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Instructional Explanations in History and Mathematics

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Abstract

This paper extends a dialogue on the nature of explanations and instructional discourse to include instructional explanations. The paper examines instructional explanations at three levels: (a) the distinctions between specific types of explanations (common, disciplinary, self, and instructional) with respect to specific features (problem type, initiation, evidence, form, and audience); (b) the occasions within history (events, structures, themes, and metasystems) and mathematics (operations, entities, principles, and metasystems) that prompt explanations; and (c) critical goal states present in successful explanation (representations known, verbal discourse complete, nature of problem understood, principles accessed). Using these three levels, three examples of shared instructional explanations are explored, two in history and one in mathematics.

Distinctions

Explanations occur in response to an implicit or explicit query. The form, location, and sense of the query shape the explanation. This research focuses on instructional explanations as distinguished from common explanations, disciplinary explanations, and self explanations. While all explanations share some features, there are also important distinctions.

Common explanations occur in response to a direct question about how to do something or proceed with an event; they are local and event or information-based. Common explanations rest heavily on at least two speakers' shared sense of how such questions are to be asked and answered. (How do I re-light the pilot light in my instantaneous water heater? Why are the registration forms due now?) The query is focussed, short, and directed. These are common because they are a frequent and vital part of our everyday life. The resulting explanations are also notable because of their lack of grounding in subject matter, their brevity (one to four exchanges are usually sufficient), lack of intimacy, and their lack of a deep sense of inquiry on the part of the asker. They are also notable for their reliance on a sharing of the social setting and rather surface features of cultural exchange.

Disciplinary explanations, on the other hand, arise from overt as well as tacit queries; they tend to be lengthy, formal, and can exist across distances of time and place. The explanational form is ritualized, maximizing communication in the absence of face and voice. Disciplinary explanations are built around a core of rhetorical conventions within the discipline that help to determine form and completion. Conventions include identification of important questions, what constitutes evidence, what may be assumed, and what the agenda is for the discipline. Disciplinary explanations have rules that help the community of thinkers in that field focus on the task of constructing new knowledge and reformulating extant knowledge. In one sense, these are socially constrained events because explanations for one group are not particularly recognizable, let alone acceptable, to another--a physics explanation is quite different from one in history or literary criticism. (The futile attempt to make them similar is documented in the writing of Hempel [Hempel & Oppenheim, 1948, 1988; Collingwood, 1946].) Disciplinary explanations contain an element of justification within the bounds of shared rules. In another sense, these explanations are non-social because the presence of a physical audience is not a requirement. Disciplinary explanations do not provide practical information in the sense of the everyday but they do provide "real" information in a real form, in the sense of information highly valued by a particular community. Disciplinary explanations serve the function of proving the legitimacy of new knowledge, reinterpretations of old knowledge, or challenges and answers to existing knowledge. From an educational standpoint, when students approximate disciplinary explanations, they are engaged in authentic disciplinary activity. When they can use the knowledge in such an explanation in other explanations outside the discipline, they engage in a different form of authenticity.

Disciplinary explanations differ from each other because different disciplines use different principles or theories, make dramatically different uses of evidence, have different rules for refutation, tend to permit different levels of self reflection or layering, and, most importantly, ask very different kinds of questions. What is the nature of the circulatory system? How does it work? What is the nature of baboon maternal behavior? How do baboon troops develop and maintain structure? When did the universe begin? How many regions are created when five planes cut a space? What is the balance between liberty and power reflected in the Constitution? What is the role of the peasant intellectual in defining national identity? Explanations in answer to these different questions look different from each other.

Self explanations are explanations given to the self to establish meaning or to incorporate new knowledge with old. The "self" may be a group that is facing a shared task, but more commonly the self explanation is intra-individual. The verbal trace of a self explanation is idiosyncratic, local, and fragmentary, leaving unsaid the known bits of information and stopping when the end is in view rather than when it is actually arrived at. The query is prompted by a sense of incongruity or desire for generative cohesion. Self explanations have been shown to be fundamental to learning complex material (Chi, Bassok, Lewis, Reimann, & Glaser, 1989). The sense of asking and answering ones' own questions is perhaps the most powerful indicator of meaningful learning behaviors, as contrasted with so-called mechanical or rote learning (Leinhardt & Putnam, 1987; Chi et al., 1989).

In younger students, this sense of self-explanation inquiry has been directly taught, and when the inquiry matches the eventual demands of the situation the instruction has been very fruitful and facilitating (Palincsar & Brown, 1984). In slightly older children (12-14), such inquiry and explanation is often an indicator of academic success, although it is a feature of learning that educators want to see more often than they actually do (Leinhardt & Putnam, 1987). At the high school level, when dealing with complex material, the form and intent of such inquiry while studying is a powerful predictor of knowledge acquisition. As Chi and her colleagues assert, self explanations are "the students' contribution to learning." (Chi, et al., 1989, p. 146)

Instructional explanations are the teachers' and texts' contributions to learning and these explanations are designed specifically for communication of a particular aspect of subject matter knowledge; they are designed to teach. Instructional explanations teach in three ways. First, instructional explanations convey, structure, convince, and demonstrate; second, instructional explanations model questioning and explaining in the discipline; and third, instructional explanations give clues to meta-cognitive behavior. Such explanations occur in response to an actual query, an anticipated or probable query, or perceived puzzlement. Instructional explanations are designed to explain concepts, procedures, events, ideas, and classes of problems in order to help students understand, learn, and use information in flexible ways. They are complete with respect to their verbal trace, but their form is more spontaneous than it is in a disciplinary explanation. They are decidedly social (computers notwithstanding) and local in time and place. Instructional explanations tend to be elaborate and reflect both the rules of communication and the rules of the discipline. Thus, they make use of examples from both the personal, shared experiences of the community of learners and from the discipline itself. They can draw on representations that are colloquial and familiar and on abstract or intermediate representations (White, in press).

Disciplinary explanations are a part of teaching and learning because of the role they play in establishing "real questions" (Lampert, 1990) and in guiding rules of evidence and justification. Self explanations are the processes of developing meaning for the self and, as such, are also a vital part of learning (Chi, et al., 1989). Instructional explanations build from their particular contexts both physically and socially; they tend to have redundancy as a conspicuous feature. The good teacher strives in the explanation for vividness and distinction from the surrounding mass of information. Disciplinary explanations are admired, on the other hand, for their elegance and parsimony.

The differences among disciplinary, self, and instructional explanations consist of differences in what gets explained, how an explanation is initiated, what constitutes rules of evidence and legitimate authority, when an explanation is complete, and the audience for an explanation. To review, what gets explained in a disciplinary explanation is an answer to an extant question or a reformulation of a position in light of a disciplinary dialogue. The explanation is initiated by a public challenge or as an act of completion to a previously incomplete understanding. What gets explained when one is explaining to oneself in a learning setting is how to work around a problem, how to connect a new piece of information to existing information, or how to restructure or rearrange existing information. This type of explanation is initiated when an incongruity is noted or an integration needed. In instructional explanations, what gets explained is information not understood, or which may not be understood or which may have future value. They are initiated in response to overt or covert queries or when a piece of information has been flagged by a teacher as important. (N.B., instructional explanations may be given by teachers, students, or both together).

The rules of evidence for discipline-based explanations are ritualized and shared, and the form tends to be formal, almost coded (especially in spoken presentations), striving towards parsimony and elegance. Completeness is required, especially in the written forms. In contrast, the rules of evidence for self explanations are either completely unrestricted, (including folklore, personal experience), or overly rule-bound. The verbal trace of self explanations as reported by Chi, et al. (1989) is fragmented, idiosyncratic, brief, and sometimes redundant. Instructional explanations use the discipline-based rules of evidence, shared personal experiences, and texts. Instructional explanations also make rich use of external representations and analogies. Such explanations are complete when consensus is reached about a shared level of understanding. This leads both to redundancy in the form and an informality of spoken presentation.[1]

Instructional explanations in mathematics, history, geography, and writing, have been the focus of an ongoing program of research. This research has addressed two issues: the objects or occasions for an explanation; and the development of a model of instructional explanations. Research addressing the occasions for instructional explanations has focussed on history (Leinhardt, 1993; Leinhardt, Stainton, & Virji, 1992; Leinhardt, Stainton, Virji, & Odoroff, 1992; Stainton & Leinhardt, 1992). Work on a model of instructional explanations in mathematics has developed to the point of "testing" a probable model of such explanations (Leinhardt, 1989), and specifies a system of goals and actions. The object of the research is an integrated discussion of instructional explanations across several different subjects.

Occasions

Instructional explanations are given in response to specific pedagogical triggers such as confusion, or direct questions. They are also given when the teacher has planned to teach a particular portion of a subject matter because that content is significant. The teaching plan may be as simple as an unordered list of ideas to be mentioned when the educational dialogue gets to it, or as complex as a detailed set of goals, examples, manipulatives, and practices. Both kinds of plans include an explanation. In history and in mathematics, there are four general occasions that seem to warrant instructional explanations. Table 1 shows these occasions by discipline.

Table 1. Occasions for instructional explanations in history and mathematics.

HISTORY	MATHEMATICS
Events short narrative episodes wars, treaties, biographies causality and change	**Operations** actions of mathematics functions, algorithms, mapping production
Structures long expository systems government, economy relational, hierarchy, flow	**Entities** elements descriptive or analytic number systems, geometric forms properties
Themes interpretive cohesive devices power vs. freedom, capital control patterns	**Principles** constraints and affordances associativity, transitivity, proof application
Metasystems tools analysis, synthesis, interpretation strategy	**Metasystems** tools heuristics, notations strategy

Events are the paradigmatic, short narrative episodes characteristic of history. Events include migrations, revolutions, changes of people and offices; their connections are causal. Events have actors, purpose, motives, consequences and are narrative in flavor. Students learn details, time, location, as well as cause and consequences of events. Operations in mathematics are comparable to events in history. These "verbs" of mathematics include mathematical functions,

[1] This discussion has avoided engaging in more than a passing reference to the intense philosophical debate surrounding "explanation." The 1971 volume by Georg Henrik von Wright, *Explanation and Understanding*, the 1976 volume by Manninen and Tuomela, *Essays on Explanation and Understanding: Studies in the Foundations of Humanities and Social Sciences*, and the 1988 volume edited by Joseph Pitt, *Theories of Explanation*, capture many of the critical moments in the debate over the last 45 years. The avoidance of the debate here has been due to three issues: first, the debate has centered out a layer from the issues of interest here, namely, a debate about the science of knowing science; second, the level of analysis is too fine-grained to be helpful currently to the coarser-grained problem of instructional explanations; third, the debate has focussed on "What counts as an explanation." The debate strategy for defining or generating explanations of explanations has been a winnowing one. However, the strategy pursued here is closer to those of archeology: cast a net in an environment of explanations and work with those captured phenomena; if some elements are judged to be inappropriately included, that is probably less harmful than never getting started on the enterprise at all. With these caveats, the discussion continues.

procedures, iterations, pattern searches, and algorithms. The performance of an operation is a simple or complex action that produces change in a manner analogous to events in history. But explanations of events in history contain agents and actions and causal connections; operations in mathematics operate on mathematical components.

Societal and physical structures in history are slow moving or static entities whose connections are relational (hierarchical, embedded, bi-directional) rather than causal. They include economic, political and social systems. While structures rise and fall in significance, their time frame is centuries or millennia as contrasted to days, months, and years. Where wars and biographies are the archetypal events, governments and social organizations (e.g., church) are the archetypal structures. Structures may or may not impact or be impacted by events. But, teaching and learning about them makes use of expository descriptive language as opposed to narrative.

Mathematical entities such as the various number systems are similar but not identical to structures. They may be operated upon, but they need to be understood in themselves. In fact, understanding a particular system such as decimal fractions imposes constraints on the operations, and the use of operations with entities. Entities also include forms, such as geometric shapes and their properties, and graphics and their properties. Many mathematical entities can play operational roles, for example, fractions and percent. Explanations of mathematical entities tend to rely on embodiments, envisioning, and definition. An aspect of entities not comparable to historical structures is that within a form (number system) each set has a defined relationship to other sets in the form, so integers are a subset of the real numbers, a square is a rectangle.

Historical themes are the interpretive principles for historians. They help build the rhetorical stance for any particular circumstance. For example, discussions of power and freedom (a political theme), or tensions between mercantile and agrarian interests (an economic theme), are themes used to account and recount historical systems and actions. It could be argued that themes are the devices of historical explanation not the objects; just as scientific principles are the devices of scientific explanation. However, instructional explanations must also include either explicitly or implicitly themes as the target for an explanation.

Mathematical principles form the constraints and affordances that impinge on operations and entities and make certain actions possible or illegal. These principles include concepts like associativity and communitivity, and they include clusters of concepts such as proof. As with history, particular proofs might easily be understood as a natural occasion for an explanation, but the idea of proof itself seems more elusive because it is both a device of mathematical explanation and an object of it.

Metasystems of history and mathematics are the tools and strategies of their respective discipline and students not only learn them but also learn the circumstances of their use. The metasystems of history include analysis, synthesis, hypothesizing, perspective taking, and interpretation. Analysis, synthesis, and hypothesis are specific historiographic actions, whereas perspective taking and interpretation are more organizational and dispositional -- a sense of slant on a topic. Instructional explanations of metasystems of history or mathematics tend to be through examples in use. The metasystems of mathematics have been the focus of intensive analysis and discussion by Polya (1973) and Schoenfeld (1985). They include problem solving heuristics, such as simplification, extreme cases, analogy, and pattern identification. A second area not usually included, however, are the notational structures of mathematics. As occasions for instructional explanations, notations could be considered entities or a property of operations. However, notations are arbitrary, frequently confusing, and have a flavor unlike the more mathematically enriched concepts. Consider, for example, the fact that Ramanujan's ideas about mathematical series were insightful and accurate, but his notations were so inventive and obscure that they made it impossible for all but Hardy to recognize their validity (Kanigel, 1991).

I view these occasions in mathematics and history as the disciplinary circumstances that prompt an explanation, not the simple categorization of topics. The reason for distinguishing among them is that the plans for a particular explanation differ depending on the occasion. For example, the role of analogy in history is far more central and frequent in explaining events and structures than in explaining themes and metasystems. Further, the character of the analogies is quite different on different occasions (McCarthy & Leinhardt, 1993). Having said that occasions are different, it is important to point out that instructional explanations often target more than one occasion in a simple explanation. In general, however, there is a tacit demand on the learner to shift when thinking about properties of a number or economic system as contrasted to thinking about division or the Civil War.

Plans of goals and actions

If occasions are the opportunities, then plans of goals and actions are the enactments of instructional explanations. We have developed a model of instructional explanations in mathematics that takes the form of a planing net (Leinhardt, 1987; Sacerdoti, 1977; VanLehn & Brown, 1980). This model has been used to describe the actions and goals present in a mathematics explanation across a wide range of instructional levels from second grade subtraction to a college level course taught by Polya (Leinhardt, 1987; Schwarz & Leinhardt, 1993). It has been shown that expert teachers regularly incorporate most of the major features in the model while novice teachers do not (Leinhardt, 1989). In this paper, I extend the model to include history. This model includes those goal states present in the majority of instructional explanations in both history and mathematics, but does not include those unique to each subject. (A complete discussion would require an examination of the non-overlapping goals in the disciplines; however, space precludes that discussion for now.) Four goal states commonly present in both history and mathematics explanations are: identification of nature of the problem; knowing representational base used to explain; completion of the verbal explanation in a manner that connects it to both the nature of the problem and the representation(s); and the principles accessed or addressed in the explanation. The action schemas that support these goal states are unique for different instructional subject matters and different occasions within subject matter.

Each subject matter has its own family of questions or problems that it addresses. There is an obvious relationship between the occasions for instructional explanations and goal of identifying the nature of the problem being explained. The occasions are the classes of likely problems in a domain. Examples of problems in history include: Was there a reasonable adjustment between liberty and power when the Constitution was written to replace the Articles of Confederation? or Was Reconstruction constructive? Examples of problems in mathematics include: What rule (function) underlies these patterns of numbers? Why is 1/4 times 3 equal to 3/4? How many spaces are created when a region is cut by five planes? Understanding what constitutes a problem requires more than a teacher stating it; it requires that, as an explanation unfolds, the teacher and students connect the representations, principles, and verbal discourse back to the problem to assure coherence. Students gradually learn how to answer particular questions in subject areas. Less frequently are they given the opportunity to learn how to frame questions. In a good instructional explanation, the question or problem being answered is as clear as the answer. In explanations given by student teachers, the problem being explained is frequently invisible (Leinhardt, 1989). This leaves students with an answer to an unasked question and the knowledge is decidedly inert (Bereiter & Scardamalia, 1985; Leinhardt, 1988a, 1988b). The specific problem being explained constrains all other goals and action in an explanational plan.

Knowing the representational base used in an explanation is one of the shared goals in history and mathematics. Representations in mathematics include microworlds, manipulatives, diagrams, and gestures. In history, representations are most frequently analogies. In both mathematics and history, examples can also be considered representations. Representative systems serve as simplifying bridges. They can only do this if they are simplifying, if the representation fits the problem being explained, and if they are known to the students. In an instructional explanation, expert teachers flag the connections between prior knowledge and the target information as well as negative entailments and misalignments between the representation and the target (Leinhardt, 1988a). That is, both the fit and the lack of fit are clear to the student. Instructional explanations do not require a secondary representational system or examples, but the majority contain them. In poor explanations, the representational system sometimes becomes the target and the explanation gets lost in its own analogy.

Any devices used in an explanation, such as representations or micro-worlds, must be known to the learner and the operations in that space must be accessible, before the device is used to build an explanation. In the parlance of Gentner (1983), attempting to build a base and a target at the same time is difficult. For example, a set of paper strips, which might be used for explaining equivalent fractions but which require uniformity of length and alignment to operate "correctly," cannot be introduced as a new representation at the same time as it is being used to explain equivalent fractions. Further, the operations in a particular representational world must also be known and not assumed. For example, in one classroom, we observed a student using base ten blocks to support subtracting with regrouping. The student had made the "trade", had taken away the subtrahend which was in her hand - she then paused clearly puzzled. She had one set of blocks in her hand (subtrahend) and two sets of blocks on her desk (difference and "bank" from trading). Which set was the answer set? (Leinhardt, 1987). In explanations given by student teachers, they often assume that the "concrete" world is definitely easier, so easy that it is self evident and can be assumed (Leinhardt, 1988a; Leinhardt, 1989).

Completing the verbal explanations would appear to be a self evident goal. However, student teachers frequently fail to complete an explanation because they get lost in the representation and "solve" the problem in the microworld but not the symbolic world (Leinhardt, 1988a). Likewise, students in history also fail to complete an explanation by not connecting the evidence back to the initial problem (Leinhardt, 1993). In part, this is because, for both teachers and students, rules for completion are less than clear.

In both mathematics and history, principles of the discipline are the mechanism of disciplinary explanations. In instructional explanations, these are both occasions for explanations (see Table 1) and goals in the plan of an explanation. Principles in an explanation help to justify why something is so -- or help to identify the rule governing a claim. In history, thematic shifts such as changes in the control of capital or the location of power are explanatory principles, so are the definitions of terms. In mathematics, principles enable legitimate moves to be identified. In both cases, principles connect representations and verbal descriptions of how a problem is being analyzed and explained.

Examples

To investigate the nature and form of instructional explanations we have been building a data base of instructional episodes across a wide range of grades, second grade through college, and a wide range of subjects -- mathematics, history, geography, writing, and psychology. The data consist of videotapes of consecutive lessons from two weeks to an entire year. In addition, student work and semi-structured interviews have been collected. From 1988 to 1990, we observed every class in the first semester in one Advanced Placement U.S. history classroom and audio or video taped all of the classes. All of the lessons were transcribed, and the transcriptions are treated as protocols. During the fall of 1988, ten consecutive math lessons taught by Lampert were likewise taped and transcribed. These two data bases were analyzed for examples of instructional dialogues that produce an explanation. The explanations are in the form of group dialogues. The first example deals with a theme in the history course, liberty and power. The second is a student in the course who had learned to explain. The third example is an explanation to justify the answer when a particular function had a fractional input value.

The first example comes from a class taught by an expert teacher, Sterling. She often led discussions that merged several occasions together, such as themes and structures. Sterling led a discussion for several days about the Constitution. As background, the U.S. Constitution was written in 1887 by 55 men who met over the long summer in Philadelphia. It was written because of the shortcomings of the interim government, Articles of Confederation, founded at the beginning of the Revolution. There exist many versions and interpretations of the motivations, background conditions, and values that shaped this critical historical period. (For excellent modern discussions, see Bailyn, 1990; Morgan, 1956; Wood, 1992.) Sterling's discussion took four class periods; the example comes from the third.

The explanation dealt with the question of the balance of liberty and power in the Constitution. The deeper question was, "Why is the Constitution as it is, and how did it become so?" The theme of liberty and power was one that had floated in and out of the course throughout the year. Sterling started this part of the explanation with the embedded question of what were the weaknesses (power weaknesses) in the Articles of Confederation that lead to the Constitutional Convention, or restated, what was the imbalance in power? The answer that she constructed with help from the students was that the Articles gave sovereignty to the states, thus dividing power; there was no overarching judicial structure, leaving a power vacuum; and there was no centralized power of taxation, producing economic powerlessness. This answer set up the expectation that the 'new' government would repair these. The second embedded query was, "On what did the delegates agree?" The answer was popular sovereignty (a shift in power) and a stronger national government, one that presumably could tax and had a judiciary (a new power). On the way there was a definition of liberty and power and a flagging of this relationship as a theme for the course.

The framing questions were models of the kinds of historical heuristics students were learning to use. The central question posted on day two was "Do you think the Founding Fathers made a good adjustment between liberty and power in the Constitution or do you think that they were too fearful that the majority of the people might loose their liberty?" (thematic query). This question started the segment on day two and ended it on day three. The question is fundamentally one that requires an explanation about the structure of the Constitution, and about the events of its writing.

The explanation was built around a circle of eight subquestions that led back to legitimize the initial question. Sub-questions posed by the teacher to help construct the explanation of the question included: "Was it necessary to go to Philadelphia in the summer of 1987 to write a new constitution?" (metasystem analysis). "What were these three weaknesses in the Articles that need to be changed?" (structure). "What were some of the areas of agreement of the delegates?" (structure). "What kind of government do we want?" (structure). "Where in this document that we are writing in Philadelphia does it say that the power in this country is with the people?" (identification). "Can you clarify those concepts [liberty and power]?" (theme). "In the period of the revolution from let's say 1781-83 what instrument of government has the power?" (identification in structure). "Now, what does Madison say about the purpose here of going to Philadelphia and writing this Constitution?" (theme). A set of nine questions at various levels of specificity framed the dialogue that answered the initial question. They continued on to discuss the various historians interpretations (metasystems) of the writing of the Constitution.

The explanation that the class constructed centered on rediscovering the legitimacy of and explaining the original question, not on answering it. The rediscovery rested in the structural discussion of the Constitution. It also included references to events and most strongly to themes as the question was itself a thematic one. Small ikat-like explanations of the metasystems of history were also given. When one student was unable to understand the second question (What were three weaknesses), Sterling had two other students extend the first student's answer so that the question could be answered. The goal of knowing what the problem was that was being explained was reached through a definite direct statement and a restatement of the question. It was supported by sub-questions. The answers to those sub-questions led into the issue of power and liberty.

Figure 1 shows a net of the central concepts that emerged as the class developed the explanation. Bold lines show student ideas. To check that the class had in place the necessary prerequisite understanding of the situation that led to the writing of the Constitution, Sterling had the students review the weaknesses shown on the

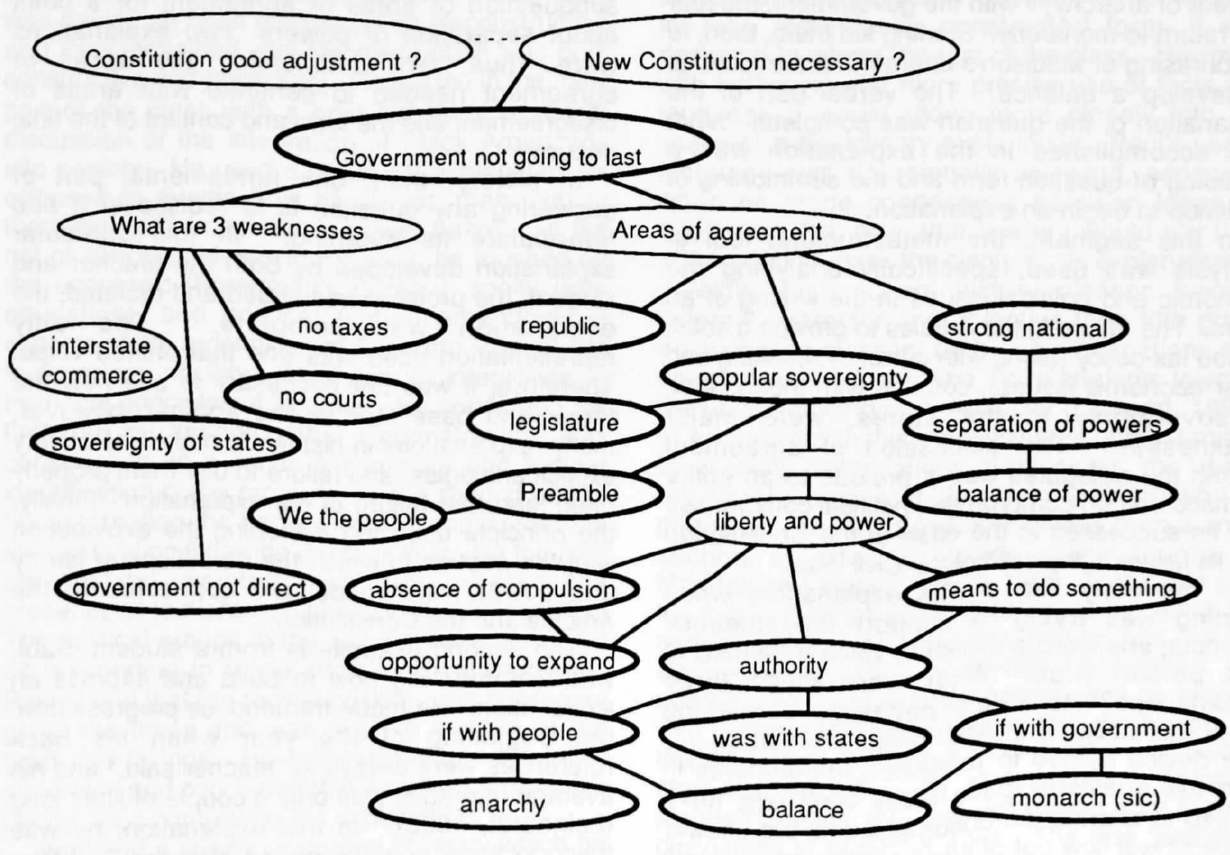

Figure 1. Collaborative History Explanation

left. These were the failures of extant power relationships. On the right hand side, the three areas of agreement include republican form of government, the location of power, and the need for a stronger central government. The last two constructs are in tension with each other because a strong national government implied less power residing with the people. However, the government existed only to serve the people and that was clear from the preamble in the Constitution.

Students identified the change from the Articles, in which the power was not with the people but with the States, to the Constitution. Both the assignment of power, and its limitation by separation of powers and checks and balances, completed the first part of the explanation. Sterling requested and got a clarification of the "concepts" of liberty and power. A student replied by offering a double layered definition for both concepts: freedom is not just the absence of compulsion but the opportunity to expand; not just authority, but the means to do something. Sterling pushed towards the issue of power and the students walked through a second historical tension -- if power was with the people, there was the fear of anarchy; if with the government, the fear of a return to monarchy. Sterling led them, then, to a rephrasing of Madison's charge that the job was to develop a balance. The verbal part of the explanation of the question was complete. What was accomplished in the explanation was a modeling of question form and the summoning of evidence to begin an explanation.

In this segment, the metastructural tool of analysis was used, specifically analyzing the economic and political issues in the writing of an event. The failure in the Articles to provide a solid, unified tax policy (along with currency, banking and other economic issues), coupled with the location of sovereignty in the states, were major weaknesses. The discussion of agreement among the delegates was a prelude to an entire thematic unit on compromise, political compromise and its successes in the case of the Constitution and its failure in the case of the Civil War.

In the early part of the explanation, when Sterling was trying to support the students' attempts, she used a switch in voice from third to first person (from "What were these three weaknesses?" to "We'd better do something about this document. And in fact we met at ...") This device helped to reposition the students in both the question space (Why could we have wanted to change the document) and the answer space (It will flow out of known problems we have been talking about). This latter hint may well have connected the student to informal information already existing but in an inert way. Later on in the explanation, she rephrased again, after she had asked for areas of agreement: "Everybody [third person probably], all fifty-five of us [first person] who are in the session in Philadelphia are going to say, 'Yes, Pete, that's a great idea. That's what we want in government.'" This was an analogical device of sorts. It flags the parts of the past that are accessible from personal knowledge. The danger was that it could be misapplied. Students could have gotten lost in the analogical space (is it us now, or us then?) and mismap as well (Gentner, 1983; Gick & Holyoak, 1980, 1983; Holland, Holyoak, Nisbett, & Thagard, 1986).

A second device that was used in the explanation, serving as a warning that she was going to make an aside or move on, was summarizing. Sterling summarized several times: "This is what is wrong;" "these were areas of agreement." This was both a reasonable pedagogical move and a reasonable discipline-based move. In essence, the teacher was reiterating what had been accomplished over the course of the discussion to flag a move to a new idea or an elaboration. After the discussion on power and liberty, she returned to the initial subquestion of areas of agreement for a point about separation of powers. Two explanations were, thus, left 'unfinished;' the areas of agreement needed to continue with areas of disagreement and the form and content of the final document.

In history, then, one fundamental part of answering any question is to rediscover it and reformulate its meaning. In this particular explanation developed by both the teacher and student, the problem was stated and restated, the explanation was complete. The only representation used was one that shifted voice. Therefore, it was not necessary to see how the target and base fit with one another; however, many explanations in history do make use of very explicit analogies, and failure to use them properly often lead to a failure of the explanation. Finally, the principle used in developing the explanation was the section in which the definitions of liberty and power were made and connected to the Articles and the Constitution.

The second example is from a student, Paul, who was learning how to build and express an explanation. He made tremendous progress from the beginning of the year when his back references were always to "teacher said," and his average utterance was only a couple of lines long (Leinhardt, 1993). In this explanation, he was trying to show how the period after the Civil War could be considered a constructive one in spite ofthe many negative features. He was in an

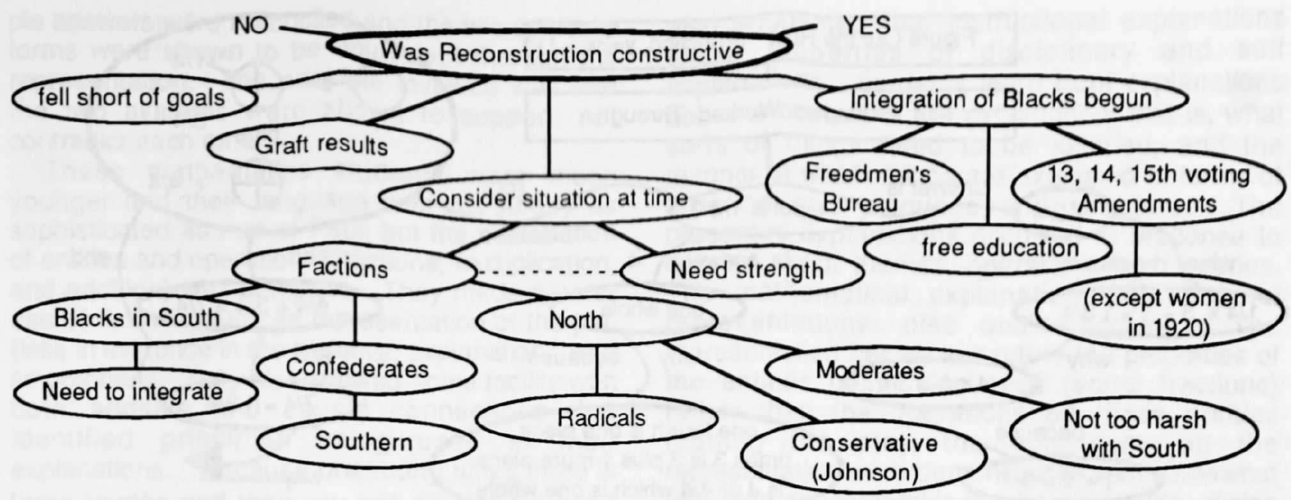

Figure 2. Student History explanation

extended dialogue with other students. Figure 2 displays the map of his explanation.

The underlying query which produced this explanation dealt with the political consequences of Reconstruction and perceptions of those conditions after the Civil War. ("We are going to question whether the period was a constructive one, whether it was a destructive one.") The class was having an open discussion on Reconstruction and Paul explained why Reconstruction was both constructive and necessary. He started in the early part of the class with a brief but fairly complete discussion of the integration of Black Americans into society. He used a structural explanation as opposed to events (this happened, then this) or thematic (shift in economic power base). As the net of concepts shows in Figure 2, he touched on the economic (Freedman's Bureau), social (free education), and political (voting amendments) aspects of the redefinition of Southern Blacks' role in American society. This was an integration of historical principles of analysis. He concluded the first part by asserting that Reconstruction was constructive, in a sense a summary and a clear identification of the problem. In the second part of his explanation, he backed up and considered the entire situation following the Civil War. He focused on political disintegration, and offered the presence of factions as evidence of disintegration. The political groups in the country were portrayed as responding to these social/political problems. Paul stated that the radical position was necessary given the times and was positive in spite of the graft and failings. He referred back to his initial statement and concluded that it was a constructive period, thus completing his explanation.

Paul had a strong command of facts, which he used as his evidence. There was no particular appeal to authority other than these facts. (He might have referred to specific authors and their positions, such as Hofstadter or Beard, something often done in the class.) He made an interesting interconnection with another student --"As Eva said" -- and then added to her statement. Later he said, "Like Arnold said." His use of voice "we" was inclusive of the other speakers and part of the form of discourse as in, 'we find.' In form, this is an instructional explanation because in its spoken, loosely, redundantly constructed form, it was designed to inform the rest of the class. However, with tightening and more precise use of facts and language it would move more closely into the space of a disciplinary explanation. Paul's use of language was occasionally awkward, especially when he made connections between ideas (in addition to this, now, that is why I would say that) that tended to mask the clarity of his explanation.

While high school history classes have a different character and objective than fifth grade mathematics classes, both use explanations that are designed to teach and move students towards the understanding of a discipline (Lampert, 1990). They accomplish this move towards understanding by invoking instantiations of principles, the use of representations, and the clear demarcation of problems. The third example comes from a small section of one 5th grade class taught by Magdelene Lampert during 1988. This class was the first in a series of ten classes during which the main focus was on functions and graphs. All ten classes were video taped and transcribed. In addition, pre- and post-interviews were conducted with Lampert and observational logs were kept. In the first class the focus was on the knowledge students had of grids and graphs. Lampert planned to move into that discussion from a quick discussion of mapping rules. The first "problem" was to find the function rule, use it, but not state it. After working through 14 ordered pairs on two

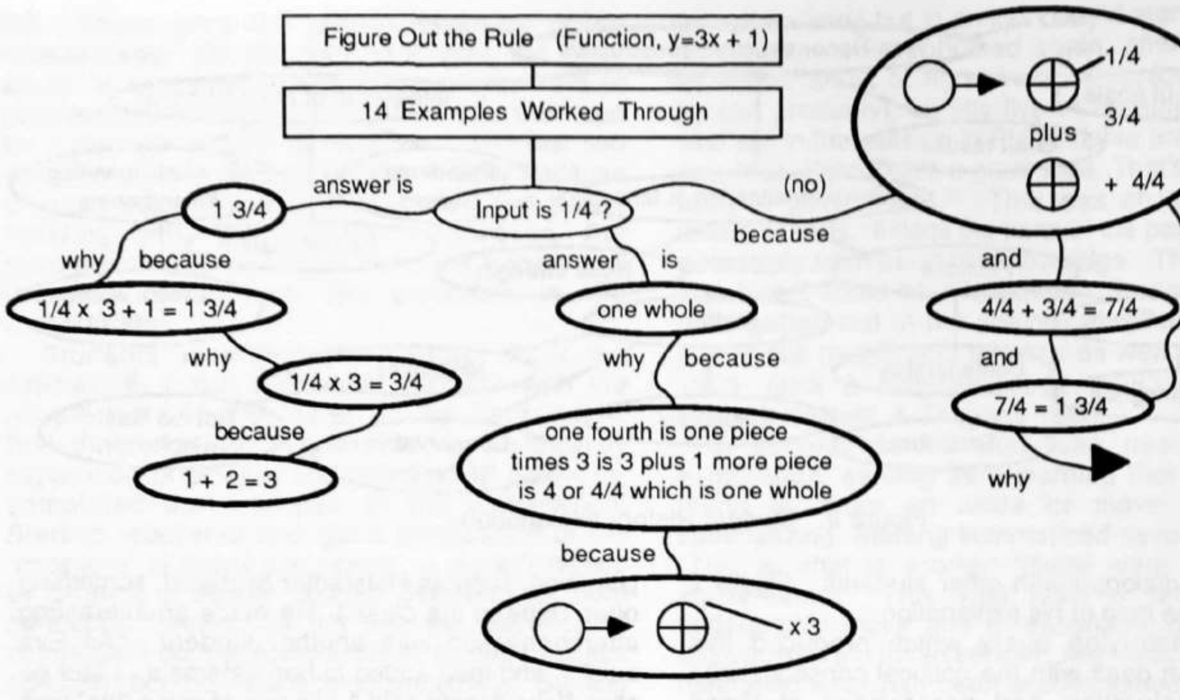

Figure 3. Collaborative Mathematics explanation

charts using whole numbers with the same rule ($y = 3x + 1$), Lampert gave 1/4 as the input number. A discussion about fractions, then referents and operations ensued. Figure 3 diagrams the resulting instructional explanation. Student contributions are in bold circles.

A student answered that if the input was 1/4 the output was 1 and 3/4. Lampert asked why and the student responded, "Because 1/4 times 3 is 3/4, and then add one." This was the "correct" answer. It did not need an explanation because of an error. Lampert again asked why ("Who can explain why one fourth times three is three fourths?"), but this time in reference to why 3 times 1/4 is 3/4.

Another student gave an ambiguous answer: if you had one piece of pie and someone brought two more you would have three pieces "that come out of four pieces of pie." (This appeared to be additive rather than multiplicative.) After a discussion of equal sizes, a third student challenged the first answer of 1 and 3/4 and claimed that one whole would also be correct. This third student's reasoning was that one fourth is one piece of pie, times three, that would be three pieces, and then you should add one more piece, which would make a whole. The last two answers were both unclear, but the second one was explained in enough detail by the student so that the confusion could be detected and discussed. The "mistake" was shifting the unit of whole pie to one piece of pie during calculation, but then reclaiming it in the answer of four fourths. The child drew a picture of a whole pie divided into fourths and pointed at one segment, showing one piece of a pie. This drawing introduced a representation of the problem for the class. Lampert put x's in three of the student's four pie pieces and asked for a label (upper right of figure). The student chose three fourths as opposed to three pieces of pie. Then Lampert said, "Plus one whole," while the student simultaneously said, "Four fourths." The student added the pictorial pieces and got seven fourths, but the picture was also of one whole and three fourths, which formed the link to the first answer which ended the particular part of the explanation, both verbally and pictorially.

The explanation continued with why 3/4 plus 4/4 is or is not 7/4. The jointly constructed explanation got to the heart of the slipping unit problem. That is one more what? One more piece/fourth or one more whole, four fourths. The explanation closed when all of the representations were shown to be consistent with each other. The initiating condition for this mini-explanation was a task in justifying a calculation, and the explanation continued with statements of procedures that were shared by all of the students. When a part was reached that was not shared a second explanation was offered. The explanation made use of a representation offered by a student, in this case, a pie. Finally, the original answer and the pie answers were reconciled and the two symbolic forms were shown to be equivalent through the representation. The principle involved was that the two systems were shown to support, not contradict each other.

These mathematics students were much younger and their language was not nearly as sophisticated as that of Paul, but the explanation of entities and operations (fractions, multiplication, and addition) was complete. They made greater use of devices, such as representation of the pie, (less in evidence in the historical explanation), with smoothness. They also showed some facility with both additive and causal connections that identified principles being used in their explanations. "Because one fourth times three is three fourths and then you just add, add a one." Later, "Cause three fourths plus four fourths equals seven fourths." A good deal of the causal part of the explanation was in fact carried by the drawing and correct marking of the pictorial representations in addition to the language. Because Lampert was a part of the dialogue there was considerable support (in the sense of scaffolding) for both identifying appropriate reasons and for using causal language to express them.

The mathematical explanation was instructional in several ways. At the most global level it was a layered discussion on a single function rule ($y=3x+1$) where the repeated use of the rule was demonstrated by mapping "inputs" to "outputs." This global explanation was in the hands of the teacher in much the same way that the choice of question for the Constitution was in the hands of Sterling. At a more focussed level, part of the explanation dealt with a specific application of the function rule which involved dealing with a fractional input. As Lampert said in her notes for the day, "...I had a big digression on fractions → I decided (off the track?) to throw in '1/4' as an input number, and we needed to do some work on the meaning of 1 3/4 also 7/4 →..." The multiplication of fractions and equivalence of improper and mixed numbers may have been "off the track" but the explanation of the procedure and its justification was not. There was a crispness to the students' public explanation; they were explaining the right thing and involved the principle of proof.

Conclusion

The discussion of explanations started with distinguishing between different kinds of explanations: common, self, disciplinary, and instructional. It was suggested that instructional explanations have a unique aspect because of both style or form and what gets explained. It was also suggested that instructional explanations share properties of disciplinary and self explanations. Students learn from explanations both the content of the explanation, that is, what sorts of things need to be justified, and the manner in which things are explained in terms of ordering and language and structure. The presented explanations occurred in response to queries about themes, operations, and entities. The mathematical explanation used several representations: pies and diagrams. The representation helped to capture the properties of the entities being discussed (equal fractions) rather than the operations on those entities (cutting the pie); the operations on the representation were demonstrated with somewhat less than an isomorphic fit (choosing a pie, slicing the pie, demarking without x's all of the pieces, demarking with x's a subset or all of the pieces, counting the pieces, using the total count of demarked -with x- and relating them to the demarked without x is comparable but not identical to adding numerators and retaining denominators). The historical explanations used the less formal representation of voice position. In many other classes there was a more complete use of analogy (McCarthy & Leinhardt, 1993). All of the explanations identified principles (equivalence, definitions of themes, analyses of conditions) as a part of the discussion, and all three completed the verbal explanation.

Explanations are a core of all teaching, didactic or dialogue based. Knowing what the aspects of good explanations are, how these aspects are learned, and how to improve on them is important for the education of teachers and for the general improvement of education. In the examples, the students were shown learning how to construct their own instructional explanations, explanations which could communicate to their fellow students in the disciplines of mathematics and history, explanations which built up from a shared known language base and were not personally idiosyncratic in ways that destroyed meaning. These explanations were reflections of the students' own emerging powers of reasoning in the respective disciplines.

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