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USE OF DIPOLE SUM RULES TO ESTIMATE UPPER AND LOWER BOUNDS FOR RADIATIVE AND TOTAL WIDTHS OF $\chi(3414)$, $\chi(3508)$ AND $\chi(3552)$*

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August 12, 1976

ABSTRACT

Upper and lower bounds on the widths for $\chi_J \to \gamma\,\psi(3095)$ can be estimated, assuming E1 transitions and approximate Russell-Saunders coupling for the $c\bar{c}$ system. Experimental widths for $\psi(3684) \to \gamma\,\chi_J$ make the lower bound more restrictive, giving radiative widths of $160 \to 240$, $230 \to 400$, $280 \to 480$ keV for 3414, 3508, 3552 MeV states, respectively. Cascade branching ratio data permit estimating the total widths to be > 1.6, 0.3-1.5, 0.6-4 MeV, respectively.

In the spectroscopy of new particle states uncovered in e^+e^- annihilation it is now rather clearly established that the three states[1,2,3] generically labelled as χ have $J^{PC} = 0^{++}, 1^{++}, 2^{++}$ for the 3414, 3508, and 3552 MeV states, respectively.[4] The spin-parity values and ordering of these states are just what is expected of the triplet p-states in any $q\bar{q}$ bound state model that parallels positronium.[5,6] The χ states are formed by the radiative decay $\psi(3684) \to \gamma\chi$. They are observed to decay into hadrons and also, for the $J = 1$ and $J = 2$ (and marginally for the $J = 0$) via the two-photon cascade, $\psi(3684) \to \gamma_1\chi \to \gamma_2\psi(3095)$. Recently, branching ratios have been reported for the $\psi(3684) \to \gamma\,\chi_J$ transitions[7,8] and also products of branching

ratios for the cascade transitions.[8,9,10] These are summarized in Fig. 1.

The view that these states are describable to a good approximation by a nonrelativistic potential model, with v^2/c^2 corrections, receives increasing support from the data.[6] We adopt this picture here. In the Russell-Saunders limit (J^2, J_z, L^2, S^2 diagonal) the states have the designations shown in Fig. 1. The details of the binding potential need not concern us, but we make the assumption from the outset that tensor forces, relativistic effects, coupled channel effects, etc. are unimportant enough that they do not vitiate our use of the second sum rule.

The branching ratios shown in Fig. 1 for $\psi(3684) \to \gamma\,\chi_J$ can be converted into radiative widths using $\Gamma_t \cong 228$ keV[11]: $\Gamma(\psi' \to \gamma\,\chi_J) = 17.5 \pm 6$ keV, 20 ± 7 keV, and 18 ± 7 keV, for the $J = 0^{++}, 1^{++}$, and 2^{++} states, respectively.[12] Values in the range from 10 to 30 keV emerge from bound state models, provided the quark charges are $e_Q = \pm 2/3$.[13,6] Furthermore, with the experimentally favored J assignments, the experimental products $\Gamma_J/(2J+1)k^3 = (10 \pm 3) \times 10^{-4}$, $(13 \pm 5) \times 10^{-4}$, and $(16 \pm 6) \times 10^{-4}$ GeV^{-2}, show constancy within errors. This indicates that the E1 rate formula,

$$\Gamma(\psi' \to \gamma\,\chi_J) \quad = \quad (4/27)\alpha\, e_Q^2 (2J+1)k^3 \left| \langle 2p|r|2s \rangle \right|^2 \tag{1}$$

is approximately valid, with a common matrix element for all three transitions. Though other multipoles are possible in principle for the $J = 1, 2$ states, we assume complete E1 dominance for the transition rates of concern here.[14]

The widths for the radiative transitions $\chi_J \to \gamma\,\psi(3095)$ can be calculated in bound-state models, but cannot be compared with the data on $(BR)^2$ without knowledge of the total widths of the χ_J states. We show here that, with E1 dominance of the $\psi' \to \gamma\chi_J$ and $\chi_J \to \gamma\psi$ transitions and the approximate validity of Russell-Saunders coupling, upper and lower limits can be set on the widths for $\chi_J \to \gamma\psi$, limits that are stringent enough to provide estimates of the total widths of the χ_J states. Given the experimental and theoretical uncertainties, these latter quantities are rather rough, but they may well be the only semi-experimental estimates available for some time.

We use two dipole sum rules. The first is the well-known Thomas-Reiche-Kuhn sum rule,

$$2\mu \sum_j \omega_{ji} |\langle j|\vec{r}|i\rangle|^2 = 3 \qquad (2)$$

where μ is the reduced mass of the two-particle system ($\hbar = c = 1$ here). With the ground state $\psi(3095)$ as the initial state, Eq. (2) permits an upper limit to be set on the E1 widths of the transitions $\chi_J \to \gamma\psi$ in the well-known way:[15]

$$\Gamma(\chi_J \to \gamma\psi) < 2\alpha e_Q^2 k^2/3\mu \qquad (3)$$

With $e_Q = 2/3$ and $2\mu = m_c = 1.65$ GeV, this gives the values shown in Table I. These upper bounds are of course dependent on our assumptions about quark charges and masses. The charge choice of 2/3 is strongly indicated by the semi-quantitative agreement of the radiative widths for $\psi' \to \gamma\chi_J$, already mentioned -- a factor of 4 smaller calculated rates seems unreasonable. The effective quark mass is perhaps less certain, but the remarkable agreement of the calculations

of De Rújula, Georgi and Glashow[16] with the observed masses and mass splittings of the charmed baryons[17] indicates that our choice cannot be appreciably wrong.

A lower bound can be obtained by use of a dipole sum rule[18] that involves only the transitions $n,\ell \to n',\ell' = \ell - 1$. We apply it to the $2p \to ns$ transitions shown in the bottom half of Fig. 1. For these the sum rule reads

$$2\mu \sum_n \omega_{ns,2p} |\langle ns|r|2p\rangle|^2 = -1 \qquad (4)$$

The beauty of this sum rule is two-fold. The minus one on the right hand side shows that the downward $2p \to 1s$ transition (γ_2 in Fig. 1) dominates the sum since it is the only term with a negative energy difference. This means we can obtain a lower bound on the γ_2 rate. Furthermore, the $2p \to 2s$ contribution is known from $\psi' \to \gamma\chi_J$. This will raise the lower bound significantly. Expressing the lower bound for the width of $\chi_J \to \gamma\psi$ as much as possible in terms of experimental quantities, we write[19]

$$\Gamma(\chi_J \to \gamma_2\psi) \geq \frac{2\alpha k_2^2}{9\mu} e_Q^2 + \frac{3}{2J+1}\left(\frac{k_2}{k_1}\right)^2 \Gamma(\psi' \to \gamma_1\chi_J) \qquad (5)$$

Comparison with Eq. (3) shows that the first term is 1/3 of the upper bound. Note that the second term sets, within our assumptions, an "absolute" lower bound, independent of quark charges and masses. With the experimental widths for the upper transition we find the lower bounds and "absolute" lower bounds shown in Table I. These values have experimental uncertainties of ~30%, at least for the "absolute" lower bound.

Table I shows that the radiative widths for $\chi_J \to \gamma\psi$ are rather closely delimited by the upper and lower bounds of Eqs. (3) and (5). In particular, the experimental branching ratios[7,8] for $\psi' \to \gamma\,\chi_J$ set relatively model-independent "absolute" lower limits of the order of 100 keV for all three transitions.

The branching ratios[8] for the cascade transitions $\psi' \to \gamma_1\chi_J \to \gamma_2\psi$ can be used, together with the bounds of Table I, to estimate the total widths of the χ_J states. With 0.08$^-$, 0.09, and 0.08 for the branching rations for $\psi' \to \gamma_1\chi_J$ [7,8,12] for $J = 0,1,2$, the $\chi_J \to \gamma_2\psi$ branching ratios are estimated to be 0.025 ± 0.025 (or 0.065 ± 0.04[10]), 0.27 ± 0.09, and 0.125 ± 0.075. The errors here are only the errors in the cascade $(BR)^2$ values. There is an additional uncertainty of ∿30% from the branching ratios for the first transition. A series of estimates for bounds on the total widths of the χ_J states are given in Table II. The "absolute" (A) lower bounds are computed by dividing the $e_Q = 0$ bound from Table I by the <u>sum</u> of the central value of the radiative branching ratio and its estimated error. Similarly, an "absolute" upper bound uses the radiative upper bound from Table I and the <u>difference</u> of the central value and its associated error for the branching ratio. The plausible (P) upper and lower bounds come from the $e_Q = 2/3$ columns in Table I, divided by the central values of the branching ratios.

The estimates in Table II for total widths are presently uncertain by ±50% or more because of experimental uncertainties in the various branching ratios, apart from theoretical uncertainties. Nevertheless, they presumably provide at least order of magnitude estimates of the total widths of the χ_J states. The relative values within

each column should be more reliable.

Predictions[20,21,6] from a $SU(4) \otimes SU(3)$ color gluon gauge theory can be compared with the ranges in Table II. The annihilation rate for $\chi \to$ gluons and/or $q\bar{q}$ is supposed to represent the annihilation into ordinary hadrons. A typical rate is $\Gamma(\chi_0 \to gg) = 96\alpha_s^2|R'(0)|^2/M^4$, with $\Gamma(\chi_2 \to gg) = (4/15)\Gamma(\chi_0 \to gg)$. For the $J = 1^{++}$ and 1^{+-} states, the formula involves an additional factor of $\alpha_s \ln\left[4m^2/(4m^2 - M^2)\right]$ and is less reliable.[21] These rates are proportional to the square of the radial derivative of the p-state wave function at the origin, a quantity that varies as the fifth power of the scale parameter of the bound state wave functions. Estimates range from $|R'(0)|^2 = 0.04\ \mathrm{GeV}^5$ [22] to $0.09\ \mathrm{GeV}^5$ [20]. A central value of $0.06\ \mathrm{GeV}^5$ and $\alpha_s = 0.19$ gives $\Gamma(\chi_0 \to gg) \simeq 1.5$ MeV, $\Gamma(\chi_2 \to gg) \simeq 0.4$ MeV, and less reliably, $\Gamma(\chi_1 \to g(\bar{q}q)) \simeq 0.13$ MeV. Including the radiative decays, we estimate the "theoretical" total widths to be ∿1.5 MeV, ∿0.2 MeV, ∿0.45 MeV for $J = 0,1,2$. These correspond roughly to the "absolute" lower bounds of Table II (unless the cascade branching ratio of ref. 10 is used). No very compelling conclusion follows from this comparison. Because of sensitivity to $|R'(0)|^2$ it may be more reasonable to use the ranges in Table II to restrict the parameters in one's model of charmonium.

The upper and lower bounds in Table I are exact statements in the limit of E1 transitions only and Russell-Saunders coupling with small splittings and no configuration mixing. The reality is that the triplet-singlet splitting of the s-states is apparently large, the p-states are relatively widely split and their successive spacings do not satisfy the Landé interval rule. To understand the p-state splittings it is necessary to include a tensor force contribution as well as the spin-orbit coupling.[23,6] There are, in addition, potential complications from relativistic effects and coupled channels above the charm threshold. For a relatively low-lying transition such as $\chi_J \to \gamma\psi$, the mixing of d-states into the s- and f-states into the p- by the tensor force may not be a serious problem. Certainly we can say that the bounds in Table I are not strict bounds. We can only hope that they provide reasonable limits on the expected radiative widths from which the rough ranges of Table II for total widths follow.

As a final, very speculative, remark, we note that there is a sum rule for E1 transition probabilities[24] that reads

$$2\mu^2 \sum_j \omega_{ji}^3 |\langle j|\vec{r}|i\rangle|^2 = \langle i|\nabla^2 V|i\rangle \qquad (8)$$

In some models of charmonium[6,23] the expectation value on the right is $(3m_c^2/2)$ times the singlet-triplet s-state mass splitting ΔM due to the Fermi interaction. We can thus get a lower bound on ΔM from the bounds on $\Gamma(\chi_J \to \gamma\psi)$ from Table I, namely, $\Delta M > 3\Gamma(\chi_J \to \gamma\psi)/4\alpha e_Q^2 = 35\text{-}65$ MeV. This suggests that the pseudoscalar partner (η_c) of the $\psi(3095)$ does not lie very close in mass to the ψ. It may indeed be the $X(2800)$.

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* Work supported in part by U. S. Energy and Research Administration.

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9. In ref. 8 four $\gamma\gamma$ events are associated with a state at 3455 MeV (or 3340 MeV). This might be the 0^{-+} partner of the $\psi(3684)$. We do not consider this state here.
10. For the J = 0 initial state, the $(B.R.)^2$ shown in Fig. 1 is based on 1 event from SPEAR. If the 2 events reported by Wiik [Proc. 1975 Int. Symposium on Lepton and Photon Interactions at High Energies, ed., W. T. Kirk, Stanford Linear Accelerator Center, p. 69] are added, this ratio becomes $0.5 \pm 0.3\%$.
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12. The quoted errors do not include the ±25% uncertainty in the total width of the $\psi(3684)$.

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14. The charge coupling involves the matrix element of $\vec{\varepsilon}\cdot\vec{r}\ \cos(\vec{k}\cdot\vec{r}/2)$. Replacement of the cosine by unity introduces errors in the rates of 15-20% at most, much less for the softest photons.[6]

15. The E1 rate for $\chi_J \to \gamma\psi$ is obtained from Eq. (1) by the substitutions $(2J+1) \to 3$, $\langle 2p|r|2s\rangle \to \langle 2p|r|1s\rangle$.

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TABLE I. Upper and lower bounds on radiative widths for $\chi_J \to \gamma_2 \psi$. (Masses and photon energies in MeV, widths in keV. $e_Q = 0$ column is second term only from Eq. (5)).

M	k_1	k_2	Upper bound	Lower bounds $e_Q=2/3$	Lower bounds $e_Q=0$
3414	260	304	240	160	80
3508	172	389	400	230	100
3552	130	428	480	280	120

TABLE II. Estimated upper and lower bounds on the total widths of the χ_J states. (All masses and widths are in MeV. The estimates in parentheses for J = 0 are based on the $(B.R.)^2$ of ref. 10. A means "absolute", P plausible.)

J	M	Lower Bounds A	Lower Bounds P	Upper Bounds P	Upper Bounds A
0	3414	1.6 (0.8)	6.4 (2.5)	9.6 (3.7)	∞ (9.6)
1	3508	0.3	0.9	1.5	2.2
2	3552	0.6	2.2	3.8	9.6

Figure Caption

Fig. 1. (Top) Observed radiative transitions through the χ states. For the first transitions the numbers are branching ratios (ref. 7; for the $J = 0$ final state the second number is from ref. 8). For the second step the numbers are the products of the branching ratios (ref. 8, 10). (Bottom) Schematic diagram showing the transitions involved in the second sum rule, used to set lower limits on the radiative widths.

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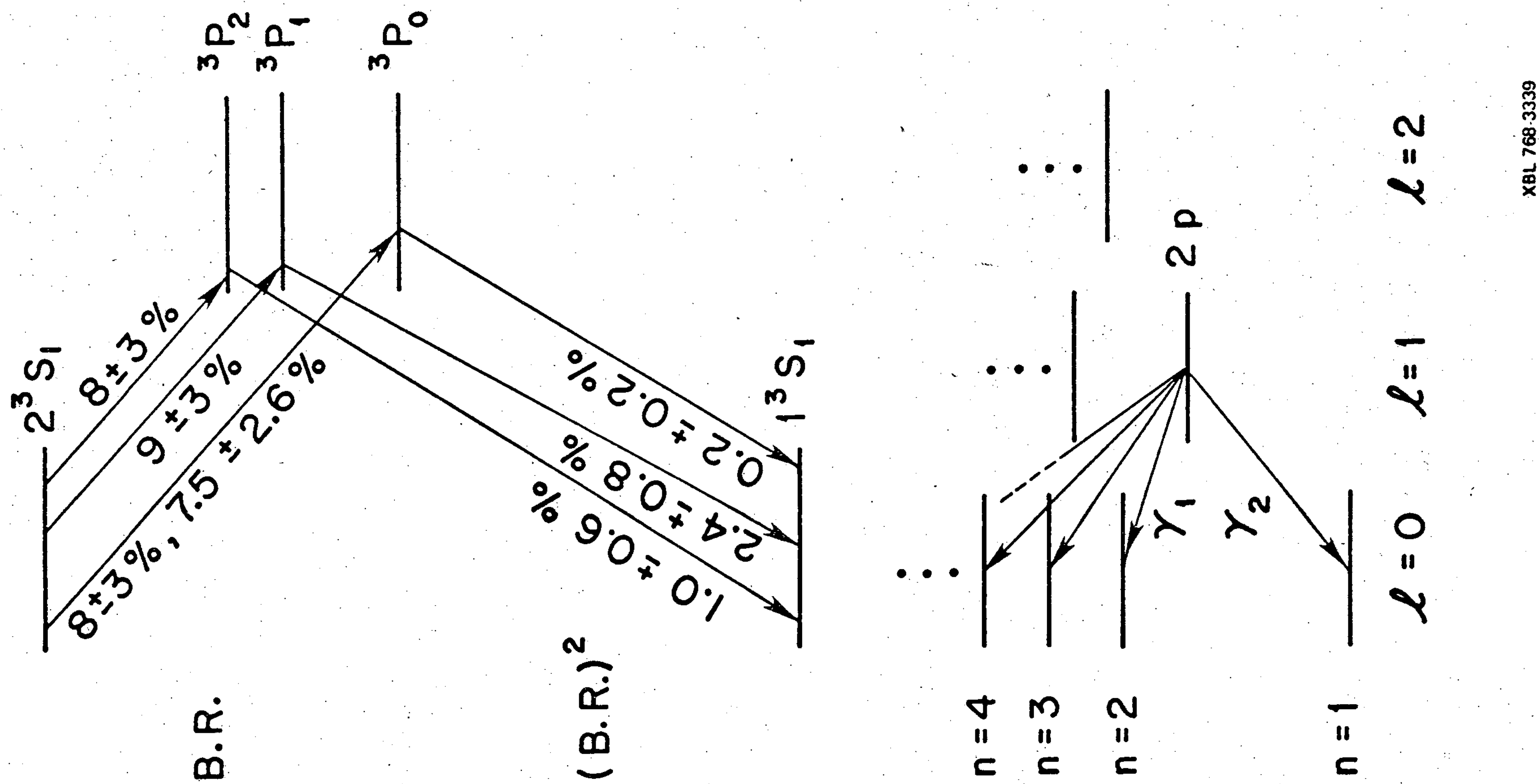

Fig. 1

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