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QCD SUM RULES AND I = 0 FINAL STATE IN D^+ AND F^+ MESONS SEMI-LEPTONIC DECAYS[†]

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ABSTRACT

QCD sum rules are used to give absolute bounds for the semi leptonic decay widths of the heavy mesons D^+ and F^+ into η and η'. This method, very stable and reliable in those cases where a heavy mass scale is involved, do not rely on any quark gluon picture of the above decays. While the F^+ decays may be important in relative value (if one relies on the present estimate of the F^+ lifetime), the very small fraction (less than 1%) obtained in the D^+ case where only the annihilation mechanism could be advocated, gives strong argument against its giving any substantiable enhancement.

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I. INTRODUCTION

We address ourselves to the question of finding absolute bounds for the semi leptonic decay rates of the D^+ and F^+ mesons into η and η', via the technique of QCD sum rules.[1] Several studies have already been devoted to those decays,[2] often connected with the so-called "annihilation mechanism", where the hadronic final state materialize from an even number of gluons. A possible substantiable enhancement was proclaimed, but at the price of somewhat doubtful assumptions.

The advantage of QCD sum rules is that they rely on the well established notion of hadronic current and do not stick to a possibly misleading interpretation of the decay in terms of quarks and gluons. One doesn't need any hypothesis about hadronic wave functions, and the only general assumption is that, at some big enough euclidian scale q^2, the current-current correlation function and its dispersive hadronic integral match at a good approximation. The fact that we can only find bounds for the decay rate is due to our incomplete knowledge of the hadronic integral, and their q^2 dependence reflects the perturbative QCD computation of the two point function of the current. However, due to the presence of a heavy mass scale (the mass of the c quark), the q^2 dependance turns out to be extremely weak in a wide range of values, say from $\sqrt{|q^2|} = 1$ GeV to $\sqrt{|q^2|} = 4$ GeV. This makes the compromise very easy between a q^2 high enough for the QCD perturbative serie and the sum rule technique to be reliable, and low enough such that the power law asymptotic behavior of the bounds is not yet set in. Consequently, the sum rule technique proves to be very trustable when applied to heavy mesons decays.

The only parameter which is left is the value of the scalar form factor at zero momentum transfer, proportional to $f_+(0)$. While in the case of the η, symmetry considerations give us an insight into its value, the η' case is known to be more tricky, and would need an understanding of the strong interactions dynamics. We shall make here the hypothesis that $f_+(0)$ in the η' case does not deviate more than $\pm$ 75% from the value predicted by a valence quark model. Whatever doubtful look this ansatz, it doesn't seem to us completely unreasonable and is supported by precedent estimates of the η' strong decay constant,[3] which has been shown to deviate surprisingly little from its quark model prediction. Moreover, the decays into η' will be shown to be less important than those into η, and an uncertainty in the first case will not affect much our conclusions. Using the estimate of the D^+ lifetime[4] $\tau_{D^+} = 9.1\ 10^{-13}$s, we show that the sum $\Gamma_{D^+ \to \eta e^+ \nu_e} + \Gamma_{D^+ \to \eta \mu^+ \nu_\mu} (\simeq 2\Gamma_{D^+ \to \eta e^+ \nu_e})$ cannot be expected to represent more than 1% of the total D^+ decays, and the sum $\Gamma_{D^+ \to \eta' e^+ \nu_e} + \Gamma_{D^+ \to \eta' \mu^+ \nu_\mu}$ a few tenths of a percent. The F^+ decays into η or η' are shown to be more important than the D^+'s. In addition to $f_+(0)$ dependant absolute bounds, we also use the data (still very unprecise) on the lifetime of the F^+ meson, $\tau_{F^+} = 2.2\ 10^{-13}$s[9] to give estimate on the branching ratios

$$\frac{\Gamma_{F^+ \to \eta e^+ \nu_e} + \Gamma_{F^+ \to \eta \mu^+ \nu_\mu}}{\Gamma_{F^+ \to all}}\ ,\quad \frac{\Gamma_{F^+ \to \eta' e^+ \nu_e} + \Gamma_{F^+ \to \eta' \mu^+ \nu_\mu}}{\Gamma_{F^+ \to all}}$$

For an $f_+(0)$ given by SU(4) symmetry, the first ratio is centered around 7%, and the second around 1% for $f_+(0)$ given by a valence quark model. We conclude that the I = 0 final states in the D^+ semi-leptonic decays can only represent a very small fraction of all the decays, while in the F^+ case, one could expect a substantiable proportion of η and η' states. As only the D^+ decays could proceed via the quark annihilation mechanism alone, we think that a substantiable enhancement by this last scheme is very unlikely. This is just in relation to the fact that the same hadronic matrix element controls both the quark decay and the quark annihilation processes; so, both are controlled by the same parameter $f_+(0)$. Consequently, we do not expect any drastic deviation for what one normally expects from Cabibbo suppressed (D^+) or Cabibbo allowed (F^+) decays.

II. SUMMARY OF THE TECHNIQUE

We shall only sketch out here the main steps of the derivation. More detailed information can be found in Ref. [5] and references therein. Let the meson M (mass M, energy momentum p) decay into M' (M', p') + lepton + neutrino, as depicted in Fig. 1. The decay rate may be written as:

$$\Gamma = G_F^2 \xi^2 \frac{1}{(2\pi)^3} \frac{1}{16 M^3} \int_{m_\ell^2}^{(M-M')^2} \frac{dt}{t} \left(1 - \frac{m_\ell^2}{t}\right)^2 \lambda^{1/2}(M^2, M'^2, t)$$
$$\times \left\{ m_\ell^2 |d(t)|^2 + (2t + m_\ell^2) \frac{1}{12} \lambda(M^2, M'^2, t) \, |f(t)|^2 \right\} \tag{2.1}$$

G_F is the Fermi constant = $1.0210^{-5}/M_p^2$ (M_p is the proton mass), ξ the Cabibbo factor, $\lambda(a, b, c)$ the phase space function,

$$\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc \tag{2.2}$$

m_ℓ the mass of the outgoing lepton. d(t) and f(t) are given by

$$d(t) = (M^2 - M'^2) \, f_+(t) + t \, f_-(t)$$
$$f(t) = 2 f_+(t) \tag{2.3}$$

in term of the form factors $f_+(t)$ and $f_-(t)$ defined by

$$\langle M'(p') | V^\mu(0) | M(p) \rangle = (p + p')^\mu f_+(t) + (p - p')^\mu f_-(t) \tag{2.4}$$

where V^μ is the hadronic current.

The bounds for $|f(t)|$ and $|d(t)|$ are respectively controlled by the J = 1 and J = 0 part of the propagator of the hadronic current (Fig. 2), $\Pi^{(1)}(q^2)$ and $\Pi^{(0)}(q^2)$, according to the decomposition:

$$\Pi^{\mu\nu}(q^2) \equiv i \int d^4x \, e^{iq\cdot x} \langle 0 | T V^\mu(x) V^{\nu\dagger}(0) | 0 \rangle$$
$$= -(g^{\mu\nu} q^2 - q^\mu q^\nu) \, \Pi^{(1)}(q^2) + q^\mu q^\nu \, \Pi^{(0)}(q^2) \tag{2.5}$$

Indeed, saturating their absorbtive parts with the $|M', n\rangle$ (or $|M', n'\rangle$) intermediate state, we end up with the inequalities:

$$\mathrm{Im}\, \Pi^{(1)}(t) \geq \frac{1}{16\pi} \frac{\left[(t-t_1)(t-t_0)\right]^{3/2}}{12\, t^3} |f(t)|^2 \, \theta(t - t_0)$$
$$\mathrm{Im}\, \Pi^{(0)}(t) \geq \frac{1}{16\pi} \frac{\sqrt{(t-t_1)(t-t_0)}}{t^3} |d(t)|^2 \, \theta(t - t_0) \tag{2.6}$$

where

$$t_0 = (M+M')^2 \quad \text{and} \quad t_1 = (M-M')^2 \; . \tag{2.7}$$

The θ functions in the r.h.s of Eq. (2.6) stress the fact that both form factors have a cut on the real axis from $t = t_o$ to $t = \infty$.

However, instead of $\Pi^{(0)}$ and $\Pi^{(1)}$, we shall rather work respectively with the propagator $\psi(q^2)$ of the divergence of the current, and the combination $\Pi(q^2) = \Pi^{(1)}(q^2) + \Pi^{(0)}(q^2)$, which corresponds to the decomposition (non orthogonal in J = 0 and J = 1 state)

$$\Pi^{\mu\nu}(q^2) = -(g^{\mu\nu}q^2 - q^\mu q^\nu)\,\Pi(q^2) + g^{\mu\nu} D(q^2) \tag{2.8}$$

The reasons are the following: in QCD, the convergent function is $\frac{\partial^2}{(\partial q^2)^2}(q^2\Pi^{(1)}(q^2))$, that is it needs 2 substractions to be defined, while the combination $\Pi^{(0)} + \Pi^{(1)}$ needs only one. This makes the upper bounds less divergent at big Q^2, as can be seen in Fig. 3, where we compare the upper bounds for the decay $D^+ \to \eta\, e^+ \nu_e$ obtained respectively with $\Pi^{(1)}$ and $\Pi^{(0)} + \Pi^{(1)}$. The choice of ψ instead of $\Pi^{(0)}$ in just for computational conveniency (ψ and $q^4\Pi^{(0)}$ have the same absorbtive part). ψ needs two derivates with respect to q^2 to be convergent in QCD.

Writing dispersion relations for $\psi''(q^2)$ $\left(= \frac{\partial^2}{(\partial q^2)^2}\psi(q^2)\right)$ and

$$\chi(q^2) = -q^2 \frac{\partial}{\partial q^2}\Pi(q^2) \tag{2.9}$$

and using the Eqs. (2.6), we obtain the inequalities:

$$\psi''(q^2) \geq \frac{1}{16\pi^2}\int_0^\infty dt\, \frac{2\sqrt{(t-t_1)(t-t_0)}}{t\,(t+Q^2)^3}\,|d(t)|^2\,\theta(t-t_0) \tag{2.10}$$

$$\chi(q^2) \geq \frac{1}{16\pi^2}\int_0^\infty dt\, \frac{Q^2\sqrt{(t-t_1)(t-t_0)}}{t\,(t+Q^2)^2}\left[\frac{|d(t)|^2}{t^2} + \frac{(t-t_1)(t-t_0)}{12t^2}|f(t)|^2\right] \times \theta(t-t_0)$$

$$(Q^2 = -q^2 \,,\; q^2 < 0) \tag{2.11a}$$

which entails

$$\chi(q^2) \geq \frac{1}{16\pi^2}\int_0^\infty dt\, \frac{Q^2[(t-t_1)(t-t_0)]^{3/2}}{12\,t^3\,(t+Q^2)^2}\,|f(t)|^2\,\theta(t-t_0) \tag{2.11b}$$

where we have used the positivity of $|d(t)|^2$ to go from Eq. (2.11a) to Eq. (2.11b). The Eq. (2.11b) is clearly non optimal as we have dropped the term $\frac{|d(t)|^2}{t^2}$. However, the price to pay (at least at high Q^2) is less than that would result from the use of $\Pi^{(1)}$ with one more substraction (see(Fig. 3)). We obtain the bounds for $|d(t)|$ respectively from the Eqs. (2.10) and (2.11b).[5,6] They write:

$$|2f_+(0)|\,\frac{\varphi_\chi(0)}{\varphi_\chi(z(t))}\left[1 - \sqrt{\frac{z^2(t)}{1-z^2(t)}}\,\sqrt{\frac{\chi(Q^2)}{4f_+^2(0)\,\varphi_\chi^2(0)} - 1}\right] \leq |2f_+(t)| \leq$$

$$|2f_+(0)|\,\frac{\varphi_\chi(0)}{\varphi_\chi(z(t))}\left[1 + \sqrt{\frac{z^2(t)}{1-z^2(t)}}\,\sqrt{\frac{\chi(Q^2)}{4f_+(0)^2\,\varphi_\chi^2(0)} - 1}\right] \tag{2.12}$$

and an analogous formula for d(t), replacing 2 $f_+(0)$ by d(0), $\chi(Q^2)$ by $\psi''(Q^2)$ and $\phi_\chi(Q^2)$ by $\phi_\psi(Q^2)$. z(t), $\phi_\chi(z)$ and $\phi_\psi(z)$ are given by

$$z(t) = \frac{\sqrt{t_0 - t} - \sqrt{t_0}}{\sqrt{t_0 - t} + \sqrt{t_0}} \tag{2.13}$$

$$\phi_\chi(z) = \frac{1}{16}\sqrt{\frac{1}{12\pi}}\sqrt{\frac{Q^2}{t_0}}\,(1+z)^2 \frac{\left(\sqrt{\frac{t_0 - t_1}{t_0}} + \frac{1+z}{1-z}\right)^{3/2}}{\left(\sqrt{\frac{t_0+Q^2}{t_0}} + \frac{1+z}{1-z}\right)^2} \tag{2.14}$$

$$\phi_\psi(z) = \sqrt{\frac{1}{8\pi}}\;\frac{1}{t_0}\;\frac{1+z}{1-z}\;\frac{\left(\sqrt{\frac{t_0 - t_1}{t_0}} + \frac{1+z}{1-z}\right)^{1/2}}{\left(\sqrt{\frac{t_0+Q^2}{t_0}} + \frac{1+z}{1-z}\right)^3} \tag{2.15}$$

The bounds for the decay rates result from plugging into Eq. (2.1) the bounds for $|d(t)|$ and $|f(t)|$ from Eqs. (2.12).

The only imput needed to implement the method is a computation of the functions $\psi''(q^2)$ and $\chi(q^2)$, and an estimate of the parameter $f_+(0)$ for each decay. This will be the subject of the next section.

III. INPUTS AND RESULTS. DISCUSSION

1. The Values of $f_+(0)$

From the Eqs. (2.1) and (2.12), the bounds for the decay rates are expected to behave roughly as the square of $f_+(0)$. Consequently, we need to give to it a reasonable range of variation.

The physical η and η' states are known to be most probably a mixture of the octet η_8 and singlet η_1 states of SU(3). However, the mixing angle is not well determined, all the more as a quark picture of those particles is certainly not sufficient.[7,8] We shall in the following neglect this mixing, encouraged in this way by the smallness of the proposed value of the mixing angle (-11°).[8] This approximation doesn't affect significantly our results (let us just keep in mind that the mixing would bring a little closer together the values of the $f_+(0)$'s in the η and η' cases).

In this framework, the magnitude of the $|f_+(0)|$ for the decays into η may be obtained by symmetry considerations:

$$\left|f_+(0)\right|_{D^+ \to \eta\, {e \atop \mu}\, {\nu_e \atop \nu_\mu}} \simeq \frac{1}{\sqrt{6}}$$
$$\left|f_+(0)\right|_{F^+ \to \eta\, {e \atop \mu}\, {\nu_e \atop \nu_\mu}} = \sqrt{\frac{2}{3}} \tag{3.1}$$

We shall allow a 50% variations on both side of those values, to take into account a possible very important breaking of the SU(4) symmetry. (The 2 decays correspond to a heavy (c) to light (d or s) mass scale transition, where we have a priori no reason to believe that SU(4) symmetry is well verified. However, up to now and surprisingly enough, the parameters $f_+(0)$ determined experimentally seem not to deviate very strongly from their symmetric values.)[9]

The η' case is more tricky, due to its singlet nature, and a possible strong mixing with gluonic states.[3,7,8] A naive valence quark model would give $f_+(0)_{\eta'} = \frac{1}{\sqrt{3}}$ but we have a priori no control on this hypothesis. We are however encouraged in this way by the evaluation of the strong decay constant $f_{\eta'}$ via QCD sum rules for the gluonic current $\alpha_s FF$.[3] It has been estimated to around one half of the quark model prediction, surprisingly near this a priori unjustified estimate. This supports the fact that, in spite of its intrinsically cumbersome nature, the η' behaves in some respects not very differently from a pure quark state. We shall take into account all these uncertainties by allowing $f_+(0)_{\eta'}$ to have a 75% variation on both sides from its quark model prediction.

The resulting range of variations for the $|f_+(0)|$'s are given in Table 1.

2. The QCD Scale q^2

The second parameter that we must fix is the scale $|q^2|$ at which one can trust the perturbative QCD expansion of the functions $\psi''(q^2)$ and $\chi(q^2)$, and the sum rule technique. From this point of view, the bigger $|q^2|$, the safer we are. However, at big $|q^2|$, the bounds diverge as a power of Q^2, becoming uninteresting (more precisely, the upper bounds diverge, while the lower ones go to 0). The remarkable fact here is that, due to the big mass scale (charm quark mass) involved, the bounds present a wide range of stability, from

$$\sqrt{|q^2|} = 1\ \text{GeV} \quad \text{to} \quad \sqrt{|q^2|} = 4\ \text{GeV} .$$

This phenomenon has already been emphasized in Ref. [6]. We shall choose Q^2 at the upper limit of this domain of stability, say $Q^2 = 16\ \text{GeV}^2$. This choice, well above the scale $(M\{^D_F + M\{^{\eta}_{\eta'})^2$ ensures undoubtedly the reliability of the sum rule technique. let us check in the following that we can also trust the perturbative QCD series.

The functions $\psi''(Q^2)$ and $\chi(Q^2)$ are given (in the so called $\overline{MS}$ renormalization scheme) by:[5,10]

$$\psi''(Q^2) = \frac{3}{8\pi^2} \frac{(\bar{m}_i(Q^2) - \bar{m}_j(Q^2))^2}{Q^2} \times$$
$$\Bigg\{ 2Q^2 \Bigg(\int_0^1 dx \frac{4x^2(1-x)^2}{[Q^2 x(1-x) + \bar{m}_i^2(Q^2)x + \bar{m}_j^2(Q^2)(1-x)]} - \frac{Q^2 x^3 (1-x)^3}{[Q^2 x(1-x) + \bar{m}_i^2(Q^2)x + \bar{m}_j^2(Q^2)(1-x)]^2} \Bigg)$$
$$+ \frac{\bar{\alpha}_s(Q^2)}{\pi} \left(\frac{11}{3} + O\left(\frac{\bar{m}^2}{Q^2} \ln \frac{Q^2}{\bar{m}^2} \right) \right)$$
$$+ \frac{16\pi^2}{3} \frac{(\bar{m}_i + \frac{1}{2}\bar{m}_j)\langle \bar{\psi}_j \psi_j \rangle + (\bar{m}_j + \frac{1}{2}\bar{m}_i)\langle \bar{\psi}_i \psi_i \rangle}{Q^4}$$
$$+ \frac{2\pi}{3} \frac{\langle \alpha_s FF \rangle}{Q^4} + O\left(\frac{1}{Q^6}\right) \Bigg\} , \tag{3.2}$$

$$\chi(Q^2) = \frac{1}{4\pi^2}\left\{ 6Q^2 \int_0^1 dx \frac{x^2(1-x)^2}{\left[Q^2 x(1-x) + \bar{m}_i^2(Q^2)\,x + \bar{m}_j^2(Q^2)(1-x)\right]} \right.$$

$$+ \frac{\bar{\alpha}_s(Q^2)}{\pi}\left(1 + \mathcal{O}\left(\frac{\bar{m}^2}{Q^2}\ln\frac{Q^2}{\bar{m}^2}\right)\right)$$

$$+ 8\pi^2 \frac{\bar{m}_i\langle\bar{\psi}_i\psi_i\rangle + \bar{m}_j\langle\bar{\psi}_j\psi_j\rangle}{Q^4}$$

$$\left. + \frac{2\pi}{3}\frac{\langle\alpha_s FF\rangle}{Q^4} + \mathcal{O}\left(\frac{1}{Q^6}\right)\right\} \tag{3.3}$$

$\bar{\alpha}_s(Q^2)$ is the QCD running coupling constant[11]

$$\bar{\alpha}_s(Q^2) = \frac{2\pi}{-\beta_1 \ln\frac{Q^2}{\Lambda^2}} \tag{3.4}$$

the $\bar{m}\,(Q^2)$ are the QCD running masses

$$\bar{m}(Q^2) = \frac{\hat{m}}{\left(\frac{1}{2}\ln\frac{Q^2}{\Lambda^2}\right)^{-\gamma_1/\beta_1}}\left\{1 - \frac{\gamma_1\beta_2}{\beta_1^3}\,\frac{\ln\ln Q^2/\Lambda^2}{\frac{1}{2}\ln Q^2/\Lambda^2}\right.$$

$$\left. + \frac{1}{\beta_1^2}\left(\gamma_2 - \frac{\gamma_1\beta_2}{\beta_1}\right)\frac{1}{\frac{1}{2}\ln Q^2/\Lambda^2}\right\} \tag{3.5}$$

with, for 3 colors and 4 flavors:

$$\beta_1 = \frac{-25}{6}\,,\quad \gamma_1 = 2\,,\quad \beta_2 = \frac{-77}{12}\,,\quad \gamma_2 = \frac{263}{36} \tag{3.6}$$

[12] [13]

We take Λ = 150 MeV and the following values for the invariant quark masses[14]

$$\hat{m}_c = 1.8\ \text{GeV}\,,\quad \hat{m}_s = .220\ \text{GeV}\,,\quad \hat{m}_d = .020\ \text{GeV} \tag{3.7}$$

We have kept in the formulas (3.2) and (3.3) the exact parametric one-loop expressions, normalized at their asymptotic values $\frac{3}{8\pi^2}\frac{(\bar{m}_i - \bar{m})^2}{Q^2}$ and $\frac{1}{4\pi^2}$ respectively, more accurate than an expansion in powers of $\frac{\bar{m}^2}{Q^2}$, in the presence of a heavy quark. To estimate the magnitude of the corrections, we use the PCAC estimates for the light quark condensates

$$(\bar{m}_u + \bar{m}_d)\left(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle\right) = -2f_\pi^2 m_\pi^2$$

$$(\bar{m}_u + \bar{m}_s)\left(\langle\bar{u}u\rangle + \langle\bar{s}s\rangle\right) = -2f_K^2 m_K^2 \tag{3.8}$$

taking $\langle\bar{u}u\rangle \sim \langle\bar{d}d\rangle \sim \langle\bar{s}s\rangle$, $\langle\bar{q}q\rangle = 0$ for heavy quarks, and the recent corrected estimate[15,16]

$$\langle\alpha_s F_{\mu\nu}F^{\mu\nu}\rangle \sim 10^{-1}\ \text{GeV}^4$$

In the case where i and j denote c and d quarks (D^+ decays), we obtain for ψ'' and χ the following expansions:

$$\psi''(Q^2) = \frac{3}{8\pi^2}\frac{(\bar{m}_i - \bar{m}_j)^2}{Q^2}\left[1\,\text{loop} + .27 + 2\;10^{-3} + O(10^{-3})\right]$$

$$\chi(Q^2) = \frac{1}{4\pi^2}\left[1\,\text{loop} + 7.3\;10^{-2} + 4\;10^{-5} + O(10^{-3})\right] \tag{3.10}$$

The only non negligible correction in formula (3.10) is the big α_s correction for ψ''. However, ψ'' controls only the bounds for $|d(t)|^2$, which gives a very small contribution to the decay rate, even when the outgoing lepton is a muon, as shown by explicit numerical computations. Consequently, $\psi''(Q^2)$ and $\chi(Q^2)$ can be very safely truncated to their 1 loop expressions.

3. Results and Discussion

The results for the absolute bounds on the decay rates

$$\Gamma_{D^+\to\eta e^+\nu_e},\ \Gamma_{D^+\to\eta\mu^+\nu_\mu},\ \Gamma_{D^+\to\eta' e^+\nu_e},\ \Gamma_{D^+\to\eta'\mu^+\nu_\mu},$$
$$\Gamma_{F^+\to\eta e^+\nu_e},\ \Gamma_{F^+\to\eta\mu^+\nu_\mu},\ \Gamma_{F^+\to\eta' e^+\nu_e},\ \Gamma_{F^+\to\eta'\mu^+\nu_\mu}$$

are shown on Fig. 4, 5, 6 and 7. We have also plotted the curves corresponding to keeping in Eq. (2.1) $|f(t)|$ and $|d(t)|$ constant and equal to their values at $t = 0$. Note that the difference between the electronic and muonic decays is very small, and that the decays into η' are very likely to be less important than those into η' according to what one expects from phase space. For $f_+(0)$'s given by their SU(4) or valence quark model predictions, we obtain:

$$\begin{cases} .16\;10^{-15}\,\text{GeV} \le \Gamma_{D^+\to\eta e^+\nu_e} \simeq \Gamma_{D^+\to\eta\mu^+\nu_\mu} \le 2.4\;10^{-15}\,\text{GeV} \\ .2\;10^{-15}\,\text{GeV} \le \Gamma_{D^+\to\eta' e^+\nu_e} \simeq \Gamma_{D^+\to\eta'\mu^+\nu_\mu} \le .46\;10^{-15}\,\text{GeV} \end{cases} \tag{3.11}$$

and

$$\begin{cases} .30\;10^{-13}\,\text{GeV} \le \Gamma_{F^+\to\eta e^+\nu_e} \simeq \Gamma_{F^+\to\eta\mu^+\nu_\mu} \le 1.8\;10^{-13}\,\text{GeV} \\ .08\;10^{-13}\,\text{GeV} \le \Gamma_{F^+\to\eta' e^+\nu_e} \simeq \Gamma_{F^+\to\eta'\mu^+\nu_\mu} \le .2\;10^{-13}\,\text{GeV} \end{cases} \tag{3.12}$$

Taking the main values from the recent averages for the D^+ and F^+ lifetimes[4]

$$\tau_{D^+} = 9.1^{+2.2}_{-1.5}\;10^{-13}\,s$$
$$\tau_{F^+} = 2.2^{+2.8}_{-1.1}\;10^{-13}\,s \tag{3.13}$$

we obtain the ranges of variations for the semi-leptonic branching ratios shown in Table 2.

From Table 2, we conclude that the sum of the semi-leptonic decay rates of the D^+ into η, η' and $e,\mu + \nu_e, \nu_\mu$, is likely to represent only less than 1 percent of all the D^+ decays, and that those of the F^+ may represent 5 - 15% (However, the uncertainty on the F^+ total decay width is still very big and makes our relative predictions affected with a large incertitude). Consequently, we cannot account for a big proportion of η and η' final states in the D^+ semi-leptonic decays. They could be more important in the F^+ decays. So far, our method has been completely independant of such or such under-lying quark-gluon picture, quark decay or quark annihilation. Both look to be controlled by the same hadronic matrix element and the same parameter $f_+(0)$. As only in the case of the D^+ could the annihilation mechanism be advocated to work alone, our numerical results lead us to think that this last scheme is very unlikely to produce any substantiable enhancement.

IV. CONCLUSION

We have given absolute bounds for the semi leptonic decay rates of the D^+ and F^+ heavy mesons into the I = 0 state η and η'. We used the technique of QCD sum rules, taking advantage of the great stability and reliability of this method in the presence of a big mass scale. The bounds depend on the parameter $f_+(0)$ which we have varied in a wide range around their SU(4) (for the decays into η) or valence quark model (for the decays into η') predicted values, and do not stick to any quark-gluon picture of the decays. While for the F^+, those decays could represent an important fraction (5,15%) in the case of the D^+, the small percentage obtained (less than 1%) very likely rules out a substantiable enhancement due to a quark annihilation mechanism.

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	-75%	-50%	SU(4) or quark model	+50%	+75%
$D^+ \to \eta \begin{matrix} e\nu_e \\ \mu\nu_\mu \end{matrix}$		.2	$\frac{1}{\sqrt{6}} \sim .4$	.61	
$F^+ \to \eta \begin{matrix} e\nu_e \\ \mu\nu_\mu \end{matrix}$		.41	$\frac{\sqrt{2}}{\sqrt{3}} \sim .82$	1.22	
$D^+ \to \eta' \begin{matrix} e\nu_e \\ \mu\nu_\mu \end{matrix}$	.15		$\frac{1}{\sqrt{3}} \simeq .58$		1
$F^+ \to \eta' \begin{matrix} e\nu_e \\ \mu\nu_\mu \end{matrix}$	.15		$\frac{1}{\sqrt{3}} \approx .58$		1

Table 1

	-75%	-50%	SU(4) or va - lence quark model	+50%	+75%
$\frac{\Gamma_{D^+ \to \eta e^+ \nu_e} + \Gamma_{D^+ \to \eta \mu^+ \nu_\mu}}{\Gamma_{D^+ \to all}}$		$[10^{-4}, 3.8 10^{-3}]$	$[4.4 10^{-4}, 6.7 10^{-3}]$	$[1.4 10^{-3}, 10^{-2}]$	
$\frac{\Gamma_{D^+ \to \eta' e^+ \nu_e} + \Gamma_{D^+ \to \eta' \mu^+ \nu_\mu}}{\Gamma_{D^+ \to all}}$	$[6 10^{-5}, 2 10^{-4}]$		$[4 10^{-4}, 1.3 10^{-3}]$		$[2 10^{-3}, 3.3 10^{-3}]$
$\frac{\Gamma_{F^+ \to \eta e^+ \nu_e} + \Gamma_{F^+ \to \eta \mu^+ \nu_\mu}}{\Gamma_{F^+ \to all}}$		[.4%,5.4%]	[2%, 12%]	[6%,20%]	
$\frac{\Gamma_{F^+ \to \eta' e^+ \nu_e} + \Gamma_{F^+ \to \eta' \mu^+ \nu_\mu}}{\Gamma_{F^+ \to all}}$	$[10^{-4}, 2.2 10^{-3}]$		[.53%,1.2%]		[1.9%,3.2%]

Table 2

FIGURE CAPTIONS

Fig. 1 The decay $M(m, p)$ into $M'(M', p') + \ell + \nu_\ell$ at lowest order of weak interactions.

Fig. 2 The propagator $\Pi\mu\nu\ (q^2)$.

Fig. 3 Q^2 dependance of the upper bounds for $\Gamma_{D^+} \to \eta\ell^+\nu_\ell$ using $\Pi(Q^2)$ or $\Pi^{(1)}(Q^2)$ and fixing $f_+(0) = 1/\sqrt{6}$.

Fig. 4 Upper and lower bounds for $\Gamma_{D^+} \to \eta\ell^+\nu_\ell$.

Fig. 5 Upper and lower bounds for $\Gamma_{D^+} \to \eta'\ell^+\nu_\ell$.

Fig. 6 Upper and lower bounds for $\Gamma_{D^+} \to \eta\ell^+\nu_\ell$.

Fig. 7 Upper and lower bounds for $\Gamma_{F^+} \to \eta'\ell^+\nu_\ell$.

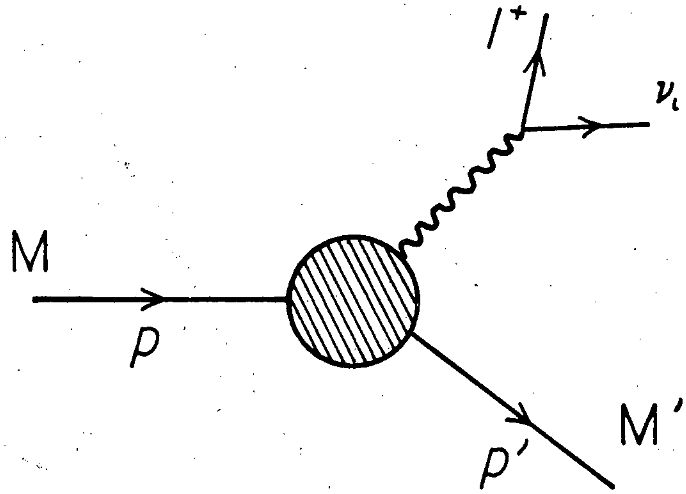


fig. 1

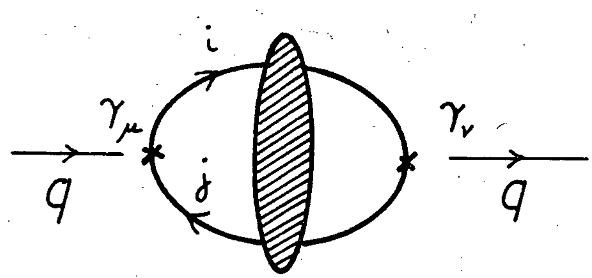


fig. 2

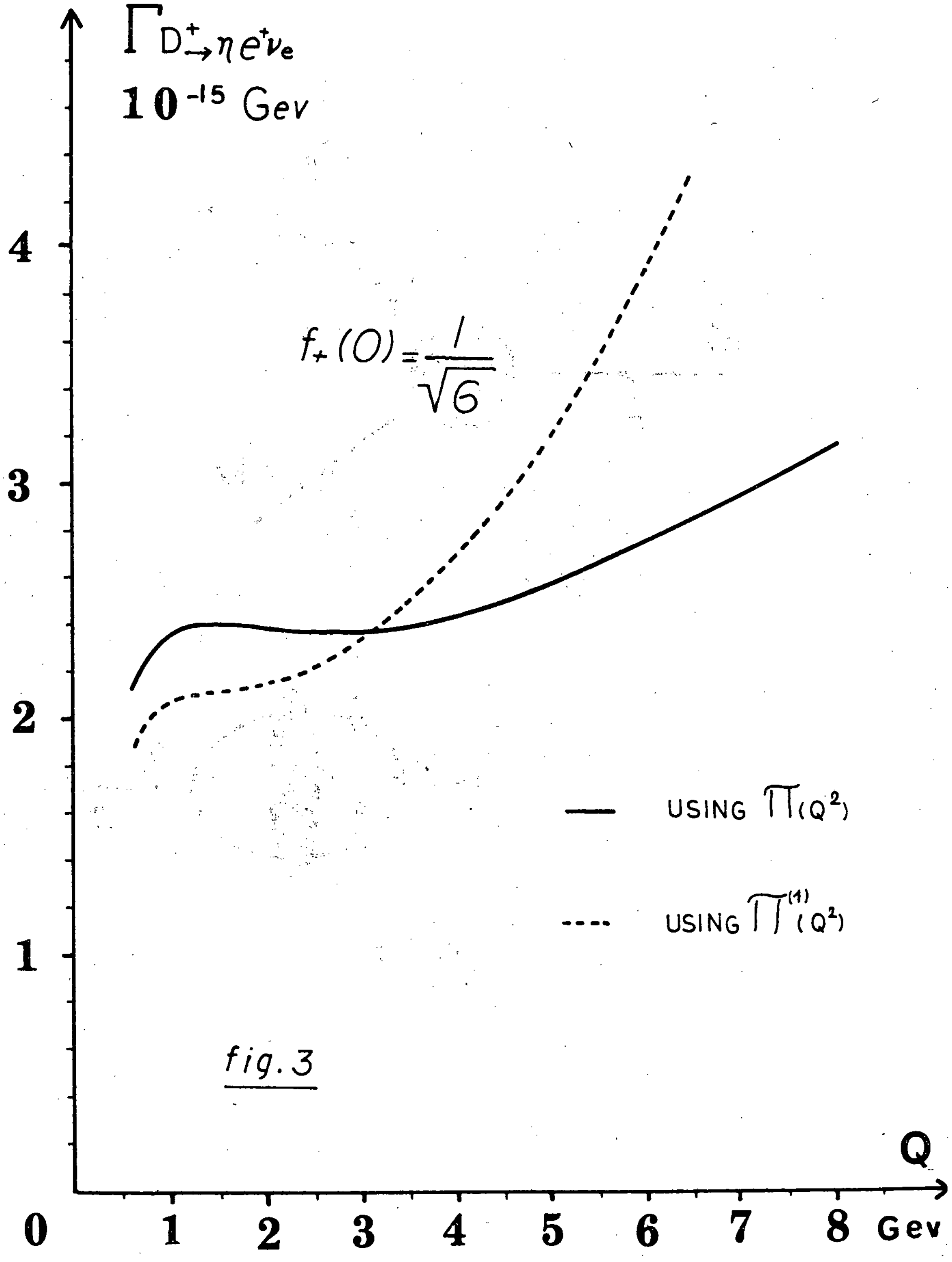
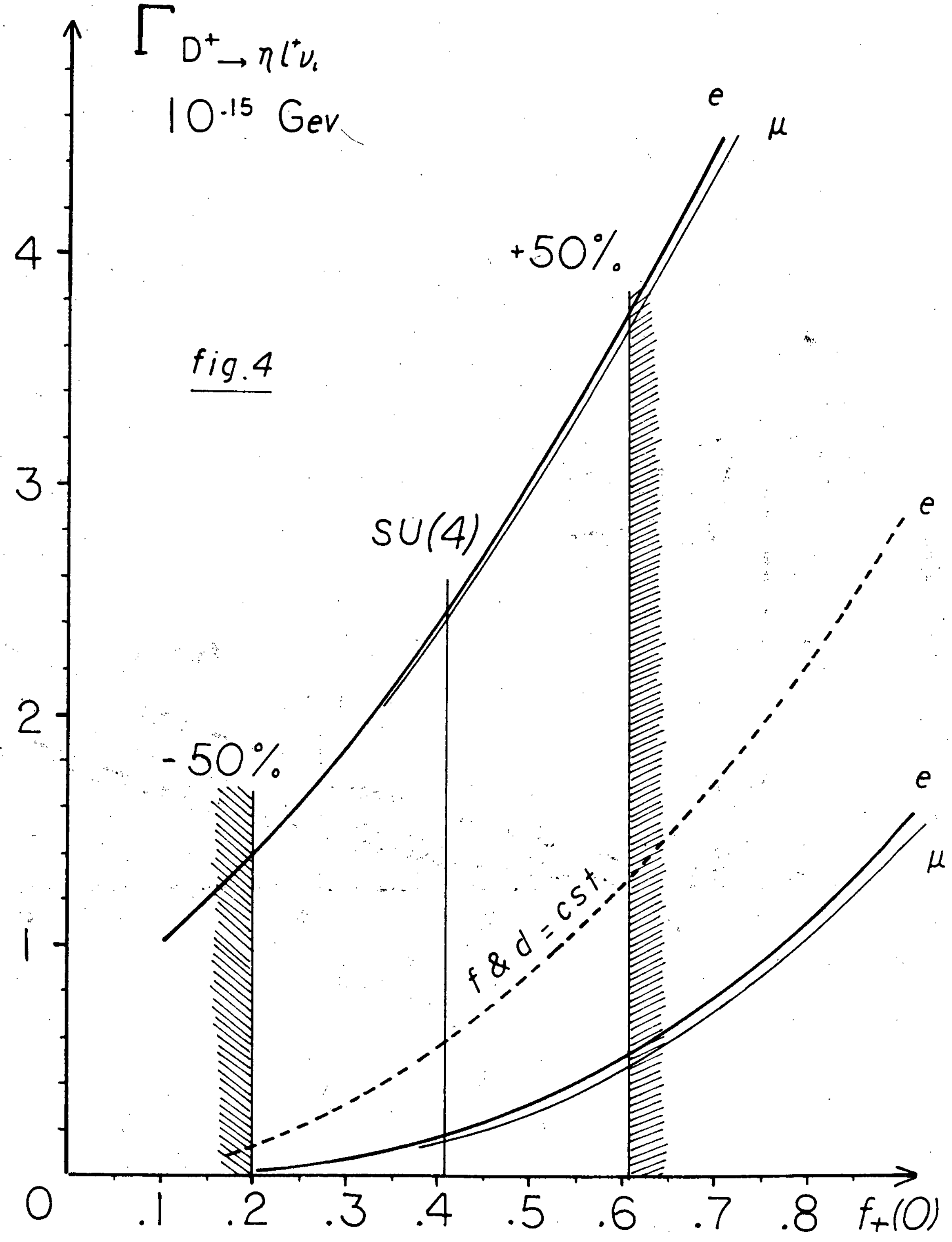
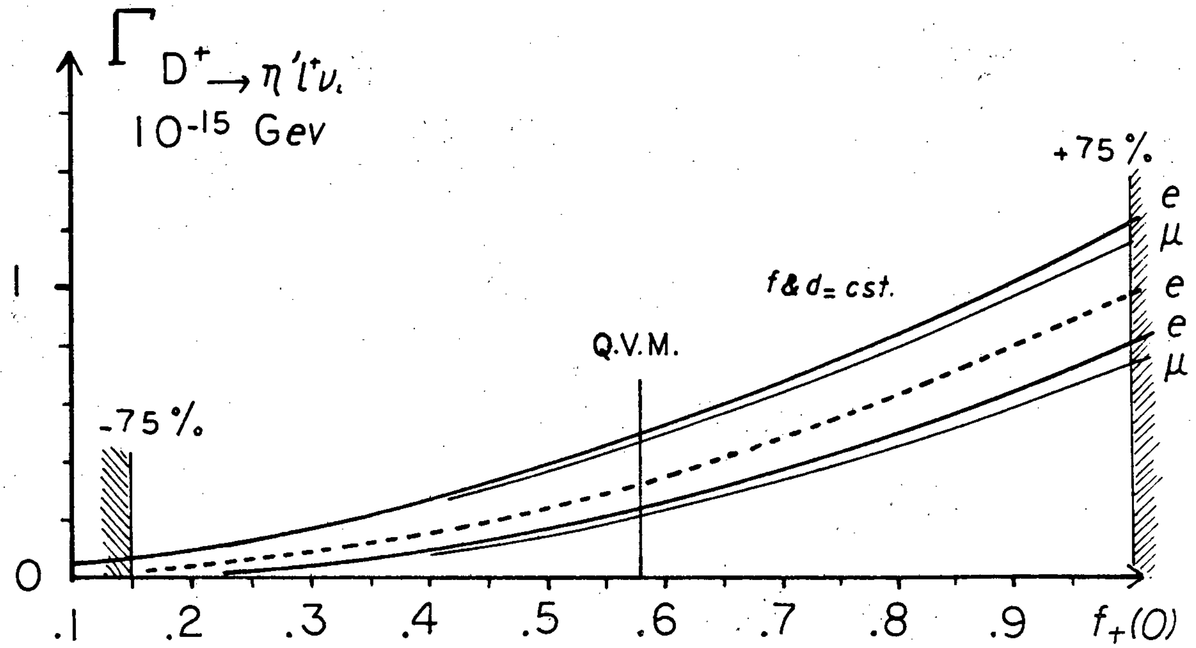

$\Gamma_{D^+ \to \eta e^+ \nu_e}$
10^{-15} Gev
4
3
2
1
0
$f_+(0) = \frac{1}{\sqrt{6}}$
USING $\Pi(Q^2)$
USING $\Pi^{(1)}(Q^2)$
Q
1
2
3
4
5
6
7
8
Gev

fig. 3

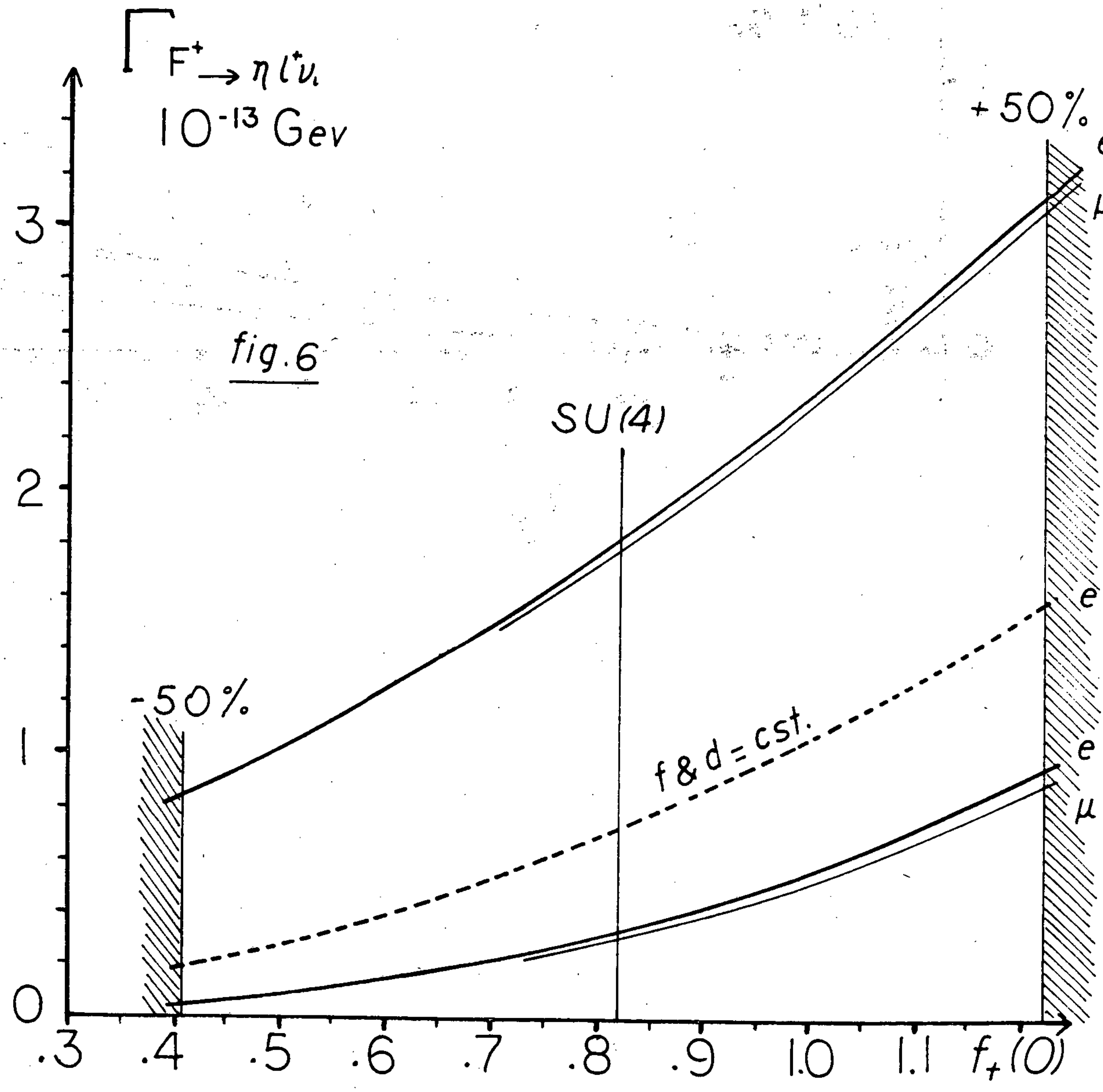


$\Gamma_{D^+ \to \eta l^+ \nu_l}$
10^{-15} Gev
fig.4
4
3
2
1
0
+50%
SU(4)
-50%
f & d = cst.
e
μ
e
e
μ
.1
.2
.3
.4
.5
.6
.7
.8
$f_+(0)$



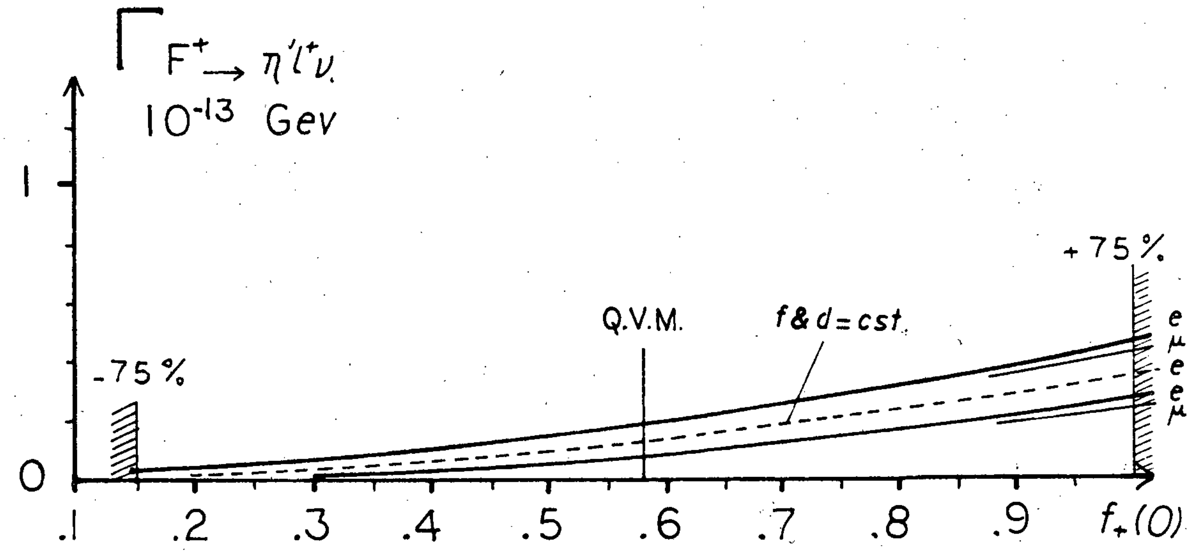
$\Gamma_{D^+ \to \eta' l^+ \nu_l}$
10^{-15} Gev
1
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
$f_+(0)$
-75 %
+75 %
Q.V.M.
f & d = cst.
e
μ
e
e
μ

fig. 5



$\Gamma_{F^+ \to \eta l^+ \nu_l}$
10^{-13} Gev
3
2
1
0
SU(4)
+50%
-50%
e
μ
e
e
μ
f & d = cst.
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
$f_+(0)$

fig.6



$\Gamma_{F^+ \to \eta' l^+ \nu}$
10^{-13} Gev
1
0
-75 %
Q.V.M.
f & d = cst
+75 %
e
μ
e
e
μ
.1
.2
.3
.4
.5
.6
.7
.8
.9
$f_+(0)$

fig. 7

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