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Permalink

<https://escholarship.org/uc/item/7q88v5nf>

ISBN

9781405162074

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Publication Date

2007-11-13

DOI

10.1002/9780470696682.ch9

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Can Students Re-Invent Fundamental Scientific Principles?: Evaluating the Promise of New-Media Literacies^{*}

Preprint of: diSessa, A. A. (2008). Can students re-invent fundamental scientific principles?: Evaluating the promise of new-media literacies. In T. Willoughby, & E. Wood (eds.), *Children's learning in a digital world* (pp. 218-248). Oxford, UK: Blackwell Publishing.

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*****6-11-07*****

^{*}This work was supported, in part, by a grant from the Spencer Foundation to Andrea A. diSessa. The conclusions and interpretations drawn here are those of the author, and not necessarily those of the Foundation. The author has a financial interest in PyxiSystems LLC, which is the owner of the Boxer software in which simulations and models were produced for this work.

1. Introduction

This chapter combines two rather different intellectual lines. The first line involves considering several cases of students re-inventing fundamental ideas in science and mathematics, such as Newton's laws and the idea of Cartesian graphs. I take up the question of how it is possible and in what ways is it a legitimate learning activity for students to accomplish such apparently prodigious feats. In answering those questions, the main (although only partial) answer has to do with the power of representations, in particular computational "new media" representations. The fact of intellectual power in the use of computational representations constitutes an empirically tractable aspect of the second main intellectual line of this chapter: how can we conceptualize and actually test promised riches in new-media literacies? The main goal here is to provide a more sharply focused and better grounded view—separating hype from legitimate hope—of some of the best intellectual possibilities offered by new media. I begin closest to the main track of the conference that gave rise to this book: How can we most productively think about the promise of new-media literacies?

a. Perspectives on Literacy

New-media literacies are touted as opening fantastic vistas for future intellectual and expressive power. I have my own version of that story, a part of which will unfold here. However, an equally exciting prospect (exciting for very different reasons) is to test and extend existing theories of literacy in the face of developing possibilities. My conviction is that past theories of literacy have implicitly assumed properties of the medium to be just like text, that central prototype for possible literacies. But, new media are not just like text, in small and large ways. Looking at nascent forms of new-media literacies, and, more importantly, anticipating and fostering the best possible literacies will require a more general framing than that which may suffice for textual literacy. I sketch here just enough of my views on a revised theory of literacy to serve as an introduction to some of the important foci in this chapter.

Oversimplifying, the modern theory of literacy has gone through two broad phases. The earlier phase (although it extends essentially to the present), involving researchers such as Goody (1977) and Ong (1982), might be characterized as cognitive and in some measure techno-centric. Phase I was cognitive in positing that the power of literacy came from attributes of transformed thinking fostered by literacy. People became more logical, capable of abstract thinking, "distanced," dispassionate, critical, and so on, in virtue of the discovery of alphabetic text as a way of encoding verbal language. The phase has been uncharitably characterized as embracing an "autonomous" view of literacy (Street, 1995), assuming that the effect of literacy arises from the very fact of using the textual representational system. I prefer the more generic and descriptive term "techno-centric" to capture a central focus on properties of the "technology," the medium, in providing "enhanced thinking powers."¹

The second phase has been described by adherents as "ideological" (Street, 1995.) or "new literacy studies" (Gee, 1996). This phase has a characteristic focus on the multiple social

¹ I want to make clear that I do not consider these fair and complete characterizations of Phase I researchers. They are rather homogenizing descriptions, closer to caricatures by critics. For my purposes, they are intended as backdrop oversimplifications that, nonetheless, provide a ground on which to position the points that follow.

functions of literacy, such as identifying members of social groups and multiple roles in, for example, producing and reproducing social strata and in allocation of social goods. In this view, there is no homogeneity in the effects of literacy—hence it is more natural to think of “literacies”—and certainly no technological determinism, as might have been attributed to the earlier work. Studies by Cole and Scribner (1981) and others pointed out that the effects of literacy depend on the social practices (e.g., forms of schooling) in which it is embedded. In the early to mid 1990s, Phase II literacy studies seldom made any reference at all to representational forms. Of course, with the increasing visibility of new media in literacy studies, a focus on the medium, however vague, has returned to a greater prominence.

My own perspective can be drawn out of this abbreviated discussion, and it is largely synthetic of the two phases.

1. Both social and cognitive perspectives are necessary for a deep understanding of the nature of literacies.
2. As emphasized much more in Phase I of literacy studies, representational forms and their properties are heavily involved in creating at least some of the powers of literacy. However, representational form is far from determining; techno-centricity must be moderated. Most “powers” of literacy must be seen as emergent in a joint material/cognitive/social system.
3. While the social viability and social functioning of literacies depend on more things than individual intellectual power, my personal interest here is, in fact, specifically in intellectual powers that might emerge from new-media literacies. I am most interested in transformations of the way in which science and mathematics are taught—indeed, in the way in which they are conceptualized—that may come about from use of new media (diSessa, in press). I do not deny more characteristically social functions, but I claim that a focus on intellectual power is legitimate.
4. Unlike the primary emphasis of Phase I studies, I believe that some of the most important intellectual powers conveyed by new-media literacies might manifest only in episodes of person-plus-medium or group-plus-medium activity. While I do not deny power in “intellectual residue” conveyed to people not in the presence of media, I also think analysis of media-plus-people systems might be more systematically productive.
5. While I focus on intellectual powers, as do Phase I theorists, times have changed with respect to theoretical and empirical accountability for the reality and efficacy of claimed powers. Most of the general powers named in Phase I literacy studies seem underspecified, undemonstrated, and even implausible given modern standards, especially with respect to empirical validation.

Statement of Goals

Bridging toward the analyses that I show here:

1. I look toward specific empirical accountability in validating the powers I attribute to literacies, intending to do better in this regard than older Phase I studies could manage, and probably more than much Phase II research, as well. (However, this review chapter

will, at best, “point to” the kind of data and argument that validate claims; I cannot afford the space to draw out important empirical details in the several cases discussed.)

2. I look specifically into human-material “cooperation” (that is, thinking in the context of representations) rather than at purely intellectual residues of literacy.
3. While I look, somewhat in passing, at social considerations in the form of human-human collaborative patterns (mediated by representational forms), I will not reach essentially large-scale social issues involved in the creation of new-media literacies. For some of that, consult diSessa (2000).

b. “Re-inventing” Fundamental Science and Mathematics

This chapter is driven by a series of empirical results over years of work in my research group. In each of the studies cited, students ended by reproducing, in some approximation, fundamental scientific or mathematical results, from Cartesian graphing to Newton’s laws of motion. Of course, they did not construct these in the fullest sense, reproducing historical genius. However, we have maintained the legitimacy and importance of what students accomplished. A main part of my task here will be to expose just a little more than the barest bones of the process and results of these re-inventions.

That students can reproduce (even partially) results of the greatest geniuses in the history of science is a challenging claim. To many researchers, it is immediately implausible and, possibly, dismissible. For example, I quote two highly respected and high profile contributors to mathematics and science education.²

“Models [laws] are things that scientists make and students use.”

“It is neither reasonable nor efficient to expect students to invent for themselves the content of ... disciplines.”

A main part of my task is to take some of the mystery out of student re-invention (but not all of the mystery—I hope that students’ work will remain impressive!). In this regard, many other researchers have participated in advocacy and explanation of “re-inventing” basic mathematics and science. For example, the Dutch realistic mathematics work (e.g., Gravemeijer, 1994) speaks of scaffolded reconstruction as a core instructional method. Several other people, mainly specialists in the learning of mathematics (e.g., Bamberger, in press; Hughes, 1986; Nemirovsky & Tierney, 1994), have explored students’ abilities to design representations, which topic will figure prominently here.

2. Principles

To help unify and focus later discussion of student re-inventing, I produce here a general set of principles by which I believe re-inventing became possible. The set is grouped into three subsets. The first principle (a singleton set) is special in that it does not concern power emerging from

² I do not cite the individuals because I have reason to believe that both have moderated or changed their views since they spoke these words.

new-media representations. Instead, it provides an important perspective on what I believe to be a likely fundamental characteristic of new-media literacies. The second set includes specific ways in which representations might be helpful in these re-inventing activities. We have data, at various levels of convincingness, on all of these principles. The third set consists of non-representational principles. These are token in illustrating a larger set of non-representational principles that must also be invoked to explain success in our re-inventing activities. They also seem salient enough in our data that it would be unfair to omit them.

The Meta-representational Principle

1. Modern students—even elementary school students—come to class with a lot of exposure to representations in many forms. A large literature shows that students come to know basic things about, for example, pictures. They learn that pictures are usually or always made by people, that they refer to a non-present reality, and that they are intended to convey or display something to an “audience” (Freeman, 1985). Our own work documents extensive creative capacities that seem commonplace in even sixth graders (diSessa, 2004). One special case of this principle is the mere idea of mathematizing the world with the expectation of understanding it better for having done so. This is such a commonplace in the modern world, which includes clocks, thermometers, scales, and numbers like “miles per gallon” at every turn, that it would be easy to miss the fact that this is a culture-specific experience, not a universal inclination. Before Galileo’s time, for example, it cannot have been true that ordinary people, much less children, would spontaneously mathematize in expectation of using thus exposed regularities in order to understand better. We see this instinctive mathematizing in our data.

Representational Principles

2. *The power of abstract, well-adapted representations:* Much of modern science relies, near its core, on representations such as algebra, calculus (which is both a conceptual and representational accomplishment), differential equations, etc. A particular representation can pick the “right” level of abstraction, and thus constrain the search for valid and comprehensible laws and models. Certain (mainly) representational entities, such as vectors, are perspicuously adequate to encode exactly what the history of science has found out about certain domains, notably many aspects of physics. Thus, providing a representational form can also provide important hints and constraints in “re-inventing” basic science. Of course it is critical that students understand those representations enough to use them creatively. This is precisely a key advance of new media, extending traditional scientific representations with ones that are as expressively powerful, but much easier to learn. A big and perhaps surprising lesson here is that writing simple programs turns out to be much easier than writing, for example, algebraic expressions for important science, and it is arguably as or more expressive of the important underlying ideas (see the discussion in diSessa, 2000, and Sherin, 2001).
3. *Representational task tuning; conceptual/representational pump-priming:* Many of our re-invention tasks are framed in representationally partially formed terms. For example, we often provided a template that gives a hint or starting place for inquiry, helping students by putting the focus on a key conceptual point, or by providing an organization of the task. A related strategy, pump-priming, involves prior exploration or instruction in related tasks, or in a task that is a sub-task of the full re-invention task. This can take the

form of an inquiry that results in a model that is related to, or part of the target re-invention. Considerations in formulating a starting template or in designing a prior exploration can tune tasks to an appropriate level of difficulty.³

4. *Dynamic representations*: Paper and pencil, algebraic and similar representations lack a pair of characteristics that are an important part of the story for new media: dynamics and interactivity. These have implications both for expressiveness and for learning. With respect to expressiveness, laws of nature expressed as simple programs can be run, and students can see the effects of the laws, for example in actual motion of objects. In developing their own versions of laws, students can see, consider, and react to their symbolic hypotheses. This is more than supporting feedback loops; it also invokes intuitive and perceptual knowledge that might otherwise be locked out of student work.⁴
5. *Support for social collaboration*: External, perceptually shared representations support good collaboration. Of course, this depends on the representation's being properly expressive with respect to the ideas at issue. Thus, adaptation to both science's best ideas (item 2, above), and to student ideas (in item 4, above) are important. A positive evaluation of a representational form depends on how the medium meets these requirements. Support for collaboration has at least the following aspects:
 - a. *"Reading" student ideas* – Words are notoriously vague, and in many cases new-media and computational representations provide a better expressive language for students to display, and teachers to "read," student ideas.
 - b. *Goal and hypothesis clarity* – Creating computational representations has an evident end-state in many cases: a program runs and shows adequate behavior. Contrast this to "producing a theory" or even creating an equation, which has uncertain implications. At earlier stages, students can express hypotheses often in terms of particular, unambiguous expressions.
 - c. *Summary representation* – Like famous equations that litter the history of science, from Newton's $F = ma$ to Einstein's $E = mc^2$, computational representations can provide compact and precise summaries of key ideas. This compactness and precision are often in sharp contrast to natural language or pictures. Many who extol the virtues of new media completely ignore the fundamental and transformative power of symbolic formulations.
 - d. *A focus for teacher intervention* – We have seen teachers use external computational representations in many ways to help students along productive intellectual pathways (conceptual support) and, more broadly, to provide practical and discussion/organizational guidance (pragmatic support). For example, a teacher can focus attention on a particular part of a program, or, more strongly, suggest a possible direction of pursuit (either a known-to-be-productive direction, or one chosen for interesting contrast) in concrete, representational terms.

³ Surprisingly, however, many of our tasks seem to work as well for sixth or seventh grade students as for advanced high schoolers.

⁴ diSessa (1993) describes the nature of some intuitive knowledge that is virtually perceptual in form. A fair amount of work in experimental psychology has shown that modality of presentation (text, pictures, moving pictures) makes a difference in conceptualization.

Non-representational Principles

6. *Building on productive intuitive ideas*: Our project has been strongly committed to understanding and building on students’ “naïve,” intuitive resources. This, of course, is scarcely unique to our project, but it has been important, in our judgment, in the success we have had.
7. *Designing a good task framing and setting a proper task context*: I will mention some particulars in examples, but, in general, we feel that task design and teachers’ skills in supporting student accomplishment are always important.
8. *Knowing it is possible*: It is highly non-trivial support to ask students to accomplish achievable tasks, framed in ways we know can succeed. This contrasts with the task that faced the first scientists in discovering fundamental science; they formulated the tasks themselves. I will say no more about this evident principle.

3. Examples

This section presents a series of mostly successful examples of students’ re-inventing or re-designing fundamental mathematics and science. I sketch the tasks as framed and the contexts in which the students worked, including some of the “pump-priming” we did. I also describe outcomes, including variation and limitations we have discovered. These examples make a prima facie case that students’ re-inventing fundamental mathematics and science is possible, and show some of the “how” and “why.”

The main new-media point of these examples is partial validation of intellectual powers granted by the use of new media in these “exceptional” accomplishments. “Exceptional” is quoted because our attitude is that these accomplishments are surprising mainly in that they are much more difficult—if they are possible at all—without new-media support. We take them to be examples of what can become everyday accomplishments in the future. Consistent both with our general statement of goals and with each of the relevant principles, we are concerned with powers and advantages conveyed by representations when they are partners in thinking by individuals and groups, not with respect to any residue in changed thought in the absence of the representations (contra Phase I modal claims). Finally, while some social features of representational use are highlighted, these remain at the classroom level. Essential large-scale issues concerning new literacies are simply not addressed, owing to the nature of the empirical work described here.

a. “Inventing Graphing”

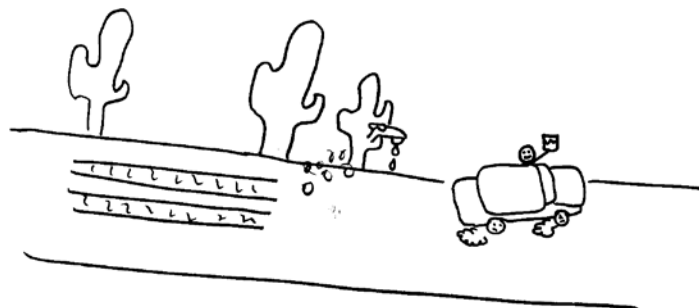
Task	Students are given the task of creating paper-and-pencil depictions that are “as simple as possible” to show various concrete motions. The initial motion-to-be-depicted was called the “desert motion”: a car drives through a desert, but the driver stops to get a drink from a cactus, and then he slowly speeds up and drives away.
Mode	Students engage in iterative design, where individuals or small groups propose or refine representations, alternating with full-class critique sessions. The teacher prompts consolidation of representations, group-consensus criteria, and proposes

	new motion problems with greater difficulties, such as reversed motion.
Outcome	Students in our first edition produced a plethora of representations, gradually moving toward more abstract and scientific representations. In the end, their favorite representation was in essence Cartesian graphing. Students were exceptionally motivated and engaged.
Replications	We have done about a half dozen replications. All showed substantial competence to design and critique representations, and substantial further development of that competence in students from sixth grade into high school. Each replication revealed new kinds of representations. A related series of studies asked students to design representations of “landscapes,” which, while not as rich, generally converged on approximations of topographical maps (Azevedo, 2000).

The first example of students’ re-invention, which we call “inventing graphing,” is exceptional in that it relates in a very different way to new literacies, compared to the examples that follow. In essence, we discovered that students have remarkable meta-representational competences (MRC). That is, they know much more about representations, in general, than most would expect. In particular, (a) they know how to generate a wide range of new representations; (b) they can effectively critique particular representations as adequate, or not, for particular uses. Although not highlighted here, (c) students also are surprisingly articulate about representation (much intuitive knowledge is more tacit and implicit); (d) they know a significant amount about how representations work; and (e) they can quickly learn some classes of representations with little instruction. These five aspects of representation form the core of our definition of meta-representational competence.

The first edition of inventing graphing (diSessa, Sherin, Hammer, & Kolpakowski, 1991) took place in a sixth grade class over four and a half 45 minute periods (roughly a week of “math class”). What attracted us to do analysis of this sequence of classes was the exceptionally high level of engagement shown by the students. But, what emerged from analysis was an impressive competence to design and critique representations. diSessa (2004) reviews the extended research program that evolved from the original episode.

a.



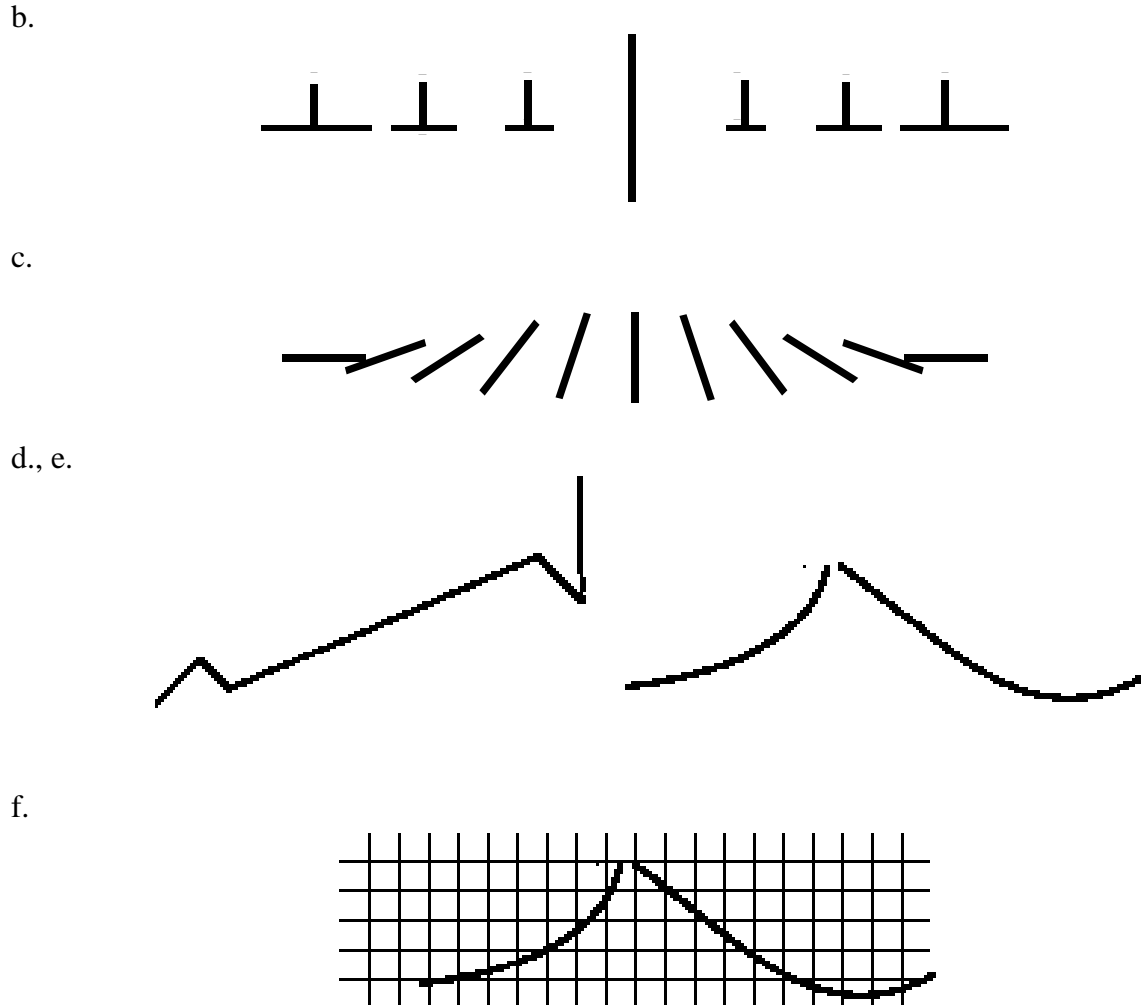


Figure 1. A sequence of student-designed representations of motion.

Figure 1 suggests some of the richness of MRC shown by students, selected from our first edition of “inventing graphing.” It also unpacks some of the steps that, in this case, led to Cartesian graphing. Figure 1a is a fairly typical “concrete” early representation. Figure 1b shows a representation that responded to a teacher’s challenge to represent the length of a stop. Here, the horizontal line represents the speed of the object, and the vertical line represents “how long it was going at that speed.” The sequence of discrete icons is a typical and important early phase of representation. Figure 1c was a brilliant innovation; speed is represented by the slope of the line, freeing up length to represent another aspect, such as duration or distance. Figures 1d and 1e represent follow-up innovations of, first, connecting the slanted lines, and then representing continuously changing speeds. Finally, a student suggested adding a grid so that different aspects (speed and time or distance) could be read off numerically.

We do not question the fact that these students likely had seen graphing before (although not, as far as we could determine, in their mathematics classes). But, the main point is not whether they reproduced graphing, per se, and “by themselves.” The really important point is that they were building on and developing a flexible, well-articulated meta-representational expertise in these

activities. The students explored many alternatives to graphing, and they explicitly considered multiple criteria for evaluating them, such as clarity, simplicity, completeness, and so on. (See, for example, diSessa, 2002). Other classes that never re-invented graphing, per se, worked just as well in this regard. In complementary manner, when students introduced graphing as a way of representing motion without critical reflection of its advantages and disadvantages with respect to alternatives, the meta-representational agenda might have been only marginally advanced, if at all.

Our take, from the extended research we have done in this area, is that the main reason for students' excellent accomplishment is a culturally well-prepared MRC, a nearly-invisible current literacy. This parallels our finding and building on sometimes invisible competences (principle 6) in some of the other examples, below. In the intellectual line of remarkable "re-inventing," the work here stands without need for elaboration. In the line of new literacies, the connection is indirect and somewhat speculative. However, we believe the existence of MRC provides a fundamental insight into possible new literacies.

To reach this insight, one must consider the fact that, unlike text, new media, especially in the service of mathematics and science, do not provide just one or a few new representational forms. Instead, they provide a basis for hundreds of new representations crafted to special purposes. Exploratory statistical displays, image processing for astronomical analysis, internet-available displays of economic growth, and so on, all make the following point: Learning representations one at a time (such as graphs, algebra, tables, etc.) is no longer a tenable approach, if it ever was. Instead, new-media literacies in the techno-scientific niche (at least) will be permeated with meta-representational capacities to understand and control myriad representations. That is, new media literacies will have a far larger component of meta-representational competence than is necessary now, with conventional literacy. Inventing graphing shows that we have opulent student resources to build on, and that the newly necessary MRC is a plausible target of instruction in future literacies.

b. Designing Newton's Laws

Task	Students are given a sketchy template for a program that simulates the motion of a rocket ship in outer space. The ship is controlled by an engine that provides individual bursts of thrust in a direction that is controllable by the captain of the ship.
Mode	Students engage in full-class cooperative design of the program, guided by the teacher.
Outcome	The students produced a program that is the programming equivalent of Newton's laws, algebraically represented as $F = ma$. In particular, the thrust of the space ship (F) adds to the existing velocity of the ship [in precisely the way that an acceleration (a , determined by F) defines the amount of change to existing velocity]. See later discussion and Appendix A.
Replication	None

This and the following examples all involve an unusual representational form that has often been completely neglected in new-media studies. We examine the power of simple computer programs to enhance the learning and understanding of science.⁵ Here, I do not make any special point about the particular language used, which is called Boxer (www.PyxiSystems.com; diSessa, 2000). Indeed, I consider this only an early-stage example of programmability of new media, some versions of which may be more graphical and less explicit (e.g., “programming by example”). However, two larger points are important to mention. First, it is the explicit and analytic nature of programming that here provides its power to extend algebra (Sherin, 2001), which is one of the core old-media representations in science and mathematics. Intuitions concerning very different, possibly easier-to-approach new media must be tested against their genuine capability to provide intellectual power. In the case of programming, we have data that assures learnability at much earlier ages than algebra, and that suggests at least as much intellectual power, compared to a difficult-to-challenge forbear, algebra.

The second point comes from considering whether new-media literacies will be truly democratic and empowering *two-way literacies*, where everyone gets to both read *and* write. Or will they be *one-way literacies*, where production is limited to an elite class (say, “media companies”), and “the masses” are, at best, consumers? I cannot take the time for an extended discussion. However I can state my position. One-way literacies are intellectually and politically suspect; some version of programming is the only way that members of a wider society can enter into the production side of dynamic and interactive representations, which hold a key ground in the promise of intellectual power in (at least some) important new-media literacies (diSessa, 2000). Programming may change its form in the future, but the function of designing dynamic and interactive representations is, in my view, not negotiable.

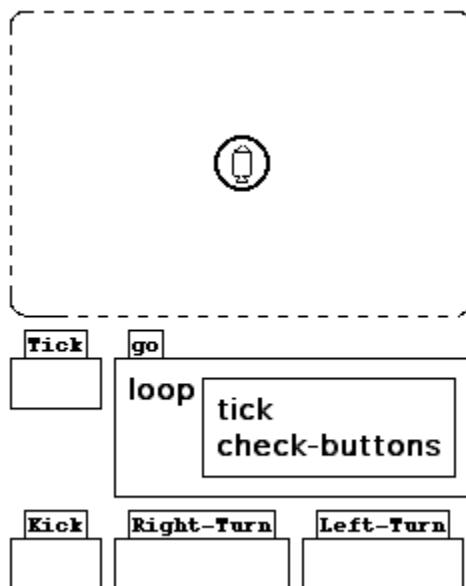


Figure 2. A template for the rocket ship program.

⁵ The role of such representations in professional science is a critical complementary study. In short, we believe new media of exactly this type are revolutionizing science. That is, however, not a topic for this chapter.

Figure 2 shows the template that we provided an experimental class of eight high-school students. The graphic at top shows the (circular) space ship and the aimable rocket engine. The “top level” program **go** simply repeats over and over **tick** (**tick** names what is executed at each “tick of the clock”) and checks the control buttons that execute **kick**, **right-turn**, and **left-turn**, according to which buttons are pressed.

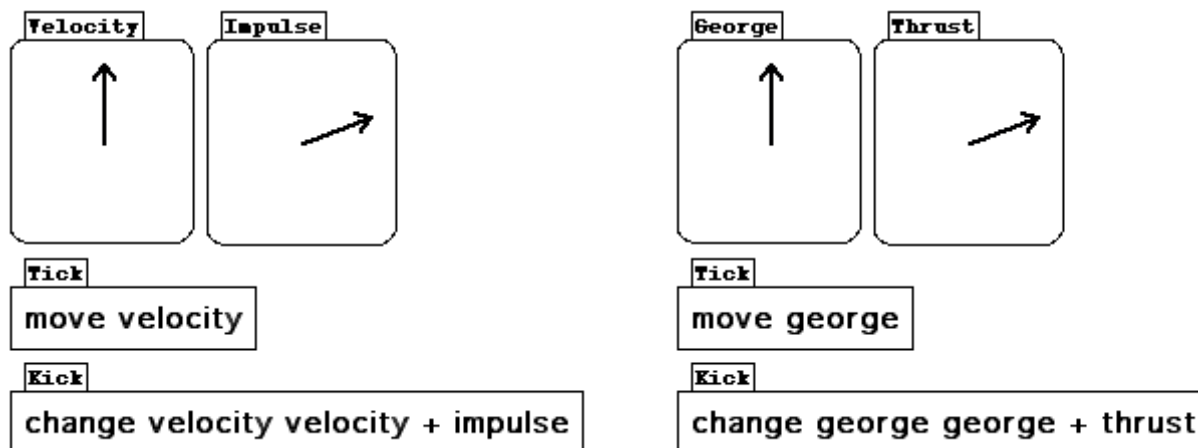


Figure 3. Left, a normative programming model, equivalent to $F = ma$ (suppressing the role of m). Right, the relevant part of the space ship program, developed by students.

Figure 3 (left) is a normative model of Newton’s laws. The **tick** procedure represents Newton’s first law. In the absence of thrust (force), an object merely continues moving in a straight line at a constant speed: On each **tick** of the clock, the object is commanded to **move** in the direction of the **velocity** vector, and a distance proportional to the velocity’s length. The **kick** program contains the essence of Newton’s third law, which specifies what forces do to existing velocity. In short, a bit of acceleration (technically, **impulse**), which is by $F = ma$ proportional to force, is added to the existing velocity each instant while the force is acting. (**Change** is the generic Boxer command to change a variable, here **velocity**, to a new result. In general, square-corner boxes represent procedures, things that are run, and rounded-corner boxes represent variables, that is, data.) Appendix A is an extended discussion of how this program represents Newton’s laws, mainly for less technically sophisticated readers. The appendix also discusses some of the more important conceptual issues in understanding Newton, and how representational systems relate to them.

The right part of Figure 3 shows the isomorphic program produced by the students. Of course, the important issue is whether the work of the students was legitimate and powerful, and how the representational form contributed to their work. Sherin, diSessa, & Hammer (1994) and diSessa (1995) show the details of the empirical analysis, of which we report an abbreviated version organized by our representational principles from Section 2. See these papers and, for example, Sherin (2001) for a justification of the legitimacy of the programming version of Newton’s laws. While this work has not been replicated—so its generality is uncertain—its importance is in specific, data-based analyses that show each of the principles in action.

2. *The power of abstract, well-adapted representations*: We trace much of the success of this activity to the fact that we framed the task in terms of programming. In particular,

students had learned about vectors, which therefore constituted a ready language in which to express problems of motion. Vectors are precisely the right level of abstraction for expressing Newton's laws, although traditionally algebra and calculus is used in place of programming constructs. In this activity, students spontaneously introduced vectors into the program, although they named them whimsically (e.g., "**George**" as the name of the velocity vector). An important remark is that learning vectors and some relations to motion was remarkably easy. Students had seen simple programs that moved objects with a vector velocity. The meaning of vector velocity became intuitively clear in dragging the tip of the velocity vector around, showing that the object moved always in the direction of the vector, with a speed proportional to its length. Vectors, in fact, were also easily learned in this way (according to principle 4, of dynamic representations) by much younger (sixth grade) students. They became very popular as user interface controls in game programs. While another pre-requisite to this task, vector addition, might be deemed an abstract principle in traditional instruction, its meaning was easily visible and important in manipulating motion in the context of such games.

A productive level of abstractness is one hint that a representation can provide. Programming representations have another advantage, mathematical precision, which we saw working to students' advantage. For example, in a preliminary activity a few days before, students discussed the effect of a "hit" (an impulse) on a moving object. In that natural language context, students accepted as adequate a simple statement that the hit "combined with" previous motion. In the context of formulating a program, however, the idea of combination was just the start of an extended inquiry into exactly how impulse and velocity should combine and be situated in the overall program.

An absolutely critical point in all of these examples is whether the students understand their accomplishment, or whether the representational (and other support) they are given led them on an un-illuminating garden path, however correct we might view the result. In this case, I report, too briefly, that students struggled substantially with the main conceptual element here, how thrust combines with existing velocity. Several proposals were tried and eventually rejected. Even after the idea of vector addition was introduced as a possibility, how it should be used in this case was unclear. Finally, one student tentatively proposed that **thrust** combined by adding with existing velocity (**George**) to create the new, self-perpetuating velocity. Several students immediately reacted with enthusiasm at the proposed solution, and this generated the final form of the program. Consult Sherin, diSessa, & Hammer (1994) and diSessa (1995) for details.

3. *Representational task tuning; conceptual/representational pump-priming*: It should be clear that we prepared the way for students' work by introducing vectors as a way to describe motion, and also by introducing them to vector addition. In fact, they had used and experimented with vector addition as a way to represent combined, simultaneously present motions, such as the motion of a boat in the current of a river. Students introduced the idea of vector addition in the spaceship context by explicitly invoking the boat-and-river-current model. But they still had more work to do to adapt it to this problem. In particular, they had to understand how the *transient thrust* becomes *permanently incorporated* into motion (via vector addition).

The template we provided, Figure 3, scaffolded focus on the key conceptual parts. The **tick** procedure represents Newton's first law. The **kick** procedure is the locus of Newton's second law. As the group seemed just at the limit of its competence in accomplishing this task, we feel the activity would likely have failed without this simple, but effective partitioning of concerns.

4. *Dynamic representations*: I already mentioned the effectiveness that we felt dynamic representation had in conveying an understanding of vectors in simple programs that controlled the real-time motion of objects. In developing their version of Newton's laws, the students often ran the program in their imagination to evaluate hypotheses for filling in slots in the template. In addition, once the program was complete, students played with it to solidify its meaning and to develop an intuitive fluency with its implications.
5. *Support for social collaboration*: As a full-class exercise, managing student-student and student-teacher collaboration is a critical need. In this case, a single, hour-long episode that targeted a particular result depended in important measure on how the members of the class coordinated their efforts. Here, I mention a few examples of the types of coordination that we listed under the general topic of collaboration.
 - a. *"Reading" student ideas* – The evident currency for the discussion was "programming code." While perhaps as ambiguous as words at first mention, code fragments can be disambiguated by imagining or actually running the program. While nuances of student intuition were not necessarily captured directly in code, it constituted a public, stable, and precise backdrop on which the more subtle job of physical reasoning and judgment could be played out. In this activity students seemed to participate with a feeling that code-fragments could stand for their ideas. For example, students pointed to code when referring to "my idea" or to the ideas of colleagues.
 - b. *Goal and hypothesis clarity* – How does one know when one has a theory of motion? On the other hand, the concrete goal of this exercise was to produce a complete, working, and sensible program. Although "sensible" may be an important topic for debate, the completeness of the program for accomplishing the given task, and the fact of its "working," are easily observed by the whole community.

In managing the discussion, the teacher often asked for clarification concerning what students meant in terms of code. He asked, "what does that mean for our program?" "What does that say about what's in **kick**?"

- c. *Summary representation* – The end-state of this design was a simple, memorable program, similar in function to mnemonic recitation of equations like $F = ma$. We know students could easily reproduce these little templates; for example, the river-and-boat code fragment that introduced vector addition into this problem came in as an easily re-produced code fragment. Of course, mnemonic symbol sequences are not the same as understanding. But they are a part of it. Scientists

think, sometimes, by drawing in equations when appropriate, and manipulating them. Student understanding of Newton’s laws works, in some measure, as a person-plus-symbol-system interaction, which is, as mentioned, my prototype for the very meaning of literacy.

- d. *A focus for intervention* – Conducting this discussion was a challenging task, even for the talented (graduate student) instructor involved in designing Newton’s laws. In our analysis, we saw him using focus on the representational form of the problem to bring students together, and to move the discussion forward. He often asked students to focus on a particular place in the program, or how their ideas related to particular expressions and places. He sometimes intervened in suggesting more mnemonic terms for elements in the program. **Thrust**, for example, emerged from a negotiation with students. (**George** would be re-named in later work, following this activity.) As a last resort, teachers can suggest particular expressions, which can then be examined for their meaning and sensibility. In this case, the instructor did not provide any substantial suggestions about what should be in the program. In general, teachers’ competence at these skillful interventions would be an important part of *their* new literacy.

With respect to non-representational principles, the success of this case seemed to us most sensitive to the framing of the task (principle 7). We believe the semi-concrete, but also non-earthly context provided enough remove from familiar contexts to allow consideration of some of the non-intuitive aspects of Newton’s law—for example, the idea that motion would continue forever without a thrust. At the same time, students evaluated the resulting motion in terms of productive “earthly” intuitions (principle 6). See Sherin, diSessa, & Hammer (1993) for details.

c. Galileo’s Model of a Falling Body

Task	Students are given a simple but defective program that simulates the motion of a dropped ball. The students are asked to improve it.
Mode	This exercise has been run in many variations, usually in a few iterations of small group design, interspersed by sharing and discussion in the full class.
Outcome	Students (and even teachers, performing the task as a part of professional development) engage in a fairly regular development involving (a) recognition of increasing velocity in a fall, (b) recognizing the regularity of the motion, (c) almost always producing the two models that were discussed by Galileo.
Replication	This task has been replicated in many classrooms, from late elementary school through high school, including being run by teachers new to the idea of teaching motion, and new to the use of programming representations. Results generally validate similar developments and outcomes. However, none of the replications has undergone the kind of detailed analysis we reported, above, for “designing Newton’s laws.” Of all our re-inventing activities, this one has had the most work in “real” classrooms, with “real” teachers.

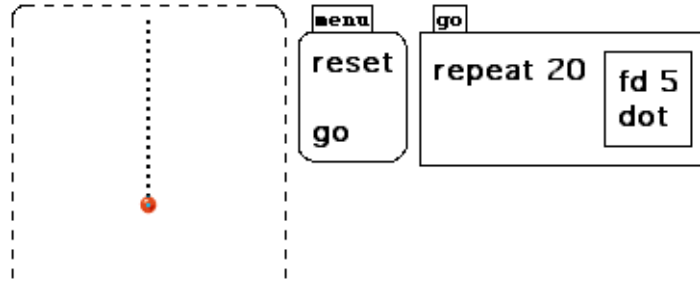


Figure 4. The starting “prompt” program with its visual result at left. **Fd** means “move forward” (downward), and **dot** means “draw a dot.” **Reset** and **go** (in the **menu**) are clicked on to reset and activate the simulation.

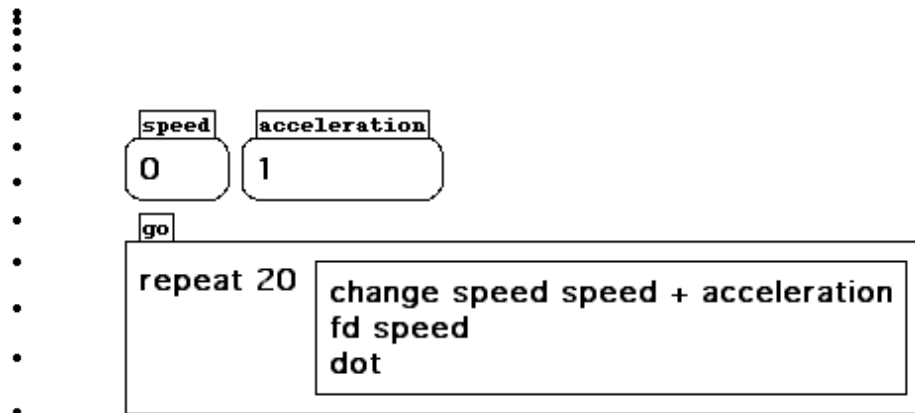


Figure 5. The normative model (Galileo’s model); a constant amount (acceleration) is added to speed each tick of the clock. Speed is thus proportional to time.

Figure 4 shows the initial faulty program. Figure 5 shows a normative model. In our experience, almost all editions have exposed a diversity of ideas and a fairly extended development, but almost always two competing models have emerged. One is the “additive” model, shown in Figure 5, and the other is the “multiplicative” model, shown in Figure 6. Students produce many variations in form and syntax that are identical to these, but we will not display them here.

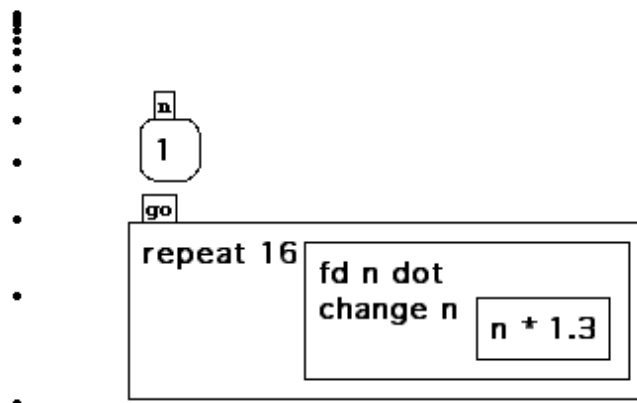


Figure 6. The multiplicative model of a fall.

The multiplicative model is highly attractive. It has many intuitively plausible qualities, and the fall is dramatic in its speedup. The “correct” model is less dramatic. Galileo proved that the multiplicative model (which is equivalent to falling at a speed proportional to distance fallen) is impossible. His strategy was to show that, according to the multiplicative model, a fall of twice as far would take the *same time*, an impossible condition. An “easier” counter to the multiplicative fall is that the model can’t get started: Distance, and therefore speed, are zero at the start, so the falling object can’t move at all. Another counter is more subtle. Reversing the model to make a toss, the multiplicative model becomes a dividing model, which never gets to the peak of the toss. However, our experience has been that these arguments are too subtle to regularly work with students, even high school students with a fair amount of scaffolding. So, we have come to be happy with convergence to these two models, using other work, such as taking data and matching to the models, to settle the issue.

Steps along the way are informative of the work students actually do in constructing these models. For example, for elementary school students, the very fact of falling at an increasing speed is often problematic, and it takes some time to work through. Here, augmenting the task by “feeling” the impact of objects falling from different heights is usually convincing. Galileo, also, suggested that experiment. Another interesting development among students is toward an assumption of uniformity in the fall. Figure 7 shows the initial models produced by a high school student. I return in a moment to his first model, **go**, but notice that in both **go** and **ho**, the incremental (decremental) distances are non-uniform.; they switch between an increment (decrement) of 1 and one of 2 half way through the motion. In our experience, almost all classes produce such non-uniform models (or “step-wise” models, where the object falls a constant distance for some number of steps, and then a new, greater constant distance for the same number of steps). Group discussion of the simplicity of a uniform model, or of a smoother one, have systematically won out in student discussion. Uniformity is an aesthetic consideration that Galileo emphasized, and it seems that groups of students can regularly come to appreciate its attractiveness in teacher-scaffolded discussion. Here, we see important commonalty between scientists and students, not in the models that they have or produce, but in the aesthetics displayed in ultimate choice of model.

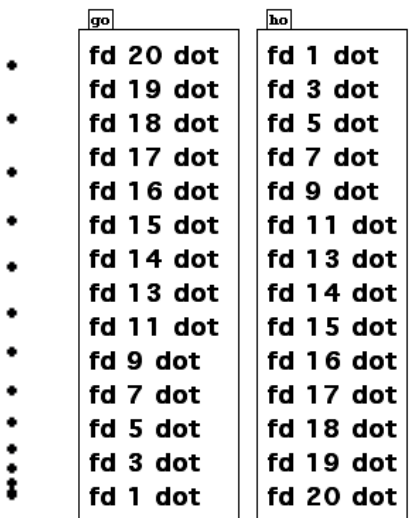


Figure 7. Two models by a high school student. The trace at left is of the first (**go**) model.

Younger students sometimes include a phase of slowing down at the end of their falls. The rationale is surprisingly strong. If the ball stops at the bottom, surely it must slow down before it stops! Indeed, the ball does slow down, but only after it contacts the ground. That can be the subject of a good discussion of why and when a ball actually slows down in a drop.

Representational considerations often arise in this modeling activity. The first model in Figure 7, **go**, shows a surprisingly common issue. When the program is run, the ball noticeably slows down at the bottom! However, the first time we saw this in a sixth grade classroom, one of the pair of boys who produced it explained that it was intended to show speeding up. He said, essentially, “we didn’t want to show the ball speeding up, we wanted to show *that it sped up*.” The problem in our interpretation was that, like many other students, and also some adults, students may chose to display “increasing speed” as “increasing numbers (density) of dots.” This is non-conventional, but not wrong in any sense. We can summarize by saying that these students intended the program to produce a representation, not a simulation (!), and that they used an unusual, but not wrong convention (greater density of dots represents more speed).

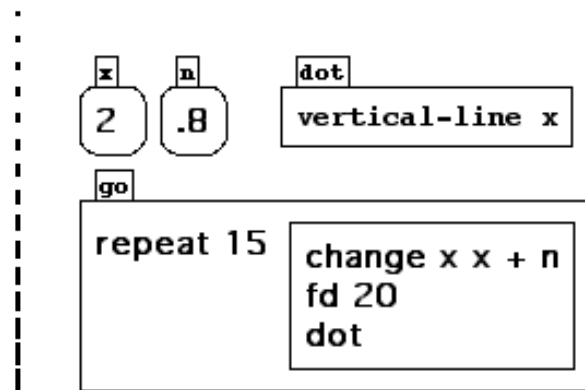


Figure 8. A student model showing speed proportional to distance in the fall.

Another representational issue is shown in Figure 8. Here, the student is showing increased speed not by using the distance between dots, but by using the length of the “dots.” He re-defined **dot** to draw a line, and then systematically changed the length of the line, which he called **x**. As mentioned, this is identical, but not transparently so, to the multiplicative model.

In net, scaffolded development of models of a dropped object regularly converge to two plausible scientific models. Along the way, student models open up excellent discussions of possibilities and plausibilities. A lot of development toward the two models arises naturally and by near consensus, but not instantly, out of discussion. Obviously, the richness and productivity of the discussion depends substantially on teacher preparation (for what s/he will see in the students’ ideas and programs, and more technical competence, for example, in recognizing programs with identical outcomes but different forms), and on negotiating the variations produced in a particular class. (Again, these examples forecast new-media literacy for teachers, as well as students.) There is no need to invoke magic in these students’ accomplishments. The convergence to cogent models emerges because of (a) good student intuitions, which, however, need cultivating, articulating and considering, and (b) the use of an apt representational form that

allows the expression of intuitive student ideas at the same time that it can carry the development to a respectable and precise normative model.

d. Temperature Equilibration

Task	Students are asked to think about how a glass of cold milk warms up when it is taken out of the refrigerator and placed on the kitchen table. Later, students take data, and then collectively try to produce a program, from a sketchy template, to reproduce the data.
Mode	The initial discussion is run by the teacher in full-class mode. Small groups of students take data, and then the teacher scaffolds the full class in making a model that can reproduce the data.
Outcome	Students seem regularly to produce a model that is correct, but not in a normative form. Students seem sensitive to the advantages of the normative model.
Replication	We have run two instances of this activity with high school students in small classes (6-8).

We report our final case briefly, as it is still in formative stages; detailed results have not been published (but see diSessa, in press). The power of representational scaffolding seemed very evident in the introductory discussion of heating of a cold glass of milk. Open discussion brings out many hypotheses about how and why the milk warms, some of them quite fanciful, and usually with no emerging consensus. When students are coaxed to use (old media!) graphs, however, we see the typical constraint representational forms can productively apply; predictions of how heating occurs converge to a few possibilities.

In one edition, one student suggested that the temperature followed a simple pattern, with increases of 1, 2, 4, 8, 16, ... degrees. This reflects the simplification that mathematization can convey, but also the lack of feedback that simply numerical or algebraic representations are prone to. We are confident that, if this student produced a program that “warmed up” according to his pattern, he would have judged it implausible. (All students in our three classes seemed convinced that the rate of heating slows down, if not in the beginning, then at least toward the end of the heating. Numbers on a page do not seem to be instinctively interpreted in a process-oriented way.)

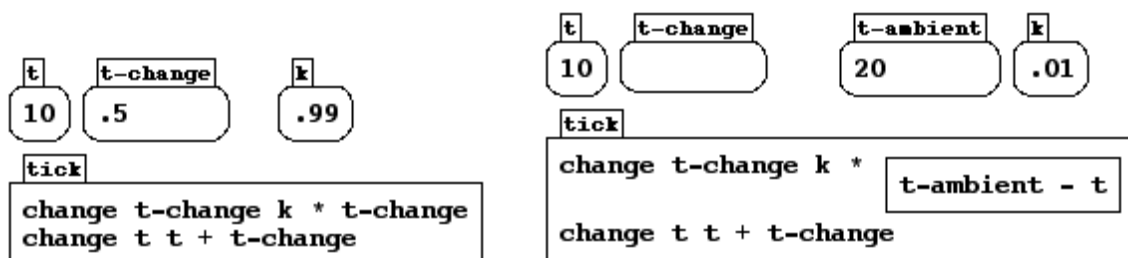


Figure 9. Left: The student-generated model of temperature equilibration. Right: The normative model (Newton’s law of heating).

Both of our experimental classes converged on the model at left in Figure 9. A normative model is shown for contrast at right. The positive outcome is that the student-generated model is very similar to the normative model. In fact, although it is not evident, the two models produce identical outputs, and can perfectly match (within experimental error) the data students took. Even more promising, students in both groups voiced advantages of the normative model over the student-generated model when the normative model was shown to them. For example, the normative model shows the ambient temperature, a measurable parameter, explicitly. In addition, the role of ambient temperature in “driving” temperature change is transparent. (Consult the **change t-change** line in the normative model on the right in Figure 9.) In the student model, the ultimate temperature, ambience, is a complex function of *both* the constant **k** and the initial **t-change**. (In the normative model, **t-change** is computed, not externally set, even for the first step.)

A significant failure of this re-invention of the law of heating/cooling became evident in the second class. When the students were introduced to the task, they quickly converged on the model at left. The next week, when we engaged them in discussion, they seemed at first not even to remember creating the model. But, within a few minutes, they created it again by virtually the same set of steps they had used the week before!⁶ In retrospect, we believe that the students were following a representational garden path of trial and error. In contrast to all of the other re-invention tasks, it seems the representation and template make this one too easy, cutting away so many possibilities that essential conceptual discussion is short-circuited. This, of course, is an expected possibility, at least in some cases. Computational representations cannot always produce exactly the right level of scaffolding to make the task of re-invention possible but appropriately challenging conceptually. Here, at least as we proposed the task, the representational scaffolding is too strong. Undoubtedly there are cases to be discovered where scaffolding is too weak.

Our judgment is that we can relatively easily fix the problem here by enriching and “complexifying” the task. That, however, remains to be demonstrated. The meta-lesson is, again, that there is little magic in these re-inventions. They depend on a good choice of representation and apt task design. They depend on teacher scaffolding and on a helpful diversity in student ideas. Use of a computational representation does not guarantee, certainly not by itself, that students will engage in important conceptual work and converge on acceptable scientific principles. On the other hand, all of the re-inventions we described provided exceptional contexts in which to engage and develop students’ scientific competencies.

Summary and Future Research

It is natural to be enthusiastic about new-media literacies. Beyond doubt, new possibilities are in the offing. It is exciting to see wide-spread, grass-roots use of new media in, for example, blogging as a new medium for news and commentary, MySpace and even simple web pages as expressions of self and new social possibilities, computer games as an emerging part of common culture or as a promising basis for future learning, and video as a wide-spread new critical and

⁶ This set of steps were reported to us by the teacher. Unfortunately, we did not video the first session.

artistic medium. But in calling new-media uses “literacies,” we must carefully separate fads or even more permanent changes in fashion from civilization-changing possibilities. In this chapter, I have applied a high threshold of validation in considering *empirically supported analyses* that show consequences in terms of things that have nearly *incontestable social value*: science and mathematics learning. The work discussed here suggests that students can learn mathematics and science effectively at much younger ages and with much more agency and pleasure than previously. We have seen that students, with support from new-media representations, can accomplish tasks such as re-inventing fundamental mathematics and science. Just as important, I believe that the ways these remarkable accomplishments are achieved can be “pinned” on properties of the new-media representations used—not to ignore the roles of good task design, good collaborative patterns, and student-sensitive teaching. To sum up, a cornerstone of this chapter is a high-level of empirical accountability for the actual impact of the representation or literacy in producing results we understand and value as important.

The perspective on literacy that frames this work draws equally from early and newer theories of it. From early “cognitive” work, we insist that all representations—the technology of literacies—are not equal, and that the specifics of the representations have strong consequences in the powers we realize from literacies. For example, I do not believe multi-modality (Kress, 2003), per se, explains the success of programming in the re-inventings discussed here.⁷ But, we reject technological determinism. We also focus, not on general intellectual powers that are supposed to result from literacies, but on specific, cogently-formulated and falsifiable consequences of thinking-and-collaborating-in-the-presence-of-external-representations.

From newer theories of literacy, I accept the importance of studying particular literacies in particular social practices. One must recognize that powers are conveyed in particular uses, not in the mere presence of a technology. However, socially oriented literacy studies have backgrounded or even dismissed the contributions of the “props” of literacy, its technology. With the intellectual powers for students that we discuss here, this seems a foolish slight, in need of redress.

This work motivates future research at several levels. Proximally, the practical and theoretical possibilities of re-invention as an important mode of mathematics and science learning are open. Study of the micro-structure of learning with dynamic and interactive representations, both by individuals and in groups, has barely begun.

At a mid level, of course, changing representational infrastructure should be as relevant to other learning foci, from other topics in mathematics and science, to history and social studies. I hope our work focuses attention on specific issues, such as the expressiveness of particular representations for particular ideas, and the exact nature of scaffolding that they can supply. Over the longer term, multiple representations and literacies will come to equilibrium with each other. For example, I forecast that computational representations will not replace algebra or calculus.

⁷ For example, multi-modality does not necessarily entail the level of abstraction, the precision, the degree of explicitness, the grain-size of articulation, the “fit” to relevant ideas, or the compactness of final description that programming offered.

However patterns in schooling⁸ and in professional use will change dramatically. How will different representations feed into or complement each other's learning and use? To my knowledge, there is yet no research specifically on this issue.

Larger-scaled, especially social issues concerning the new-literacy possibilities explored here remain unaddressed. Can "programming" come to be seen by society at large as a masterable and powerful tool? In this regard, it is illuminating to remember that algebra and even text were both historically regarded as the province of technical experts, and certainly of no value to ordinary folk. How long will it take for schools to realize the promise of new-literacy possibilities, *especially* in core, high-stakes learning where conservatism is likely to be strongest, such as mathematics and science? How do intellectual powers, such as competence in mathematics and science, interact with social functions of literacy, especially destructive ones, such as social class divisions, and pipe-line preservation of privilege and power? No one has good answers to those important questions. In coming to address them, we will need better and more refined theories of literacy that take into account (a) the nature of humans as knowers, learners, and actors, (b) the extensions in thinking and doing made possible by particular new media, and (c) the social embedding of new literacies, including the dynamics that might bring them into existence.

Acknowledgments

The studies cited were accomplished by and with the help of many members of the Boxer Research Group at the University of California, Berkeley. Authors of cited papers were primary contributors. Current members of the group, who contributed to the temperature equilibration modeling work, include Jeanne Bamberger, Lauren Barth-Cohen, Janet Casperson, Karen Chang, Michael Leitch, Katie Lewis, Orit Parnafes, and Jed Stamas. Jeanne Bamberger provided helpful comments on an earlier draft.

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⁸ For example, I think we will generally be able to teach with computational representations long before one can use traditional representations, like algebra.

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Appendix A: How Programming Representations Relate to Algebraic Ones

I aim here to show how the programming representation for Newton’s laws relates to the usual, algebraic representation. It is intended to help those less comfortable with the physics. In addition, I want to expose and talk more about how particular representation does, or does not, alter the conceptual landscape. In general, the footnotes in this appendix are for physics experts.

Start with the conventional representation of Newton’s laws:

$$F = ma$$

First, we drop the m (or, equivalently, set it to one). Mass (m) is just simply another thing to learn about. In fact, mass is not important in terms of the main conceptual problems students have with Newton’s laws. Hence:

$$F = a$$

We can use the definition of acceleration as the rate of change of velocity, the ratio of the change of velocity to the change in time:

$$F = \frac{\Delta v}{\Delta t}$$

In traditional physics, this ratio is taken *in the limit*, as Δt approaches zero. (If Δt is large, this formula give the *average* force.) But, in the programming “world,” time is not continuous; it jumps, one tick of the clock (one iteration of the program’s “loop”) at a time.⁹ The best we can do for small times is to take the smallest Δt possible, one tick.¹⁰ Furthermore, for convenience, we can choose units of time so that Δt is just 1—one tick of the clock.¹¹ So we have:

$$F = \Delta v$$

⁹ People often complain that the world is not discrete, so students still have to learn limits. However, the continuous Newtonian world, as usually represented, is an idealization, too. In addition, are you really so sure there is no quantum of time, much smaller than a second? In fact, no everyday happenings would be different if time were discrete with a tiny quantum! All science is idealization. Programming-represented Newton’s laws is just a different idealization.

¹⁰ In a discrete world, the finite (one tick) ratio is not *approximately* the acceleration; it is the *definition* of acceleration.

¹¹ In a world with no obvious, fundamental “yardsticks,” it makes sense to write formulae so that they do not depend on the units you chose. But, in a programming world, it is so obvious to measure time by counting ticks, that writing in unit-independent terms is an unnecessary burden. In advanced physics, one often takes the speed of light to be 1, which is a similar simplifying move. In our physics curriculum, the boring stuff of units is an advanced topic. Students get to understand and use Newton’s laws long before they can finesse units.

Force is precisely the change in velocity (of the object to which the force is applied) for each tick of the clock. If force is an impulse, which occurs in brief (one-tick) bursts like the rocket engine proposed in our designing Newton's laws task, one gets a change in velocity equal to the magnitude of the force during a tick precisely when the rocket fires, and none otherwise. The programming form for a thrust is thus:

change v v + F ; that is, “change v to the *old* velocity, **v, + F**”

This is precisely the **kick** part of the program spaceship program. If the force persists, this command would be executed each tick of the clock.

Conceptual issues:

One consequence of this program fragment is that, if there is no force, velocity never changes. This is called Newton's first law, and it is counterintuitive. In everyone's experience, (earthly) motion always dies away. The representation makes no difference as far as this existence of the issue is concerned.

A second typical problem is that, like Aristotle, many students feel the speed (magnitude of the velocity) of an object should be proportional to the force that you apply to it. A bigger engine makes a car go faster; in order to run faster, you have to “do more work.” But this would correspond to a law where velocity, not change in velocity, is proportional to force, **change v F**, rather than **change v v + F**. Again, the conceptual problem remains.

For these two conceptual problems, the programming representation strips away a lot of complications, including units and limits (calculus), so that the core conceptual problems are exposed. For engaging those problems, students have the advantage of seeing exactly how different laws ($F = v$ or $F = \Delta v$) correspond to different behaviors. They can make a simple edit to the program, and can see (or, perhaps better, run in their imagination to infer) the consequences. In net, the programming representation does not finesse the conceptual problems, it “purifies” so as to expose them more clearly, and it embeds them in a richer analytical/empirical set of experiences (which are, of course, dependent on the social practices engendered in the classroom).

One sees the same pattern in learning again when forces and velocities become vectors. A principal conceptual problem is that, when force is perpendicular to velocity, the speed (magnitude or length of velocity) does not change at all, but the velocity vector rotates. This is how it can be that objects orbit in a circle (roughly like the moon around the earth) at a constant speed, even as gravity is “accelerating them” toward the center of their orbit (accelerating the moon downward, toward the earth). Students in high school or freshman physics classes memorize “in circular motion, the acceleration is toward the center of the circle,” but they seldom understand it. The computational representation allows students to see exactly what happens to the vectors, one step at a time. With standard representations, seeing the consequences of a given force law involves a technically advanced task, “solving a differential equation.” Furthermore, with a simple program, students can play with the phenomenology. They can see how a force not-quite perpendicular to velocity speeds the object up or slows it

down, while still re-aiming it (rotating the velocity). At the other extreme, a force aligned with velocity increases or decreases speed, but there is no change in direction. Forces not-quite aligned with velocity rotate it a little, in addition to changing speed. All together, these different experiences lend a fabric of sensibility to the initially counter-intuitive fact that a force perpendicular to a velocity maintains speed, but rotates the velocity. Here, the contribution of the representational form is that simple experiments can be transparently notated, conceptualized (in seeing each step in the program), and experienced.