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Authors
Dafalias, Y. F
Taiebat, M.
Rollo, F.
et al.

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Convergence of rotational hardening with bounds in clay plasticity

Y. F. DAFALIAS*†‡, M. TAIEBAT§, F. ROLLO|| and A. AMOROSI||

Convergence of the rotational hardening variable α used in the anisotropic models of clay plasticity, with a constitutively defined attractor/bound $\alpha_b(\eta)$, function of the stress ratio η, under fixed η loading, is analytically investigated. It is analytically shown that depending on various parameters of the rate equation of the evolution of α, such convergence may or may not occur despite the apparent necessity of convergence stemming from the form of the evolution equation of α.

KEYWORDS: anisotropy; clays; constitutive relations

NOTATION

c	rotational hardening (RH) parameter for equation (4)
e	void ratio
$f = 0$	yield surface (YS)
$g = 0$	plastic potential
K_0	coefficient of earth pressure at rest
L	plastic multiplier
M	critical-state stress ratio in the $p-q$ plane
n	RH parameter for equation (5)
p	mean effective stress
p_{at}	atmospheric pressure
p_0	size of YS along the p-axis
q	deviatoric stress
x	RH parameter for the saturation limit of anisotropy
α	RH variable in the $p-q$ space
α_b	bound of α in the $p-q$ space
α_∞	another bound of α in the $p-q$ space
ε	strain
η	stress ratio
κ	slope of the rebound line in the e– ln p plane
λ	slope of the compression line in the e– ln p plane
v	Poisson's ratio

INTRODUCTION

Rotational hardening (RH) is a term indicating the rotation of the yield surface (YS) $f = 0$, and plastic potential surface (PPS) $g = 0$, in stress space as a result of non-hydrostatic loading, reflecting the developing fabric anisotropy. Sekiguchi and Ohta (1977) introduced RH for clay plasticity constitutive modelling, followed by a plethora of various other propositions which are beyond the scope of this paper. RH is expressed by an evolving deviatoric stress-ratio-type tensor $\boldsymbol{\alpha}$, which becomes a scalar-valued dimensionless stress-ratio quantity α in the triaxial $p-q$ space, exclusively dealt with in this work.

The scope of this study is to address the issue of convergence of the evolving α with a constitutively defined bound $\alpha_b(\eta)$, function of the stress ratio $\eta = q/p$, under fixed stress ratio η loading. This type of loading is encountered in some practical applications, such as K_0 loading in the field, but the motivation of this analysis in addressing fixed η loading is not a practical application per se, but the fact that its laboratory realisation provides useful data for constitutive modelling calibration. It is shown analytically that depending on various parameters of the rate equation of the evolution of α, such convergence may or may not occur, and in the latter case, convergence with a different bound is obtained. The subject matter is presented using a specific form of the YS but the conclusions can be readily extended to various other forms of YS employing RH.

THE RH CONSTITUTIVE MODEL AND ITS APPLICATION TO FIXED η LOADING

In Dafalias (1986) and Dafalias and Taiebat (2013, 2014), where further references to RH can be found, the YS and PPS for associative plasticity are given analytically by

$$g = f = (q - p\alpha)^2 - \left(M^2 - \alpha^2\right)p(p_0 - p) = 0 \tag{1}$$

and plotted in the $p-q$ space of Fig. 1. M is the critical-state stress ratio, α is the stress ratio-type RH variable that must always be less than M for equation (1) to have real solution for p and q, and p_0 is the value of p at $\eta = \alpha$; clearly, $p_0 \geq p \geq 0$. The plastic volumetric and deviatoric strain increments are given by

$$\mathrm{d}\varepsilon_v^p = \langle L\rangle \frac{\partial g}{\partial p} = \langle L\rangle p\left(M^2 - \eta^2\right) \tag{2a}$$

$$\mathrm{d}\varepsilon_q^p = \langle L\rangle \frac{\partial g}{\partial q} = \langle L\rangle 2p(\eta - \alpha) \tag{2b}$$

respectively, where the loading index (or plastic multiplier) L depends on the increments dp, dq and its exact form is obtained from the consistency condition $df = 0$. A non-associative flow rule can be adopted if one uses N instead of M in equation (1) for the YS $f = 0$ (Jiang *et al*., 2012).

The increment of p_0 follows from the usual e–ln p normal consolidation line with slope λ and the rebound (elastic) line with slope κ for clays, as $\mathrm{d}e^p = -(\lambda - \kappa)(\mathrm{d}p/p_0)$ with $\mathrm{d}e^p$ the plastic increment of the void ratio related to the plastic volumetric strain increment by $\mathrm{d}e^p = -(1 + e_{in})\,\mathrm{d}\varepsilon_v^p$ with e_{in},

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*Department of Civil and Environmental Engineering, University of California, Davis, USA.
†Department of Mechanics, Faculty of Applied Mathematical and Physical Sciences, National Technical University of Athens, Athens, Greece.
‡Institute of Thermomechanics, Czech Academy of Sciences, Prague, Czech Republic.
§Department of Civil Engineering, University of British Columbia, Vancouver, BC, Canada (Orcid:0000-0003-2067-8161).
||Department of Structural and Geotechnical Engineering, Sapienza University of Rome, Rome, Italy (Orcid:0000-0002-5390-4447).

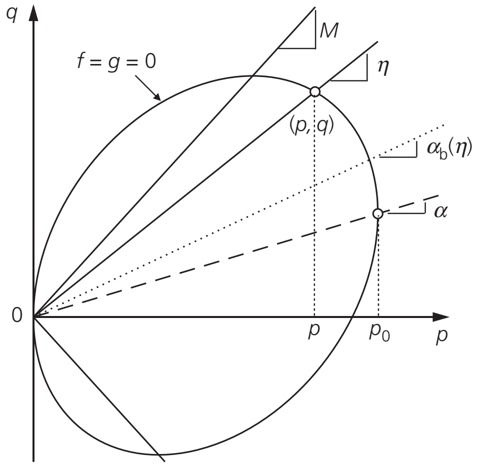

Fig. 1. Schematic diagram of the anisotropic yield and PPS in the $p-q$ space and the lines with slopes RH variable α, loading stress ratio η and bound $\alpha_b(\eta)$ (using the simplest form of η/x here)

the initial value of e. A combination of the above in conjunction with equation (2a) yields

$$\mathrm{d}p_0 = \frac{1+e_{\text{in}}}{\lambda-\kappa} p_0 \mathrm{d}\varepsilon_v^p = \langle L\rangle \frac{1+e_{\text{in}}}{\lambda-\kappa}\, p_0 p\left(M^2-\eta^2\right) \tag{3}$$

The increment of α is given by Dafalias and Taiebat (2013)

$$\mathrm{d}\alpha = \langle L\rangle c p_{\text{at}} \frac{p}{p_0}(\alpha_b(\eta)-\alpha) \tag{4}$$

where $c > 0$ is a dimensionless model constant controlling the pace of evolution, p_{at} is the atmospheric pressure for non-dimensionalising c and $\alpha_b(\eta)$ is the bound, function only of η such that $\alpha_b(\eta) < \eta$. Dafalias and Taiebat (2013, 2014) proposed various forms of $\alpha_b(\eta)$, the simplest one being the original suggestion by Dafalias (1986) with $\alpha_b(\eta) = \eta/x$ and $x \geq 1$, a model constant. In this work, the exact form of $\alpha_b(\eta)$ is irrelevant.

According to equation (4), α evolves towards $\alpha_b(\eta)$ because its increment dα is along $\alpha_b(\eta) - \alpha$. The name 'bound' for $\alpha_b(\eta)$ is used for historical reasons as one can have $\alpha > \alpha_b(\eta)$, which suggests that $\alpha_b(\eta)$ acts for α as an attractor rather than a bound. This can be visualised in Fig. 1 for the case $\alpha < \alpha_b(\eta)$, where a line of a fixed slope η is necessarily associated with a line of fixed slope $\alpha_b(\eta)$ (irrespective of the definition of the latter), towards which the line of slope α evolves. Under extensive fixed η loading, α is expected to merge asymptotically with $\alpha_b(\eta)$ as per equation (4).

But will it merge? The answer to the above question was first investigated numerically by carrying out fixed η loadings and using the simple definition of $\alpha_b(\eta) = \eta/x$. Two such cases are shown in Figs 2(a) and 2(b), where the numerical implementation of equation (4) used the parameters and the initial values presented in Table 1, with the initial value of α set at $\alpha_{\text{in}} = 0{\cdot}10$, while η maintained its initial value of 0·6 throughout the loading; the only difference between the two cases was the value of c.

It was found that for a large value of c, the merging of α with $\alpha_b(\eta)$ seems to have taken place, the latter being $\alpha_b(0{\cdot}6) = 0{\cdot}6/1{\cdot}71 = 0{\cdot}351$, while for a small value of c, no such merging is seen to occur. One may then argue that the small value of c naturally delays convergence with $\alpha_b(\eta)$, but this is not really the case shown in Fig. 2(b) because the α seems to converge asymptotically with another bound lower than $\alpha_b(\eta)$ and denoted by α_∞, as p goes to infinity. It appears therefore that equation (4) yields the asymptotic convergence of α with $\alpha_b(\eta)$ only for certain values of c despite the fact that this does not follow at first glance from the form of equation (4) that suggests α evolves towards $\alpha_b(\eta)$ irrespective of the value of positive c.

ANALYTICAL INVESTIGATION OF RH CONVERGENCE UNDER FIXED η LOADING

The numerical results may suggest but cannot provide a rigorous interpretation of obtained graphs, notwithstanding the reasons of observed differences. To reveal the reasons behind the foregoing unexpected numerical observations, equation (4) must be integrated in closed analytical form. However, it will be expedient to slightly generalise equation (4) to include any power of p_0 with the corresponding adjustment of the power of p_{at} for the non-dimensionalisation of c, such that equation (4) reads

$$\mathrm{d}\alpha = \langle L\rangle c p_{\text{at}}^n \frac{p}{p_0^n}(\alpha_b(\eta)-\alpha) \tag{5}$$

Clearly, for $n = 1$, equation (4) is retrieved from equation (5). Solving equation (3) for L and substituting into equation (5) one obtains

$$\frac{\mathrm{d}\alpha}{\alpha_b(\eta)-\alpha} = C(\eta)\frac{\mathrm{d}p_0}{p_0^{n+1}} \tag{6a}$$

$$C(\eta) = \frac{\lambda-\kappa}{1+e_{\text{in}}}\frac{1}{M^2-\eta^2} c p_{\text{at}}^n \tag{6b}$$

If based on the definition of $\alpha_b(\eta)$, it happens that $\alpha_b(\eta) - \alpha = 0$, equation (5) yields d$\alpha = 0$ and equations (6a) and (6b) are irrelevant. The task of integrating equations (6a) and

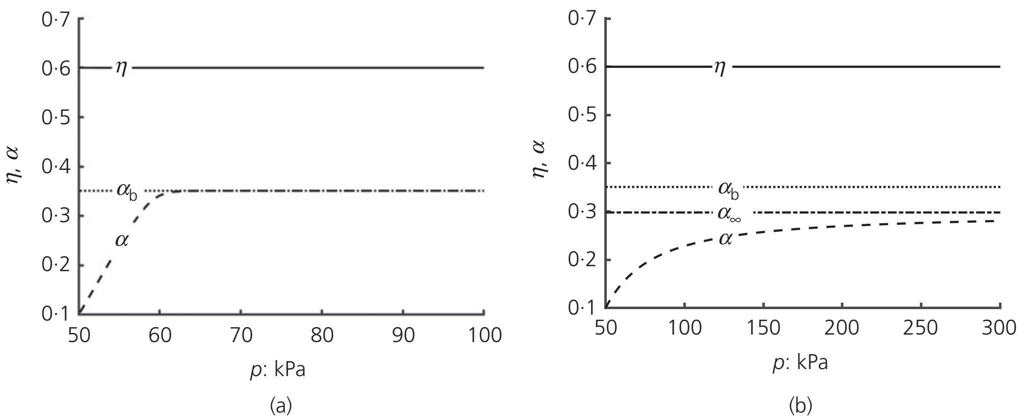

Fig. 2. Illustration of the effect of parameter c on the evolution of the RH variable α: (a) $c = 200$; (b) $c = 5$

Table 1. Parameters and initial values of the model

Symbol	M	λ	κ	ν	x	p_{in} : kPa	η_{in}	e_{in}	$p_{0,\mathrm{in}}$: kPa	α_{in}
Value	1·1	0·44	0·04	0·20	1·71	50	0·6	1·5	60·42	0·1

(6b) is to address the convergence of α with $\alpha_b(\eta)$ under drained fixed η loading starting from $\alpha = \alpha_{\mathrm{in}} \neq \alpha_b(\eta)$ with the sub 'in' meaning 'initial'. Notice that for fixed η loading, the quantity $C(\eta)$ is constant. With the foregoing understanding integration of equations (6a) and (6b) from α_{in} to α and from $p_{0,\mathrm{in}}$ to p_0 yields two distinct families of solutions

$$\frac{\alpha}{\alpha_b(\eta)} = 1 - \left(1 - \frac{\alpha_{\mathrm{in}}}{\alpha_b(\eta)}\right)\left(\frac{p_{0,\mathrm{in}}}{p_0}\right)^{C(\eta)} \text{for } n = 0 \tag{7}$$

$$\frac{\alpha}{\alpha_b(\eta)} = 1 - \left(1 - \frac{\alpha_{\mathrm{in}}}{\alpha_b(\eta)}\right) \times \exp\left[-\frac{C(\eta)}{n}\left(\frac{1}{p_{0,\mathrm{in}}^n} - \frac{1}{p_0^n}\right)\right] \text{for } n \neq 0 \tag{8}$$

During the integration of equations (6a) and (6b), the quantity $\ln[|(\alpha_b(\eta) - \alpha)|/|(\alpha_b(\eta) - \alpha_{\mathrm{in}})|]$ appears. However, $(\alpha_b(\eta) - \alpha)/(\alpha_b(a) - \alpha_{\mathrm{in}}) > 0$ because α and α_{in} are both either greater or smaller than $\alpha_b(\eta)$, hence, the absolute values of the foregoing ln function can be deleted. The obtained equations (7) and (8) satisfy the imposed initial condition $\alpha = \alpha_{\mathrm{in}}$ at $p_0 = p_{0,\mathrm{in}}$. However, an investigation of the limit of α is necessary, denoted as α_∞, for the various cases of continued fixed η loading as $p_0 \to \infty$ for consolidation, or $p_0 \to 0$ for dilation (even if $p_0 \to 0$ still the limit of α is denoted as α_∞). The objective will be to find out if the limit of the left-hand side $\alpha/\alpha_b(\eta)$ of equations (7) and (8) – that is, $\alpha_\infty/\alpha_b(\eta)$, is 1 or not; in the former case α converges with its bound $\alpha_b(\eta)$, while in the latter it does not.

Loading at fixed $\eta > M$

This is the most common case and implies that $C(\eta) > 0$ from equations (6a) and (6b) and that $p_0 \to \infty$ due to consolidation with the increasing size of the YS. The following cases then may occur according to the sign of n.

Case $n = 0$: it follows from equation (7) that as $p_0 \to \infty$ one obtains (recall $C(\eta) > 0$)

$$\frac{1}{p_0^{C(\eta)}} \to 0 \Rightarrow \frac{\alpha}{\alpha_b(\eta)} \to \frac{\alpha_\infty}{\alpha_b(\eta)} = 1 \tag{9}$$

Case $n > 0$: it follows from equation (8) that as $p_0 \to \infty$ one obtains

$$\frac{1}{p_0^n} \to 0 \Rightarrow \frac{\alpha}{\alpha_b(\eta)} \to \frac{\alpha_\infty}{\alpha_b(\eta)} = 1 - \left(1 - \frac{\alpha_{\mathrm{in}}}{\alpha_b(\eta)}\right)\exp\left[-\frac{C(\eta)}{n}\frac{1}{p_{0,\mathrm{in}}^n}\right] \tag{10}$$

Case $n < 0$: it follows from equation (8) that as $p_0 \to \infty$ one obtains (recall $C(\eta) > 0$)

$$\frac{1}{p_0^n} \to \infty \Rightarrow \frac{\alpha}{\alpha_b(\eta)} \to \frac{\alpha_\infty}{\alpha_b(\eta)} = 1 \tag{11}$$

Loading at fixed $\eta > M$

This is the less common case and implies that $C(\eta) < 0$ from equations (6a) and (6b) and that $p_0 \to 0$ due to dilation on decreasing the size of the YS. The following cases then may occur according to the sign of n.

Case $n = 0$: it follows from equation (7) that as $p_0 \to 0$ one obtains (recall $C(\eta) < 0$)

$$\frac{1}{p_0^{C(\eta)}} \to 0 \Rightarrow \frac{\alpha}{\alpha_b(\eta)} \to \frac{\alpha_\infty}{\alpha_b(\eta)} = 1 \tag{12}$$

Case $n > 0$: it follows from equation (8) that as $p_0 \to 0$ one obtains (recall $C(\eta) < 0$)

$$\frac{1}{p_0^n} \to \infty \Rightarrow \frac{\alpha}{\alpha_b(\eta)} \to \frac{\alpha_\infty}{\alpha_b(\eta)} = 1 \tag{13}$$

Case $n < 0$: it follows from equation (8) that as $p_0 \to 0$ one obtains

$$\frac{1}{p_0^n} \to 0 \Rightarrow \frac{\alpha}{\alpha_b(\eta)} \to \frac{\alpha_\infty}{\alpha_b(\eta)} = 1 - \left(1 - \frac{\alpha_{\mathrm{in}}}{\alpha_b(\eta)}\right)\exp\left[-\frac{C(\eta)}{n}\frac{1}{p_{0,\mathrm{in}}^n}\right] \tag{14}$$

From equations (9)–(14) it follows that $\alpha/\alpha_b(\eta) \to \alpha_\infty/\alpha_b(\eta) = 1$ in all cases except for equations (10) and (14). For these two cases if the absolute value $C(\eta) \gg 1$, which can be achieved by the choice of a large value of c in equations (6a) and (6b), one has approximately $\alpha/\alpha_b(\eta) \to \alpha_\infty/\alpha_b(\eta) \approx 1$ as the exponential term that includes $C(\eta)$ tends towards zero for large $|C(\eta)|$. A more careful inspection of the signs of $C(\eta)$ and n in equations (10) and (14) reveals that the exponential term is always smaller than 1, thus, $\alpha_\infty/\alpha_b(\eta) \leq 1$ or ≥ 1 depending on whether $\alpha_{\mathrm{in}}/\alpha_b(\eta) \leq 1$ or ≥ 1, respectively. $\alpha_b(\eta)$ is defined in such a way that always $\alpha_b(\eta) < M$ (Dafalias & Taiebat, 2013), thus, $\alpha < M$ and any such α serves as $\alpha_{\mathrm{in}} < M$. Therefore, accounting for the aforementioned effect of $\alpha_{\mathrm{in}}/\alpha_b(\eta)$ on $\alpha_\infty/\alpha_b(\eta)$, it follows that always $\alpha_\infty < M$ as both $\alpha_b(\eta) < M$ and $\alpha_{\mathrm{in}} < M$. Hence, convergence of α with $\alpha_\infty \neq \alpha_b(\eta)$ does not cause any concern that one may end up with $\alpha \geq M$ which is not allowed according to equation (1), while the limit $\alpha_\infty/\alpha_b(\eta)$ is well defined for calibration purposes. Nevertheless, there is an issue with equations (10) and (14) that renders the $\alpha_\infty/\alpha_b(\eta)$ function of the initial values α_{in} and $p_{0,\mathrm{in}}$ of α and p_0, respectively. This is counterintuitive to the expectation that extended fixed η loading as $p_0 \to \infty$ or 0 should provide the same $\alpha_\infty/\alpha_b(\eta) \neq 1$ irrespective of these initial values but this happens only for the cases where $\alpha_\infty/\alpha_b(\eta) = 1$.

INTERPRETATION AND ILLUSTRATION OF THE ANALYTICAL SOLUTIONS

As for $n = 1$ equation (5) becomes equation (4), the analytical solution of the latter for fixed η loading is given by equation (8) for $n = 1$. Consequently, the limit of α in this case is given by equation (10) for $n = 1$ and reads

$$\frac{\alpha}{\alpha_b(\eta)} \to \frac{\alpha_\infty}{\alpha_b(\eta)} = 1 - \left(1 - \frac{\alpha_{\mathrm{in}}}{\alpha_b(\eta)}\right)\exp\left[-C(\eta)\frac{1}{p_{0,\mathrm{in}}}\right] \tag{15}$$

Equation (15) solves the puzzle of the numerical simulations of Fig. 2. While equation (4) suggests at first glance that α must converge asymptotically with $\alpha_b(\eta)$ as there is always an increment $\mathrm{d}\alpha$ along $\alpha_b(\eta) - \alpha$, it hides the fact that another bound α_∞ exists with which α converges asymptotically, and which acts as a blockage preventing convergence

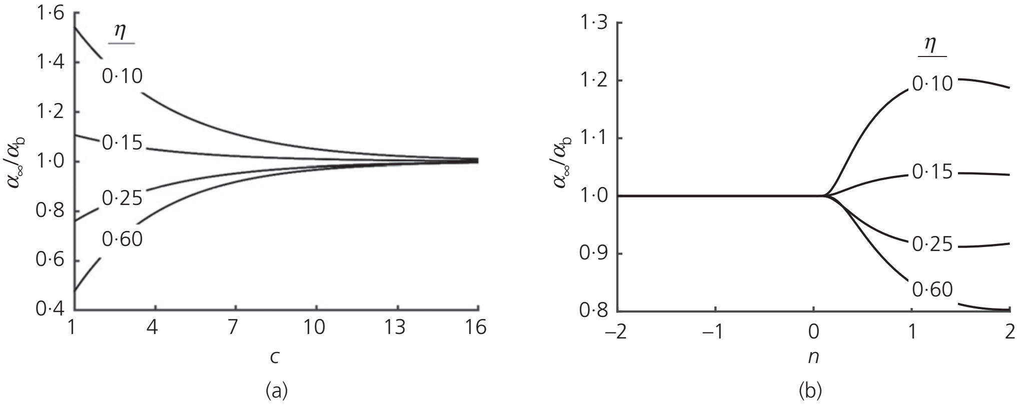

Fig. 3. Illustration of the effects of (a) parameter c for the case of $n = 1$ (equation (9)) and (b) parameter n for the case of $c = 5$ (equations (9)–(11)), on the ratio of $\alpha_\infty/\alpha_b(\eta)$

with $\alpha_b(\eta)$. This $\alpha_\infty \neq \alpha_b(\eta)$ is given by equation (15) for continued fixed $\eta < M$ loading. In concrete numerical terms and referring to the values of various quantities presented in Table 1, $\eta = \eta_{in} = 0{\cdot}6$ and $x = 1{\cdot}71$ yields $\alpha_b(0{\cdot}6) = 0{\cdot}6/1{\cdot}71 = 0{\cdot}351$, hence, it follows from equations (6b) with $n = 1$ and (15) that (a) for $c = 200$, the $\alpha_\infty/\alpha_b(0{\cdot}6) = 1$ (approximately) $\Rightarrow$ $\alpha_\infty = 0{\cdot}351$, as shown in Fig. 2(a), and (b) for $c = 5$, the $\alpha_\infty/\alpha_b(0{\cdot}6) = 0{\cdot}8496 \Rightarrow$ $\alpha_\infty = 0{\cdot}2982$, as shown in Fig. 2(b). The seemingly correct optical observation about the convergence of α with $\alpha_b(\eta)$ for the large value of c in Fig. 2(a) stems only from the infinitesimally small numerical deviation of $\alpha_\infty/\alpha_b(\eta)$ from 1 in this case.

A graphical illustration of the analytical results reached in relation to equations (9)–(15) is provided in the plots of Fig. 3. Figure 3(a) refers to equation (15) and illustrates the effect of various values of c and the stress ratio $\eta < M$ on the ratio $\alpha_\infty/\alpha_b(\eta)$. The values of Table 1 are used, including $\alpha_{in} = 0{\cdot}1$ and the value $x = 1{\cdot}71$ that defines $\alpha_b(\eta) = \eta/x$. In all cases, one has $\alpha_\infty \neq \alpha_b(\eta)$ as per equation (15) but observe that for $c > 15$, $\alpha_\infty \approx \alpha_b(\eta)$. For smaller values of c, the ratio $\alpha_\infty/\alpha_b(\eta)$ differs considerably from 1 and based on the discussion following equation (15), one can observe that depending on the values of η, which defines the corresponding $\alpha_b(\eta) = \eta/x$, $\alpha_\infty/\alpha_b(\eta)$ is smaller or greater than 1 depending on whether $\alpha_{in}/\alpha_b(\eta)$ is also smaller or greater than one, respectively. Figure 3(b) refers to both equations (10) and (11) and illustrates the effect of various values of the exponent n and the stress ratio η on the ratio $\alpha_\infty/\alpha_b(\eta)$ for a relatively small value of $c = 5$. For $n \leq 0$, one obtains $\alpha_\infty/\alpha_b(\eta) = 1$ for any value of $\eta < M$ but large deviations of $\alpha_\infty/\alpha_b(\eta)$ from 1 are seen for $n > 0$.

CONCLUSION

For a large class of simple anisotropic clay models, the key incremental (or rate) evolution equation for the evolving RH stress ratio internal variable α is of the form of equation (5) that includes an exponent n for p_0, suggesting that under continued fixed η drained loading, α is expected to converge with the RH bound $\alpha_b(\eta)$ that acts as an attractor on α. This work showed that if $\eta < M$ such convergence occurs during consolidation as $p_0 \to \infty$ when $n \leq 0$, while for $n > 0$, α converges with a known value $\alpha_\infty \neq \alpha_b(\eta)$, although close to it for sufficiently large values of the RH parameter c. Similarly, for $\eta > M$ the convergence of α with $\alpha_b(\eta)$ as $p_0 \to 0$ during the corresponding dilation occurs when $n > 0$, but again convergence with $\alpha_\infty \neq \alpha_b(\eta)$ takes place when $n < 0$. The foregoing conclusions are independent of the specific form of $\alpha_b(\eta)$. These conclusions show that the statement $\alpha_\infty = \alpha_b(\eta)$ as a consequence of $d\alpha = 0$ made in Dafalias & Taiebat (2013, 2014) is not always accurate. However, this does not imply any error in the development presented in the foregoing references because they addressed directly the role of the choice of specific $\alpha_b(\eta)$ and not the convergence of α with it.

It must be mentioned that these results were obtained in conjunction with the well-known and widely used specific form of a YS and plastic potential given by equation (1), as well as the specific form of RH of the bounding/attractor-type given by equation (5), thus, cannot be assumed to have universal validity. However, they act as precaution and provide guidelines not to assume a priori such convergence of α with an explicitly stated (as done here) or an implied $\alpha_b(\eta)$, in several other cases with different YSs.

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