$, otherwise they need to borrow the efficient market value of work experience $\delta (\omega + \phi_2)$, as was seen in (5).

Constrained case Again the price of output depends on whether or not there is education in equilibrium. If there is, then the workers' entry condition defines the price of output as a function of the borrowing ability b:

$$\tilde{p}^E(b) = p \ \text{st} \ \ u(-\phi_1 + b) + u(p(1+e) - \phi_2 - b) = 2u(\omega), \qquad b \in (\phi, \omega + \phi_1]. \quad (32)$$

This implies that the price of output (and thus also the wage $\tilde{w}^{E,2}$) is decreasing in the borrowing ability. The reason is simple: for a given output price, higher borrowing b allows a smoother consumption and so results in higher utility; to keep the lifetime utility equal with the outside opportunity, the price of output must then decrease. However, education is not feasible unless $b > \phi_1$, because first period consumption $b - \phi_1$ must be positive. It is easy to show (proof is omitted) that the output price under education is continuously decreasing in b: it "starts" from infinity at ϕ_1 and reaches the efficient price p^{E*} at $b = \phi_1 + \omega$, and is flat thereafter.[17]

Given that some borrowing is possible, workers can use it to smooth their income whether or not they choose to obtain an education. The possibility of limited borrowing will therefore reduce the price and wages also in the absence of education. The price is again defined by the free entry condition of workers:

$$\tilde{p}(b) = p \ \text{st} \ \ u(p(1-\delta) - \phi_2 + b) + u(p(1+\delta) - \phi_2 - b) = 2u(\omega), \qquad b \in [0, \delta(\omega + \phi_2)]. \quad (34)$$

It is easy to check that this is decreasing in b until the unconstrained level of borrowing $\delta(\omega + \phi_2)$ is reached. The polar cases of $\tilde{p}(0) \equiv \tilde{p}$ and $\tilde{p}(\delta(\omega + \phi_2)) = \omega + \phi_2 = p^*$ were analyzed in Section 2.[18]

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[17]For example, with log-utility, the closed-form price of output can be solved from (32) as

$$\tilde{p}^E(b) = \frac{1}{1+e}\left(b + \phi_2 + \frac{\omega^2}{b - \phi_1}\right). \quad (33)$$

[18]With log-utility, the equilibrium price under limited borrowing ability b and no education can be solved from (34) as

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Education is efficient if, in the absence of any borrowing constraints, there would be education, i.e., if $p^{E*} < p^*$. In the constrained case, the choice of career regime (i.e., education or no education) is still constrained efficient because it results in the lowest possible output price for the given level of the constraint b. To see this, note that the lifetime utility from any career is increasing in the price of output. Suppose, for example, that there were education in equilibrium but $\tilde{p}^E(b) \equiv p' > \tilde{p}(b)$. This would imply that, while price p' holds, a career without education would result in higher than outside utility, since it results, by definition, in exactly the outside level of utility if the price were only $\tilde{p}(b)$—which contradicts p' being an equilibrium price. Therefore, given a level of borrowing ability b, there is education in equilibrium if and only if $\tilde{p}^E(b) < \tilde{p}(b)$. This implies that a self-financing educational loan program that crowds out technically more efficient on-the-job learning can only increase efficiency—although a loan program that were not earmarked towards in-the-school learning would be even better,

Proposition 10 *If education is efficient, then there is education for values of borrowing ability that are above a certain threshold. If education is inefficient, then there is education in equilibrium for intermediate values of borrowing ability if the cost of education is lower than the cost of production and the learning curve at work is sufficiently steep.*

The proof is in the Appendix. The first part is very simple to show: a borrowing ability that only covers the tuition (ϕ_1) would result in zero consumption for the young, so they would have to opt for learning-by-doing, which at least results in positive novice consumption in equilibrium, thanks to the ability of the output price to be distorted upwards. Borrowing ability has to be significantly above the cost of education before it makes education more attractive than learning-by-doing.

To see how limited borrowing can induce technically inefficient education, recall that the unconstrained level of borrowing when learning-by-doing is efficient (i.e., when education is inefficient) is increasing in the steepness of the learning curve: $b^* = \delta(\phi_2 + \omega)$. On the other hand, whether education is efficient or not is independent of the steepness of the learning curve (δ) because it does not affect the lifetime output of a worker. There can be education in equilibrium if the cost of education is relatively low, and the borrowing ability is high enough to more than cover that cost but far from covering the cost of learning-by-doing.

As mentioned before, there are two good reasons for education: either it is a more economical way to increase worker productivity than actual work experience, or it is sufficiently more

$$\tilde{p}(b) = \frac{\phi_2 - \delta b + \sqrt{(b - \delta\phi_2)^2 + (1 - \delta^2)\,\omega^2}}{1 - \delta^2}. \tag{35}$$

effective than work experience to justify the opportunity cost of fewer years worked. With constrained borrowing, there is also a third reason that can cause education: the learning curve on the job may be so steep that the workers are not able to absorb the wage profile associated with on-the-job learning.

5   Retirement

In this section I study the effect of the worker liquidity constraint on retirement age in a single production sector with learning-by-doing. To make this choice meaningful, I assume that careers can last up to three periods and that productivity declines at the end of the career. More specifically, a worker's output in the first two periods is, as before, $1 - \delta$ and $1 + \delta$, and then $\xi \in (0, 1 + \delta)$ in the third period. The opportunity cost of labor is ω in any period; in addition to being the value of leisure or "retirement" this could also be a "bridge job" sector without learning-by-doing.[19] For brevity, individuals in their third period are referred to as "old," and retirement after two or three periods as "early" and "late" respectively.

When is early retirement efficient? Equivalently, when would the equilibrium with unconstrained worker borrowing entail early retirement? Under early retirement, nothing really changes for unconstrained workers compared to the model with two-period careers, so the equilibrium price $p^* = \omega + \phi$ is the same as in the two-period setup of Section 2. (The additional period simply adds ω to both sides of the workers' entry condition.) The wage on offer to an old worker would be $p^*\xi - \phi$, and workers would indeed choose to retire early if this were less than the outside income:

$$
\begin{aligned}
(\omega + \phi)\,\xi - \phi \ &< \ \omega \iff \\
\xi \ &< \ 1. \tag{36}
\end{aligned}
$$

Thus early retirement is efficient if the old worker's output is below the average output over the career so far. By contrast, $\xi > 1$ describes the case in which worker productivity never falls below the preceding lifetime average. When late retirement is efficient then the equilibrium price is below p^* because workers' average output per period is no longer unity but something higher, namely $(2 + \xi)/3$. Setting the lifetime income equal to the outside opportunity and solving for the equilibrium price of output under late retirement gives

$$
\begin{aligned}
[p(1 - \delta) - \phi] + [p(1 + \delta) - \phi] + [p\xi - \phi] \ &= \ 3\omega \iff \\
p\,(2 + \xi) - 3\phi \ &= \ 3\omega \iff \\
p_L^* \ &= \ \frac{(\omega + \phi)}{(2 + \xi)/3}. \tag{37}
\end{aligned}
$$

---

[19]Recall from Section 2 that in a sector with $\delta = 0$ the wage is a constant ω regardless of the liquidity constraint.

As always in the absence of a liquidity constraint, the price of output is the average opportunity cost of production.

Proposition 11 *Efficiency requires that old workers retire if their output falls below the preceding lifetime average (i.e., if $\xi < 1$). When young workers are borrowing constrained, the output has to decline further before old workers retire: there is early retirement only if $\xi < \tilde{\xi}$, where $\tilde{\xi} \in (1 - \delta, 1)$.*

Proof. There is early retirement if and only if the equilibrium price associated with early retirement results in a wage offer to old workers that is below the outside wage, $p\xi - \phi < \omega$. The efficient retirement decision was already analyzed above. For the liquidity constrained case, note first that workers are able to equalize consumption between their second and third career periods by saving a part of their second period income. This equalized consumption is the average of $p(1 + \delta) - \phi$ and ω, so the entry condition defining the equilibrium price under early retirement, $\tilde{p}_E$, is p such that

$$u\left(p(1 - \delta) - \phi\right) + 2u\left(\frac{p(1 + \delta) - \phi}{2} + \frac{\omega}{2}\right) = 3u(\omega). \tag{38}$$

It is simple to check that the left side of this entry condition must be below the right side at $p = p^* = \omega + \phi$, due to concavity of u. Thus $\tilde{p}_E > p^*$ at $\xi = 1$, so there would definitely not be early retirement at what is the efficient retirement threshold. On the other hand, $\tilde{p}_E \xi - \phi < \omega$ must be true for ξ close enough to zero. In fact, it must be true for all $\xi < 1 - \delta$, because then old workers would earn less than novices; this is impossible in equilibrium, because novices always earn less than ω but old workers would never accept a wage below ω.[20] ∎

Inefficiently delayed retirement can be expected to be a problem in occupations where the value of learning-by-doing is high. Define the retirement threshold as the highest level of old worker productivity ξ at which which there is early retirement in equilibrium.

Proposition 12 *The efficient retirement threshold is independent of the steepness of the learning curve δ and the complementary production cost ϕ. If young workers are liquidity constrained, then the retirement threshold is declining in both.*

The proof is in the Appendix. A steep learning curve and high production cost both increase the value of acquired skills and thus the upwards distortion in output price caused by workers' inability to pay the full value for the skills up-front. The increased output price in turn makes staying at work more attractive for the old by increasing the market value of their output. If

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[20]The reason why novices accept a wage below the outside wage is that it brings a higher wage in the future, but an old worker would have no such payoff to look forward to.

$\xi < 1$ then it is technically efficient for old workers to step down and make room for younger workers.[21] But when novices are contemporaneously less productive then the old then, in a competitive labor market, they have to earn a lower wage to exactly reflect their disadvantage in the market value of output. This difference, $p(\xi - (1 - \delta))$, can be thought of as an indirect "buy-out" price that novices pay to induce old workers to retire. This difference is large when the increasing part of the learning curve is steep (high δ) and when the complementary cost is high (high ϕ). Again, when learning is not too valuable, then the "payment" to employers is merely a wage discount relative to the outside wage.

Could mandatory early retirement be a solution to inefficient late retirement? Mandatory retirement would indeed increase output per worker, but it would not address the root cause of the inefficiency. As was the case with intersectoral turnover in Section 3, the decisions to switch sectors are constrained efficient: there can be late retirement in equilibrium only if it results in a lower price of output then early retirement. While mandatory retirement would increase the average output per worker, it would increase the wages required to attract liquidity constrained workers into this sector as they would have less time to earn returns to their initial investment of consuming less than the outside level.

6 Conclusion

This paper has analyzed the efficiency of learning-by-doing with a model of a perfectly competitive labor market where the demand for labor derives from two perfectly competitive product markets ("sectors") that use and produce skills with a common element. As is well known, in the absence of enforceable long-term wage contracts, firms have no incentive to invest into workers' human capital, except to the extent that it is unusable at other firms.[22] Here it was shown that, in a setup with learning-by-doing, the analogous problem is the delayed entry of workers into professions where learning is most valuable. At the same time, some of the learning is (from social efficiency point of view) needlessly wasted through "career-hopping" between jobs and sectors. However, holding constant the labor market institutions, the resulting inefficient career patterns

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[21]The feature that the efficient retirement threshold is exactly unity is due to the constant opportunity cost of labor. If it were lower for the old then the "discount" would show up on the right side of (36), and would result in $1 > \xi^* > \tilde{\xi}$.

[22]This problem can be is mitigated by imperfect competition and frictions that allow firms to retain some rents even from general training. See e.g., Katz and Ziederman (1990) and Stevens (1994), and Acemoglu and Pischke (1999) for a review. On the other hand, with perfect capital markets such frictions could only hurt by making it harder to convince outside employers that training has indeed taken place (Chang and Wang 1996). These arguments would apply also to general learning-by-doing, but for reasons of focus, I assumed away additional imperfections.

would just look like a reasonable job ladder in the data—workers switching jobs and getting pay increases in the process—even when they could in fact be excessive "career-hopping" resulting from financial constraints. To measure the magnitude of the inefficiency would ideally require an introduction or abolition of indentured servitude, or less ideally and more realistically, significant exogenous changes to the extent of the market imperfections. The prediction is that better liquidity or longer wage commitment by young workers would show up as steeper wage profiles with lower lifetime average wages and as less turnover between categories of jobs.

The key to understanding the source of inefficient learning-by-doing is to understand how the implicit prices of learning opportunities are determined in the interaction of labor and product markets. In a spot labor market equilibrium, employers have to be indifferent between employees of different productivity levels, so the wage differentials must reflect the differences in the market value of their contemporaneous output. At the same time, *someone* must hire novices in equilibrium, and the product markets may have to adjust for this to be possible. The wage profiles are steep when the learning curve is steep and when the complementary costs of a job are substantial (e.g., when it involves operating expensive machinery or directing many subordinates). When the wage profile is very steep, then novice workers should accept very low, easily even negative wages, for the lifetime average wage to be equalized across different careers. The expensive up-front cost of learning in one sector can be avoided by learning less effectively in another sector that has a flatter learning curve and therefore a flatter wage profile in equilibrium.

In the absence of liquidity constraints, workers' career choices would be based on their effects on lifetime income, and the breakdown of the value of output into its age-profile would be inconsequential (even though it would show up in the age profile of wages). Lack of wage commitment forces workers to absorb the early, increasing part of the age-profile of productivity into their age profile of consumption. Under a steep learning curve and/or high production costs, the consumption profile implied by the technically efficient career may be too steep for individuals to handle. Technically efficient learning is displaced by career paths that begin in jobs that are "cheaper" for workers to start with and result in less steep wage profiles.

The problem of workers caring about the time profile of consumption is closely related to the problem of risk aversion, as they both stem from the diminishing marginal utility of consumption. As is well known, when individuals face uninsurable risk they have the incentive to make socially inefficient investment decisions if this results in a more equal consumption across the states of the world. The inefficiency here is similar: workers make career choices that result in flatter income profiles over their career. However, due to the perfect competition setup of the model, the inefficiency is ultimately borne by consumers in the form of a higher output price. Output and learning were modeled as completely deterministic to highlight that informational

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issues are not at the core of inefficient learning-by-doing. The extension to uncertain learning would be natural and would provide an additional pressure towards inefficient career patterns. (The imperfect information equivalent of partially transferable learning would be job performance that is less informative about productivity in other sectors; flatter expected wage profiles would then be achieved by basing selection into high value industries on performance in jobs with noisier signals.) One unrealistic side-effect of modeling learning as deterministic is that there is turnover only into one direction; a model with noisy learning would generate similar results about net turnover.

This paper also analyzed the efficiency of the timing of moving into and out of a labor market with learning-by-doing. Liquidity constraints can cause both under- and overeducation, depending on what the alternatives are. If education is less effective than actual job experience in enhancing productivity, then a general way to state the results is that entry to and exit from a sector with learning-by-doing both tend to be delayed relative to the efficient benchmark. When learning on the job is very effective, and therefore carries a high implicit price, then a less effective education can be a way to avoid the high entry cost into the industry. While relaxing worker liquidity constraints is always helpful, subsidies for education can reduce efficiency by crowding out on-the-job learning. The exit side of the coin is also bad: retirement is delayed from sectors where learning-by-doing is most valuable. The liquidity constraint is not binding on the old workers who should retire, but it is binding on the young workers who should displace them. While novices are less productive than old workers, they have more to benefit from the learning-by-doing that the job offers. This benefit has an implicit price that the young workers should pay to indirectly "buy out" older workers from their jobs—the latter have no learning benefit to look forward to, although they may be contemporaneously more productive. Delayed retirement results when the buy-out price is too high for liquidity-constrained novices to bear.

The straightforward but typically unrealistic remedy to the problem of inefficient learning-by-doing is long-term wage commitment. This could mean much longer and broader enforcement of non-compete clauses, which would then also have to prevent (unilateral) turnover to different industries, and not just to competing firms in the same product market. However, this idea conflicts with the conventional justification and even with the term "non-compete" clause.[23] Note that having the same firms offer jobs in both sectors or "job categories" does not help internalize the public good aspect of novice-hiring, as the prices and wages would still be derived from a competitive market. However, a labor market monopsony would be able to offer an efficient career pattern or "job ladder" if the job experience that it generates is sufficiently useless in outside sectors. (For socially efficient hiring, it is enough that the sectoral monopsony

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[23]A non-compete clause can prevent a quitting worker from working for immediately competing firms for up to one year—but even these modest clauses are in practice very hard to enforce, see Adler (1999).

competes for workers at the entry stage where they choose between careers.) This may have implications for the public sector, especially in some occupations in the military. Interestingly, the U.S. government has recently willingly relinquished its monopsony in the infantry sector by the recent outsourcing of security operations, which could be expected to result in inefficient turnover and higher total costs of production.

Appendix

Proof of Proposition 7. First compare the prices while assuming that the equilibrium regime is not affected by the liquidity constraint (LC). For regime A, Proposition 2 states that $\tilde{p}_i > p_i^*$, which also covers sector 1 price under regime B and sector 2 price under regime D because they involve single-sector careers. For regime B, the result $\tilde{p}_2^B > p_2^{*B}$ follows from p_2^B (equation 15) being increasing in p_1 and $\tilde{p}_1 > p_1^*$ by Proposition 2. For regime C, first note that (21) implies $\tilde{s}_1^C < s_1^{*C}$ by the same argument that was used in proving Proposition 3, with smaller s here being analogous to higher p there. The result $\tilde{p}_i^C > p_i^{*C}$ then follows from combining $\tilde{s}_1^C < s_1^{*C}$ with (19) and (20).

Sector–1 price under regime D is the exception: it can be either higher or lower in the constrained case. To see this, first recall that $\tilde{\beta}_A < \beta_A^*$ by Proposition 5. Now suppose that δ_1 is (arbitrarily close) to zero. This means that $\tilde{p}_1 = p_1^* < p_2^* < \tilde{p}_2$. Consider a small $\sigma' < \sigma_B^*$, so that at threshold β_A, the regime switches from A to D. Since prices are equal to the isolated prices at the threshold, and $\tilde{\beta}_A < \beta_A^*$, and because $\tilde{p}_1^D$ is decreasing in β (see equation (22)) while p_1^* is independent of β, it must be the case that $\tilde{p}_1^D < p_1^*$ at $\{\sigma', \beta_A^*\}$, where also the unconstrained regime switches to D. By continuity, $\tilde{p}_1^D < p_1^{*D}$ at $\{\sigma', \beta_A^* + \epsilon\}$ for small enough ϵ. However, for β close enough to one, $\tilde{p}_1^D > p_1^{*D}$. To see this, notice that at $\beta = 1$ the veteran wage is the same both for switchers and for those who started in sector 2, so the novice wage must also be equal in both sectors. Then p_1^D can be solved from $p_1^D(1-\delta_1)-\phi_1 = p_2(1+\delta_2)-\phi_2$ in both the constrained and unconstrained cases. Thus $\tilde{p}_1^D > p_1^{*D}$ follows from $\tilde{p}_2 > p_2^*$, and, due to continuity, this must also hold for some $\beta < 1$.

Then consider the possible changes in equilibrium regime. First, suppose that $\sigma > \tilde{\sigma}_B$, so that the LC causes the regime to change from A to B if $\beta \in (\beta_A^*, \tilde{\beta}_A)$. Now only sector-2 price is affected by the change. Combining (15) with $\tilde{p}_2^B > p_2^*$ and rearranging we get $\tilde{p}_1(1-\delta_1)-\phi_1 > p_2^*(1+\delta_2)\beta - \phi_2$; but this is always true due to $\tilde{p}_2 > p_2^*$ and $\beta > \tilde{\beta}_A$. Second, LC will cause the regime to change from B to C if $\sigma \in [\sigma_B^*, \tilde{\sigma}_B]$ and $\beta > \beta_A^*$. At $\{\sigma_B^*, \beta\}$ for any $\beta > \beta_A^*$ the price under LC is higher, because regime-C prices hold both with and without LC, and $\tilde{p}_i^C > p_i^{*C}$ as shown above. Beyond this border, as $\tilde{p}_2^C$ is decreasing in σ and $\tilde{p}_1^C$ is increasing in σ, while p_2^{*B}

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is independent of σ but. But $\tilde{p}_2^C$ decreases towards $\tilde{p}_2^B$ (which it reaches at $\tilde{\sigma}_B$) and $\tilde{p}_2^B > p_2^{*B}$, so both prices are again higher under LC. However, for some $\sigma < \sigma_B^*$ regime D holds, and we saw that $\tilde{p}_1^D > p_1^{*D}$ is possible. All the same results for wages follow from wages being increasing in the output prices.

Proof of Proposition 9. Without loss of generality, fix the characteristics of sector i at $(\delta_i, \phi_i) \in (0,1) \times (0,\infty)$. Under sectoral isolation, the veteran wage in sector i is then fixed at $w_{i2} = p_i(1 + \delta_i) - \phi_i$, where p_i is the sector-i output price under isolation. The direction of possible turnover will be from sector i to sector j if and only if (δ_j, ϕ_j) are such that $w_{j2} > w_{i2}$, i.e., if

$$p_j(1 + \delta_j) - \phi_j > w_{i2}. \tag{39}$$

The corresponding equality defines the isoquant $f(\delta_j | w_{i2})$, such that if sector j has characteristics $(\delta_j, f(\delta_j))$ then the veteran wage is equal in both sectors and there is no turnover. If $\phi_j > f(\delta_j | w_{i2})$ then sector j is the recipient of turnover. The isoquants are different in the constrained and unconstrained case, as p_j takes the form (1) or (6).The proof follows from showing that, for the isoquants that cover (δ_i, ϕ_i), the one corresponding to the constrained case is everywhere steeper, i.e. that $\left| \partial \tilde{f} / \partial \delta \right| > |\partial f^* / \partial \delta|$. Ignoring the subscript j, the implicit differentiation of (39) yields

$$\frac{\partial f}{\partial \delta} = -\left(\frac{(\partial p / \partial \delta)(1 + \delta) + p}{(\partial p / \partial \phi)(1 + \delta) - 1} \right). \tag{40}$$

For the unconstrained case, $\partial p^* / \partial \delta = 0$ and $\partial p^* / \partial \phi = 1$, and this simplifies to $\partial f^* / \partial \delta = -p^* / \delta$. For the constrained case, after plugging in the implicit derivatives of the price from (8) and (9), this simplifies to $\partial \tilde{f} / \partial \delta = -\tilde{p} / \delta$. Since $\tilde{p} > p^*$, it is the case that $0 > \partial f^* / \partial \delta > \partial \tilde{f} / \partial \delta$, and there must be a region where the direction of turnover is different in the constrained case. Figure 4 illustrates the argument.

[Figure 4]

Proof of Proposition 10. Recall that, thanks to constrained efficiency, there is education if and only if $\tilde{p}^E(b) < \tilde{p}(b)$, and that both prices are decreasing in b. There are three cases to consider.

1. Education is efficient. Note that $\tilde{p}^E(b)$ must start above $\tilde{p}(b)$ because it starts from infinity at ϕ_1, due to Assumption 5. It will eventually reach the unconstrained level $\tilde{p}^E(\phi_1 + \omega) = p^{E*}$, which is by definition below $\tilde{p}(b)$ for any b when education is efficient. Therefore, there exists a threshold $b^e \in (\phi_1, \phi_1 + \omega)$ above which workers obtain an education.

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2. Education is inefficient and there is no education for any b. The proof for this case is trivial (consider δ near zero).

3. Education is inefficient but there is education for some b. Since $\tilde{p}^E(b)$ cannot be defined for $b \leq \phi_1$, and it has a left limit at infinity at $b = \phi_1$, it must initially be above $\tilde{p}(b)$ which is finite even at $b = 0$. On the other hand, $\tilde{p}^E(b)$ must be above p^* at $b = \delta(\phi_2 + \omega)$ since $p^* < p^{E*} = \tilde{p}^E(\delta(\phi_2 + \omega))$ by the definition of education being inefficient. Now suppose that the cost of education is lower than the cost of production: $\phi_1 < \phi_2 \Rightarrow \phi_1 + \omega < \delta(\phi_2 + \omega)$. Consider the prices at $b = \phi_1 + \omega$, so that borrowing for education is unconstrained, but borrowing for job experience is constrained for sufficiently high δ. Then $\tilde{p}(\phi_1 + \omega) > p^{E*} = \tilde{p}^E(\phi_1 + \omega) > p^*$ is a sufficient condition for there to be education in equilibrium; this means that at the level of borrowing at which career with education results in full consumption smoothing, the price under learning-by-doing would be above the price under education. It remains to show that $\tilde{p}(b)$ is decreasing in δ. But the proof of Proposition 3 implies that $\tilde{p}(b)$ is increasing in the parameter δ, because the proof is not affected by adding constant terms (b and $-b$ respectively) inside the utility functions. So, even though education is technically inefficient, there is education for intermediate values of $b \in [b_0, b_1]$, where $\phi_1 < b_0 < b_1 < \delta(\phi_2 + \omega)$.

Proof of Proposition 12. It was already shown that the efficient retirement threshold is $\xi^* = 1$. In the liquidity constrained case, there is early retirement if $\tilde{p}_E \leq \tilde{p}_L(\xi)$, with the equality defining the retirement threshold $\tilde{\xi}$. To define the equilibrium price under late retirement, $\tilde{p}_L(\xi)$, note that workers can equalize their consumption in their last two periods through saving. Thus $\tilde{p}_L(\xi)$ is p such that

$$u\left(p(1 - \delta) - \phi\right) + 2u\left(\frac{p(1 + \delta + \xi)}{2} - \phi\right) = 3u(\omega), \tag{41}$$

while $\tilde{p}_E$ was already defined in (38). It is straightforward to show that $\tilde{p}_L(\xi)$ is decreasing in ξ, so crosses $\tilde{p}_E$ exactly once, at $\tilde{\xi}$. The comparison of the equalized second and third period consumption in (38) and (41) shows that $\tilde{p}_E \leq \tilde{p}_L(\xi)$ is equivalent with the "incentive compatibility" of voluntary early retirement.

Finally, to analyze the comparative statics of $\tilde{\xi}$, implicitly differentiate both (41) and (41) to find the changes with respect to δ.

$$\frac{d\tilde{p}_E}{d\delta} = \frac{p\left(u'(x_1) - u'(x_2)\right)}{u'(x_1)(1 - \delta) + u'(x_2)(1 + \delta)} \tag{42}$$

$$\frac{d\tilde{p}_L}{d\delta} = \frac{p\left(u'(x_1) - u'(x_2)\right)}{u'(x_1)(1 - \delta) + u'(x_2)(1 + \delta + \xi)} \tag{43}$$

These are both positive, due to $x_1 < x_2$ and $u'' < 0$, the only difference being the larger denominator of $d\tilde{p}_L/d\delta$. Note also that the consumption profiles are the same at $\tilde{\xi}$. This means

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that an increase in δ would shift the line $\tilde{p}_E$ up more than $\tilde{p}_L(\xi)$ in the neighborhood of previous $\tilde{\tilde{\xi}}$, causing a decrease in $\tilde{\tilde{\xi}}$. The proof for ϕ is the same, with only the common numerator in $d\tilde{p}_E/d\delta$ and $d\tilde{p}_L/d\delta$ being $u'(x_1) + 2u'(x_2)$.

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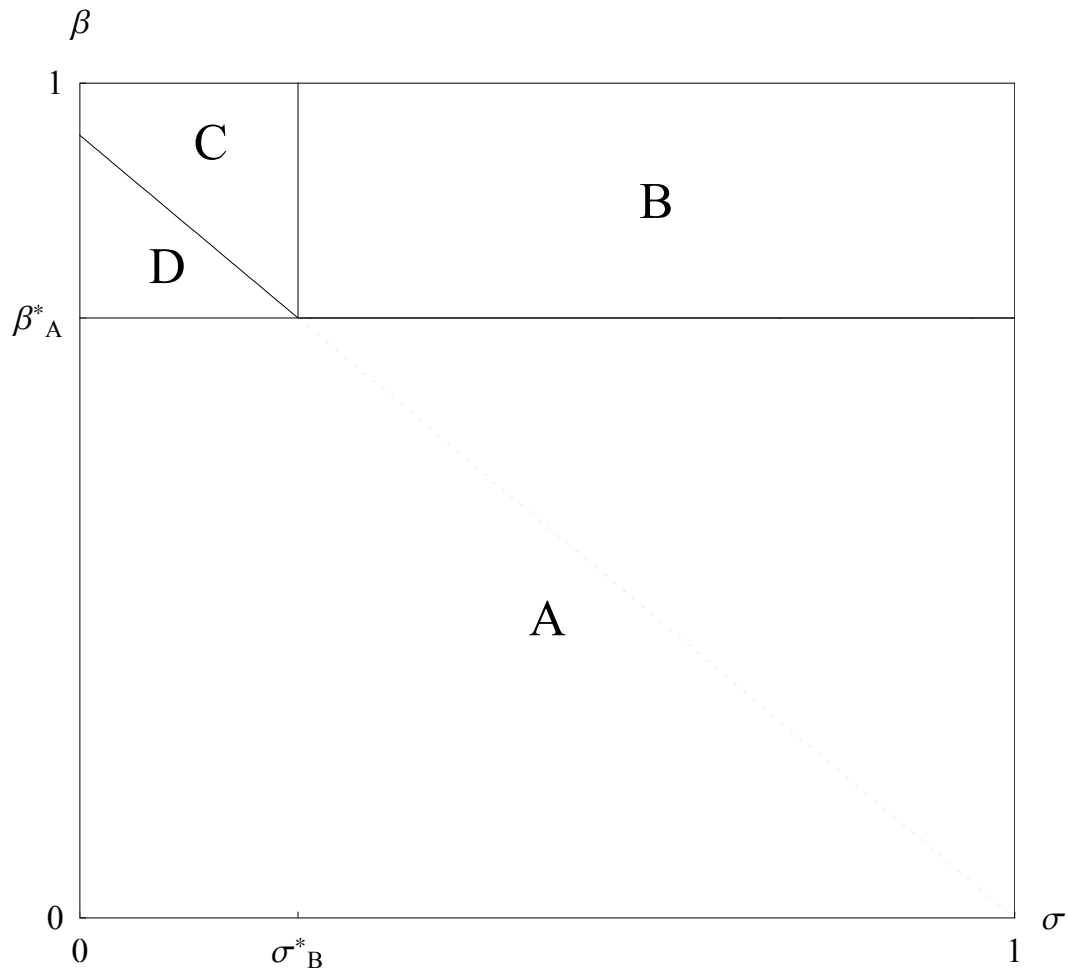Figure 1.

A schematic representation of the career regimes.
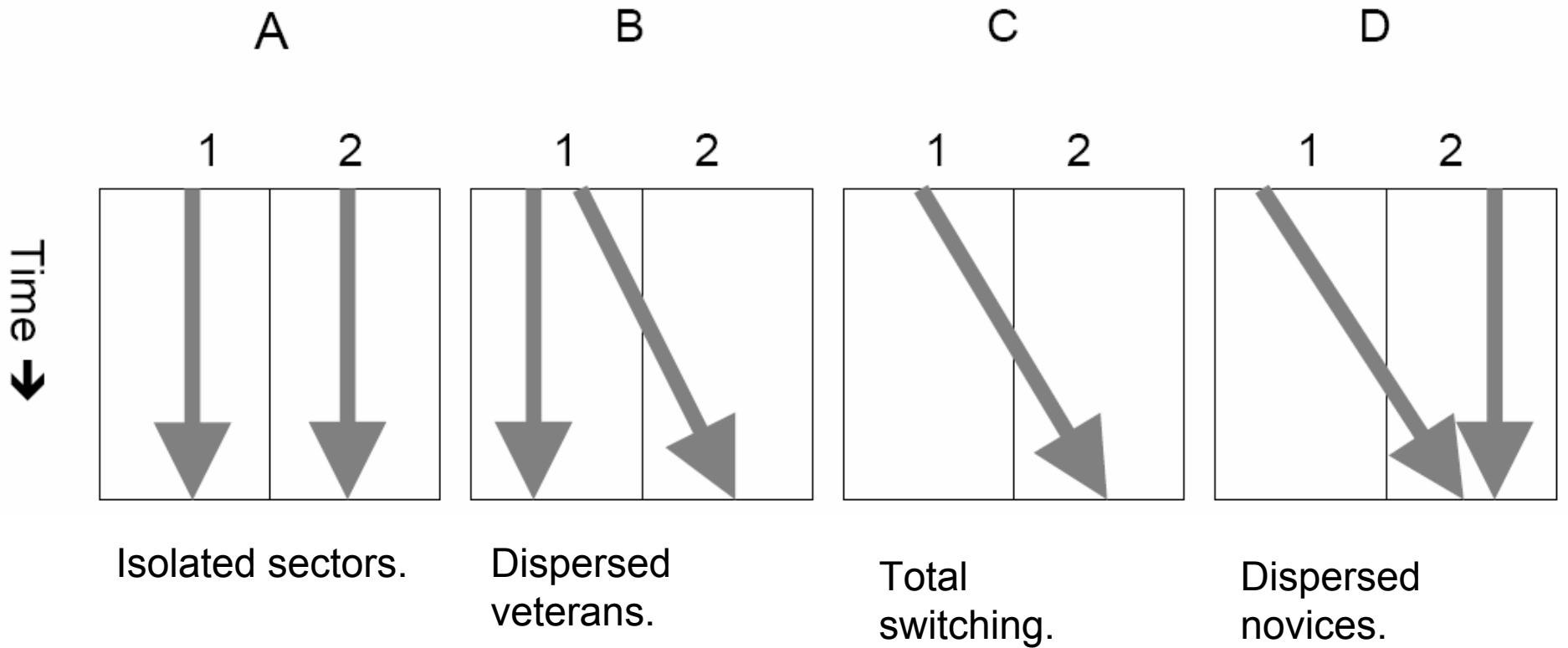
Figure 2: The dependence of the equilibrium career regime on σ, the relative demand for sector-1 output, and β, the transferability of learning across sectors, β.
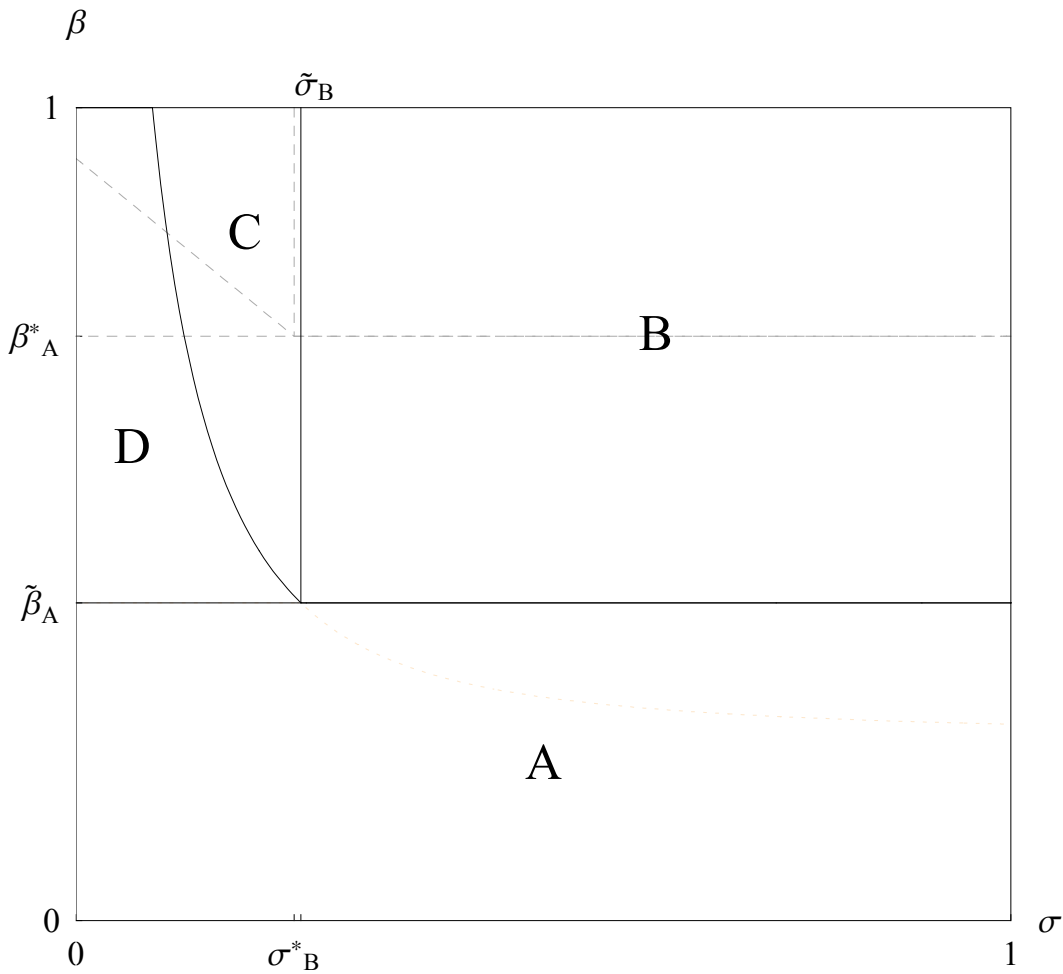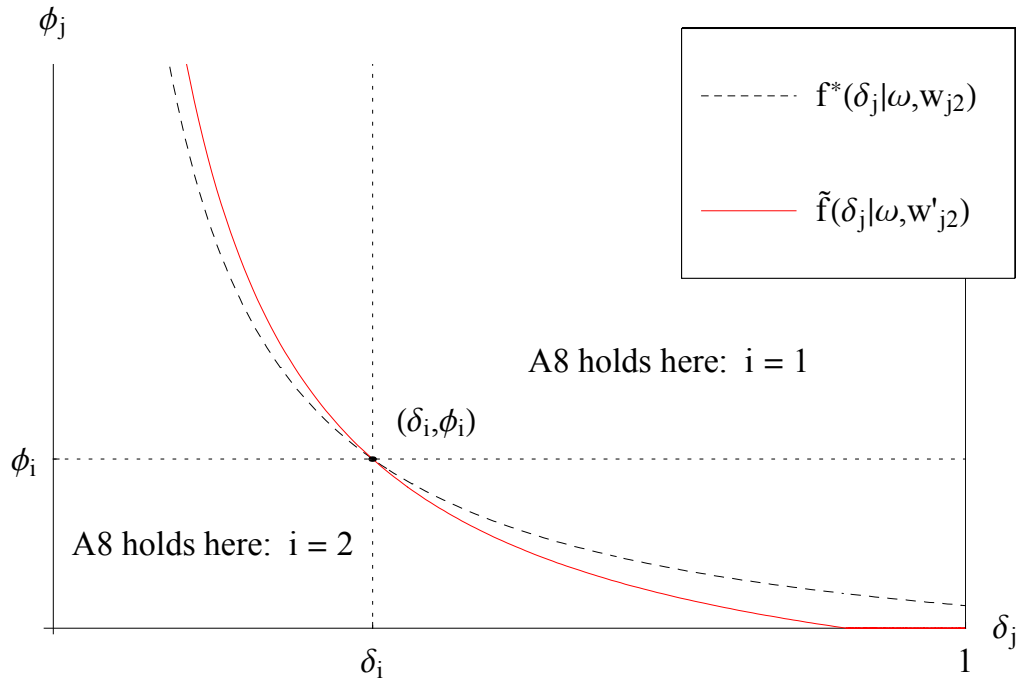
Figure 3: The effect of a liquidity constraint on the equilibrium regime. Solid lines refer to the constrained case, dashed lines to the unconstrained case.

Figure 4. Direction of turnover. Here the characteristics of sector i are fixed, while the characteristics of sector j can take on any values $(\delta,\phi)\in[0,1)\times[0,\infty)$. The direction of turnover is from i to j if (δ_j, ϕ_j) is located above the isoquant f, i.e., when $\phi_j > f(\delta_j)$. The isoquant covers all values (δ_j, ϕ_j) at which the wage profiles would be the same in both sectors (then there would be no turnover even at $\beta = 1$). The isoquant is different for constrained ($\tilde{f}$) and unconstrained (f^*) cases, so the region between the isoquants represents the possible values (δ_j, ϕ_j) for which the direction of turnover would differ between the cases.