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Effective potential for highly relativistic neutrinos in a weakly magnetized medium and their oscillation

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Abstract We have calculated the effective potential experienced by highly relativistic neutrinos in a weakly magnetized electron–positron plasma, where a momentum-dependent finite-width correction to the propagator of W is considered to account for the threshold effect. Magnetars are believed to be sources of TeV–PeV neutrinos which are produced due to photomeson and proton–proton interactions in their atmosphere. We have studied the resonant-oscillation process $\nu_e \leftrightarrow \nu_{\mu,\tau}$ of the highly relativistic neutrinos in the atmosphere of SGR 1806-20, which is a magnetar. It is shown that, for high-energy neutrinos propagating within the magnetar atmosphere, the resonance condition can never be satisfied. On the other hand, if GeV neutrinos are produced deep inside the magnetar atmosphere, where the temperature is about 50 keV or more, then these neutrinos can undergo resonant oscillation.

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1 Introduction

The coherent forward scattering of low-energy neutrinos in a medium will experience an effective matter potential, which is given by

$$V_{\nu_e} = \sqrt{2} G_F (N_e - N_{\bar{e}}). \tag{1}$$

This is derived by considering the contact interaction and assuming that momentum transfer is much smaller than the vector-boson mass. On the other hand, when the neutrino-momentum transfer is comparable to the vector-boson mass ($q^2 \simeq M_W^2$), the approximation under which (1) is derived no longer holds. For example, near the W production threshold in a non-relativistic electron background, the neutrino energy necessary is

$$E_\nu \simeq \frac{M_W^2}{2m_e} \simeq 10^7 \text{ GeV}, \tag{2}$$

and close to the above neutrino energy range, there can be a contribution due to W-boson production.

The potential of a propagating neutrino is determined by the matrix element of the forward-scattering amplitude, which is basically the real part of the neutrino self-energy. Keeping this in mind in Refs. [1, 2], the authors multiplied the real part of the Breit–Wigner type propagator [3] for the W-boson, i.e.

$$W_{\mu\nu}(q) = \frac{-g_{\mu\nu}}{q^2 - M_W^2 + \mathrm{i}\Gamma_W M_W}, \tag{3}$$

with the one in (1) and the threshold effect on high-energy neutrino propagation in AGNs and GRBs are studied.

In a recent paper [4], the momentum-dependent W-width in the propagator is considered to calculate the propagator function, and the propagator used is [3, 5]

$$W_{\mu\nu}(q) = -\frac{(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2})}{q^2 - M_W^2 + \mathrm{i} q^2 \gamma_W} = -\frac{(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2})}{q^2 \gamma_d - M_W^2}, \tag{4}$$

where $\gamma_W = \Gamma_W / M_W (\simeq 0.0266)$; Γ_W and M_W are respectively the decay width and the mass of the W-boson and $\gamma_d = 1 + \mathrm{i}\gamma_W$. Here they have used the finite-temperature field-theory method to calculate the effective potential and the threshold correction to it due to the finite decay width of the vector boson. The propagator functions in the case of Refs. [1, 2] and the case of Ref. [4] are different, because in the former one there is no momentum dependence compared

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to the latter one, so they have a different limiting behavior. On the other hand, there is a minor change in the result.

From the astrophysical point of view, the possible sources for the production of very high-energy neutrinos are supernova remnants [6], magnetars [7–10], AGNs [11, 12] and GRBs [13–15] etc. The high-energy neutrinos produced when propagating in their respective surrounding matter environments will experience the threshold potential, depending on the energy of the neutrinos. All these astrophysical objects as well as our sun and the earth have magnetic fields. So neutrinos propagating in these media will be affected by the presence of a magnetic field. The combined effect of matter and the magnetic field on the propagation of low-energy neutrinos ($q^2 \ll M_W^2$) is an active field of research from the point of view of neutrino oscillations and an MSW type effect [16–22] and also from the point of view of the study of helicity flipping of the neutrinos in the presence of a magnetic field. To our knowledge, the combined effect of matter and the magnetic field on the high-energy neutrinos when the energy is close to the vector-boson production threshold has not yet been studied. We believe that both the matter and the magnetic field have an interesting effect on the high-energy neutrinos propagating in the vicinity of the supernova remnants, AGNs and GRBs etc. (assuming only normal matter), because of the following reason: only the electron anti-neutrinos ($\bar{\nu}_e s$) propagating through the above environments will feel the threshold effect but not other neutrinos (because of $\bar{\nu}_e + e^- \to W^-$). So we believe that the oscillation pattern for $\bar{\nu}_e \leftrightarrow \bar{\nu}_{a,s}$ will be different from $\nu_e \leftrightarrow \nu_{a,s}$, where a and s are respectively the active and the sterile neutrino species.

In this work, we have used the finite-temperature field-theory method with linearized electron propagator in a constant magnetic field ($|e|B \ll m_e^2$) to calculate the linear order in the magnetic-field correction to the effective potential due to the threshold effect. We have also shown that the propagator function used in Ref. [1] can also be derived by considering the vector-boson propagator to be of Breit–Wigner type, where the decay width does not depend on the momentum transfer. The magnetic-field correction can be important when the high-energy neutrinos are propagating in the vicinity of highly magnetized objects like magnetars or anomalous X-ray pulsars (AXPs). We have studied the $\nu_e \leftrightarrow \nu_{\mu,\tau}$ resonant oscillation in the atmosphere of SGR 1806-20.

2 Effective potential

In thermal field theory, the effective potential of an elementary particle is calculated from the real part of the self-energy diagram. Using the formalism of real-time finite-temperature field theory [23–25] we calculate the effective potential of the neutrino in an e^+e^- background. To leading order, the momentum-dependent W-propagator from (4) is

$$W_{\mu\nu}(q) = -\frac{g_{\mu\nu}}{q^2\gamma_d - M_W^2}. \tag{5}$$

The neutrino self-energy can be written as

$$-\mathrm{i}\Sigma(k) = \frac{g^2}{2}\int \frac{\mathrm{d}^4 p}{(2\pi)^4} R\gamma_\mu S_l(p)\gamma_\nu L W^{\mu\nu}(q), \tag{6}$$

where the four-momentum $q = k - p$. Putting (5) in (6) we obtain

$$\Sigma(k) = -g^2 \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{R\gamma^\mu S_l(p)\gamma_\mu L}{q^2\gamma_d - M_W^2}\eta_F(p.u). \tag{7}$$

The exact electron propagator in the presence of a constant magnetic field is given by [23–25]

$$\mathrm{i}S_0(p) = \int_0^\infty \mathrm{d}s\, \mathrm{e}^{\Phi(p,s)} G(p,s), \tag{8}$$

with

$$\Phi(p,s) = \mathrm{i}s\left(p_0^2 - m_e^2\right) - \mathrm{i}s\left(p_3^2 + p_\perp^2\right)\frac{\tan z}{z} - \epsilon|s|, \tag{9}$$

where m_e is the mass of the electron and we have defined $z = |e|Bs$ and $p_\perp^2 = p_1^2 + p_2^2$. By using the following two four-vectors:

$$u^\mu = (1, \mathbf{0}); \qquad b^\mu = (0, \hat{\mathbf{b}}), \tag{10}$$

with magnetic-field vector given by $B\hat{\mathbf{b}}$, we have written the electron propagator in a covariant form. The function $G(p,s)$ in (8) is given by

$$G(p,s) = \sec^2 z\left[\not{A} + \mathrm{i}\not{B}\gamma_5 + m\left(\cos^2 z - \mathrm{i}\Sigma^3 \sin z \cos z\right)\right], \tag{11}$$

where A_μ and B_μ stand for

$$A_\mu = p_\mu - \sin^2 z(p\cdot u u_\mu - p\cdot b b_\mu), \tag{12}$$

$$B_\mu = \sin z \cos z(p\cdot u b_\mu - p\cdot b u_\mu), \tag{13}$$

and

$$\Sigma^3 = \gamma_5 \not{b}\not{u}. \tag{14}$$

The electron propagator in a medium is given by

$$S_e(p) = S_0(p) - \left(S_0(p) - \bar{S}_0(p)\right)\eta_e(p.u), \tag{15}$$

with

$$\eta_e(p.u) = \frac{\theta(p.u)}{\mathrm{e}^{\beta(p.u-\mu_e)}+1} + \frac{\theta(-p.u)}{\mathrm{e}^{\beta(-p.u+\mu_e)}+1}, \tag{16}$$

where β is the inverse temperature and μ_e is the electron chemical potential. In the weak field approximation (up to order $|e|B$) the fermion propagator in the background medium is given by

$$S_e = S_0 + S_B + S_T + S_{TB}, \tag{17}$$

where S_0 and S_B are the zero-temperature and the magnetic-field parts, respectively. S_T and S_{TB} are the temperature-dependent and the magnetic-field and temperature parts, respectively. Our interest here is to study the neutrino propagation in the electron plasma in the presence of a magnetic field, so we are only concerned with the last two terms (i.e., S_T and S_{TB}) and these are given by [25]

$$S_T = 2\pi \mathrm{i}(\not p + m_e)\delta\big(p^2 - m_e^2\big)\eta_e(p.u), \tag{18}$$

and in the weak-field approximation the S_{TB} part is given by

$$S_{TB}(p) = 2\pi \mathrm{i}\delta'\big(p^2 - m_e^2\big)\eta_e(p.u)|e|BG_B, \tag{19}$$

where the prime is for the derivative with respect to the argument in the delta function and

$$G_B = \gamma_5[p.b \not u - p.u \not b + m_e \not u \not b]. \tag{20}$$

With the last two terms, the self-energy is given by

$$\Sigma(k) = -g^2 \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{R\gamma^\mu(S_T(p) + S_{TB}(p))\gamma_\mu L}{q^2\gamma_d - M_W^2}\eta_e(p.u). \tag{21}$$

On simplifying this we get

$$\Sigma(k) = -g^2 \int \frac{\mathrm{d}^4 p}{(2\pi)^3}\Bigg[\frac{R\not pL}{q^2\gamma_d - M_W^2}\delta\big(p^2 - m_e^2\big)\eta_e - |e|B\frac{R(p.b\not u - p.b\not b)L}{q^2\gamma_d - M_W^2}\delta'\big(p^2 - m_e^2\big)\eta_e\Bigg]. \tag{22}$$

We can express the self-energy as

$$\Sigma = R\hat{\Sigma}L, \tag{23}$$

and parameterize $\hat{\Sigma}$ in terms of the available four-vectors, i.e.

$$\hat{\Sigma} = a\not k + b\not u + c\not b. \tag{24}$$

But from the structure of the self-energy in (22) we can see that there is no $\not k$ dependence in it. The scalar functions b and c are given as

$$b = -g^2 \int \frac{\mathrm{d}^4 p}{(2\pi)^3}\delta\big(p^2 - m_e^2\big)\eta_e \frac{p.u}{q^2\gamma_d - M_W^2} + g^2|e|B \int \frac{\mathrm{d}^4 p}{(2\pi)^3}\delta'\big(p^2 - m_e^2\big)\eta_e \frac{p.b}{q^2\gamma_d - M_W^2}, \tag{25}$$

and

$$c = -g^2|e|B \int \frac{\mathrm{d}^4 p}{(2\pi)^3}\delta'\big(p^2 - m_e^2\big)\eta_e \frac{p.u}{q^2\gamma_d - M_W^2}. \tag{26}$$

The derivative of the delta function in the above equations can be written as

$$\delta'\big(p^2 - m_e^2\big) = \frac{u^\mu}{2p.u}\partial_\mu \delta\big(p^2 - m_e^2\big). \tag{27}$$

Putting this in the above equations and after doing the delta-function integral we have

$$\begin{aligned} b = &-\frac{g^2}{2}\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\frac{1}{2E}\Bigg[\frac{f_e}{(k-p)^2\gamma_d - M_W^2} - \frac{f_{\bar e}}{(k+p)^2\gamma_d - M_W^2}\Bigg] \\ &- \frac{g^2}{2}|e|Bu^\mu \int \frac{\mathrm{d}^3 p}{(2\pi)^3}\frac{1}{2E}\partial_\mu\Bigg[\frac{p.b}{p.u}\frac{f_e}{(k-p)^2\gamma_d - M_W^2} - \frac{p.b}{p.u}\frac{f_{\bar e}}{(k+p)^2\gamma_d - M_W^2}\Bigg], \end{aligned} \tag{28}$$

and

$$c = \frac{g^2}{2}|e|Bu^\mu \int \frac{\mathrm{d}^3 p}{(2\pi)^3}\frac{1}{2E}\partial_\mu\Bigg[\frac{f_e}{(k-p)^2\gamma_d - M_W^2} - \frac{f_{\bar e}}{(k+p)^2\gamma_d - M_W^2}\Bigg]. \tag{29}$$

Now doing the angular integrations, we obtain

$$\begin{aligned} b = &-\frac{g^2}{2}\int \frac{p^2\,\mathrm{d}p}{(2\pi)^2}\frac{1}{2E_\nu p\gamma_d}\big(\ln|X|f_e - \ln|Y|f_{\bar e}\big) \\ &+ \frac{g^2}{2}|e|B\int \frac{p^2\,\mathrm{d}p}{(2\pi)^2}\frac{1}{2E}\frac{\partial}{\partial E}\Bigg[\frac{pf_e}{E}\Bigg\{\frac{1}{E_\nu p\gamma_d} + \frac{M_W^2 - (m_e^2 - 2E_\nu E)\gamma_d}{(2E_\nu p\gamma_d)^2}\ln|X|\Bigg\} \\ &+ \frac{pf_{\bar e}}{E}\Bigg\{\frac{1}{E_\nu p\gamma_d} + \frac{M_W^2 - (m_e^2 + 2E_\nu E)\gamma_d}{(2E_\nu p\gamma_d)^2}\ln|Y|\Bigg\}\Bigg], \end{aligned} \tag{30}$$

and

$$c = \frac{g^2}{2}\frac{|e|B}{2E_\nu\gamma_d}\int \frac{p^2\,\mathrm{d}p}{(2\pi)^2}\frac{1}{2E}\frac{\partial}{\partial E}\Bigg[\frac{f_e}{p}\ln|X| - \frac{f_{\bar e}}{p}\ln|Y|\Bigg], \tag{31}$$

where we have defined $E = \sqrt{\mathbf{p}^2 + m^2}$, and E_ν is the energy of the neutrino. The quantities X and Y are defined by

$$X = \frac{M_W^2 - (m_e^2 - 2E_\nu E + 2E_\nu p)\gamma_d}{M_W^2 - (m_e^2 - 2E_\nu E - 2E_\nu p)\gamma_d}, \tag{32}$$

and

$$Y = \frac{M_W^2 - (m_e^2 + 2E_\nu E + 2E_\nu p)\gamma_d}{M_W^2 - (m_e^2 + 2E_\nu E - 2E_\nu p)\gamma_d}. \tag{33}$$

The expressions for b and c in (30) and (31) are the exact ones valid for small magnetic field $eB \ll m_e^2$. For a non-relativistic background where the condition $T < m_e$ (T is the temperature of the background) is satisfied and we can expand the logarithms for small loop momentum p which give

$$\ln\left[\frac{M_W^2 - (m^2 \mp 2E_\nu E + 2E_\nu p)\gamma_d}{M_W^2 - (m_e^2 \mp 2E_\nu E - 2E_\nu p)\gamma_d}\right] \simeq -\frac{4E_\nu p\gamma_d}{M_W^2 - (m_e^2 \mp 2E_\nu E)\gamma_d}. \tag{34}$$

So the expressions for b and c in the non-relativistic limit are

$$b \simeq \frac{g^2}{2}\mathcal{R}\int \frac{p^2\,\mathrm{d}p}{2\pi^2} \times \left(\frac{f_e}{M_W^2 + 2E_\nu m_e\gamma_d} - \frac{f_{\bar{e}}}{M_W^2 - 2E_\nu m_e\gamma_d}\right), \tag{35}$$

and

$$c \simeq \frac{g^2|e|B}{4}\mathcal{R}\int \frac{p^2\,\mathrm{d}p}{2\pi^2}\frac{\beta}{m_e} \times \left(\frac{f_e}{M_W^2 + 2E_\nu m_e\gamma_d} - \frac{f_{\bar{e}}}{M_W^2 - 2E_\nu m_e\gamma_d}\right), \tag{36}$$

where $\mathcal{R}$ denotes the real part. Expressing this in terms of the Fermi coupling constant and density of the background particles/anti-particles, we obtain

$$b = \sqrt{2}G_F M_W^2 \mathcal{R}\left(\frac{N_e}{M_W^2 + 2E_\nu m_e\gamma_d} - \frac{N_{\bar{e}}}{M^2 - 2E_\nu m_e\gamma_d}\right), \tag{37}$$

and

$$c = \frac{G_F}{\sqrt{2}}|e|B\frac{M_W^2}{m_e T}\mathcal{R}\left(\frac{N_e}{M_W^2 + 2E_\nu m_e\gamma_d} - \frac{N_{\bar{e}}}{M_W^2 - 2E_\nu m_e\gamma_d}\right), \tag{38}$$

where the number density of electron/positron is defined as

$$N_{e(\bar{e})} = 2\int \frac{\mathrm{d}^3 p}{(2\pi)^3} f_{e(\bar{e})}. \tag{39}$$

The effective potential of the neutrino can be calculated when the determinant of the inverse propagator vanishes, i.e.

$$\mathrm{Det}\left[\not{k} - \Sigma(k)\right] = 0, \tag{40}$$

which gives

$$k_0 - |\vec{k}| = V_{\nu_\alpha} \simeq b - c\cos\phi, \tag{41}$$

where ϕ is the angle between the magnetic field and the neutrino momentum. The real parts of b and c are the propagator functions given by

$$f(\pm s_W) = \frac{1 \pm s_W}{(1 \pm s_W)^2 + s_W^2\gamma_W^2}, \tag{42}$$

where we have defined $s_W = 2E_\nu m_e/M_W^2$. Then the potential is given by

$$\begin{aligned} V_{\nu_e}(s_W) \equiv V &= \left(1 - \frac{1}{2}\frac{|e|B}{m_e T}\cos\phi\right) \\ &\quad \times \sqrt{2}G_F\left(N_e f(s_W) - N_{\bar{e}} f(-s_W)\right) \\ &= \sqrt{2}G_F N_e f(s_W)\left(1 - \frac{1}{2}\frac{|e|B}{m_e T}\cos\phi\right) \\ &\quad \times (1 - \delta N), \end{aligned} \tag{43}$$

where

$$\delta N = \frac{N_{\bar{e}} f(-s_W)}{N_e f(s_W)}. \tag{44}$$

So (43) is the desired neutrino potential up to linear order in the magnetic field. The potential for the anti-neutrino is given by $V_{\bar{\nu}_e}(s_W) = -V_{\nu_e}(-s_W)$.

The propagator functions $f(s_W)$ and $f(-s_W)$ are plotted in Fig. 1. The discontinuity in the function $f(-s_W)$ at $s_W = 1$ corresponds to W-boson production. The calculation of the effective potential of the neutrino in the e^-e^+ background implies the calculation of the matrix element of the scattering processes $\nu_e + e^- \to \nu_e + e^-$ and $\nu_e + e^+ \to \nu_e + e^+$, and for the anti-neutrino we have to replace ν_e by $\bar{\nu}_e$. For $s_W \simeq 1$ the processes $\nu_e + e^+ \to W^+$ and $\bar{\nu}_e + e^- \to W^-$ will also contribute, which corresponds to the peak of $f(-s_W)$. On the other hand, for $\nu_e + e^- \to \nu_e + e^-$ and $\bar{\nu}_e + e^+ \to \bar{\nu}_e + e^+$, there is no threshold behavior for $f(s_W)$.

To derive the propagator function used in Refs. [1, 2] we can take the Breit–Wigner type propagator for the unstable particles (W- and Z-bosons) as shown in (3). In this case, we can replace the propagator of (4) by (3) and repeat the calculation. After going through similar steps as the one above we get

$$b \simeq \frac{g^2}{E_\nu}\int \frac{p^2\,\mathrm{d}p}{(2\pi)^2}\frac{1}{2E}\left[\left(1 - \frac{M_W^{*2}}{M_W^{*2} + 2E_\nu E}\right)f_e + \left(1 - \frac{M_W^{*2}}{M_W^{*2} - 2E_\nu E}\right)f_{\bar{e}}\right], \tag{45}$$

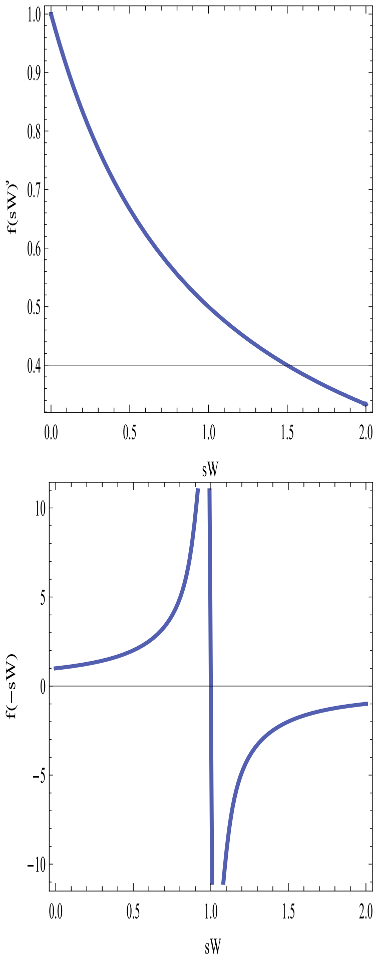

Fig. 1 The functions $f(s_W)$, the upper one, and $f(-s_W)$, the lower one, of (42)

where we have defined $M_W^{*2} = M_W^2 - i\Gamma_W M_W$. After doing the simplification we obtain

$$V_{\nu_e}(s_W) \equiv V = \sqrt{2} G_F \big(N_e f(s_W) - N_{\bar{e}} f(-s_W)\big), \tag{46}$$

with

$$f(\pm s_W) = \frac{1 \pm s_W}{(1 \pm s_W)^2 + \gamma_W^2}. \tag{47}$$

So using the finite-temperature field-theory method it is straightforward to show the origin of the propagator function in Ref. [1]. Comparison of (42) with (47) shows that the momentum dependence in the first case gives the extra factor s_W^2 in (42).

3 Applications

Compared to the zero magnetic-field case, the weakly magnetized medium has an extra contribution, as shown in (43). The correction due to a magnetic field is very small for a small magnetic field. The only place where probably it can have a substantial contribution is the magnetar and may be in supernova remnants. It is believed that magnetars have a magnetic field much above the critical value (up to $\sim 10^{15}$–10^{16} G) and also that they are sources of TeV–PeV neutrinos [7–10]. The giant flare of SGR 1806-20 on the 27th of December 2004 is believed to be a magnetar [8–10, 26, 27] and located in our galaxy approximately 15 kpc away. It had a gamma-ray energy output of $E_\gamma \sim 3 \times 10^{46}$ erg for a period of ~ 0.1 s. The polar field strength estimated from its period and the period derivation is $B \sim 1.6 \times 10^{15}$ G. The giant flares are formed due to the rearrangement of the magnetic field on the magnetar crust. These flares have many similarities with the cosmological gamma-ray bursts (GRBs) and both lead to the formation of relativistic fireballs. In both cases there is copious production of e^+e^- pairs. The presence of baryons in the fireball are responsible for the production of high-energy neutrinos through inelastic $p\gamma$ and pp interactions. The electrons associated with the baryons will also increase the opacity of the fireball. The produced high-energy (10^{14}–10^{15} eV) neutrinos in the magnetar environment can oscillate resonantly/non-resonantly depending on the nature of the fireball. So we will study the oscillation of these neutrinos in a magnetar environment. First let us estimate the effect of the magnetic-field correction to the effective potential experienced by these neutrinos away from the magnetosphere. By taking $B \simeq 0.01 m_e^2/e \simeq 4 \times 10^{11}$ G away from the magnetosphere of the magnetar, a temperature of order $T \simeq 0.05 m_e$ and $\cos\phi \sim 1$, i.e. forward-moving neutrinos, it gives $|e|B/(2m_e T)\cos\phi \simeq 0.10$, which is 10% reduction compared to the case without magnetic field. So depending on the strength of the magnetic field and the temperature of the background plasma there can be variation in the effective potential seen by the propagating high-energy neutrinos away from the magnetosphere of SGR 1806-20 or any other magnetar. We study the oscillation process $\nu_e \leftrightarrow \nu_{\mu,\tau}$ in this environment. The potential for the above oscillation process is given by (43). The neutral current part of the potential is the same for ν_e and $\nu_{\mu,\tau}$, which will cancel and for this reason the matter potential for the process $\nu_\mu \leftrightarrow \nu_\tau$ vanishes. The probability of ν_e to $\nu_{\mu,\tau}$ conversion is given by

$$\mathcal{P}(t) = \frac{\Delta^2 \sin^2 2\theta}{\omega^2} \sin^2\left(\frac{\omega t}{2}\right), \tag{48}$$

where

$$\omega = \sqrt{(V - \Delta\cos 2\theta)^2 + \Delta^2 \sin^2 2\theta}, \tag{49}$$

$\Delta = \Delta m_\nu^2/(2E_\nu)$ with E_ν the energy of the neutrino and θ is the mixing angle of the neutrino. The resonance takes place when

$$V = \Delta\cos 2\theta \tag{50}$$

is satisfied in (48). This corresponds to

$$\Delta\tilde{m}_\nu^2\cos 2\theta \simeq 2.53\times 10^{-23}\left(1-\frac{1}{2}\frac{|e|B}{m_e T}\cos\phi\right) \times (1-\delta N)E_{\nu,14}N_{e,0}f(s_W), \quad (51)$$

where $\Delta\tilde{m}^2$ is in units of eV^2, $E_{\nu,14}$ is in units of 10^{14} eV, and $N_{e,0}=N_e$ cm^3. At this point the resonance length is

$$l_{\text{res}} \simeq \frac{2.5\times 10^{10}E_{\nu,14}}{\Delta\tilde{m}^2{}_\nu \sin 2\theta}\ \text{cm}. \quad (52)$$

The number density of theγ on the surface of the magnetar can be estimated to be [8, 9]

$$n_\gamma \simeq \frac{L_\gamma}{4\pi r_0^2 m_e} \simeq 9.73\times 10^{29}L_{47.5}r_{0,6}^{-2}\ \text{cm}^{-3}, \quad (53)$$

where the luminosity we have taken is $L_\gamma \sim 3\times 10^{47}L_{47.5}$ erg/s, and the radius of the magnetar is $r_0 \sim 10^6$ cm. The temperature at this point is given by $T\sim (n_\gamma\pi^2/2\zeta(3))^{1/3} \sim 313$ keV. We have to estimate the number density of the e^+e^- pair in the fireball which is produced due to high-energy gammas. To create an e^+e^- pair, the minimum energy of the photon should be $h\nu \sim 2m_e$. So the number density of photons near this threshold in a region of radius r is $n_\gamma \simeq L_\gamma/8\pi r^2 m_e$. By equating the electron–positron annihilation rate to the local-expansion rate in the fireball, Nakar et al. in Ref. [9] have shown that the remaining number of the e^+e^- is given by

$$N_\pm \simeq 6.1\times 10^{44}E_{46.5}^{3/4}r_{0,6}^{-1/2}t_{-1}^{1/4}, \quad (54)$$

where the pair temperature is $T\sim 18$ keV at a distance $r_\pm$. The number density of photon at this temperature is $n_\gamma \simeq 1.8\times 10^{26}$ cm^{-3}. Also the number density of e^+e^- is

$$N_e = 2.2\times 10^{30}\left(\frac{T}{m_e}\right)^{3/2}\mathrm{e}^{-m_e/T}\text{cm}^{-3} \simeq 6.8\times 10^{15}\text{cm}^{-3}, \quad (55)$$

and the radius $r_\pm \sim 2.1\times 10^9$ cm. The baryons in the fireball are accelerated by the Fermi mechanism and produce high-energy neutrinos through $p\gamma$ and pp interactions in this region. We assume that after the pions are produced, they instantaneously decay to produce neutrinos.

We found that for $B\simeq 0.01me^2/e$ and $T=0.05m_e$, the correction to the effective potential due to the magnetic field is 10%, which is small. Apart from the γ and e^+e^- plasma, the magnetar environment also contains baryons and their electrons. These excess electrons will also contribute to the opacity of the medium. Due to the uncertainty in the baryon load of the fireball, there can be two different situations: (1) the total contribution of electrons to the effective potential of the neutrino as well as to the opacity of the plasma is much higher than the positron contribution, so that the latter contribution can be neglected ($\delta N \ll 1$); or (2) even if there are excess electrons due to the baryon content of the fireball, the positron will also contribute. Here we consider the $\delta N\ll 1$ case and assume that exclusion of the positron part will not change the result drastically (as excess electrons from the baryon contamination in the fireball will always dominate). With this the resonance condition in (51) is simplified to

$$\Delta\tilde{m}_\nu^2\cos 2\theta \simeq 1.7\times 10^{-7}. \quad (56)$$

By using the large and small mixing angles from the experiments, we have calculated the Δm_ν^2 as well as the resonance length. For large mixing angle [28] in the range $0.64 \le \sin^2 2\theta \le 0.96$, we obtain the Δm_ν^2 and the resonance length in the ranges 2.8×10^{-7} eV^2 $\le \Delta m_\nu^2 \le 8.5\times 10^{-7}$ eV^2 and $3.\times 10^{16}$ cm $\le l_{\text{res}} \le 1.1\times 10^{17}$ cm, respectively. For small mixing angle [29], in the range $2\times 10^{-3} \le \sin^2 2\theta \le 7\times 10^{-3}$, we obtain $\Delta m_\nu^2 \sim 10^{-7}$ eV^2 and the resonance length $l_{\text{res}} \sim 10^{18}$ cm. The resonance lengths obtained are much larger than $r_\pm$. So at l_{res} the temperature will be many orders of magnitude smaller than at $r_\pm$ and also the density of the e^+e^- will be very much suppressed, by a factor $\exp(-m_e/T)$. So the high-energy neutrinos ($E_\nu \sim 10^{14}$ eV) will never satisfy the resonant condition. The allowed neutrino mixing angles ($\sin^2 2\theta$) and Δm_ν^2 for $E_\nu = 10^{14}$ eV are shown in Fig. 2(a).

From the neutrino-oscillation experiments [28, 29] we know that $\Delta m_\nu^2 \sin 2\theta < 1$. So for resonance to take place within the magnetar atmosphere, the denominator of (52) must be small. The only way to make it small is to have low-energy (GeV or lower) neutrinos (at this energy, the threshold effect is not there and $f(\pm s_W)=1$) and these neutrinos must be produced at a radius r which should satisfy $r<r_\pm$, so that the plasma temperature as well as the number density of e^+e^- will be higher. If the plasma temperature is about 50 keV and GeV neutrinos are propagating in it, for large neutrino mixing we obtain $10^{-3} \le \Delta m_\nu^2 \le 3.1\times 10^{-3}$ eV^2, which is perfectly compatible with the atmospheric Δm_ν^2 and the resonance length lies in the range $6.7\times 10^7 \le l_{\text{res}} \le 3\times 10^8$ cm, which is also below $r_\pm$. The allowed neutrino mixing angles ($\sin^2 2\theta$) and Δm_ν^2 for $E_\nu = 10^9$ eV are shown in Fig. 2(b).

To summarize our results: we have used the real-time finite-temperature field-theory method to calculate the effective potential of high-energy neutrinos propagating in an e^+e^- background in the presence of a weak magnetic field. To account for the W-boson production threshold effect, a momentum-dependent finite-width correction to the W-propagator is considered. We found that the magnetic-field correction is small. It is believed that magnetars are potential sources of TeV–PeV neutrinos. So we have studied the $\nu_e \leftrightarrow \nu_{\mu,\tau}$ resonant oscillation for a neutrino energy

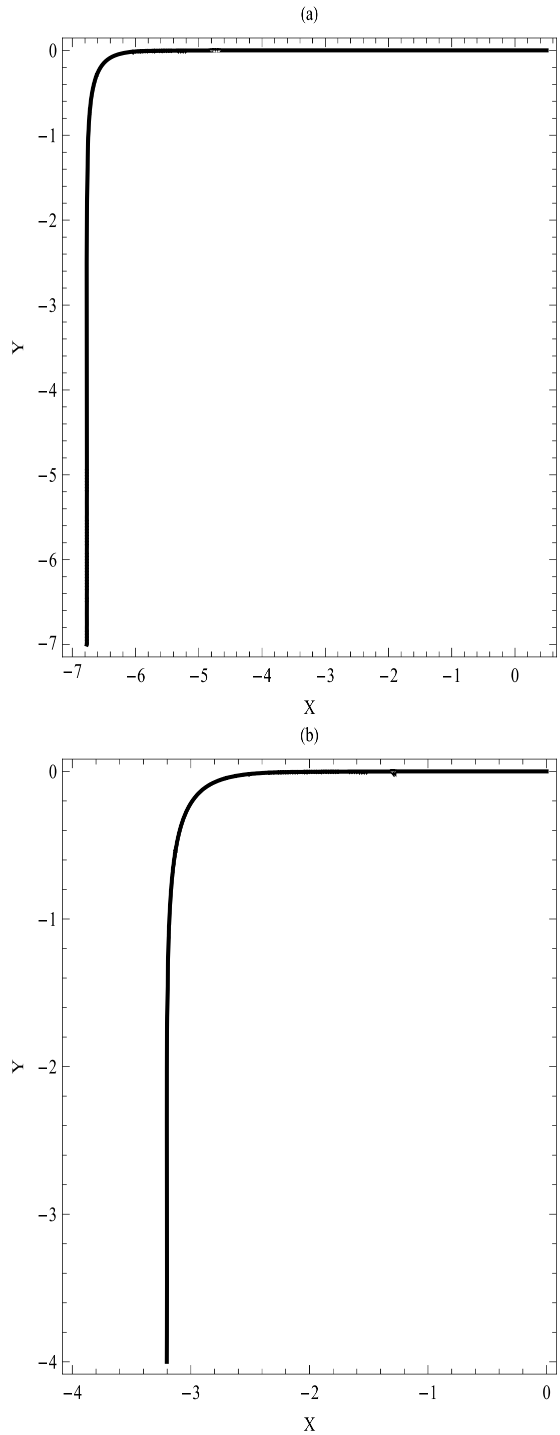

Fig. 2 We have defined $X = \log[\Delta\tilde{m}_\nu^2]$ and $Y = \log[\sin^2 2\theta]$. The plots are for neutrino energy (**a**) $E_{\nu,14} = 1$ and (**b**) $E_{\nu,14} = 10^{-5}$

of $E_\nu \sim 10^{14}$ eV in SGR 1806-20, which is a magnetar. By equating the electron–positron annihilation rate to the local-expansion rate at a radius $r_\pm$ and assuming that the very high-energy neutrinos are produced within this radius, we found that the resonance condition can never be satisfied for these neutrinos. On the other hand, with the observed atmospheric neutrino-oscillation parameters (both mixing angle and the mass square difference) [28], resonant oscillations of neutrinos can take place under the condition that their energies are to be in the GeV range and they should be produced deep inside the SGR atmosphere ($r < r_\pm$), where the plasma temperature is of the order of 50 keV or higher.

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