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Reducing the Impact of Math Anxiety on Mental Arithmetic: The Importance of Distributed Cognition

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Abstract

Mathematics anxiety negatively affects performance in simple arithmetic tasks. The experiment reported here explored the role of interactivity in defusing the impact of math anxiety on mental arithmetic. Participants were invited to complete additions presented on paper without using their hands or any artefact; in a second, interactive, condition, the same problems were presented in the form of a set of manipulable tokens. Math anxiety was significantly correlated with mental arithmetic performance only in the static condition. The results of a mediation analysis indicated that the effect of math anxiety on mental arithmetic was mediated by working memory capacity in the static condition; in the interactive condition, math anxiety and working memory did not significantly correlate with performance. Interactivity encouraged the coupling of internal and external resources to create a cognitive system that augmented and transformed working memory capacity, diffusing the resource drain caused by math anxiety.

Keywords: Mental arithmetic, interactivity, math anxiety, individual differences, distributed cognition

Introduction

A person's proficiency in mathematics and an appreciation that effort is a key determinant of math performance will likely have important consequences for his or her educational and occupational opportunities. In addition, a mathematically competent workforce is identified as a strategic driver of economic growth (National Mathematics Advisory Panel, 2008). There are indications in the US and in the UK (National Numeracy Facts and Figures, 2012) that numeracy levels are in decline.

An important factor that impedes math performance and reduces exposure to math—with the inevitably negative impact on the acquisition of math knowledge and skills—is math anxiety. Richardson and Suinn (1972) define math anxiety as "feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" (p. 551). From a processing efficiency perspective (Eysenck & Calvo, 1992), math anxiety impairs performance by using up working memory resources to maintain and retrieve negative performance-related thoughts and memories (Ashcraft & Krause, 2007). As a result, math anxious people deploy limited cognitive resources when working on a math problem, leading to poorer performance, reinforcing a cycle of anxiety and avoidance that perpetuates poor numeracy.

Mental Arithmetic

In the absence of pen and paper, mental arithmetic is a quintessential working memory task. Admittedly, for simple problems where the solution draws on long-term memory knowledge of well-rehearsed answers (e.g., 3 + 3), working memory plays a more limited role (DeStefano & LeFevre, 2004). However, for more complex problems, such as multiple number additions, working memory resources must be deployed to arrive at a correct answer (Ashcraft, 1995). These resources involve storage of interim totals and place markers as well as executive function skills that direct attention (e.g., which number to add next) or the retrieval of strategies to support more efficient and reliable performance.

The exact nature of the resources recruited depends on the context of reasoning, defined by the features of the external environment in which the problem is presented. For one, the manner of presentation (visual, auditory) would recruit different subsystems of working memory. In addition, if the numbers are visually presented, working memory would be taxed differently depending on whether the presentation is sequential or simultaneous. Even with a simultaneous presentation, the numbers' arrangement in space—columnar, linear, or random—

would also determine the extent of working memory load. More important is the opportunity to manipulate the problem presentation to facilitate thinking: Enabling participants to re-order and group numbers would likely help them remember the numbers already added, identify felicitous sub-totals and interim totals, guide attention, and encourage the development of more efficient arithmetic strategies.

Imagine a participant invited to complete an addition problem involving seven numbers, some single digit, some double digit. In one condition, the problem is presented on a piece of paper as a randomly configured array of numbers; the participant is asked to put her hands palm down on the flat surface on which the problem is presented. The mental effort required cannot be guided and supplemented with complementary actions (Kirsh, 1995) such as pointing and re-arranging. In this context, mental arithmetic performance should reflect the participant's working memory capacity, arithmetic knowledge and skill. Imagine, in turn, the same problem but, this time, presented as a set of number tokens, which the participant is invited to manipulate. The importance of arithmetic knowledge and skills remain; however, now, working memory is augmented by a modifiable problem presentation. Such a dynamic presentation unveils a shifting array of opportunities and possibilities, whether strategically engineered or fortuitously encountered. Thus, working memory is augmented not simply in terms of storage capacity, but also in terms of executive functions. That is, a shifting problem presentation cues certain strategies—for example by grouping certain numbers together—and guides attention. Hence, in a modifiable environment, the strategic control of attentional resources originates, partly, in the world.

The Present Experiment

Participants' performance in a mental arithmetic task is likely to be impaired by math anxiety, and this may be particularly apparent when the mental arithmetic task requires a larger commitment of working memory resources, such as in a static context of reasoning where participants cannot interact with numbers that compose a problem. In turn, if reasoners are given the opportunity to couple their working memory resources and arithmetic skills to a dynamic and modifiable problem presentation, the impact of math anxiety might be considerably attenuated. This is because the coupling of internal and external resources creates a more robust and resilient cognitive system that augments the participants' working memory resources, which then can more easily soak up the resource-depleting rehearsal of performance-related thoughts. Arithmetic performance might be positively correlated with math anxiety in a static reasoning environment; however when participants can extend their cognitive resources and let the environment shoulder some of the computational efforts, then accuracy may be influenced by math anxiety to a lesser extent.

Math anxious individuals cope with math anxiety by limiting their exposure to math, which further limits their levels of numeracy (Ashcraft, 2002). Hence, to get a better window on the influence of anxiety on mathematical cognition, a relatively simple task was developed for this experiment engaging basic arithmetic skills acquired and mastered by university undergraduates. Participants completed the additions in both a static, non-interactive, context and in one where tokens corresponding to the elements of the addition problems could be touched, arrayed, grouped, in whatever manner to support problem solving; hence interactivity was manipulated within-subjects.

Performance was measured in terms of accuracy (absolute error) and efficiency. Thinking efficiency was calculated as the ratio of the proportion of correct solutions for a set of problems over the proportion of time invested by that participant to complete the set out of the maximum time invested by the slowest participants. In the static condition, participants' working memory resources would likely be stretched, particularly by the long additions; in turn the coupling of internal to external resources in the interactive condition could augment the participants' working memory capacity and executive processes.

Participants' working memory capacity was assessed using a computation span task. Math anxiety was predicted to correlate negatively with working memory capacity. More important, the magnitude of error in the mental arithmetic task was predicted to correlate positively with anxiety level and negatively with working memory capacity, but only in the static condition. Thus, a key prediction was that interactivity would defuse the impact of anxiety on calculation error. In a similar manner, math anxiety and working capacity should predict thinking efficiency in the static, but not in the interactive condition. Mediation analyses were conducted to determine the direct and indirect effect of math anxiety on thinking efficiency in both conditions.

Method

Participants

Forty psychology university undergraduates (35 females, overall mean age 20.8, *SD* = 3.2) received course credits for their participation.

Material and Measures

Mathematics Anxiety. Mathematics anxiety was measured using an abridged version of the original 98-item scale (Suinn, 1972) developed by Alexander and Martray (1989). The abridged version is based on 25-items for each of which participants used a 5-point scale (1 = "not at all", 5 = "very much") to describe how anxious the event described made them feel. The 25 items assessed math anxiety in terms of test anxiety (e.g., "studying for a math test"), numerical task anxiety (e.g., "reading a cash register receipt after your purchase") and math course anxiety (e.g., "watching a teacher work on an algebraic equation on the blackboard"). Math anxiety scores could range from 25 to 125 – the higher the score, the higher the math anxiety; the mean score in the present sample was 66.0 (SD = 18.1).

Working memory capacity. Working memory was assessed using a computation-span test (Ashcraft & Kirk, 2001, p. 226). Participants solved simple arithmetic problems in blocks increasing from 2 to 6 problems (e.g., "50 + 7 = ?", "60 ÷ 2 = ?", "19 - 8 = ?" was a block of three problems). At the end of each block, participants were prompted to recall in correct order the last number of each problem in that block (for the example above, correct recall would be "7, 2, 8"). There were two blocks for each sequence length (e.g., two blocks with sequences of 3 different problems) for a total of 10 blocks. Working memory capacity was measured as the sum of all correct answers across the 10 blocks, for a maximum score of 40. The mean number of digits recalled by the participants in the present study was 24.1 (SD = 7.6).

Arithmetic Task. Participants carried out short and long additions, involving either 7 or 11 numbers (see Fig. 1), as fast and as accurately as possible. They completed the problems in blocks, five from the short set first, and five from the long set second. Performance was measured in terms of the mean absolute error and in terms of efficiency. Efficiency was measured as the ratio of accuracy (proportion correct sums) over time invested in doing the sums. The latter was measured as the proportion of actual time to complete the sums divided by the maximum time needed to complete them in that condition; this maximum was determined by taking the average of the top quartile latencies. Inefficient performance is reflected with a ratio smaller than 1 indicating that proportion accuracy was smaller than proportion time invested.

Procedure

The mental arithmetic task, working memory span task, and the completion of the 25-item mathematics anxiety scale were embedded in an experimental session that lasted approximately 40 minutes, and which included other tests of motivation and cognitive skill unrelated to the present experiment. The session always started with participants completing the math anxiety scale. During the mental arithmetic task, participants were presented with the five additions from the short set first. After a 2-min distractor task (a word search puzzle), participants were presented with the five additions from the long set; the problem order within each set was randomized for each participant. These two sets of sums were presented twice. For one presentation participants performed the additions with their hands on the table facing them (the static condition) and announced their answer out loud; for the second presentation, square numbered tokens (3cm by 3cm) were used, and participants were encouraged to touch, move or group the tokens in whatever manner to help them add the numbers (the interactive condition); as in the static condition, participants announced the solution for each problem out loud. While the long set always followed the short set, the order of condition (non-interactive, interactive) was counterbalanced across participants. With 10 different problems, involving 10 unique configurations of numbers, and 90 numbers across the two sets, it was unlikely that participants remembered the solution to each problem when presented a second time. Still, to prevent direct retrieval of solutions during the second presentation, the participants completed the computation span test between the two presentations of the arithmetic task. Problem set size (with two levels) and interactivity (with two levels) were independent variables that were manipulated in a 2×2 repeated measures design.

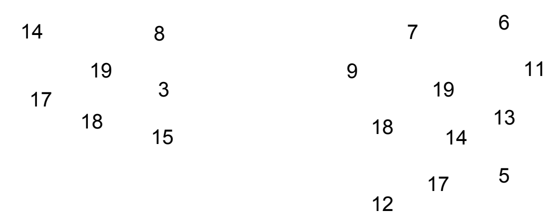

Figure 1: Examples of additions from the short set (7-number additions) and the long set (11-number additions).

Results

The correlation matrix involving the anxiety and working memory span measures along with the mental arithmetic performance measures is reported in Table 1. We note, for now, that math anxiety scores were negatively correlated with working memory span, r (38) = -.318, p = .045. The correlations with the different measures of mental arithmetic performance in the static and interactive conditions are described below.

Absolute Error

The mean absolute deviation from the correct answer or absolute error for the short and long sums in the static and interactive conditions are reported in the top half of Table 2. Mean absolute error was similar for the short sums across conditions; however, errors increased for the long sums, in a relatively more pronounced manner in the static condition. In a 2x2 repeated measures analysis of variance (ANOVA), the main effect of condition was not significant, $F < 1$, the main effect of problem size was marginally significant, $F(1, 39) = 4.02$, $p = .052$, but the interaction was not significant, $F(1, 39) = 2.26$, $p = .141$.

Table 1: Correlation matrix involving mathematics anxiety, working memory capacity, and mental arithmetic performance averaged across all 10 additions in the static and interactive condition (*df* = 38).

	1 MARS	2 SPAN	3 ERR-S	4 ERR-I	5 EFF-S	6 EFF-I
1	-	-.318 *	.427 **	.002	-.306	-.230
2		-	-.283	.030	.494 **	.341 *
3			-	.238	-.758 **	-.443 **
4				-	-.387 *	-.605 **
5					-	.725 **
6						-

Note: * $p < .05$ ** $p < .01$. **MARS** = Mathematics Anxiety Rating Scale scores; **SPAN** = Computation span scores; **ERR-S** = Average absolute error in the static condition; **ERR-I** = Average absolute error in the interactive condition; **EFF-S** = Average efficiency ratio in the static condition; **EFF-I** = Average efficiency ratio in the interactive condition.

Math anxiety was strongly correlated with absolute error in the static condition averaged across all 10 problems, $r = .427$, $p = .006$ (see Table 1), but not in the interactive condition, $r = .002$, $p = .989$. To determine the interaction between math anxiety and condition (interactive, static), the difference in the average absolute errors between the interactive and static condition were regressed on the anxiety scores mean deviation form (an alternative to dichotomising anxiety scores with a median split—which reduces power—as recommended by Brauer, 2002). In the absence of an interaction, one would expect that as math anxiety level increased, participants would not benefit from manipulating the tokens—in other words, the difference between the interactive and static condition would be constant across levels of math anxiety. However, the slope of the regression line, $\beta = -.372$, was significantly negative, $t(38) = -2.471$, $p = .018$. This confirms that participants who were more math anxious made errors of a smaller magnitude in the interactive than in the static condition.

Finally, working memory span was marginally correlated with error in the static condition, $r = -.283$, $p = .077$, but not in the interactive condition, $r = .030$, $p = .852$.

Efficiency Ratio

The mean efficiency ratios are reported in the bottom half of Table 2. Participants' efficiency exceeded 1 in the static condition for the short problems, but declined for the long sums. In turn, efficiency remained well calibrated and constant across problem size in the interactive condition. The 2x2 repeated measures ANOVA revealed that the main effect of condition was not significant, $F < 1$, the main effect of problem size was significant, $F(1, 39) = 5.24$, $p = .028$, as was the condition by problem size interaction, $F(1, 39) = 5.37$, $p = .026$.

Table 2: Mean absolute error and efficiency ratio, along with the standard deviation, for short and long sums in the static and interactive conditions.

	Set Size			
	Short		Long	
Condition	*M*	*SD*	*M*	*SD*
	Abolute Error			
Static	3.3	4.4	5.6	6.1
Interactive	3.6	4.7	4.2	3.9
	Efficiency Ratio			
Static	1.2	0.9	0.9	0.8
Interactive	1.0	0.7	1.0	0.7

Math anxiety was negatively correlated with the efficiency ratio averaged across all 10 problems in the static, $r = -.306$, $p = .055$, but not in the interactive condition, $r = -.230$, $p = .153$. The average efficiency ratios were not characterised by a significant math anxiety by condition interaction, however. In the regression of the difference in the average efficiency ratios between the interactive and static condition on the mean deviation form of the math anxiety scores, the slope of the regression line, $\beta = .161$, was not significantly different from zero, $t(38) = 1.008$, $p = .320$.

Working memory span was positively correlated with efficiency in the static, $r = .494$, $p = .001$ and to a lesser extent in the interactive condition, $r = .341$, $p = .031$. In light of the strong correlation between working memory capacity and efficiency, the mediation of the effect of math anxiety on efficiency via working memory capacity in both the static and the interactive condition was analysed using the procedure and SPSS macro developed by Preacher and Hayes (2008). A simple mediation model analysis was run with math anxiety as the independent variable (X), working memory capacity as the mediator (M) and average efficiency as the dependent variable (Y); Figure 2 depicts the results of both mediation model analyses for the static (left panel) and interactive condition (right panel). In the case of the static condition, the total effect of math anxiety on mental arithmetic performance (path **c**) was negative and significantly different from zero. Math anxiety significantly influenced working memory in a negative direction (path **a**) and working memory significantly influenced efficiency (path **b**). Finally, the effect of anxiety on efficiency after controlling for working memory (path **c′**) was no longer significant. A bootstrap analysis revealed that the 95% bias corrected interval with 5000 resamples

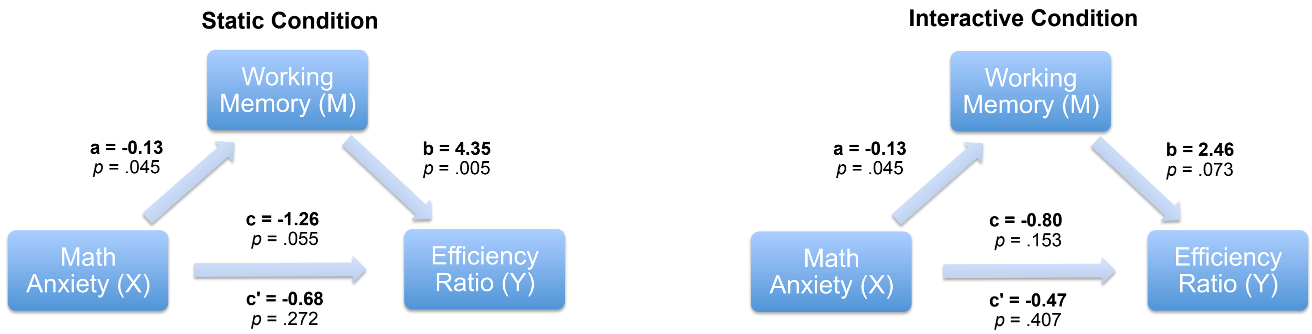

Figure 2: Results of the mediation analysis in the static (left panel) and interactive condition (right panel).

for the size of the indirect effect (-0.58; *CI* [-1.47; -0.03]) did not include a zero value and thus can be consider to be statistically significant. A traditional Sobel´s test approached significance, z = -1.73, p = .084. Thus, the effect of math anxiety on mental arithmetic efficiency in the static condition was completely mediated by working memory (see Fig. 2, left panel). In the interactive condition, the total effect of math anxiety on efficiency (path **c**, see Fig. 2, right panel) was negative but not significantly different from zero. Thus, strictly speaking, the condition for mediation analysis was not fulfilled (Baron & Kenny, 1986). However, it has sometimes been argued that the indirect effect can still be significant, and omitting this analysis could lead to the failure of detecting interesting mechanisms (Hayes, 2009). With this in mind, the mediation analysis was conducted and showed that math anxiety influenced significantly working memory in a negative direction (path **a**), while working memory marginally influenced mental arithmetic performance (path **b**). Finally, the effect of anxiety on mental arithmetic performance after controlling for working memory (path **c´**) was not significant. A bootstrap analysis revealed that the 95% bias corrected interval with 5000 resamples for the size of the indirect effect (-0.33; *CI* [-0.99; 0.02]) included zero and thus cannot be consider to be statistically significant. Finally, the Sobel´s test was not significant, z = -1.41, p = .160. Thus, there was no significant total or indirect path between math anxiety and mental arithmetic efficiency in the interactive condition (see Fig. 2, right panel).

Discussion

In this experiment participants completed short and long additions in two different contexts, one which permitted the reconfiguration of the problem through the spatial rearrangement of the number tokens, and one which did not. Participants were generally accurate—although less so for longer additions—and interactivity did not significantly enhance accuracy. However, the significant interaction between problem size and condition for the efficiency ratio measure confirmed that thinking efficiency dropped for the longer sums in the static condition, but remained stable in the interactive condition. The interaction between problem difficulty and context of reasoning (static, interactive) indicates that determining the benefits of physically re-shaping a problem presentation is an exercise done relative to the degree of task difficulty. Thus, with a relatively easy task, interactivity might not benefit the reasoning agent, but interactivity can enhance efficiency when the task is challenging and undertaken on the basis of internal resources alone.

Math anxiety was significantly correlated with working memory capacity. This has been reported previously (Ashcraft, 2002) especially when capacity is gauged with a span test that involves numbers and operations. The more important findings was the significant interaction between math anxiety level and the degree of interactivity: as math anxiety increased, participants made fewer errors in the interactive than in the static condition.

It is important to stress that this experiment employed a repeated measures design: Participants and their levels of maths anxiety were identical in the static and interactive condition. Having said this a post-task measure in each condition might have offered a better measure of how much anxiety was experienced in completing the sums. Manipulating tokens might have altered participants' experience in terms of intrinsic motivation, attentional commitment, and self-efficacy.

In turn, reasoning efficiency, as determined by the ratio of accuracy over time invested in completing the sums, was marginally correlated with math anxiety in the static condition, but not in the interactive condition. The mediation analysis confirmed that the effect of math anxiety on efficiency in the static condition was mediated by working memory capacity. In turn, in the interactive condition, math anxiety had no effect on reasoning efficiency, but working memory capacity marginally influenced performance. According to processing efficiency theory (Ashcraft, 2002) math anxiety exacts working memory resources to maintain performance-related beliefs and fears. As the static condition put a higher demand on working memory, efficiency was more directly determined by working memory capacity. In the interactive condition, however, participants have the opportunity to recruit external resources to help them complete the sums. They can group the number tokens to guide and direct attentional resources and identify congenial interim totals that facilitate more efficient addition strategies. The coupling of internal and external resources creates a cognitive system (Wilson & Clark, 2009) that augments memory storage and distributes the control of executive function in a manner that copes better with the resource drain caused by math anxiety. These findings lend support to the conjecture that for simple mental arithmetic problems, performance improvements are better supported in a learning environment that fosters interactivity.

Future research may explore the role of interactivity in helping reasoners enhance their mental arithmetic performance in contexts that can elicit higher levels of anxiety, such as under time pressured or in situations of greater accountability. One of the recommendations of the National Mathematics Advisory Panel (2008, p. 31) is to determine the etiology of math anxiety and important advances in charting its neurodevelopmental origins have recently been reported (Young, Wu, & Menon, 2012). In addition, it might be of particular interest to determine whether intervention programmes that are based on interactive training exercises enhance participants' level of instrumentality, efficacy and confidence, reducing math anxiety in more traditional situations, and encouraging greater exposure to mathematics.

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